Evaluating the Optimal Exit Point and Timing of Switching From 100% Biodiesel to Petroleum Diesel in a Carbon Priced Environment

by

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DEDICATION

I wish to dedicate not only this paper, but all the work I have completed in the past five years in the Iowa State University College of Engineering Industrial and Manufacturing Systems Engineering department, to my parents Brent and Anne. By instilling in me the virtues of hard-work, perseverance, and belief, they laid the foundation upon which my collegiate education could be, and has been, built. Without their love, support, and sacrifice along the way, this collegiate education would have remained just a dream instead of the reality that it is today. For each of them, I will always be deeply grateful.

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NOMENCLATURE

CARB	California Air Resources Board
CI	Carbon Intensity
CO_2e	Carbon Dioxide Equivalent
GBM	Geometric Brownian Motion
IRR	Internal Rate of Return
LCFS	Low Carbon Fuel Standard
MJ	Megajoule
MT	Metric Ton
UCO	Used Cooking Oil

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ABSTRACT

The carbon management policies in the United States are in flux and vary greatly from state to state. Under these circumstances, for a transportation company, we investigate an interesting problem of transitioning from 100% biodiesel to petroleum diesel as the petroleum diesel price exhibits a high degree of volatility. Specifically, using a stochastic optimal control technique called Real Options, we show how such a company can optimally determine the threshold petroleum diesel price that triggers a switch of fuel from 100% biodiesel to petroleum diesel based on economic perspectives. We will also show how the expected time to such a transition can be derived so that the company can prepared for the change in fuel. Our approach will be demonstrated by an extensive numerical example that illustrates the key features of our mathematical model and analysis. We aim to provide an interesting and relevant guidance for transportation companies that are continually facing the ever-evolving carbon management policies across the country. Finally, we present the discussion, concluding remarks, and future research areas.

CHAPTER 1. INTRODUCTION

Climate change and global warming as a result of greenhouse gas emissions is one of the most pressing issues facing the scientific and political communities today. If the world takes no action to change its current course, scientists have estimated that global average temperatures will rise about 2.8°C by the end of this century—far above the ambitious target of only 1.5°C outlined in the Paris Climate Accords. Continuing the current trajectory is expected to cause exacerbated water scarcity, widening food insecurity, increased severe weather, permanent damage to wildlife, and numerous other consequences we are both aware and unaware of today (UNEP, 2022).

The scientific community clearly identifies the burning of fossil fuels as society's main source of energy as the number one culprit of greenhouse gas emissions. From 1970 through 2011, 78% of additional global greenhouse gas emissions were attributable to the burning of fossil fuels and industrial processes (IPCC, 2014). In 2021, CO_2 emissions from the transportation sector in the United States accounted for 38% of all energy-related emissions—the largest contributor of any sector in this category and even larger than electric power generation. This means that the burning of fossil fuels as the main fuel source for the transportation sector of the United States economy contributed over 1.748 billion metric tons of CO_2 into the atmosphere in 2021 alone (Shirley et al., 2022).

For these reasons, there is now a concentrated effort to decarbonize the transportation sector from numerous stakeholders inside and outside the industry. Environmental scientists have been joined in their call for decarbonization by policy makers, vehicle manufacturers, fuel producers, companies, and consumers.

Introduction to Carbon Pricing

Despite the agreement that the transportation sector of the economy needs to be decarbonized, and quickly, the implementation of policy, tax breaks, incentives, programs, etc. to encourage the utilization of alternative fuels has not happened uniformly across the United States. With many of the decisions and formulations of decarbonization plans left to the discretion of the states, a wide array of programs have developed that very greatly in scope, method, and practice from state to state. Some states have implemented programs that do very little to properly incentivize the uptake of cleaner alternative fuels while other states have implemented more aggressive carbon pricing programs.

In the United States, the most expansive and aggressive carbon pricing program is California's Low Carbon Fuel Standard (LCFS). In 2009, the California Air Resources Board (CARB) passed the LCFS, a first-of-its-kind greenhouse gas emission reduction strategy, and implemented the system on January 1, 2011. The California LCFS system works by establishing a target carbon intensity (CI) baseline score for petroleum-based gasoline and diesel fuel used in the state each year. Given the scope of this paper, the focus will be on petroleum diesel as the established baseline fuel and 100% biodiesel as its cleaner alternative. The LCFS system also identifies CI values for 100% biodiesel depending on where and how it was made. The expansive LCFS CI database that contains these CI calculations and was used to identify the CI values employed for petroleum diesel and 100% biodiesel in the model. In order to standardize the quantity of emissions across all fuel types, the CI values of all fuels under the LCFS system are measured in units of gCO_2e/MJ which bases the emissions off the energy content instead of the units in which the fuel is typically measured or sold which can vary greatly depending on the type of fuel (CARB, 2022).

The California LCFS system works as an open market where carbon credits are constantly traded between entities—with companies that used fuels with a higher than baseline CI score, resulting in a carbon deficit, buying credits from companies that used cleaner, lower than baseline CI fuels in their fleets to develop carbon credits. This means that the price of an LCFS carbon credit is always in flux driven by supply and demand like the price of a stock.

This complexity is beyond the scope of this paper. As a result, this paper will use a static price of carbon as an incentive to a company operating with cleaner fuels. In other words, a company utilizing a fuel with a CI lower than the baseline fuel's CI will develop a carbon credit revenue. Specifically, petroleum diesel will be the baseline fuel and 100% biodiesel will be its cleaner counterpart. The price of carbon used throughout this paper will be set at \$75/metrictonCO$_2$e which is the value the International Monetary Fund has estimated is necessary to achieve the goals of the Paris Climate Agreement (Parry, 2021).

The fundamental question of this paper analyzes fuel utilization decisions in this carbon priced environment after a transition has already been made to 100% biodiesel. Essentially, for the business owner who operates a fleet currently running on 100% biodiesel and is ultimately driven by economics, what happens when the price dynamics of 100% biodiesel and petroleum diesel move in such a way that established incentives from carbon pricing no longer makes 100% biodiesel economically competitive? This question forms the basis of this paper. To begin, it is important to understand the close pricing relationship between petroleum diesel and 100% biodiesel.

