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The value of jumboization in transportation ships: A real options approach

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ABSTRACT
In the presence of budget constraints, "jumboization," has been adopted as a practical solution to meet increased transportation needs. By jumboization, we mean increasing the capacity of a ship by extending its length at a future date. There are, however, two kinds of jumboization: Fixed design (retrofitting) and flexible design. With fixed design, the initial construction cost is lower, but the subsequent jumboization cost is higher. With flexible design, the initial construction cost is higher, but the subsequent jumboization cost is lower. In this article, for both designs, we build and analyze economic decision models, and show how to value the option to jumboize. Our framework utilizes a stochastic optimal control approach that considers the volume of transportation needs (the demand) as an underlying uncertain factor. Under the criterion of cost savings maximization, we determine optimal threshold demand level to jumboize. Through analytical and numerical analyses, we obtain conditions under which the flexible design is preferred over fixed design, and *vice versa*. A comprehensive, illustrative example is also provided.

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Engineering design; flexibility in engineering design; jumboization; vehicle capacity expansion; real options

1. Introduction

In recent years, a new trend that has emerged in both industrial practice of design and academic research (Wang, 2005) involves design options that highlight that initial product design can be done in such a way that a user can modify the design in the future at a relatively small cost. In other words, by incurring an up-front cost in the initial design, the user may purchase an option to change the design in the future at relatively lower cost. Several existing real-life examples of this phenomenon include flexible building, e.g., relative ease in adding floors at a future date for parking (De Neufville *et al.*, 2006) and communications satellites (De Weck *et al.*, 2004).

Ship design is a practical area in which real design options can be addressed. Jumboization is considered to be a type of modularity in ship design (Doerry, 2014). When a decision maker (throughout this article we will refer to a single decision maker, although real-life ship design decisions are obviously carried out by a group of professionals) decides to jumboize, a ship's hull is divided into two components, a newly built midsection inserted, and the whole process terminates by combining separated hull sections (Figure 1). Jumboization fits into the definition of real options in engineering design, as the decision maker must pay an upfront cost during the initial design procedure to achieve a stronger hull structure than actually required by the initial design (Buxton and Stephenson, 2001). Moreover, the decision maker will typically have the right, but no obligation, to insert the mid-body. In the language of financial options, the upfront and jumboization costs are regarded as the purchase price of an option and the strike price, respectively.

In this study, we evaluate jumboization investments for transportation ships with a real options approach to determine the expected time of jumboization and its value, and to provide a managerial guideline regarding choice between fixed and flexible designs. In the case of flexible design, jumboization can be conducted more easily, and is thus, less costly. Transportation ships are our sole focus in this article, as cutting the hull may not be a feasible operation for other types of ships, such as warships, due to security concerns.

Various transportation ships have been jumboized so far in both the private and public sectors. For instance, our investigation reveals that replenishment oilers whose primary purpose is to transport fuel to U.S. Navy ships at sea (U.S. Navy ships whose fuel is replenished by replenishment oilers are called receiving ships in the literature) have generally been considered for jumboization. To the best of our knowledge, the U.S. Navy has jumboized 13 replenishment oilers so far, and these ships were not initially designed for jumboization. There are research questions (Doerry, 2014) concerning possible cost savings if they had initially been designed for jumboization. Doerry (2014) states that there is a need for analytically rigorous methods to evaluate flexibilities in design of U.S. Navy ships, motivating us to conduct this study for both private and public sector ships.

In this article, we embrace fuel cost saving as the financial gain resulting from jumboization. Although private sector transportation ships have larger spaces after jumboization,

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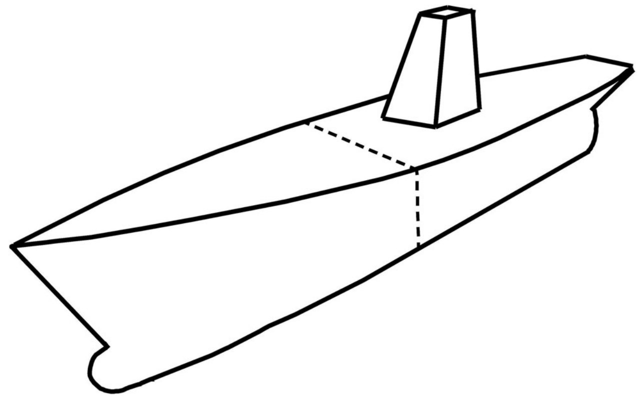

Figure 1. A model ship with a line where a new mid-section will be inserted.

which more likely translates into more revenue, we neglect this part because revenue gain is also conditional upon several external factors, such as freight rates and currencies, and it is highly fluctuating. Focusing solely on fuel cost saving as a financial outcome is also useful to derive managerial insights for the decision makers in the U.S. Navy and other non-profit organizations.

Throughout this study, we will consider only one transportation ship and evaluate its jumboization. Considering a single transportation ship and neglecting its collaboration with other transportation ships is not an abstraction from reality. The U.S. Navy currently has six fleets serving at sea and each fleet has its own commanders and missions, which reflect that they operate individually. The area of operation of each fleet can also be subdivided into smaller regions with respect to various crucial places such as port locations. This compartmentalized property of the U.S. Navy enables us to focus on a smallest unit of such a massive system of operations. Another supporting fact is found in reports published by The U.S. Navy's Military Sealift Command. They do not exhibit any real-life example of a situation where multiple replenishment oilers depart from a location while simultaneously replenishing receiving ships, see Shannon (2014) and various reports published by Military Sealift Command in different years. This implies that replenishment oilers operate individually, and that the fuel demand of receiving ships is separable and independent of other ships (see also Blackman (2012) who assumes that only one replenishment oiler departs from a port). We encourage interested readers to review Besbes and Savin, (2009) and Kim *et al.* (2012), whose models consider only one ship in the private sector. Note that, in contrast with the requirement of having a single transportation ship, our framework is applicable to the presence of multiple ports of discharge. See Assumption 2 for more explanation in this respect.

