Decision making framework in deterministic and stochastic forward/reverse logistics supply chain design

by

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The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation/thesis. The Graduate College will ensure this dissertation/thesis is globally accessible and will not permit alterations after a degree is conferred.

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DEDICATION

I would like to dedicate this thesis to my wife and to my parents without whose support I would not have been able to complete this work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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ABSTRACT

Nowadays an efficient supply chain system plays a crucial role in manufacturing production. Suppliers, manufacturers, and customers are the main stakeholders of a supply chain system. Forward and reverse networks are two main types of supply chain networks. In a forward supply chain network, raw materials are transported from suppliers to manufacturing facilities and are manufactured into final products which are then shipped to the customers. In a reverse supply chain network, the final products which their quality do not meet the minimum standards, or reached to end of their lives are transported to upstream facilities such as manufacturing facilities, recycling centers, or disposal centers. To design an efficient supply chain system, decision makers encounter a wide variety of strategic, tactical, and operational decision making problems in the forward/reverse logistics in deterministic and stochastic environments. In this dissertation, various mathematical methods have been formulated to study a forward supply chain problem in a deterministic environments and two reverse logistics problems under uncertainties.

In the first paper, two integrate mixed integer linear programming models have been proposed for a forward supply chain network. These models aim to find optimal investment on establishing distribution centers to minimize the transportation costs from suppliers to distribution centers and from distribution centers to customers. In the first model, vehicles are allowed to load different types of products to deliver to the customers (multi-product delivery) while in the second model delivery vehicles are allowed to load only one type of product (single-product delivery). Various instances with different sizes were generated to validate the introduced models. Three solution methods including deterministic mode, opportunistic mode, and benders decomposition algorithm in CPLEX have been employed to solve the proposed models. The results show that integrated model reduces total system costs by 24.72% on average. Also, multi-product delivery approach results in saving rate up to 31.27% compared to single-delivery approach. Among the solution

methods to solve the proposed models, opportunistic mode outperforms other methods on average in terms of objective function value and computational run-time.

In the second paper, a two-stage stochastic programming model has been developed for a reverse logistics network under return and demand quantity uncertainties. This model aims to minimize the total cost of network including establishing costs of sorting centers, warehouses, recycling centers, and disposal centers, and transportation cost. Decision on the number of opened facilities among some candidates are the strategic decisions and tactical decisions include lot sizing, transportation plan, level of inventory, backorder, and shortage over the planning horizon. Probability distributions of return and demand quantities are considered normal. Moment matching method was used to generate discrete sets of scenarios and fast forward selection algorithm was applied as scenario reduction method. A case study was conducted to validate the proposed model. Numerical results indicate that the stochastic model solution outperforms the expected value solution.

In the third paper, a multi-stage programming model has been formulated to address a multi-echelon, multi-period reverse logistics network in which the main uncertain parameters are primary markets' return quantity and quality, and secondary markets' demand quantity. The formulated model minimizes the total cost of establishing facilities, inventory cost, and backorder and shortage cost. Moment matching method for scenario generation and fast forward selection for scenario reduction have been adopted to generate a finite number of discrete scenarios. Extensive form of the problem is used to solve the introduced stochastic programming model. A case study has been conducted to validate and evaluate the proposed model and solution method.

CHAPTER 1. INTRODUCTION

1.1 Background

Supply chain management (SCM) is "The holistic management approach for integrating and coordinating the material, information and financial flows along a supply chain" (Handfield and Nichols Jr, 1999). Designing a supply chain network is one of the biggest challenges in supply chain management. A supply chain network is defined as a logistics network by Simchi-Levi et al. (2004). In this complex logistics system raw materials are converted into finished products and then distributed to final users (consumers or companies) (Ghiani et al., 2004). These definitions serve as the forward logistics network concept in SCM. Reverse logistics network is another type of supply chain network which aims to collect used, refurbished, or defective products from customers and then carry out some recovery activities (Govindan et al., 2017). Based on the American Reverse Logistics Executive Council, reverse logistics is defined as "The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the points of consumption to the point of origin for the purpose of recapturing value or proper disposal" (Rogers et al., 1999a). According to Hugos (2003) logistics management is different from SCM in some aspects. Logistic management as part of SCM, is involved in decision making process for activities such as inventory management, distribution, and procurement, while SCM focuses on other activities such as marketing, customer service, and finance as well.

Logistics management involves with a wide variety of decisions which can be categorized into three levels: strategic, tactical and operational decisions. The most common strategic decisions include determining the facilities locations and number of facilities and their sizes, technology and area allocation for production at different facilities, and selection suppliers (Simchi-Levi et al. (2004)). Tactical decisions consist of decisions on pricing, purchasing raw material from suppliers, production planning, transportation and routing. At the operational level, decisions related to demand fulfillment and inventory control are made. All of these decisions can be made in deterministic or stochastic environments with the presence of uncertain parameters.