CHAPTER 2. THE PRICE RELATIONSHIP BETWEEN PETROLEUM DIESEL AND 100% BIODIESEL

Biodiesel has an intimate pricing relationship with petroleum diesel. According to data collected since 100% biodiesel prices were first recorded in September of 2005, in each recording period, the average price of 100% biodiesel sold in the United States has never been recorded below the average price of 100% petroleum diesel (*see Appendix A for full data table*). In fact, upon plotting the data on the same graph (*Figure 1*), one can see the correlation in price behavior these two fuels have held over the last seventeen years. These two fuels have very similar production processes, but the price of the biodiesel inputs—plant oils, waste grease, animal fats, etc.—are more expensive than their petroleum diesel counterparts. Also, at this stage of the technologies, petroleum fuel processing capitalizes on economies of scale in way that the more infant technology of biodiesel refining has yet to achieve. These two factors are the main drivers of the price premium biodiesel holds to petroleum diesel (Fendt & Jones Prather, 2021).

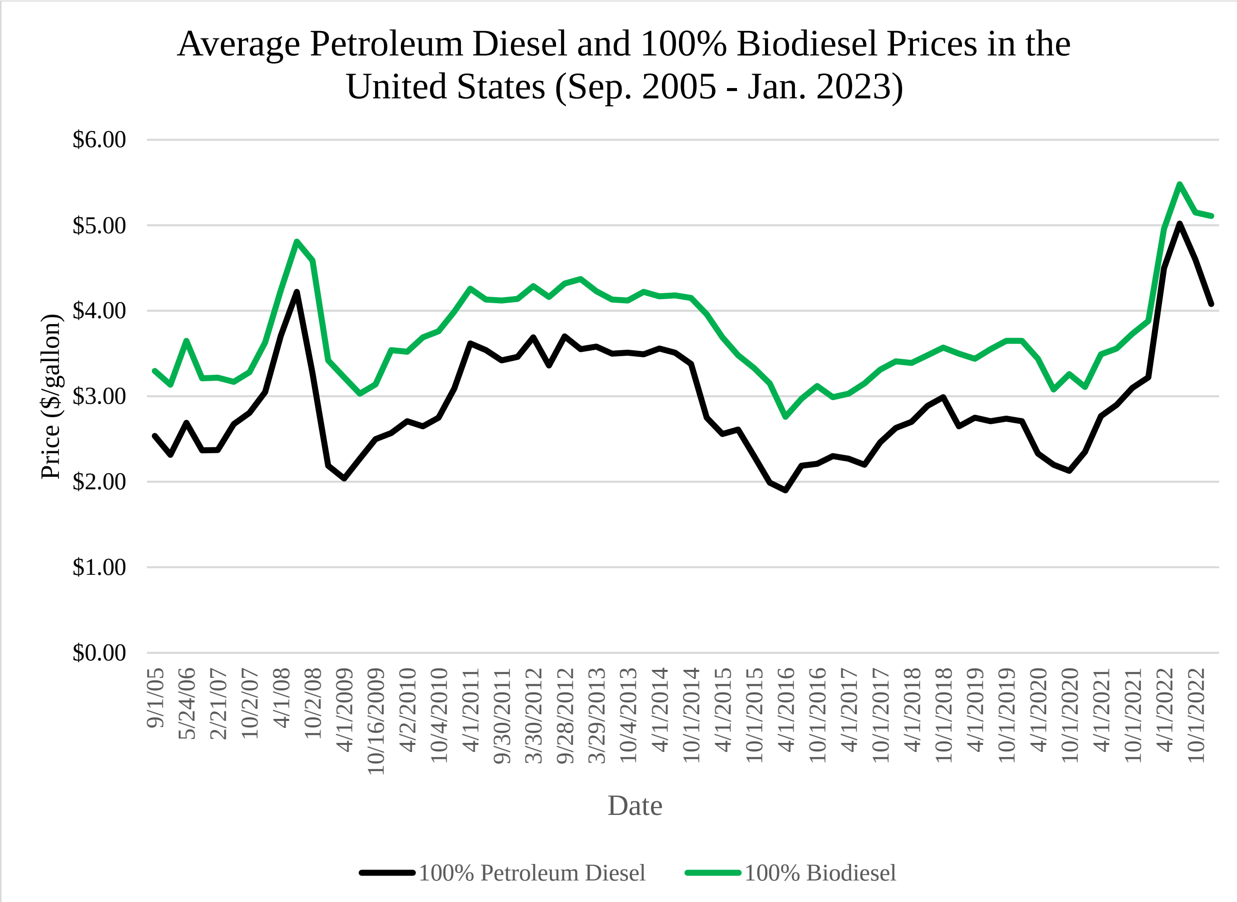


Figure 1: Average Petroleum Diesel and 100% Biodiesel Price Chart

According to data from the United States Department of Energy, over the time period of September 2005 through December 2022, 100% biodiesel was, on average, $0.79/gallon more expensive than 100% petroleum diesel. The spread between the costs of 100% biodiesel and 100% petroleum diesel was as low as $0.46/gal and as high as $1.32/gal. In percentage terms, 100% biodiesel was priced at an average premium of 29% above the price of 100% petroleum diesel. The percentage difference in the price of 100% biodiesel above the 100% petroleum diesel price over the time period was as low as 9% and as high as 58% (United States Department of Energy, 2023).

Therefore, in an environment with insufficient or nonexistent economic incentives for carbon reduction, it makes no economic sense for a company operating a petroleum diesel powered fleet to make the transition to biodiesel because of this pricing premium. This illustrates the need for an incentive program like a nationwide carbon price to adequately incentivize fleets to operate on cleaner biodiesel. If this carbon price revenue was not an option, according to the data from the United States Department of Energy, economically it makes no sense for a fleet to implement biodiesel to begin with. This also enables this paper to answer the question outlined in the introduction of when, in a carbon priced environment, is the optimal time to transition out of biodiesel and back to petroleum diesel.