The importance of this study is twofold:

1. Although jumboization is an option for the U.S. Navy, its value has not been yet estimated. Doerry (2014) argues that if this value is known, an appropriate amount of upfront cost would be spent to make ships ready for future jumboization operations, so as to save a massive jumboization cost. Our study attempts to answer such a practical question by providing a guideline regarding the trade-off between upfront and jumboization costs. Through our study, one can assess if jumboization is a viable option for specific cases. This also enables us to evaluate when a fixed design is more preferable over flexible design.
2. Application of the real option approach is not common for non-profit investments (defense, public sector, charity organization, etc.) due to a challenge arising from the irrelevance of profit – if it even exists. Through this study, we show a way of how to apply a real options approach for non-profit investments. We model the gain through jumboization as a cost saving in burning fuel. We are aware that not all public investments can be modeled from this perspective, but our study shows it to be a plausible base upon which more discussions and research can be built.

The remainder of this article is structured as follows: Section 2 describes extant literature that exemplifies jumboization. It also introduces studies that broadly investigate the value/benefit of flexibility in engineering design. Section 3 presents modeling assumptions and the analytical framework for option valuation. Section 4 proposes managerial guidelines concerning the choice between flexible and fixed designs, followed by sensitivity analyses in Section 5 revealing significant managerial insights. To exhibit the key components of our framework, in Section 6 we provide a numerical example based on a real replenishment oiler. Section 7 discusses possible generalizations of modeling assumptions, an equation used in the article, and a critical research question. Section 8 concludes this article by summarizing its key findings. An appendix is provided as Supplemental Online Materials.

2. Literature review

This study contributes to various streams of literature, some of which are reviewed below.

Evaluation of jumboization has been conducted for private sector ships in recent years, with a growing number of research studies in this area. For example, Bačkalov *et al.* (2014) study the economic feasibility of lengthening of inland vessels in Europe by focusing on two particular reference ships, proving that lengthening larger ships is more attractive than using multiple smaller vessels, due to a shorter payback period. Ericson and Lake (2014) determine the payback period by considering investment cost and additional income resulting from increased cargo capacity of an example ship. They reveal that lengthening brings about a reduction in required propelling power per cargo tonne (throughout this article, we use metric ton whenever we refer to mass. From now on, we will use t and tonne interchangeably) at a constant speed. Buxton and Stephenson (2001) conduct a simulation analysis to evaluate different design strategies for a container ship. Flexible design is proven to be the most preferable approach in terms of net present values governing the design strategies. Another simulation study is conducted by Knight and Singer (2012)

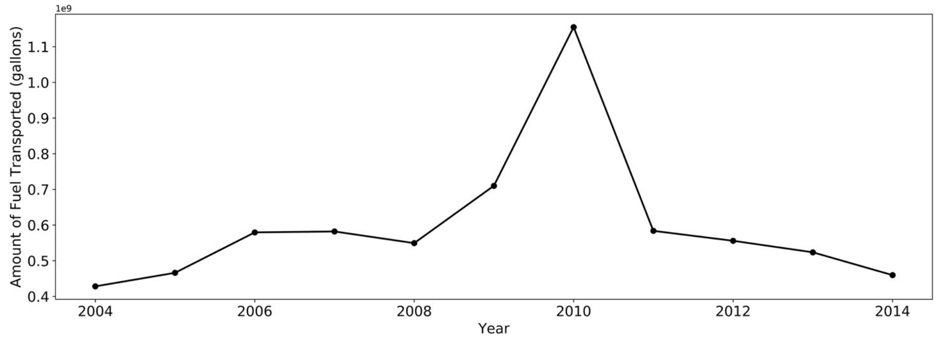

Figure 2. Fuel amount transported by all replenishment oilers in different years.

to determine the value of jumboization of a container ship by modeling the freight rate as the underlying stochastic parameter. Rehn *et al.* (2019) measure changeability levels in engineering systems and demonstrate their proposed framework on an offshore ship, typically a tanker, but configurable to a vessel carrying a light well intervention tower when necessary. Rehn *et al.* (2018) investigate trade-offs between platform flexibility and performance expectation for offshore ships and conclude that, although flexibility increases capacity, it still might be unfavorable because it may potentially harm platform operability (see also, Niese and Singer (2014) and Pettersen and Erikstad (2017)).

As for U.S. Navy ships, there is an abundance of work highlighting real options applications for evaluating the modularity concept. Gregor (2003) assesses flexibilities in naval ship design and procurement and demonstrates methods of utilizing a real options approach through a case study that emphasizes the characteristics of modular design for ships. Page (2012) presents a case study based on a destroyer-type ship and provides results related to the financial benefits of modularity. Knight (2014) develops a novel approach comprising real options, utility theory, and game theory to evaluate design flexibilities in naval ship design. Case studies focusing on different aspects of modularity are solved to demonstrate the use of the proposed approach. Knight and Singer (2014) clarify a critical question regarding how much extra power equipment should be installed at the initial design stage to increase the future capacity for the number of beds in a high-speed connector ship, one of whose primary functions is to provide medical support.

Further review of the literature is available in the Supplemental Online Materials. In the next section, we present our modeling assumptions and an analytical framework for evaluating a jumboization option on transportation ships.

3. Mathematical model

Since profit-making companies and non-profit organizations (e.g., the U.S. Navy) may have various types of transportation ships, we make simplifying assumptions to build the most fundamental model in order to facilitate the derivation of managerial insights. Our model is based on the following scenario and assumptions: Suppose the decision maker wants to purchase a new transportation ship, and s/he is asked to choose between two design alternatives: fixed design or flexible design.

Assumption 1. Demand for transported item carried by the transportation ship (tonne at a time, e.g., half a month as a unit time interval) follows a Geometric Brownian Motion (GBM) process, mathematically stated as:

$$dD_t = \alpha D_t dt + \sigma D_t dz \tag{1}$$

where dz is a Brownian increment; i.e., $dz = \epsilon\sqrt{dt}$, $\epsilon \sim N(0,1)$. In this case, $E[dz] = 0$ and $Var[dz] = dt$.

α (%/unit time) and σ (%/unit time) are defined as growth and volatility parameters of demand evolution. Note that transportation cycle may vary between days and months.

Treating demand as an uncertain factor is a not a faulty way of modeling this transportation problem. For example, demand is monitored by the decision maker to determine when jumboization has to be performed, in line with real practice followed by the U.S. Navy. History shows that demand for fuel by receiving ships has been an influential factor in deciding whether or not to jumboize replenishment oilers.