Forward logistics as the traditional form of logistic management has been studied widely in the existing body of literature. Special form of forward logistics which addresses facilities locating and vehicle routing decisions in a single problem is called location routing problem (LRP). In its basic form it can be defined as follows. "Given a set of potential facility locations and a set of customer demands to be satisfied, we have to simultaneously determine the number and position of one or more facilities (strategic location decisions); the customer-to-facility (one-to-one) assignment (strategic assignment decisions); the size of the vehicle fleet used to serve the customer and the routes to be performed by each vehicle dispatched from the located facilities (tactical routing decisions). The aim is the minimization of the total system cost, given by the sum of location and distribution costs". Most of the LRP contributions consider single-commodity flow in problem formulation. The other gap is the decision making on routing between suppliers level (pickup routing). In most cases addressed in the literature final products are distributed among the customers transported from facilities such as distribution centers, warehouses, and cross docks. However, if the customers are closer to suppliers or the capacity of facilities are not enough to meet customers' demands, a reasonable solution is to ship their demand directly from supplier.

As one of the most recent contributions in the forward logistics network, Ahkamiraad and Wang (2018) considered multiple cross docks in a capacitated cross dock vehicle routing problem with pickup, delivery, and time windows. They proposed a hybrid solving method including genetic algorithm and particle swarm optimization. The authors did not include cross dock locating decision. Also, their work does not propose exact solution methods to solve the problem. This study considers establishing multiple distribution centers by locating them among several potential candidates. In the first paper, I enriched their work by developing an integrated model including direct shipment, and distribution centers' location. I also considered multiple suppliers for each product and two types of product delivery routes: single-product delivery route and multi-product delivery route. The objective is to locate multiple distribution centers to minimize the total system costs. Different types of constraints have been incorporated including pickup routing, delivery routing, and direct shipment constraints. Three solution methods including deterministic and opportunistic modes in CPLEX solver, and benders decomposition algorithm have been employed to solve the proposed model.

The research on reverse logistics, as another type of logistics management, has been continuously growing in recent years. As per United Nations Environment Program (UNEP) on framework of Global Partnership on Waste Management (2010), Organization for Economic Co-Operation and Development (OCED) countries alone produced 1.7-1.9 billion tonnes of municipal solid waste and 490 million tonnes of hazardous waste annually (UNEP (2010)). According to another report, 44.7 million tonnes E-waste generated across the world annually (Balde et al. (2015)). The amount of waste generated across the world increases the importance of reverse logistics networks in attempt to decrease waste rate and take the leftover(s) back to supply chain.

Network design for a reverse logistics system is one of the most challenging supply chain problems (Melo et al. (2009)). Compared to the traditional forward logistics network planning, more activities are involved in reverse logistics network design. Also, there are more uncertainties both in terms of quality and quantity in reverse logistics networks. (Yu and Solvang (2018)).

To address these challenges, researchers have developed decision-making models and solution techniques for reverse logistics problems over the past decades. The second paper of this dissertation seeks to fill some gaps existing in the literature. In terms of mathematical modeling of a reverse logistics network design, I considered real world characteristics of reverse logistics such as backorder and shortage for secondary markets and outsourcing which have typically been ignored in the literature. The second paper aims to introduce a two-stage stochastic programming model for multi-period reverse logistics which includes lot-sizing (allowing backorder and shortage) and outsourcing. Moment matching method has been used to generate scenarios and fast forward selection method is used as a reduction method to select a proper subset of generated scenarios as the most representative scenarios to approximate underlying continuous distributions of stochastic parameters. The third paper generalizes the second paper by extending the problem to a multi-stage stochastic programming model. In this model return and demand quantity, and return quality have been considered as uncertain factors. Scenario generation and scenario reduction algorithms were implemented and extensive form of the model was used to solve the formulated problem.

1.2 Dissertation structure

The reminder of this dissertation is organized as follows. Chapter 2 presents the first paper with the title "Multi-product pickup and delivery supply chain design with location-routing and direct shipment". This paper has been published in International Journal of Production Economics (IJPE). Chapter 3 presents the second paper with the title "A two-stage stochastic programming model for multi-period reverse logistics network design with lot-sizing". This paper has been published in Computers and Industrial Engineering (CIE). The third paper with the title "Multi-stage stochastic programming for multi-period reverse logistic with location routing and lot-sizing" is presented in chapter 4. Finally, conclusions and future research directions are outlined in chapter 5.

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CHAPTER 2. Multi-product pickup and delivery supply chain design with location-routing and direct shipment

A paper accepted by *International Journal of Production Economics*

2.1 Abstract

This paper presents a decision making model that considers distribution centers locating, pickup and delivery vehicle routing, and direct shipment simultaneously. The goal is to optimize the distribution centers' location, the delivery, pickup, and direct shipment routes so that the total costs are minimized. Two types of delivery modes are considered. In the first type, each vehicle loads the same type of products and in the second type, each vehicle is allowed to load multiple types of products. An integrated mixed integer linear programming model was formulated for each delivery type. Computational experiments were conducted to validate the models. Three solution methods: deterministic mode, opportunistic mode, and benders decomposition algorithm in CPLEX have been analyzed.