CHAPTER 3. OPTIMAL TRANSITION POINT AND TIMING MODEL FORMULATION

This chapter outlines all assumptions made and mathematical formulation necessary to develop the optimal transition timing model that identifies the optimal price and expected time until making the transition out of 100% biodiesel and back to petroleum diesel. The model in this paper uses the established optimal timing model from chapter five of Dixit & Pindyck's *Investment Under Uncertainty* (Pindyck & Dixit, 2012).

Assumption 1: There is an investment cost, I $(I > 0)$, to transition vehicles from operation on 100% biodiesel to petroleum diesel.

Assumption 2: When a vehicle is transitioned from operation on 100% biodiesel to petroleum diesel, the vehicle requires the same amount of fuel as it did before the transition. It is also assumed that this transition can be executed with no downtime.

According to the United States Department of Energy's Alternative Fuels Data Center, one gallon of 100% biodiesel only has about 93% of the energy content of an equivalent gallon of petroleum diesel (United States Department of Energy, 2021). Initially, it seems that by switching from 100% biodiesel to petroleum diesel, the purchaser would need to purchase 7% less fuel to satisfy their original demand. However, literature published with findings collected from a real-world trial, found that running on 100% biodiesel lowered total fuel demand compared to petroleum diesel over the course of the trial by 1.7% (Optimus Technologies, 2021). 100% biodiesel's improved lubricity and reduced particulate matter when combusted compared to petroleum diesel are the two factors that make up for its energy content deficit.

Assumption 3: The price of petroleum diesel follows a geometric Brownian motion (GBM) and can be expressed as stated below:

$$dP_P = \alpha P_P dt + \sigma P_P dz \tag{1}$$

Where α ($\alpha > 0$) is the instantaneous growth rate of petroleum diesel price and σ is the volatility. Both α and σ are in units of %/unit time. This equation, Equation (1) relates the change in petroleum diesel price, dP_P, to the sum of its instantaneous drift rate, $\alpha P_P dt$, and its volatility, $\sigma P_P dz$, where dt is an increment of time and dz is an increment of a standard Wiener process where $dz = \varepsilon_t \sqrt{dt}$ and $\varepsilon_t \sim N(0, 1)$ (Dixit & Pindyck, 2012).

Assumption 4: 100% biodiesel is priced at a premium to petroleum diesel with petroleum diesel serving as its basis. Based on the data from the United States Department of Energy outlined in Chapter 2, the price of 100% biodiesel, P_B, has averaged a 29% premium above the price of petroleum diesel, P_P, over the seventeen years of data collection. This relationship is expressed by Equation (2) below:

$$P_B = 1.29P_P \tag{2}$$

Both P_B and P_P are in units of $/gallon.

This method of representing two related GBM processes through constant multiplication is well examined and employed in like literature (see Min & Jackman, 2022).

Assumption 5: Operational revenue, R_O, that revenue which comes from regular business operation, remains constant regardless of which fuel type is being utilized in the fleet. Operational revenue, R_O, will be expressed in units of $/gallon to align with the units of fuel price and the soon to be established, carbon credit revenue.

From the customer's perspective, the type of fuel used in the fleet is irrelevant so long as the expected service/product is provided.

Assumption 6: In this environment, there is a uniform and static carbon price in units of $/metrictonCO$_2$e (dollars per metric ton of carbon dioxide equivalent). In order to factor this into the model equations, it must be converted to units of $/gallon to match the units for price of fuel and operational revenue. A generic form of this conversion can be found in *Appendix B*. This calculation will be carried out in the numerical case study in the following chapter, Chapter 4.

Assumption 6 becomes relevant when the transition is made from 100% biodiesel, Phase 1, to petroleum diesel, Phase 2. This price of carbon is a carbon credit revenue, R_C, that the fleet recoups as a reward for their decision to utilize cleaner fuels in Phase 1, but will be surrendering when the transition is made to petroleum diesel in Phase 2.

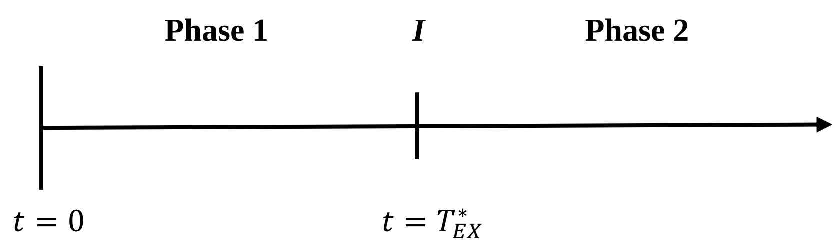
Assumption 7: In this environment, petroleum diesel is the baseline fuel. In other words, a company utilizing petroleum diesel is not punished or rewarded, but a company using a cleaner fuel such as 100% biodiesel, as in Phase 1 of the model, generates a carbon credit revenue as a reward for their carbon reduction.