Our assumption concerning demand following a GBM process requires statistical validation. We unfortunately lack data of demand amount transported by transportation ships. We, however, came across several reports annually-published by the U.S. Navy (Shannon, (2014) and other similar reports published in previous years) describing the total amount of fuel per year transported by all replenishment oilers (see Figure 2 for evolution of fuel amount transported over years, and note 1 gallon is equivalent to 3.78×10^{-3} cubic meter). We conduct statistical tests on this data set (see Supplemental Online Materials) by assuming it is representative of the fuel amount transported by a single replenishment oiler. These tests validate the GBM assumption.

Assumption 2. Ports of load and discharge of the transportation ship do not change. In other words, the transportation ship makes round trips between two specific locations. The distance in nautical miles between two ports is denoted by X (one nautical mile is equal to 1852 m).

Note that the transportation ship travels distance X twice, once while transporting the cargo to the port of discharge and once when returning to the loading port.

This assumption plays an important role when modeling transportation logistics in both the U.S. Navy and the private

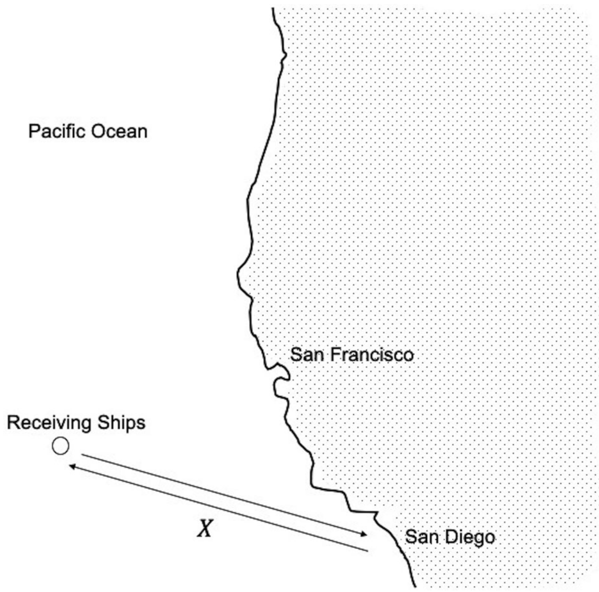

Figure 3. An example route for the replenishment oiler.

sector. As for the U.S. Navy, replenishment of receiving ships can be accomplished within a very small area (Figure 3). To confirm this point, Blackman (2012) simulates and predicts future replenishment locations in the eastern and northern Pacific, showing that replenishment locations change by less than 20 nautical miles. Historical data on replenishment locations (Blackman, 2012) supports the assumption that multiple receiving ships can be replenished within a very small region at sea. For example, receiving ships around Monterey, California were replenished more than 100 times within a 50-square mile area over a couple of weeks.

There are several studies in the ship scheduling literature that make similar assumptions. For example, Boros *et al.* (2008) consider two shipping companies with different objectives as parties in a supply chain contract, and determine the optimal cycle time of their vessels by assuming that the vessels operate between two specified ports. Another study conducted by Chen *et al.* (2007) achieves solvability of special cases of bi-directional vessel routing using a linear program; they again assume that ships operate between two specified locations.

Assumption 3. The transportation ship moves at a constant speed of S in knots (1 knot is equal to 0.514 m/s), during each round trip. In other words, it moves with a fixed speed in transporting the cargo to the port of discharge and in returning to the loading port and this speed remains constant during subsequent round trips.

This assumption can be justified in two different ways. First, speed change of the transportation ship may have a dynamic aspect on a one-way trip, but we simplify it by noting that there exists an average speed, calculated over each one-way trip. Embracing the average speed rather than dealing with its dynamic nature is a common trend in the literature. For example, Aydin *et al.* (2017) assume in their model that the ship speed (average speed) does not change during a trip from one port to the next port (see also Besbes and Savin (2009) and Kim *et al.* (2012)).

Second, our assumption implies that average speed remains constant through multiple round trips. This can be rationalized as follows: By Assumption 2, travel distances of the transportation ship do not change over the time horizon, and by Assumption 1, demand occurs at each equal time period. Furthermore, in reality, the engines of the transportation ships are of a size sufficient to carry maximum loads. Thus, no matter how large the load, the transportation ship is able to maintain the constant speed. Since it travels the same distance multiple times throughout the modeling horizon, the decision maker can choose an appropriate speed for operational purposes. In line with this justification, Fagerholt (1999) determines optimal fleet size and optimal route for each selected ship to transport its cargo from a central depot to multiple off-shore locations. The main assumption of his study is that all selected ships have a common speed that does not change over time, and this is claimed to apply to many practical problems. He also emphasizes that the model does not deal with temporal aspects of the problem, as the model does not try to consider time windows in scheduling all the ships.

In addition to the above explanations, we note that dynamic aspects of speed change may not be incorporated into our mathematical model, as how the speed changes with time is not obvious (e.g., undefined mathematical formulation).

Note that since $(1852X/0.514S)/3600 \cong X/S$ gives the number of hours needed for the transportation ship to transport cargo to the port of discharge, the unit time duration should be larger than or equal to $2X/S$. If it is larger than $2X/S$, the transportation ship completes its task and stays at a port without functioning until another call for cargo.

Assumption 4. The transportation ship is non-depreciating, thus jumboization is an infinitely-lived option.

Although this assumption seems to be impractical, we require it because our analytical framework results in closed-form solutions only if this assumption is put in place (Dixit and Pindyck, 1994).

For further discussions about Assumptions 1, 2, 3, and 4, interested readers can review the Supplemental Online Materials.

Assumption 5. Let I_{flex} and I_{fixed} be the costs incurred during jumboization operations for flexible and fixed designs, respectively. It is assumed that $I_{flex} < I_{fixed}$.

This is intuitively true because the decision maker pays less for jumboization, due to the fact that flexible design implies that preparations have already been made for lengthening; otherwise, flexible design has no competitive advantage at all, due to its additional upfront costs incurred at the initial stage of ship-building.