2.2 Introduction

Developing efficient supply chain strategy is critical to managing the material flows and logistics costs. A key player in supply chain systems is the third party logistics provider (3PL). Nowadays, companies typically outsource their logistics, warehousing, and cross docking to 3PL to improve the logistics efficiency and focus on the essential production. Such activities have been adopted successfully in many companies across multiple industry sectors such as: Wal-Mart (Stalk et al. (1992); Gue (2001)), Robert Bosch LLC (Yildiz et al. (2010)), Eastman Kodak Co. ((Cook et al., 2005)), Goodyear GB Ltd. (Kinnear (1997)), and Toyota (Witt (1998)). 3PLs usually integrate operation, warehousing, and transportation services in order to meet customers' or suppliers' needs, such as pickup and delivery services for products and materials. 3PLs consolidate incoming shipments from different suppliers, store them for a limited time, and then distribute the products with a fleet of delivery vehicles. Therefore, developing an efficient distribution system is very important for 3PLs. However, due to the computational complexity, the optimization models designed for a distribution network are often simplified and somewhat disconnected from the real industrial scenarios Ladier and Alpan (2016). Specifically, the supply routing, distribution centers' location, and customer delivery problems are often studied separately in the current literature.

There has been some recent literature on vehicle routing problem (VRP) with distribution center location. A distribution center might be a warehouse which related problem is called location routing problem (LRP) or a cross dock which related problem is vehicle routing with cross docking (VRPCD). Boccia et al. (2018) introduced a multi-commodity LRP and solved it with a branch and cut algorithm. Ferreira and de Queiroz (2018) presented two heuristics incorporating simulated annealing method to solve the capacitated Location-Routing problem. The solution methods improved overall average available in the literature. Lee et al. (2006) studied an integrated problem including VRP and cross docking. The authors proposed a mathematical model with an objective to find an optimal number of vehicles in order to minimize the overall costs. They developed a Tabu search algorithm to solve the problem. Liao et al. (2010) proposed new Tabu search algorithm to improve the solution method introduced by Lee et al. (2006). They proved that the average improvements were as high as 10-36% for various instances. Agustina et al. (2014) studied cross docking operations to ensure food to be delivered just in time and minimize total system costs. The total system costs include inventory holding costs, transportation costs, and the penalty costs of early/tardy deliveries. Hasani-Goodarzi and Tavakkoli-Moghaddam (2012) proposed capacitated vehicle routing problem for multi product cross docking with split deliveries and pickups. Goods are collected in a distribution center by a fleet of vehicles before shipped to customers. Products are then sorted according to their destinations, and finally shipped to customers.

Most of the studies in supply chain design consider single distribution center in a fixed location. Santos et al. (2013) developed a model with single distribution center and two routing types. In the first routing type, a delivery process starts at the cross dock and then visit a subset of suppliers. In the other type, namely pickup and delivery routes, delivery process starts after visiting a subset of suppliers without any stop at cross dock. Dondo and Cerdá (2015) addressed a VRPCD problem with heterogeneous vehicle fleet routing and truck scheduling in a multi-door cross dock system.

They developed an approximate sweep-based model considering a set of constraints mimicking the sweep algorithm for assigning nodes to vehicles in order to improve branch and cut search. Numerical examples have been solved for up to 50 transportation requests and a heterogeneous fleet of 10 vehicles within reasonable running time. Birim (2016) addressed vehicle routing problem with cross docking in which vehicle fleet were heterogeneous with different capacities. The author proposed a simulated annealing algorithm to solve the problem. Keshtzari et al. (2016) formulated a mixed integer programming model for scheduling of inbound and outbound trucks in a cross docking system. They also proposed a particle swarm optimization algorithm hybridized with a simulated annealing to solve large size instances. Comparisons were made with two other metaheuristics in the literature. Baniamerian et al. (2018) proposed a vehicle routing and scheduling problem with cross docking in a supply chain network with three echelons. The authors presented a mixed integer linear programming model to minimize early/tardy deliveries and transportation cost. The authors developed a two phase genetic algorithm to solve the problem. Ahkamiraad and Wang (2018) considered multiple cross docks in a capacitated cross dock VRP with pickup, delivery and time windows. However, the authors did not include cross dock locating decision. They proposed a hybrid solving method including genetic algorithm and particle swarm optimization. This study considers establishing multiple distribution centers by locating them among several potential candidates.

Direct shipment is another characteristic which has been seldomly studied with other characteristics such as warehousing or cross docking. Musa et al. (2010) addressed the transportation problem of a cross docking network where both direct shipment and indirect shipment from suppliers to customers were included. They developed an integer programming model and an ant colony optimization (ACO) to solve the model. Ma et al. (2011) studied a transportation and shipment consolidation problem with cross docking which includes direct shipment and takes into account inventory and time scheduling and transportation cost altogether. They provided a two stage solution method to solve integer programming model.

In distribution network design for a supply chain, complex issues arise such as collecting and delivering multiple products, considering capacity decision for distribution centers and vehicles. Typically, these decision models are formulated and solved without coordination due to computational tractability. It should be noted that the more integrated the resolution of these problems is, the more efficient a supply chain system can be (Santos et al., 2013). To the best of our knowledge, integrating multiple distribution centers' locating, vehicle routing and direct shipment has not been studied as a single decision making problem in the existing literature. Thus, the main motivation of this study is to develop a distribution system to address this gap and to integrate multiple decision making problems into a single framework. The problem addressed in this paper is similar to the one studied in Ahkamiraad and Wang (2018). In this study, we enriched their work by including direct shipment, and distribution centers' location. We also considered multiple suppliers for each product and single-product delivery route. The objective is to locate multiple distribution centers to minimize the total system costs. Different types of constraints are incorporated including pickup routing, delivery routing, and direct shipment constraints. Three solution methods including deterministic and opportunistic mode in CPLEX solver, and benders decomposition algorithm have been employed to solve the proposed model. Numerical results show that opportunistic mode outperforms two other solution methods. The main contributions of this study are as following:

- Proposing integrated models including capacitated distribution center location, pickup and delivery vehicle routing and direct shipment.
- Introducing multi-product and single-product delivery concepts and developing two different models based on them.
- Employing deterministic and opportunistic solution methods to solve the proposed problem.