This assumption is important for identifying the cash flows of both Phase 1 and Phase 2 of our model, and frankly, is what gives this model its value. Without this assumption, there is no incentive to make the transition to 100% biodiesel in the first place because of its price premium to petroleum diesel.

m	Quantity of fuel used by the fleet (gallons/year)
R_O	Revenue from regular business operation ($/gallon)
R_C	Revenue from carbon credit generation ($/gallon)
P_P	Price of petroleum diesel ($/gallon)
I	Investment cost necessary to make the transition from 100% biodiesel to petroleum diesel ($)
ρ	Annual discount rate (%/unit time)
α	Instantaneous growth rate of petroleum diesel price (%/unit time)
σ	Instantaneous volatility of petroleum diesel price (%/unit time)
V_1	Business value in Phase 1 ($)
V_2	Business value in Phase 2 ($)
P_P^*	Petroleum diesel price at which the transition back to petroleum diesel is optimal ($/gallon)

Table 1: Optimal Timing Model Parameters and Variables

These assumptions enable this problem to be solved via an optimal transition timing model. A visualization of the model objective is displayed below:



Where Phase 1 is the current state of fleet operation on 100% biodiesel and Phase 2 is the future state, after investment, which utilizes petroleum diesel as the fleet's fuel. Also, $t = T^*$ is identified as the optimal time of investment to maximize the value of the business operation. This problem can also be seen as a maximization of the total expected discounted value through identifying T^* according to the following:

$$\max_{T} E\left[\int_0^T (mR_O - 1.29mP_P + mR_C)e^{-\rho t}dt - I\, e^{-\rho t} + \int_T^{\infty} (mR_O - mP_P)e^{-\rho t}dt\right] \tag{3}$$

Using this objective equation, Equation (3), as a foundation, the progression toward identifying the optimal petroleum diesel price, P_P^*, at which a transition to petroleum diesel should be made can begin. Using the stochastic optimal control methods outlined in Dixit and Pindyck's *Investment Under Uncertainty* (2012), this optimal switching point, P_P^*, and the corresponding expected time, T_{EX}^*, until that optimal switching point can be identified (Dixit & Pindyck, 2012).

Business Value in Phase 2: After Transitioning to Petroleum Diesel

Once the business has made the investment, I, necessary to transition to petroleum diesel from 100% biodiesel, the business recognizes a cash flow of $(mR_O - mP_P)$. The value of the business in Phase 2 can be represented via Equation (4) below (*see Appendix C for full derivation*):

$$V_2(P_P) = \frac{mR_O}{\rho} - \frac{mP_P}{\rho - \alpha} \tag{4}$$

Simply put, Equation (4) displays the business value via the discounted cash flow. This solution is in accordance with the solution of Zhao & Min (2020) which examines a scenario of similar circumstances.

Business Value in Phase 1: Utilizing 100% Biodiesel

In the current state of 100% biodiesel utilization, Phase 1, the business has a cash flow of $(mR_O - m1.29P_P + mR_C)$. At any time before a transition has been made back to petroleum diesel, Phase 2, the value of the business must satisfy the Bellman Optimality Equation below (Dixit & Pindyck, 2012).

$$\rho V_1 dt = (mR_O - m1.29P_P + mR_C)dt + E(dV_1) \tag{5}$$

The Bellman Optimality Equation states that the return for holding the option to transition, $\rho V_1 dt$, should be equal to the current profit while holding the option—the Phase 1 cash flow we defined above—plus the expected appreciation of the business value (Dixit & Pindyck, 2012).

By applying Ito's Lemma to the variable, V_1, the ensuing second order differential equation is derived (Dixit & Pindyck, 2012).

$$\frac{1}{2}\sigma^2 V_1{}'' P_P^2 + \alpha V_1' P_P - \rho V_1 + (mR_O - m1.29P_P + mR_C) = 0 \tag{6}$$

In order to ensure the convergence of Equation (6), the following two conditions must be satisfied:

$$V_1(P_P^*) = V_2(P_P^*) - I \tag{7}$$

$$V_1{}'(P_P^*) = V_2{}'(P_P^*) \tag{8}$$

Equation (7) and Equation (8), are the value matching and smooth pasting conditions respectively. The value matching condition ensures that at the point of transitioning, the value of the business in Phase 1 must be equal to the value of the business in Phase 2 minus the cost of the investment to transition. The smooth pasting condition ensures that at the point of price optimality the slopes from the left side and the right side approach the same value and guarantees continuity.

Upon defining the technical conditions of $(\rho - \alpha) > 0$ and $\left(\alpha - \frac{\sigma^2}{2}\right) > 0$, the general solution to the second order differential equation, Equation (6), can be derived. Below, Equation (9) returns the business value in Phase 1 (*see Appendix D for full derivation*) (Dixit & Pindyck, 2012):

$$V_1(P_P) = A_1 P_P{}^{\beta_1} + \frac{mR_O}{\rho} - \frac{m1.29P_P}{\rho - \alpha} + \frac{mR_C}{\rho} \tag{9}$$

where,

$$\beta_1 = \frac{\frac{1}{2}\sigma^2 - \alpha + \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + (2\sigma^2\rho)}}{\sigma^2} \tag{10}$$

With the business value equations established for both Phase 1 and Phase 2, they can be plugged into Equations (7) and (8) to identify the optimal price of petroleum diesel, P_P^*, at which the company should transition out of 100% biodiesel and back into petroleum diesel (*see Appendix E for full derivation*). Upon executing this operation, the following formulas to return P_P^* and A_1 are determined:

$$P_P^* = \left(\frac{\beta_1}{(\beta_1 - 1)}\right)\left(\frac{(\rho - \alpha)}{0.29}\right)\left(\frac{R_C}{\rho} + \frac{I}{m}\right) \tag{11}$$

$$A_1 = -\frac{0.29m}{\beta_1(\rho - \alpha){P_P^*}^{\beta_1 - 1}} \tag{12}$$

As stated above, executing Equation (11) will return the optimal transition point, and executing Equation (12) returns a value for A_1, a constant used in determining the business value in Phase 1 as expressed in Equation (9). With the equation for P_P^* now established, this solution can be applied to the expected time until optimal transition point equation, T_{EX}^*, as below in Equation (13) (Dixit & Pindyck, 2012):

$$T_{EX}^* = \frac{\ln(P_P^*) - \ln(P_0)}{(\alpha - \frac{1}{2}\sigma^2)} \tag{13}$$

where P_0 is the current price of petroleum diesel and T_{EX}^* is the time from today, in years, that the company can expect to make the optimal transition out of 100% biodiesel and back to petroleum diesel.