The upfront cost of flexible design, denoted by I_0, can arise from a stronger hull structure by more advanced scantlings. The hull of a jumboized ship must undergo strengthening due to it experiencing larger bending moments and shear forces (Buxton and Stephenson, 2001). A bending moment is defined as the amount of bending applied to the hull by external forces, measured in tonne-meters (Bulk

Table 1. Notations associated with ship design and their definitions

Notation	Definition	Explanation
$\mathcal{L}$	Light ship mass	Mass of the ship's hull and other permanent items (including ballast water) in the ship (tonnes)
Δ	Displacement	Light ship mass plus the maximum amount of cargo that the ship can carry (tonnes). In other words, displacement refers to maximum tonnes that the ship can carry.
P	Power	The maximum power required to propel the ship (kW)
L	Length	Length of the ship (m)
B	Breadth	Width of the ship (m)
$\mathbb{D}$	Draft	Vertical distance between the waterline and the bottom of the hull (m)
$\mathcal{D}$	Depth	Vertical distance between the top and the bottom of the hull (m)

Carrier Guide, 2010). It is basically caused by two different forces: weight on the hull (acting downwards) and buoyancy (acting upwards). If the weight distribution exceeds the buoyancy in the mid-section of the hull, the bending moment is called sagging. On the other hand, if the weight distribution is greater than the buoyancy in the stern (backward part of the hull) and the bow (forward part of the hull) sections, this is called hogging. In addition to weight and buoyancy, the forces created by waves can also result in bending moments (González, 2013). The shear force (measured in tonnes), is defined as the force applied at any point of the length of the ship and tends to move one part of the hull to an adjacent position vertically. In other words, it is the tendency of the hull to break apart, and is basically caused by uneven load distribution and unbalanced vertical forces. To reduce bending moments and shear forces, use of higher strength steels is suggested. If at the initial design the decision maker decides to use higher strength steels by paying upfront cost I_0, the effects of higher bending moments and shear forces resulting from jumboization can be balanced.

In the subsequent subsections, we first introduce the benefits gained through jumboization, the main objective to be maximized in our model. We then present a way of determining the value of the jumboization option, as well as its expected time by means of an analytical framework.

3.1. Fuel cost saving gained through jumboization

The literature describing the mechanical design of ships reveals that lengthening a ship generally decreases its wave-making resistance (ABS, 2017). Since resistance against the ship is directly proportional to fuel consumption (Ericson and Lake, 2014), we assert that jumboization usually leads to savings in fuel costs. (Replenishment oilers in the U.S. Navy transport fuel. In the replenishment oiler context, demand refers to fuel transported to the receiving ships. Bunker fuel refers to the fuel consumed to propel the replenishment oiler. To better reflect this difference, we use tonnes as the units of cargo and gallons as the units of bunker fuel).

A question may arise as to whether adopting saving in fuel costs as the main performance measure for jumboization is too limited. In other words, it may seem that saving in fuel costs is a secondary objective of jumboization investment, or their natural ramification. However, there are private transportation companies who have introduced jumboization operations into their fleets for the sake of energy consumption. Finnlines, a transportation company located in Finland, jumboized its four large vessels in 2017 to reduce energy consumption per unit transported cargo (Finnlines, 2017). This certainly strengthens our assertion that savings in fuel costs should be taken into account as a performance measure for jumboization investment.

Sen and Yang (1998) indicate that power (required to propel the transportation ship) and fuel consumption are proportional. In the literature, several expressions for power that approximate the real power required by a ship can be found. In this study, we will use the most elaborate and precise approximation. Table 1 shows the notation and corresponding definitions of basic parameters utilized in ship design. Other parameters used throughout the study and their definitions are given in the text.

Sen and Yang (1998) express power as follows:

$$P = \frac{\Delta^{2/3} S^3}{m + n\frac{0.514S}{\sqrt{gL}}} \tag{2}$$

where g is the gravitational constant (m/s^2) and $m > 0$ and $n < 0$ are coefficients (recall that S is speed). Equation (2) is subject to the constraints $L/B \geq 6$, $L/\mathcal{D} \leq 15$ and $L/\mathbb{D} \leq 19$, stemming from mechanical principles. (They also represent boundaries of empiricism because Equation (2) is an empirical relationship. See the Supplemental Online Materials for more explanation.) For example, increasing the length causes a greater chance of rolling down. In addition to mechanical constraints, topological barriers of routes require ships not to exceed certain levels in these dimensions. Note that the power expressed in Equation (2) represents the maximum power (often called installed power) required to propel the ship, because Δ, by definition, is the maximum number of tonnes that a ship can carry. Equation (2) captures many realities. For example, at constant displacement, if the length of the ship increases, then the maximum power required in moving it decreases; this supports the fact that a longer hull creates less resistance and leads to a smaller power requirement.

Sen and Yang (1998) give expressions for m and n as well. Their analysis results in $m := 4977C_b^2 - 8105C_b + 4456$ and $n := -10847C_b^2 + 12817C_b - 6960$ where C_b is a block coefficient (note that b is not a subscript; C_b is generally used to denote block coefficient in the ship design literature), defined as follows: Imagine that a rectangular prism is built around the submerged part of the ship. The proportion of the actual volume of this part to the volume of the rectangular prism is defined as a block coefficient. Block coefficients are said to increase as a result of jumboization (Ericson and Lake, 2014). It is a simple mathematical fact that a block coefficient of a to-be-added midsection is greater than those of bow and stern sections, so that the overall block coefficient of the lengthened ship is increased.