The rest of the paper is organized as follows. Problem statement and decision models are described in Section 2.3. The solution methods are detailed in Section 2.4. Numerical results are discussed in Section 2.5 and conclusions are outlined in Section 2.6.

2.3 Problem statement and decision models

This paper addresses a decision making problem in a distribution network involving suppliers, distribution centers, and customers. Goods are collected from suppliers and delivered to customers either directly or after consolidation in distribution centers. The decision maker aims to minimize the total cost of distribution centers' investment and products transportation. In this integrated problem, each delivery route can consist of a subset of customers with demand of multiple products, as illustrated in Figure 2.1, or a subset of customers with demand of single product as shown in Figure 2.2. Implementing these two scenarios results in two different models. This section describes mixed integer linear programming models for both scenarios. The first formulation considers multi-product delivery scenario and the second one addresses single-product delivery scenario.

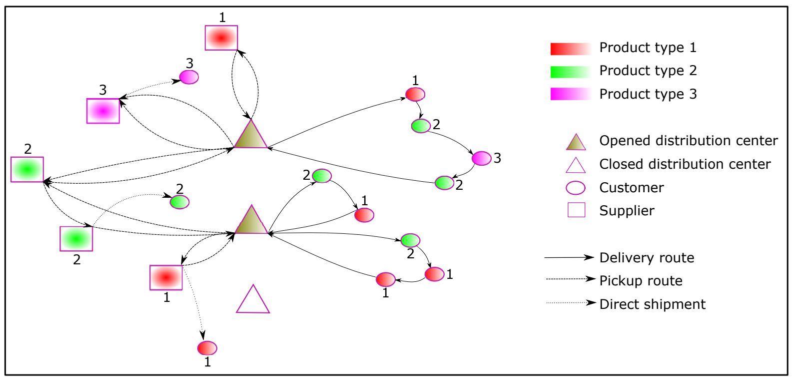


Figure 2.1: Multi-product delivery illustrating

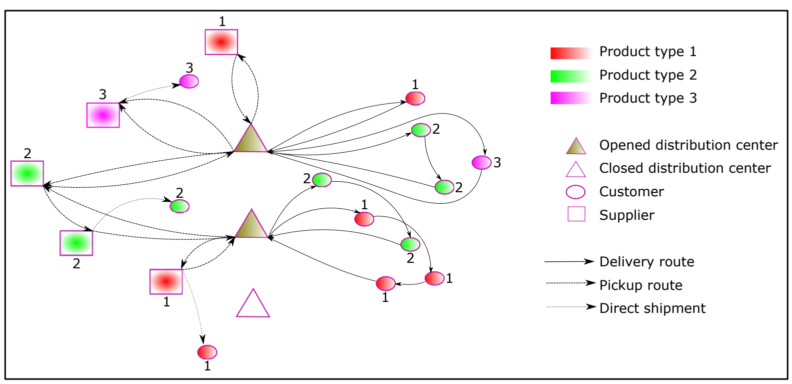


Figure 2.2: Single-product delivery illustrating

2.3.1 Model assumptions

The following assumptions have been adopted in the model formulation:

– Each node is either pickup node (supplier), distribution center node, or delivery node (customer).

- All delivery requests must be fulfilled either by delivering from distribution centers or direct shipments.
- Every delivery node must be visited just once but pickup nodes are allowed to be visited more than once.
- Every pickup or delivery vehicle must come back to its originated distribution center after completing pickup or delivery route.
- The pickup and delivery are performed by a fleet of homogeneous capacitated vehicles.
- All distribution centers and vehicles are capacitated.
- Every supplier has a supply capacity.

2.3.2 Mathematical notations

- **Indices and sets**

$N_C = \{1, 2, .., n_c\}$: Set of distribution centers' nodes

$N_D = \{n_c + 1, n_c + 2, .., n_c + n_d\}$: Set of delivery nodes (customers)

$N_{CD} = N_C \cup N_D$: Set of distribution centers and delivery nodes.

$N_P = \{n_c + n_d + 1, n_c + n_d + 2, .., n_c + n_d + n_p\}$: Set of pickup nodes (suppliers)

$N_{CP} = N_C \cup N_P$: Set of distribution centers and pickup nodes.