CHAPTER 4. NUMERICAL CASE STUDY

With the optimal timing model equations established, the model can be executed in a numerical study to ensure its validity. The first step toward model execution is identifying the model inputs necessary to carry out the case study. CyRide, a large transportation fleet on the Iowa State University campus, was chosen as the subject of this numerical case study. The data used to calculate the values of the parameters for this numerical case study were sourced from the International Monetary Fund, the United States Department of Energy, the CARB Current Fuel Pathways database, and information gathered directly from CyRide officials. A complete table displaying all model parameters and their values for this specific CyRide case study can be found below (*Table 2*).

Model		
Parameter	**Variable**	**Value**
Price of Petroleum Diesel per Gallon	P_0	$4.08/gal
Annual Fuel Demand	m	340,000 gal
Investment Cost	I	$1,200,000
Annual Discount Rate	ρ	0.12
Annual Growth Rate of Petroleum Diesel Price	α	0.09
Annualized Volatity of Petroleum Diesel Price	σ	0.18
Carbon Credit Revenue per Gallon	R_C	$0.91/gal

Table 2: Numerical Example Input Parameters

The price of petroleum diesel, P_0, comes from the United States Department of Energy's most recent recording on January 1st, 2023. Annual fuel demand, m, information came from CyRide officials. Investment cost, I, is calculated based on an estimated cost to transition of $15,000/vehicle multiplied by the 80 vehicles in the CyRide fleet. The annual discount rate, ρ, is an assumed value based on a typical IRR of the transportation industry. Both annual growth rate, α, and annual volatility, σ, are also derived from the United States Department of Energy's petroleum diesel price data over the time period of January 2018 – January 2023. The last model parameter requires a series of unit conversions to identify the carbon credit revenue per gallon, R_C, from the carbon price of $75/metrictonCO$_2$e. This calculation can be identified below:

First, identify the CI scores of both the baseline fuel, petroleum diesel, and the alternative fuel, 100% biodiesel using the CARB database and notes (CARB, 2022);(CARB, 2020).

$$CI\ score\ Petroleum\ Diesel = 100.54\ \frac{gCO_2e}{MJ}$$

$$CI\ Score\ 100\%\ UCO\ Biodiesel = \ 23\ \frac{gCO_2e}{MJ}$$

Next, identify the energy content in MJ/gallon of both fuels.

$$Energy\ Content\ of\ Petroleum\ Diesel = 146\ \frac{MJ}{gal}$$

$$Energy\ Content\ of\ 100\%\ UCO\ Biodiesel = \ 135\ \frac{MJ}{gal}$$

Calculate the emissions per goal of both fuels.

$$Emissions\ per\ Gallon\ of\ Petroleum\ Diesel = 100.54\ \frac{gCO_2e}{MJ} \times 146\frac{MJ}{gal} = 14{,}678.84\ \frac{gCO_2e}{gal}$$

$$Emissions\ per\ Gallon\ of\ 100\%\ UCO\ Biodiesel = 23\ \frac{gCO_2e}{MJ} \times 135\frac{MJ}{gal} = 2{,}531.25\ \frac{gCO_2e}{gal}$$

Identify the emissions reduction on a per gallon basis.

$$Emissions\ per\ Gallon\ Reduction = \ 14{,}679\ \frac{gCO_2e}{gal} - 2{,}531\ \frac{gCO_2e}{gal} = 12{,}148\ \frac{gCO_2e}{gal}$$

Convert this result to metric tons CO_2e per gallon.

$$12{,}148\frac{gCO_2e}{gal} \times \frac{1\ metric\ ton\ CO_2e}{1{,}000{,}000gCO_2e} = 0.012148\frac{metric\ tons\ CO_2e}{gal}$$

Now, using the carbon price per metric ton, and the emissions reduction in metric tons CO_2e above, the carbon credit revenue per gallon, R_C, can be calculated.

$$0.012148\frac{metric\ tons\ CO_2e}{gal} \times \frac{\$75}{metric\ ton\ CO_2e} = \frac{\$0.91}{gal}$$

$$R_C = \frac{\$0.91}{gal}$$

With all model parameters established, the values can be plugged into Equations (10), (11), and (12) to calculate β_1, P_P^*, and T_{EX}^*, respectively.

$$\beta_1 = \frac{\frac{1}{2}\sigma^2 - \alpha + \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + (2\sigma^2\rho)}}{\sigma^2} \tag{10}$$

$$\beta_1 = \frac{\frac{1}{2}(0.18)^2 - (0.09) + \sqrt{(\frac{1}{2}(0.18)^2 - (0.09))^2 + (2(0.18)^2(0.12))}}{(0.18)^2}$$

$$\beta_1 = 1.271$$

$$P_P^* = \left(\frac{\beta_1}{(\beta_1 - 1)}\right)\left(\frac{(\rho - \alpha)}{0.29}\right)\left(\frac{R_C}{\rho} + \frac{I}{m}\right) \tag{11}$$

$$P_P^* = \left(\frac{(1.217)}{((1.271) - 1)}\right)\left(\frac{((0.12) - (0.09))}{0.29}\right)\left(\frac{(0.91)}{(0.12)} + \frac{(1{,}200{,}000)}{(340{,}000)}\right)$$

$$P_P^* = \frac{\$5.39}{gal}$$

$$T_{EX}^* = \frac{\ln(P_P^*) - \ln(P_0)}{(\alpha - \frac{1}{2}\sigma^2)} \tag{13}$$

$$T_{EX}^* = \frac{\ln(5.39) - \ln(4.08)}{((0.09) - \frac{1}{2}(0.18)^2)}$$

$$T_{EX}^* = 3.77 years$$

CHAPTER 5. DISCUSSION

The CyRide numerical study returns results with interesting insights. Despite a carbon credit revenue of nearly \$1/gallon for the use of 100% UCO biodiesel under a carbon price of \$75/MTCO_2e, the model suggests that in just under four years, the fleet can expect to make the transition out of biodiesel and back into petroleum diesel. In an era when alternative, clean fuels are being adopted at their quickest rate, it seems strange that according to this model a company can expect to make the switch from biodiesel back to petroleum diesel in the relatively near future. This is because the price of petroleum diesel has seen a significant increase since the beginning of the conflict in Ukraine in early 2022, and because the price of petroleum diesel has increased at a rapid rate, the price of biodiesel, at a 29% premium has also hastened. As the value of petroleum diesel price climbs, so does the impact of a 29% premium for biodiesel and, at a point, it no longer makes economic sense to use 100% biodiesel because the price premium is outpacing the investment cost and revenue form carbon credits.