Since Sen and Yang (1998) state that the maximum daily consumption of bunker fuel is a linear function of P, i.e., $0.0046P + 0.2$, the maximum amount of bunker fuel consumption in a one-way voyage can be written as

$[(0.0046P + 0.2)/24](X/S)$. The amount of bunker fuel consumed per unit displacement (gallon/tonne) in a one-way voyage is derived as $\mathcal{F} := [(0.0046P + 0.2)X]/(24S\Delta)$. Since Δ includes both $\mathcal{L}$ and D (recall that D denotes demand for the cargo carried by the transportation ship and note that we drop the subscript t in D_t because it is irrelevant in this discussion), separation of round trip voyages of the transportation ship turns out to be important. Whereas it carries $\mathcal{L}$ and D to the port of discharge in one direction, it carries only $\mathcal{L}$ when returning to the loading port, so the fuel cost (\$/unit time) is given as

$$\mathcal{F}C(\mathcal{L} + D) + \mathcal{F}C\mathcal{L} \tag{3}$$

where C is the cost of a unit of bunker fuel (\$/gallon). Since jumboization changes Δ, $\mathcal{L}$, L and C_b, $\mathcal{F}$ and $\mathcal{L}$ in expression (3) vary between the pre-jumboization and the post-jumboization cases. Let $\mathcal{F}_1$ and $\mathcal{F}_2$ have the same definitions as $\mathcal{F}$, but denote pre-jumboization and post-jumboization cases, respectively (Make the same definitions for $\mathcal{L}$ as well.) Note that, since $\mathcal{F}$ is a function of Δ, L and C_b, these parameters can also be given subscripts 1 and 2. Therefore, fuel cost saving per unit time due to jumboization is expressed as $[\mathcal{F}_1C(\mathcal{L}_1 + D) - \mathcal{F}_2C(\mathcal{L}_2 + D)] + [\mathcal{F}_1C\mathcal{L}_1 - \mathcal{F}_2C\mathcal{L}_2]$, with simplification

$$2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C + (\mathcal{F}_1 - \mathcal{F}_2)CD \tag{4}$$

Note that $2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C$ may be negative, whereas $\mathcal{F}_1 - \mathcal{F}_2$ should be positive, so that expression (4) can be positive for large values of D. This emphasizes that there is a level for D above which expression (4) is positive and jumboization is effective in bringing about a saving in fuel costs.

3.2. Option valuation for jumboization in the analytical framework

Since jumboization is an option for the decision maker to be exercised when financial benefits from jumboization become justified, this problem can be treated as an optimal stopping problem. In other words, there exists a D^* (threshold demand level), above which the decision maker decides on jumboization and below which, he/she does not so decide. When the transportation ship is jumboized, the decision maker begins to gain all future savings in fuel costs immediately after jumboization. Assuming that jumboization is done at the level of D_x (note that x does not denote time, rather D_x is just a notation used to denote demand level at which jumboization is done), the value of project (project in this context is the jumboized transportation ship) is expressed as

$$V(D_x) = E\left\{\int_0^\infty \left[2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C + (\mathcal{F}_1 - \mathcal{F}_2)CD_t\right]e^{-\rho t}\mathrm{d}t\right\} \tag{5}$$

where ρ (%/unit time) is the risk-adjusted discount rate exogenously specified. Note that the lower bound of the integral is taken as zero, corresponding to the demand level denoted by D_x. Due to the assumption that the transportation ship has an infinite life, the model turns out to be time-invariant (see Dixit and Pindyck (1994) and Gryglewicz *et al.* (2008) for discussions). This implies that every time point behaves as a time point 0 with the consideration that there is literally no end to the ship's useful life.

It is assumed that $\rho > \alpha$ (recall that α is a drift parameter of the underlying demand process), else otherwise, waiting longer for the investment would always become a better policy (Dixit and Pindyck, 1994). Equation (5) is simplified as

$$V(D_x) = \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} + (\mathcal{F}_1 - \mathcal{F}_2)CE\left[\int_0^\infty D_t e^{-\rho t}\mathrm{d}t\right] \tag{6}$$

To perform the integration in Equation (6), we need to change the order of the integration and expectation. According to Fubini's theorem, certain conditions should hold when changing this order (Klebaner, 2005). Changing the order of integration and expectation is viable in our case (we omit the details due to the space limits), leading to the solution

$$V(D_x) = \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho} + \frac{(\mathcal{F}_1 - \mathcal{F}_2)C}{\rho - \alpha}D_x \tag{7}$$

The value of the option to jumboize the transportation ship, denoted by F, has a value that evolves as $\rho F\mathrm{d}t = E[\mathrm{d}F]$, meaning that the option gains capital appreciation before jumboization. It has no term related to saving fuel costs, as it is realized after jumboization. Since F is a function of D, one can derive the explicit form of $\mathrm{d}F$ by applying Ito's lemma, i.e., $\mathrm{d}F = \left[\alpha DF' + (1/2)\sigma^2D^2F''\right]\mathrm{d}t + \sigma DF'\mathrm{d}z$. It gives $E[\mathrm{d}F] = \left[\alpha DF' + (1/2)\sigma^2D^2F''\right]\mathrm{d}t$. When $E[\mathrm{d}F]$ is plugged into $\rho F\mathrm{d}t = E[\mathrm{d}F]$, one obtains $(1/2)\sigma^2D^2F'' + \alpha DF' - \rho F = 0$, and this second-order homogenous differential equation has the general solution $F(D) = AD^\beta$ that can be explicitly written as $F(D) = A_1D^{\beta_1} + A_2D^{\beta_2}$. The parameters β_1 (with +) and β_2 (with −) are found as

$$\beta_{1,2} = (1/2) - \left(\alpha/\sigma^2\right) \pm \sqrt{\left[(1/2) - (\alpha/\sigma^2)\right]^2 + 2\rho/\sigma^2}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are verified in Dixit and Pindyck (1994). To solve $F(D) = A_1D^{\beta_1} + A_2D^{\beta_2}$, boundary conditions must be established. One boundary condition, $\lim_{D\to 0} F(D) = 0$, is justifiable, because when the demand level approaches zero, the option to jumboize the transportation ship becomes ineffective, resulting in $F(D) = A_1D^{\beta_1}$. Other boundary conditions can be written for threshold demand value. At D^*, one writes

$$F(D^*) = V(D^*) - I \tag{8}$$

$$F'(D^*) = V'(D^*) \tag{9}$$

where I (can be either I_{fixed} or I_{flex}) is the investment cost incurred during jumboization operations. Equation (8) is a value-matching condition indicating that the decision maker receives benefits from jumboization via savings in fuel costs in exchange for jumboization cost. Equation (9) is smooth-pasting condition that guarantees optimality at D^*. The following proposition shows our main result:

Proposition 1. *Under conditions $0 < \alpha < \rho$ and $\mathcal{F}_1 - \mathcal{F}_2 > 0$ as well as $I - 2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C/\rho > 0$; Equations (7), (8), and (9) lead to*

$$D^* = \left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(\mathcal{F}_1-\mathcal{F}_2)C} \tag{10}$$

$$F(D) = \left[\frac{(\mathcal{F}_1-\mathcal{F}_2)C}{(\rho-\alpha)\beta_1}D\right]^{\beta_1}\left[\left(I - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)\frac{1}{\beta_1-1}\right]^{1-\beta_1} \tag{11}$$

It will be noted that $I - 2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C/\rho > 0$ should hold in order to obtain $D^* > 0$. As will be seen in the numerical example, design parameter values of a real replenishment oiler satisfy $\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2 < 0$, causing no problem in this respect, but if numerical values cause $\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2 > 0$ and this results in $D^* < 0$, we must enforce $I - 2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C/\rho > 0$ by adjusting the numerical values.