$V_D = \{1, 2, .., ndv\}$: Set of delivery vehicles

$V_P = \{1, 2, .., ndv\}$: Set of pickup vehicles

$R = \{1, 2, .., npr\}$: Set of products

i, j, k: Distribution centers and delivery nodes $i, j, k \in N_{CD}$

$p, p^{'}, e$: Distribution centers and pickup nodes $p, p^{'}, e \in N_{CP}$

v: Delivery vehicles $v \in V_D$

$v^{'}$: Pickup vehicles $v \in V_P$

r: Products $r \in R$

- **Parameters**

n_c: Number of available distribution centers

n_d: Number of customers

n_p: Number of suppliers

ndv: Number of available delivery vehicles in each distribution center

npv: Number of available pickup vehicles in each distribution center

npr: Number of products

d_i: Demand of customer i

cd_{ij}: Cost of traveling from node i to node j per product unit

$cp_{pp'}$: Cost of traveling from node p to node $p^{'}$ per product unit

cdr_{pi}: Cost of traveling from node p to node i per product unit

CC_j: Capacity of distribution center j

CS_p: Maximum amount of supply by supplier p

CDV: Capacity of delivery vehicles

CPV: Capacity of pickup vehicles

FC_j: Fixed cost of opening of distribution center j

FDV: Fixed cost of using a delivery vehicle

FPV: Fixed cost of using a pickup vehicle

FDr: Fixed cost of using a vehicle for direct shipment

$a_{pr} = 1$ if supplier p supplies product type r, 0 otherwise

$b_{ir} = 1$ if customer i demands product type r, 0 otherwise.

- **Decision variables**

$w_j = 1$ if distribution center j is open, 0 otherwise.

$z_{ij} = 1$ if customer i is assigned to distribution center j, 0 otherwise.

$x_{ijv} = 1$ if delivery vehicle v travels from node i to node j, 0 otherwise.

$y^i_{pp'v'} = 1$ if pickup vehicle v' travels from node p to p' in a route originated from distribution center i, 0 otherwise.

$D_{pi} = 1$ if a direct shipment vehicle travels from supplier p to customer i, 0 otherwise.

u_{ijv}: Load on delivery vehicle v during traveling on arc (i, j).

$s^i_{pp'v'}$: Load on pickup vehicle v' during traveling on arc (p, p') in a route originated from distribution center i.

$g^i_{prv'}$: Amount of pickup load of product type r from node p by vehicle v' during a route originated from distribution center i.

2.3.3 Model formulation

As discussed in Section 2.3, two different delivery scenarios have been studied in this paper. This section includes mathematical formulations based on both delivery scenarios.

- **Multi-product delivery scenario**

In multi-product delivery scenario, each delivery vehicle is allowed to load and deliver various types of products. The following formulation addresses an integrated model including pickup, delivery and direct shipment constraints considering multi-product delivery. The objective function is expressed as following:

$$
\begin{aligned}
\text{Min } f = &\sum_{j\in N_C} FC_j w_j + \sum_{i\in N_C}\sum_{j\in N_{CD}}\sum_{v\in V_D} FDV x_{ijv} + \\
&\sum_{i\in N_{CD}}\sum_{j\in N_{CD}}\sum_{v\in V_D} cd_{i,j} u_{ijv} + \sum_{i\in N_C}\sum_{\substack{p\in N_C\\ p=i}}\sum_{p'\in N_P}\sum_{v'\in V_P} FPV y^i_{pp'v'} + \\
&\sum_{i\in N_C}\sum_{p\in N_{CP}}\sum_{p'\in N_{CP}}\sum_{v'\in V_P} cp_{pp'} s^i_{pp'v'} + \sum_{p\in N_P}\sum_{i\in N_D} (FDr + cdr_{pi} d_i) D_{pi}
\end{aligned} \tag{2.1}
$$

The first term of objective function represents fixed costs of distribution centers. The second and third terms represent setup costs for delivery vehicles and transportation costs for delivery routes, respectively. Similarly, the fourth and fifth terms represent setup costs for pickup vehicles and transportation costs for pickup routes, respectively. The last term denotes the setup and transportation costs for direct shipment vehicles.

Constraints (2.2)-(2.14) formulate delivery process:

$$z_{ij} \le w_j \quad i \in N_D, j \in N_C \tag{2.2}$$

$$\sum_{j\in N_D} x_{ijv} \le 1 \quad i \in N_C, v \in V_D \tag{2.3}$$

$$\sum_{i\in N_D} d_i z_{ij} \le CC_j \quad j \in N_C \tag{2.4}$$

$$x_{ijv} \le z_{ij} \quad i \in N_D, j \in N_C, v \in V_D \tag{2.5}$$

$$x_{jiv} \le z_{ij} \quad i \in N_D, j \in N_C, v \in V_D \tag{2.6}$$

$$\sum_{\substack{k\in N_{CD}\\ k\neq i}}\sum_{v\in V_D} x_{ikv} = \sum_{j\in N_C} z_{ij} \quad i \in N_D \tag{2.7}$$

$$\sum_{\substack{j\in N_{CD}\\ j\neq i}} x_{jiv} - \sum_{\substack{j\in N_{CD}\\ j\neq i}} x_{ijv} = 0 \quad i \in N_{CD}, v \in V_D \tag{2.8}$$

$$x_{ijv} + z_{il} + \sum_{\substack{k\in N_C\\ k\neq l}} z_{ik} \le 2 \quad i \in N_D, j \in N_D, l \in N_C, v \in V_D \tag{2.9}$$

$$\sum_{j \in N_{CD}} \sum_{v \in V_D} u_{jiv} - \sum_{j \in N_{CD}} \sum_{v \in V_D} u_{ijv} = d_i \sum_{j \in N_C} z_{ij} \quad i \in N_D \tag{2.10}$$

$$\sum_{i \in N_D} \sum_{v \in V_D} u_{jiv} = \sum_{i \in N_D} d_i z_{ij} \quad j \in N_C \tag{2.11}$$

$$u_{ijv} \leq (CDV - d_i) x_{ijv} \quad i \in N_{CD}, j \in N_{CD}, v \in V_D \tag{2.12}$$

$$u_{ijv} \geq d_j x_{ijv} \quad i \in N_{CD}, j \in N_{CD}, v \in V_D \tag{2.13}$$

$$\sum_{j \in N_D} \sum_{v \in V_D} x_{ijv} \leq ndv \quad i \in N_C \tag{2.14}$$