The potential transition away from biodiesel in favor of more emissive petroleum diesel can be combatted in a few ways. First, the government or entity that wishes to ensure companies continue the use of biodiesel in their fleets, can increase the price of carbon which will increase the revenue per gallon of carbon credits, R_C. According to Equation (11), as the revenue from carbon credits increases, so too does the optimal transition price point, P_P^*, at which a company should exit biodiesel. This also means that as the optimal transition price point increases, the expected time until that transition point, T_{EX}^*, increases which means companies will delay their exit from biodiesel. A very similar result is found when analyzing investment cost, I. If a government entity decides to leave the price of carbon steady, they can instead make the investment to switch to petroleum diesel more expensive by levying additional taxes/fees to deter a company from making this decision. Once again, as Equation (11) displays, as I increases, so too does P_P^*. Finally, the last action a government could take would be the implementation of incentives to biodiesel producers or taxes/fees on petroleum producers to tighten the gap in price between the two fuels. As Equation (11) shows, if the price premium of biodiesel to petroleum diesel is decreased from 29%, in effect signaling that the two fuels are closer in price, the optimal transition price point, P_P^*, increases as well.

CHAPTER 6. CONCLUSION

This paper provides interesting insights into the price dynamics of biodiesel and petroleum diesel withing a carbon priced environment and how a company operating a fleet of vehicles on one of these fuels can make the optimal economic decision of when to employ one in place of the other. Analyzing these decisions from a carbon priced environment perspective allows companies to think ahead about the decisions that they will likely have to make as carbon pricing continues to spread across the country. Armed with these equations, a fleet operator or business leader can be confident they are making the best financial choice for an engineering decision such as fuel utilization.

This paper also informs the government entities who are in control of the carbon priced environment. It provides insight into how the decisions they make in regards to incentives and regulations can impact the decisions of the fleet owner who ultimately is driven by ensuring their company is successful. Manipulating items like the price of carbon, the investment costs of transitioning from one fuel to another, and the pricing dynamics of both biodiesel and petroleum diesel allow them to ensure cleaner fuels are continually implemented in their jurisdiction. Ultimately, the power really lies in their hands instead of the business owner who is simply reactive to what is best for his business given their established policies.

Extensions for Future Research

This work has limitations in both its scope and complexity. First, one will notice that this work analyzed the situation after a company has made the transition from petroleum diesel into cleaner 100% biodiesel and is considering making the transition back to petroleum diesel. An immediate extension of this work would be to apply this optimal timing model to a situation in which a company is considering the move from petroleum diesel to 100% biodiesel. Furthermore, it is possible to generate a model that oscillates between 100% biodiesel and petroleum diesel depending on the fuel price dynamics and volatility within an optimal control theory framework.

Another advancement or extension of this work is to apply its principals, first in this simpler form and then in a more complex manner, to other clean fuels beyond strictly biodiesel. Also, representing carbon price as its own geometric Brownian motion would be an excellent extension to more closely resemble the California LCFS which is subject to a dynamic open market and is not a static price as was assumed in this paper.

The price premium was also assumed to be a static 29% based on the historical price data of both biodiesel and petroleum diesel, but this is not the case. The price premium of biodiesel has varied greatly in both percentage and nominal value; to provide a more precise recommendation for optimal transition point, P_P^*, and time until the optimal transition point, T_{EX}^*, this dynamic price premium should be employed over the static method used in this work. In another way, we observed that the fluctuation of the difference between the 100% biodiesel price and petroleum diesel price may potentially be a separate GBM process (see *Figure 1*). This implies that a new framework of decision making is possible purely based on this new GBM process (see *Figure 2* below).

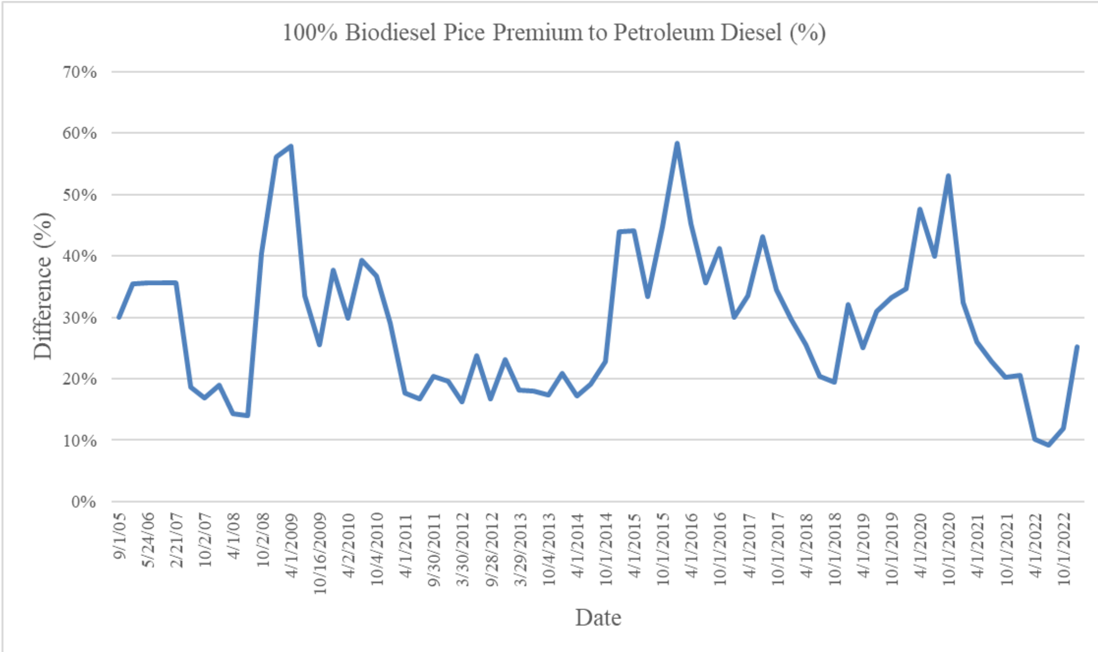


Figure 2: Plot of 100% Biodiesel Price Premium to Petroleum Diesel

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APPENDIX A: Petroleum Diesel and 100% Biodiesel Price Data