Given that D^* has the form presented in Equation (10), the expected time for a demand process to pass from an arbitrary D_0 (demand value at time 0) to D^* (under the condition that $D_0 < D^*$ because D has positive drift) is given by $\tau := (\ln D^* - \ln D_0)/(\alpha - \sigma^2/2)$. Note that we must assume $\alpha - \sigma^2/2 > 0$ if τ is to be positive.

In the next section, we provide a managerial guideline concerning the choice between flexible and fixed designs, and we propose conditions under which flexible design becomes financially superior to fixed design.

4. Choice between flexible and fixed designs

Since the decision maker does not necessarily need to accept flexible design, a question may arise as to conditions under which a flexible design would be more preferable than a fixed design. Upfront cost incurred for flexible design (I_0) plays a significant role in trading-off the options.

As stated previously, a transportation ship with fixed design can also be jumboized (retrofitting). Such a vessel has also an option value that is contingent upon demand uncertainty. To compare flexible and fixed designs, option values of both designs at time 0 must be taken into account. Let $F_{flex}(D_0)$ and $F_{fixed}(D_0)$ denote the option values of flexible and fixed designs, respectively, at time 0. A flexible design should be preferred over a fixed design in a case where the difference between $F_{flex}(D_0)$ and $F_{fixed}(D_0)$ is larger than the upfront cost. In other words, a flexible design should be preferred if $I_0 < F_{flex}(D_0) - F_{fixed}(D_0)$. Simplifying and rearranging terms gives

$$\begin{aligned} I_0 < {} & \left[\frac{(\mathcal{F}_1-\mathcal{F}_2)C}{(\rho-\alpha)\beta_1}D_0\right]^{\beta_1}\left(\frac{1}{\beta_1-1}\right)^{1-\beta_1} \\ & \left[\left(I_{flex} - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1}\right. \\ & \left. - \left(I_{fixed} - \frac{2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1}\right] \end{aligned} \tag{12}$$

The right-hand side of inequality (12) can be defined as the upper bound of the upfront cost. If I_0 is less than this upper bound, then the flexible design would be preferred, otherwise, the decision maker ought to adopt a fixed design.

Another guideline can be derived in a similar way by solving inequality (12) for D_0. Instead of tracking option values, the decision maker can track D_0 and make a decision accordingly. In other words, if the decision maker knows I_0, I_{flex}, and I_{fixed} *a priori*, he/she would prefer flexible design under the condition that

$$\begin{aligned} & \left[\frac{I_0}{\left(I_{flex} - \frac{2(\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1} - \left(I_{fixed} - \frac{2(\mathcal{F}_1\mathcal{L}_1-\mathcal{F}_2\mathcal{L}_2)C}{\rho}\right)^{1-\beta_1}}\right]^{\frac{1}{\beta_1}} \\ & \left(\frac{1}{\beta_1-1}\right)^{\frac{\beta_1-1}{\beta_1}}\left(\frac{(\rho-\alpha)\beta_1}{(\mathcal{F}_1-\mathcal{F}_2)C}\right) < D_0 \end{aligned} \tag{13}$$

It is inferred from inequality (13) that there is a certain level of initial demand above which a flexible design turns out to be more profitable. This is intuitively correct, as a relatively higher demand at time 0 more than likely rises to much higher levels in the future (since drift is positive), creating a need for extra space for transported cargo. Hence, the chance of jumboization of the transportation ship increases.

5. Sensitivity analysis and corresponding managerial insights

In this section, we present the result of sensitivity analyses derived from Equation (10) and the right-hand side of inequality (12) that give rise to significant managerial insights. The following results hold when $0 < \alpha < \rho$ and $\mathcal{F}_1 - \mathcal{F}_2 > 0$ as well as when $I - 2(\mathcal{F}_1\mathcal{L}_1 - \mathcal{F}_2\mathcal{L}_2)C/\rho > 0$.

Proposition 2. *For D^*, it can be inferred that $\partial D^*/\partial\mathcal{L}_1 < 0$ and $\partial D^*/\partial\mathcal{L}_2 > 0$. With respect to option value differences, it results in $\partial[F_{flex}(D_0) - F_{fixed}(D_0)]/\partial\mathcal{L}_1 > 0$ and $\partial[F_{flex}(D_0) - F_{fixed}(D_0)]/\partial\mathcal{L}_2 < 0$.*

If $\mathcal{L}_1$ is larger, D^* decreases (and the flexible design becomes more favorable), as the decision maker tends to gain more savings in fuel cost and should jumboize the transportation ship earlier because a larger mass results in greater fuel costs. On the other hand, if $\mathcal{L}_2$ is larger, then the decision maker should wait for higher demand values (and the flexible design becomes less desirable) to jumboize because the larger mass has less impact on savings in fuel costs.

We show other sensitivity analyses in the Supplemental Online Materials. In the following section, we demonstrate our mathematical model by solving a numerical example based on a real replenishment oiler. To reiterate, our framework is also applicable to transportation ships operated in the private sector, but finding relevant data may be a challenging task.

6. Numerical example based on a real replenishment oiler

In the Supplemental Online Materials, we list the annual amount of fuel transported by all replenishment oilers (Table A1) from 2004 to 2014. Although we are aware that

Table 2. Parameters and their values used in the numerical example.