Constraints (2.2) ensure each customer is assigned to one open distribution center. Constraints (2.3) guarantee that a vehicle traveling from a customer toward distribution centers is allowed to end its route in one of the distribution centers. Constraints (2.4) state that total demand of assigned customers to a distribution center should not exceed capacity of the distribution center. Constraints (2.5) state that traveling from a customer to a distribution center is allowed only if the customer is assigned to that distribution center. Similarly, constraints (2.6) ensures that traveling from a distribution center to a customer is allowed only if the customer is assigned to that distribution center. Constraints (2.7) state that traveling to customer i from other customers or distribution centers is allowed only if customer i is assigned to one of distribution centers. In constraints (2.8), arrivals to each customer or distribution center node must be equal to departures. Constraints (2.9) prohibit illegal routes, e.g. routes do not start and end at the same distribution center. Constraints (2.10) state that the amount of unloading in each node is equal to its demand. Constraints (2.11) ensure total amount of loads on vehicles starting routes for a distribution center is equal to total demand of customers assigned to that distribution center. Constraints (2.12) are related to vehicle capacity such that the total load on each arc does not exceed the vehicle capacity. Constraints (2.13) state that total load on each arc arriving to a customer must meet demand of the customer. Constraints (2.10)-(2.13) ensure customers' demands are satisfied. Constraints (2.14) set the limit on the number of delivery vehicles based on the fleet size.

Constraints (2.15)-(2.26) formulate pickup process:

$$y^i_{pp'v'} \leq w_i \quad i \in N_C, p \in N_C, p^{'} \in N_P, v^{'} \in V_P, p = i \tag{2.15}$$

$$\sum_{p' \in N_P} \sum_{v' \in V_P} a_{p'r} y^i_{pp'v'} \leq 1 \quad i \in N_C, p \in N_C, r \in R, p = i \tag{2.16}$$

$$\sum_{p' \in N_P} \sum_{v' \in V_P} a_{p'r} y^i_{pp'v'} \geq b_{jr} z_{j,i} \quad i \in N_C, p \in N_C, j \in N_D, r \in R, p = i \tag{2.17}$$

$$\sum_{p' \in N_P} y^i_{pp'v'} \leq 1 \quad i \in N_C, p \in N_C, v^{'} \in V_P, p = i \tag{2.18}$$

$$\sum_{\substack{p \in N_{CP} \\ p \neq p^{'}}} y^i_{pp'v'} - \sum_{\substack{p \in N_{CP} \\ p \neq p^{'}}} y^i_{p'pv'} = 0 \quad i \in N_C, p^{'} \in N_P, v^{'} \in V_P \tag{2.19}$$

$$y^i_{pp'v'} + y^i_{p'pv'} \leq 1 \quad i \in N_C, p \in N_P, p^{'} \in N_P, v^{'} \in V_P, p \neq p^{'} \tag{2.20}$$

$$y^i_{pp'v'} \leq \sum_{r \in R} a_{pr} a_{p'r} \quad i \in N_C, p \in N_P, p^{'} \in N_P, v^{'} \in V_P, p \neq p^{'} \tag{2.21}$$

$$g^i_{p'rv'} \leq \sum_{p \in N_{CP}} CPV y^i_{pp'v'} \qquad i \in N_C, p^{'} \in N_{CP}, r \in R, v^{'} \in V_P \tag{2.22}$$

$$\sum_{p \in N_{CP}} g^i_{prv'} \leq CPV \quad i \in N_C, r \in R, v^{'} \in V_P \tag{2.23}$$

$$\sum_{p \in N_P} \sum_{v' \in V_P} g^i_{prv'} = \sum_{j \in N_D} b_{jr} d_j z_{ji} \quad i \in N_C, r \in R \tag{2.24}$$

$$\sum_{r \in R} g^i_{prv'} + \sum_{e \in N_{CP}} s^i_{epv'} \leq (1 - y^i_{pp'v'})CPV + s^i_{pp'v'} \quad i \in N_C, p \in N_P, p^{'} \in N_{CP}, v^{'} \in V_P \tag{2.25}$$

$$s^i_{pp'v'} \leq CPV y^i_{pp'v'} \quad i \in N_C, p \in N_P, p^{'} \in N_{CP}, v^{'} \in V_P \tag{2.26}$$

Constraints (2.15) state a pickup route is allowed to start from a distribution center only if the distribution center has been opened. Constraints (2.16) ensure each pickup route is allowed to start by meeting only one supplier. Constraints (2.17) ensure pickup route starting a distribution center, must visit suppliers which are capable to provide products for assigned customers to the distribution center. Constraints (2.18) guarantee each pickup route is allowed to start by meeting only one supplier. Constraints (2.19) indicate the number of arrivals must be equal to the number of departures in each distribution center or supplier node for every pickup route. In constraints (2.20), subtours are eliminated. Constraints (2.21) ensure a pickup vehicle travel among pickup nodes which supply same type of product. Constraints (2.22)-(2.24) determine the amount of pickup loads which should be picked up from each supplier. Constraints (2.25)-(2.26) are flow inequalities that update pickup load on each vehicle through the route.