Date	Petroleum-Based Diesel	100% Bio-Based Diesel
9/1/05	$2.54	$3.30
1/1/06	$2.32	$3.14
5/24/06	$2.69	$3.65
9/4/06	$2.37	$3.21
2/21/07	$2.37	$3.22
7/3/07	$2.67	$3.17
10/2/07	$2.81	$3.28
1/21/08	$3.05	$3.63
4/1/08	$3.71	$4.24
7/21/08	$4.22	$4.81
10/2/08	$3.27	$4.59
1/12/09	$2.19	$3.42
4/1/2009	$2.04	$3.22
7/20/2009	$2.27	$3.03
10/16/2009	$2.50	$3.14
1/19/2010	$2.57	$3.54
4/2/2010	$2.71	$3.52
7/12/2010	$2.65	$3.69
10/4/2010	$2.75	$3.76
1/24/2011	$3.09	$3.99
4/1/2011	$3.62	$4.26
7/14/2011	$3.54	$4.13
9/30/2011	$3.42	$4.12
1/13/2012	$3.46	$4.14
3/30/2012	$3.69	$4.29
7/13/2012	$3.36	$4.16
9/28/2012	$3.70	$4.32
1/10/2013	$3.55	$4.37
3/29/2013	$3.58	$4.23
7/12/2013	$3.50	$4.13
10/4/2013	$3.51	$4.12
1/1/2014	$3.49	$4.22
4/1/2014	$3.56	$4.17
7/1//14	$3.51	$4.18

10/1/2015	$3.38	$4.15
1/1/2015	$2.75	$3.96
4/1/2015	$2.56	$3.69
7/1/2015	$2.61	$3.48
10/1/2015	$2.30	$3.33
1/1/2016	$1.99	$3.15
4/1/2016	$1.90	$2.76
7/1/2016	$2.19	$2.97
10/1/2016	$2.21	$3.12
1/1/2017	$2.30	$2.99
4/1/2017	$2.27	$3.03
7/1/2017	$2.20	$3.15
10/1/2017	$2.46	$3.31
1/1/2018	$2.63	$3.41
4/1/2018	$2.70	$3.39
7/1/2018	$2.89	$3.48
10/1/2018	$2.99	$3.57
1/1/2019	$2.65	$3.50
4/1/2019	$2.75	$3.44
7/1/2019	$2.71	$3.55
10/1/2019	$2.74	$3.65
1/1/2020	$2.71	$3.65
4/1/2020	$2.33	$3.44
7/1/2020	$2.20	$3.08
10/1/2020	$2.13	$3.26
1/1/2021	$2.35	$3.11
4/1/2021	$2.77	$3.49
7/1/2021	$2.90	$3.56
10/1/2021	$3.10	$3.73
1/1/2022	$3.22	$3.88
4/1/2022	$4.50	$4.96
7/1/2022	$5.02	$5.48
10/1/2022	$4.60	$5.15
1/1/2023	$4.08	$5.11

APPENDIX B: Carbon Credit Revenue per Gallon Calculation

$$Emissions\ per\ Gallon\ of\ Petroleum\ Diesel = CI\ score\ Petroleum\ Diesel \times\ Energy\ Content\ of\ Petroleum\ Diesel$$

$$Emissions\ per\ Gallon\ of\ 100\%\ Biodiesel = CI\ score\ 100\%\ Biodiesel \times\ Energy\ Content\ of\ 100\%\ Biodiesel$$

$$Emissions\ per\ Gallon\ Reduction = Emissions\ per\ Gallon\ of\ Petroleum\ Diesel - Emissions\ per\ Gallon\ of\ 100\%\ Biodiesel$$

$$Emissions\ per\ gallon\ reduction\ \left(\frac{gCO_2e}{gal}\right) \times \frac{1MT}{1{,}000{,}000g} = Emissions\ per\ gallon\ reduction\ (\frac{MTCO_2e}{gal})$$

$$Carbon\ Price \left(\frac{\$}{MTCO_2e}\right) \times Emissions\ per\ gallon\ reduction\ \left(\frac{MTCO_2e}{gal}\right) = Carbon\ Credit\ Revenue\ per\ Gallon\ (\frac{\$}{gal})$$

APPENDIX C: Derivation of Phase 2 Business Value

$$V_2 = E\left[\int_T^{\infty} (mR_O - mP_P)e^{-\rho(x-t)}dt\right]$$

$$= \int_T^{\infty} e^{-\rho(x-t)}(mR_O)dt - E\left[\int_T^{\infty} e^{-\rho(x-t)}mP_P\, dt\right]$$

$$= mR_O \int_T^{\infty} e^{-\rho(x-t)}dt - m\int_T^{\infty} e^{-\rho(x-t)}E(P_P)\, dt$$

$$= mR_O \int_T^{\infty} e^{-\rho(x-t)}dt - m\int_T^{\infty} e^{-\rho(x-t)}P_P e^{\alpha(x-t)}\, dt$$

$$= mR_O \int_T^{\infty} e^{-\rho(x-t)}dt - mP_P\int_T^{\infty} e^{-(\rho-\alpha)(x-t)}\, dt$$

$$= \frac{mR_O}{\rho}e^{-\rho(x-t)}|_T^{\infty} - \frac{mP_P}{\rho-\alpha}e^{-(p-a)(x-t)}|_T^{\infty}$$

$$V_2 = \frac{mR_O}{\rho} - \frac{mP_P}{\rho-\alpha}$$

APPENDIX D: Derivation of Phase 1 Business Value

The foundation for this derivation comes from chapter 5 of *Investment Under Uncertainty* (Dixit & Pindyck, 2012). Specifically, the foundational derivation, in its entirety, spans pages 142-144. This applied solution draws on the derivation provided in similar scenarios of Zhao & Min (2020) and Sadat et al., (2023).