Parameter	*Numerical Value*	*Parameter*	*Numerical Value*
Δ_1	26,417 t	Δ_2	36,977 t
$\mathcal{L}_1$	5790 t	$\mathcal{L}_2$	11,645 t
L_1	180 m	L_2	214 m
C_{b1}	0.88	C_{b2}	0.91
S	20 knots ($\approx$ 10.3 m/s)	σ	0.307 per year
ρ	0.089 per year	α	0.054 per year
C	\$2.46 per gallon ($\approx$ \$651 per cubic meter)	X	1400 nautical miles ($\approx$ 2593 km)
I_{fixed}	\$20,000,000	I_{flex}	\$15,000,000

this is an aggregated data set, we intend to estimate drift and volatility parameters of this data set for use of a single replenishment oiler in our model. Table 2 lists all parameters and their numerical values.

Let $\theta_y := \ln\phi_y - \ln\phi_{y-1}$ where ϕ_y is the amount of fuel transported in year y, i.e., $2005 \le y \le 2014$. The standard deviation of θ_y is accepted as te volatility of this data set ($\sigma = 0.307$ per year). The drift parameter ($\alpha = 0.054$) is estimated from the relation $\alpha - \sigma^2/2$, which represents the average of θ_y (for more details on the estimation of parameters, see Xu *et al.* (2012)).

We use as many actual values associated with a replenishment oiler's design parameters as possible. We take the replenishment oiler USS Willamette as an example; its construction began in 1978 and it was jumboized in 1991 (Pike, 1999), with the following design characteristics: $\Delta_1 = 26,417$ t, $\Delta_2 = 36,977$ t, $\mathcal{L}_1 = 5790$ t, $\mathcal{L}_2 = 11,645$ t, $L_1 = 180$ m and $L_2 = 214$ m.

Since the decision maker selects between fixed and flexible designs in 1978, it is proper to use a risk-adjusted discount rate estimated for 1978. Checking historical nominal treasury interest rates in White House (2016), we find that the risk-free interest rate was 8.9% in 1978. Since the U.S. Navy is a government organization, there is no market for jumboization investment, so we postulate that the risk-adjusted discount rate is equal to the risk-free interest rate ($\rho = 0.089$ per year).

Let us assume that receiving ships call for demand every 0.04 years (14.6 days), resulting in $\sigma = 0.0123$, $\alpha = 0.0022$ and $\rho = 0.0036$ per 0.04 years. With these values, the conditions $\alpha - \sigma^2/2 = 0.0021 > 0$ and $\rho - \alpha = 0.0014 > 0$ hold and β_1 is calculated as 1.613.

Pike (1999) gives the replenishment oiler's speed as 20 knots. We suppose that the block coefficients of the replenishment oiler before and after jumboization are $C_{b1} = 0.88$ and $C_{b2} = 0.91$ (reasonable numbers for large ships such as replenishment oilers, ro-ro ships and tankers), respectively. Therefore, we find that $m_1 = 1178$, $n_1 = -4081$, $m_2 = 1202$ and $n_2 = -4279$. Equation (2) gives $P_1 = 39,659$ kW and $P_2 = 36,786$ kW. The maximum amounts of bunker fuel consumption in 0.04 year (recall $\left[(0.0046P + 0.2)/24\right](X/S)$) are calculated as 532 t and 494 t before and after jumboization, respectively, by assuming that the replenishment oiler traverses the distance $X = 1400$ nautical miles in one direction each 0.04 year. Note that these values are expressed in tonnes and must be converted to gallons using the density value of bunker fuel. The type of bunker fuel used is Navy Special Fuel Oil (NSFO). Environmental Technology Centre (2018) gives NSFO density as 0.9349 g/mL (or, 0.9349 kg/L). Since one barrel of oil is equal to 159 L (42 gallons), the density of bunker fuel is found to be 0.1486 tonnes/barrel. We obtain the maximum consumptions of bunker fuel per 0.04 years as $(532/0.1486) \cdot 42 = 150,500$ gallons and $(494/0.1486) \cdot 42 = 139,610$ gallons before and after jumboization, respectively.

Finally, $\mathcal{F}_1$ and $\mathcal{F}_2$ are obtained as $150,500/26,417 = 5.69$ gallons/tonne and $139,610/36,977 = 3.77$ gallons/tonne, respectively, indicating that jumboization is effective in bringing about savings in fuel costs for the given demand values. Kucuksayacigil and Min (2017) quantify the value of the jumboization option for replenishment oilers as well, but with some simplifications. One of these simplifications is the use of a function by which we estimate power required to propel the replenishment oilers. Kucuksayacigil and Min (2017) uses a function in which the so-called Admiralty coefficient is replaced with a constant. Later work shows that the Admiralty coefficient actually depends on the block coefficient, speed, and length of the ships (we use this more accurate formulation in the current work). Due to this simplification, the formulations that Kucuksayacigil and Min (2017) use give rise to $\mathcal{F}_1$ and $\mathcal{F}_2$ as 11.18 gallons/tonne and 9.66 gallons/tonne, respectively. It is evident that simplified formulations may lead to perturbations in the calculation.

The remaining parameters are the cost of bunker fuel and jumboization cost. NYSERDA (2018) states that $C = 2.46$ \$/gallon. We assume jumboization costs for the flexible design and fixed design are $I_{flex} = 15,000,000$ and $I_{fixed} = 20,000,000$. Using Equation (10), we obtain $D^* = 23,502$ t per 0.04 years. Option value at time 0 is obtained using Equation (11) as $F_{flex}(D_0) = 6,966,584$ and $F_{fixed}(D_0) = 6,341,082$ with the assumption $D_0 = 7000$ t. Hence, upfront cost for the flexible design should not exceed $F_{flex}(D_0) - F_{fixed}(D_0) = 625,502$ if the flexible design is to be preferred. If I_0 is given as \$2,000,000, the initial demand value should not be less than 14,386 t if the flexible design is to be preferred, as derived by inequality (13). Given that $D_0 = 7000$ t, the expected time of jumboization is calculated as $\tau = 23.24$ years.