Constraints (2.27)-(2.28) formulate direct shipment process:

$$\sum_{j \in N_C} z_{ij} + \sum_{p \in N_P} D_{pi} (\sum_{r \in R} b_{ir} a_{pr}) = 1 \quad i \in N_D \tag{2.27}$$

$$\sum_{i \in N_C} \sum_{r \in R} \sum_{v' \in V_P} g^i_{prv'} + \sum_{i \in N_D} d_i D_{pi} \leq CS_p \quad p \in N_P \tag{2.28}$$

Constraints (2.27) ensure customers either are assigned to distribution centers and delivery routes or direct shipment routes. Constraints (2.28) limit pick up loads to supply capacity of each supplier. Direct shipment is viable because of following reasons. Firstly, if a customer is closer to a supplier, it is less costly to ship quantities directly than to ship to a distribution center, and then transport to the customer. Secondly, if the optimal number of opened distribution centers' capacity cannot meet all customers' demand, direct shipment from suppliers to some customers is necessary.

Constraints (2.29)-(2.36) include additional constraints:

$$x_{ijv} = 0 \quad i \in N_C, j \in N_C, v \in V_D \tag{2.29}$$

$$u_{ijv} = 0 \quad i \in N_D, j \in N_C, v \in V_D \tag{2.30}$$

$$s^i_{pp'v'} = 0 \quad i \in N_C, p \in N_C, p^{'} \in N_P, v \in V_P \tag{2.31}$$

$$g^i_{prv'} = 0 \quad i \in N_C, p \in N_C, r \in R, v^{'} \in V_P \tag{2.32}$$

$$y^i_{pp'v'} = 0 \quad i \in N_C, p \in N_C, p^{'} \in N_C, v^{'} \in V_P \tag{2.33}$$

$$y^i_{pp'v'} = 0 \quad i \in N_C, p \in N_C, i \neq p, p^{'} \in N_{CP}, v^{'} \in V_P \tag{2.34}$$

$$w_j, z_{ij}, x_{ijv}, y^i_{pp'v'}, D_{pi} \in \{0, 1\} \tag{2.35}$$

$$u_{ijv}, g^i_{prv'}, s^i_{pp'v'} \geq 0 \tag{2.36}$$

Constraints (2.29) indicate travel among distribution centers by delivery vehicles are not allowed. Constraints (2.30) ensure delivery vehicles are empty when after visiting last customer in route. Constraints (2.31) show pick up vehicles are empty before visiting first customer in their route. Constraints (2.32) indicate pick up from distribution centers is not allowed. In Constraints (2.33) travel among distribution centers by pick up vehicles are prohibited. Constraints (2.34) state pickup vehicle leaving a distribution center is not allowed to visit suppliers which are assigned to different distribution centers. Constraints (2.35) and (2.36) show the binary and non-negative variables, respectively.

- **Single-product delivery scenario**

Mathematical model for single-product delivery would have the objective function (2.1), constraints (2.2)-(2.36) plus following set of constraints:

$$x_{ijv} \leq \sum_{r \in R} b_{ir} b_{jr} \quad i \in N_D, j \in N_D, v \in V_D \tag{2.37}$$

Constraints (2.37) ensure delivery vehicles meet customers with demand of same product.

2.4 Solution techniques

Since vehicle routing problem is NP-hard, the models formulated in this paper are NP-hard as well. Therefore, efficient solution methods are necessary to solve the problems. This section discuses the solution methods to obtain optimal or near optimal solutions. In this study, CPLEX solver is used to solve the linear programming models and three different solution methods including deterministic mode, opportunistic mode, and benders decomposition algorithm have been analyzed. Most of the problems in the literature solved by CPLEX use deterministic mode and the performance of opportunistic mode has not been explored. So, this paper evaluates the performance of using opportunistic mode in the introduced problem. Benders decomposition method as another well-known solution method is used to compare the results.

- **Deterministic mode**

By default, CPLEX uses parallel algorithms only when the optimization remains deterministic. Therefore, deterministic means that repeated solving of the same model at the same parameter settings on the same computing platform results in exactly the same solution path, the same level of performance and the same values in the solution (IBM, 2018).

- **Opportunistic mode**

Opportunistic parallel optimization needs less synchronization between threads. Therefore, it leads to better performance on average. In opportunistic mode the differences in timing among threads, or the order in which tasks are executed in different threads may result in a different solution path and consequently different solution vectors or different timings during optimization with parallel threads. In opportunistic mode, the actual optimization may differ from run to run, including the the path traveled in the search and solution time (IBM, 2018).

- **Benders decomposition algorithm**

In benders decomposition algorithm the model was decomposed into master problem and subproblem. This method aims to find optimal solution by solving both problems repeatedly and adding feasibility and optimality cuts to master problem.