We begin by rewriting our second order differential equation as below:

$$\frac{1}{2}\sigma^2 V_1{''} P_P^2 + \alpha V_1' P_P - \rho V_1 = -(mR_O - mP_P + mR_C)$$

This solution has both a general solution and a particular solution. First, a particular solution can be verified using the same method as *Appendix C*:

$$V_1(P_P) = \frac{mR_O}{\rho} - \frac{1.29mP_P}{\rho - \alpha} + \frac{mR_C}{\rho}$$

given the technical condition of $(\rho - \alpha) > 0$.

Also, a general solution can be identified as:

$$V_1(P_P) = A_1 {P_P}^{\beta_1} + A_2 {P_P}^{\beta_2}$$

Under technical conditions of $(\rho - \alpha) > 0$ and $\left(\alpha - \frac{\sigma^2}{2}\right) > 0$, where the roots of the equation are:

$$\beta_1 = \frac{\frac{1}{2}\sigma^2 - \alpha + \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + (2\sigma^2\rho)}}{\sigma^2} > 1$$

$$\beta_2 = \frac{\frac{1}{2}\sigma^2 - \alpha - \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + (2\sigma^2\rho)}}{\sigma^2} < 0$$

and A_1 and A_2 are constants to be determined.

Let us further examine the general solution for business value in Phase 1 that we defined above:

$$V_1(P_P) = A_1 {P_P}^{\beta_1} + A_2 {P_P}^{\beta_2}$$

If we allow P_P to go to infinity, the first term, $A_1 {P_P}^{\beta_1}$, also goes to infinity because $\beta_1 > 1$. This makes sense because as the price of petroleum diesel approaches infinity, there is no incentive to switch back to petroleum diesel and the switch will be delayed. However, when P_P is allowed to approach 0, the second term, $A_2 {P_P}^{\beta_2}$, approaches infinity suggesting the switch should be delayed. This does not make sense because as the price of petroleum diesel decreases to zero, there is high incentive to switch back to petroleum diesel. Therefore, the second term, $A_2 {P_P}^{\beta_2}$, is eliminated from the general solution and upon combining the general solution with the particular solution, the business value of Phase 1 is determined:

$$V_1(P_P) = A_1 {P_P}^{\beta_1} + \frac{mR_O}{\rho} - \frac{1.29mP_P}{\rho - \alpha} + \frac{mR_C}{\rho}$$

APPENDIX E: Derivation of Optimal Transition Price Equation

Begin by plugging the preestablished equations for Phase 1 business value, $V_1(P_P^*)$, and Phase 2 business value, $V_2(P_P^*)$ into the value matching condition of Equation (7):

$$V_1(P_P^*) = V_2(P_P^*) - I$$

$$A_1 P_P^{*\beta_1} + \frac{mR_O}{\rho} - \frac{1.29mP_P^*}{\rho - \alpha} + \frac{mR_C}{\rho} = \frac{mR_O}{\rho} - \frac{mP_P^*}{\rho - \alpha} - I$$

$$A_1 P_P^{*\beta_1} = \frac{1.29mP_P^* - mP_P^*}{\rho - \alpha} - \frac{mR_C}{\rho} - I$$

$$A_1 P_P^{*\beta_1} = \frac{mP_P^*(1.29 - 1)}{\rho - \alpha} - \frac{mR_C}{\rho} - I$$

$$A_1 P_P^{*\beta_1} = \frac{0.29mP_P^*}{\rho - \alpha} - \frac{mR_C}{\rho} - I \qquad \text{(a)}$$

Upon simplifying as far as possible, the smooth pasting condition of Equation (8) is used:

$$\beta_1 A_1 P_P^{*\beta_1 - 1} = \frac{0.29m}{\rho - \alpha}$$

$$A_1 P_P^{*\beta_1 - 1} = \frac{0.29m}{\beta_1(\rho - \alpha)} \qquad \text{(b)}$$

Dividing Equation (a) by Equation (b):

$$\frac{A_1 P_P^{*\beta_1}}{A_1 P_P^{*\beta_1 - 1}} = \frac{\frac{0.29mP_P^*}{\rho - \alpha} - \frac{mR_C}{\rho} - I}{\frac{0.29m}{\beta_1(\rho - \alpha)}}$$

$$P_P^* = \left(\frac{0.29mP_P^*}{\rho - \alpha} \times \frac{\beta_1(\rho - \alpha)}{0.29m}\right) - \left(\frac{mR_C}{\rho} \times \frac{\beta_1(\rho - \alpha)}{0.29m}\right) - \left(I \times \frac{\beta_1(\rho - \alpha)}{0.29m}\right)$$

$$P_P^* = \beta_1 P_P^* - \frac{R_C \beta_1 (\rho - \alpha)}{0.29\rho} - \frac{I\beta_1(\rho - \alpha)}{0.29m}$$

$$\beta_1 P_P^* - P_P^* = \frac{R_C \beta_1 (\rho - \alpha)}{0.29\rho} + \frac{I\beta_1(\rho - \alpha)}{0.29m}$$

$$P_P^*(\beta_1 - 1) = \beta_1 \left(\frac{R_C(\rho - \alpha)}{0.29\rho} + \frac{I(\rho - \alpha)}{0.29m}\right)$$

$$P_P^* = \left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{(\rho - \alpha)}{0.29}\right)\left(\frac{R_C}{\rho} + \frac{I}{m}\right)$$