As stated previously, USS Willamette was jumboized after 13 years although our numerical results reveal that optimal time for jumboization was approximately 23 years. Our model approaches evaluation of jumboization from the perspective of fuel cost savings. However, the savings in fuel costs due to jumboization may, in reality, not be the sole and primary motivation. Replenishment oilers were most likely jumboized based on immense need for extra capacity under a critical budget cut. The U.S. Navy also had a tendency to extend the service life of replenishment oilers by jumboizing them. In reality, also, there is no evidence that the decision makers in the U.S. Navy followed any optimality criteria to decide on jumboization (or they have been aware that, in any sense, their decision has been optimal). We think that perhaps this observation explains the discrepancy between jumboization times of USS Willamette in reality and in our quantitative framework.

Data Availability Statement: For any further information on data availability, please see Supplemental Online Materials.

7. Discussions on assumptions, equations, and research questions

The demand evolution modeled in Equation (1) may not always follow such a smooth process. Demand for transported items may abruptly jump to lower or higher levels. This may result from a change in the number of receiving ships due to scrapping, for both the private sector and the U.S. Navy. For the U.S. Navy solely, another reason may be uncertainties in political relations between nations in high-tension areas. History has shown us that U.S. Navy ships become more active during a military crisis. For instance, it is empirically evident that replenishment oilers transported oils to U.S. Navy ships in much larger amounts during 2010 when the U.S. was involved in resolution of the Libyan crisis (see Figure 2). In these cases, a diffusion process, infinitesimal changes within infinitesimal time period such as Equation (1), is shocked by a jump event, which can be defined as sudden abnormal changes. Our framework is also applicable under such a case, but an analytical solution is no longer available, due to the embedded jump process. Equation (1) can be rewritten with a jump process as

$$\mathrm{d}D_t = (\alpha - \lambda \mathscr{k})D_t \mathrm{d}t + \sigma D_t \mathrm{d}z + (\mathscr{k} - 1)dq \qquad (14)$$

where $\mathscr{k}$ is the random jump magnitude (percentage change in D_t) if a jump occurs, dq is an increment in the jump process (equal to one if a jump occurs or zero otherwise), and λ is the arrival rate of the jump events. Equation (14) is called a jump-diffusion process in the literature. Discretization of Equation (14) should be approached since our framework with a jump-diffusion process is not amenable to analytical tractability. There exist different examples of discretization of jump-diffusion process in the literature (Amin, 1993; Martzoukos and Trigeorgis, 2002; Hilliard and Schwartz, 2005; Dai *et al.*, 2010; Kucuksayacigil and Min, 2020). According to Hilliard and Schwartz (2005), a discrete version of this process is

$$D_{t+1} = D_t e^{\pm\sigma\sqrt{\mathbb{t}}} e^{\omega \mathbb{k}} \qquad (15)$$

where $\mathbb{t}$ is the duration of a period in the discrete tree, $\sigma\sqrt{\mathbb{t}}$ is the magnitude of the diffusion movement in up (+) or down (-) directions, and $\omega \in \{0, \pm 1, \ldots, \pm\zeta\}$ denotes points to discretize the random jump magnitude, which are positioned in vertical order with distance $\mathbb{k}$ between two successive points (Figure 4, see Hilliard and Schwartz (2005) for details of discretization). The number of points to discretize the jump process is ζ (an odd number).

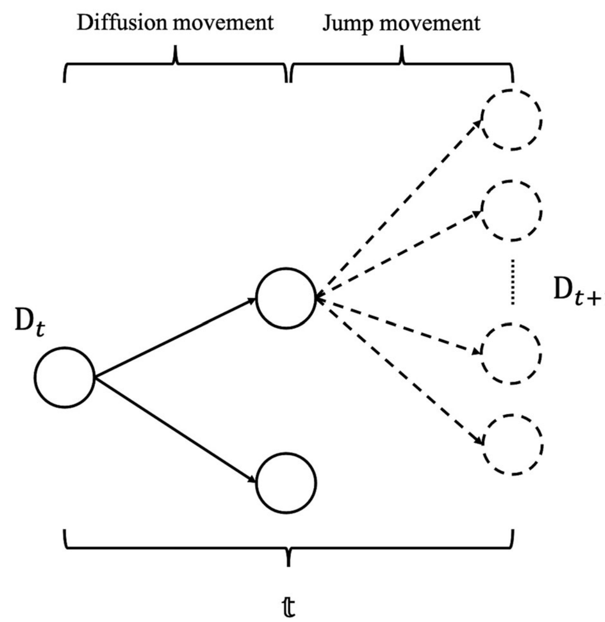

Figure 4. Discretization of GBM and jump process.

After constructing the lattice, typical backward induction is performed. At each node of demand, the fuel cost saving can be computed by following the framework drawn in this article. This approach suffers from the curse of dimensionality. Kucuksayacigil and Min (2020) propose an approach to mitigate the computational burden. According to the proposed approach, jump branches are replaced at every certain number of periods, rather than putting them at every period. This approach saves a considerable amount of computation time without sacrificing significant solution quality.

We discuss other assumptions, equations, and research questions in the Supplemental Online Materials.

8. Concluding remarks and future research

In this article, we demonstrated how to quantify the value of a jumboization option for transportation ships in both the private and public sectors (e.g., replenishment oilers of the U.S. Navy). Having shown that jumboization brings about savings in fuel costs, we derived the expected time for jumboization investment and its value contingent on the uncertain demand factor. A managerial guideline regarding the choice between flexible and fixed designs was provided, pointing out that relatively low-demand values at the initial stages of a design should be accepted as a signal to adopt a fixed design. One of the limitations of this article is that the proposed framework may not be applicable to combat ships, as adding a midsection to crucial ships such as war-fighting ships may not be possible due to security issues. Future extensions of this article could involve abandonment and purchasing options for transportation ships. Another uncertain factor and its corresponding stochastic process (e.g., jump-diffusion process to model sudden changes in a random demand path) could also be taken into account to build an underlying framework. We could also take into account revenue gain due to larger cargo spaces in private sector transportation ships after jumboization. Finally, in the context of jumboization, where the practical value of jumboization is not known by U.S. Navy practitioners (Dr. Doerry of the U.S. Navy), we note that a benchmark/reference point is highly desirable. As the closed-form solution is not influenced by the particular choice of numerical values, our study can be served as a basis for further discussion and exchange of ideas in this important topic.

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