The mathematical formulation presented in Section 2.4 can be represented with:

$$
\begin{aligned}
\text{Min} \quad C_1\Psi + C_2\Phi & \\
A\Psi \qquad\qquad & \geq a \\
B\Phi & \geq b \\
D\Psi + E\Phi & \geq f \\
\Psi & \in \mathbb{Z}_2^m \\
\Phi & \in \mathbb{R}_+^n
\end{aligned}
$$

Where Ψ includes binary variables $w_i, z_{i,j}, x_{ijv}, y^i_{pp'v'}$ and D_{pi}. In addition, Φ represents continuous variables including $u_{ijv}, g^i_{prv'}$ and $s^i_{pp'v'}$.

Therefore the master problem becomes:

$$
\begin{aligned}
\text{Min} \;\; C_1\Psi + z \\
A\Psi \geq a \\
Feasibility\ cuts \\
Optimality\ cuts \\
\Psi \in \mathbb{Z}_2^m \\
z \geq 0
\end{aligned}
$$

The subproblem becomes:

$$
\begin{aligned}
\text{Min} \;\; C_2\Phi \\
B\Phi \geq b \\
E\Phi \geq f - D\Psi \\
\Phi \in \mathbb{R}_+^n
\end{aligned}
$$

In the master problem, feasibility cuts are formed by $r_1 b + r_2(f - D\Psi) \leq 0$. Feasibility cut will be added to the master problem if dual of subproblem is unbounded. In this set of constraints $r = \{r_1, r_2\}$ is a direction of unboundedness. Optimality cuts are formulated by $\pi_1 b + \pi_2(f - D\Psi) \leq z$. Optimality cut will be added if dual of subproblem has an optimal solution. Considering this concept, $\pi = \{\pi_1, \pi_2\}$ is an extreme point and optimal solution of dual of subproblem.

2.5 Numerical results and analysis

This section provides a numerical example to validate and illustrate the introduced models. Then, three different analyses are provided. Firstly, comparisons between the integrated and separated problems are presented. Secondly, multi-product delivery scenario is compared with the single-product delivery. The last analysis evaluates the implementation of the proposed model with single-product delivery scenario on problem sets.

To test the models and solution methods, small, medium and large sets of problem instances were generated. Parameters have been adapted from Lee et al. (2006). The values of parameters for different size of instances are given in Table 2.1.

Each problem set consists of 10 randomly generated instances. The MIP models have been implemented in CPLEX 12.8 on a PC with two 2.6 GHz 8-Core Intel and 100 GB of RAM memory.

Table 2.1: Problem parameter values

	Small	Medium	Large
$\lvert N_C\rvert$	2	3	3
$\lvert N_D\rvert$	10	24	30
$\lvert N_P\rvert$	5	6	10
$\lvert N_D \cup N_P\rvert$	15	30	40
$\lvert V_D\rvert$	2	3	3
$\lvert V_P\rvert$	2	3	3
$\lvert R\rvert$	2	3	3
d_i	U(5,30)	U(5,30)	U(5,30)
cd_{ij}	U(0.1,3.1)	U(0.1,3.1)	U(0.1,3.1)
$cp_{pp'}$	U(0.1,5.1)	U(0.1,5.1)	U(0.1,5.1)
cdr_{pi}	U(0.1,7.1)	U(0.1,7.1)	U(0.1,7.1)
FDV	100	100	100
FPV	200	200	200
FDr	250	250	250
FC_j	U(500,1000)	U(500,1000)	U(500,1000)
CC_j	U(250,500)	U(250,500)	U(250,500)
CS_p	U(150,300)	U(300,500)	U(400,700)
CDV	100	100	100
CPV	150	150	150

2.5.1 A numerical example

The applicability of the proposed model is demonstrated with a numerical example of supply chain network. The potential supply chain network consists of 17 nodes including 2 potential candidates for distribution centers (DC), 5 suppliers (S), and 10 customers (C) with demand of two different types of products.

Each distribution center has two homogeneous pickup vehicles with the capacity of 150 units which costs 200 and two homogeneous delivery vehicles with the capacity of 100 units and cost of 100. Fixed cost of using a vehicle for direct shipment is 250. Transportation cost for each unit of product between network's nodes are listed in Table 2.2 and Table 2.3. Other parameters of the example are shown in Table 2.4.

Table 2.2: Transportation cost from all nodes to distribution center or supplier nodes

	DC1	DC2	S1	S2	S3	S4	S5
DC1	0	2.27	3.44	4.37	3.97	3.81	1.53
DC2	1.72	0	4.71	4.95	2.38	3.20	2.50
S1	2.25	2.26	0	3.57	0.51	4.46	2.99
S2	1.25	3.31	3.26	0	1.46	4.94	1.58
S3	1.29	0.94	4.87	1.93	0	5.07	2.58
S4	4.98	1.16	4.30	1.87	1.47	0	1.85
S5	4.65	4.80	0.11	1.14	3.09	3.89	0
C1	1.71	2.84	0.15	0.79	4.12	1.56	0.98
C2	0.19	1.26	0.39	2.94	1.83	2.32	0.59
C3	1.39	0.78	6.41	5.46	2.29	6.41	5.21
C4	0.39	3.05	4.94	3.05	0.2	3.99	0.77
C5	2.54	2.43	1.59	5.78	1.07	6.24	6.15
C6	1.54	0.81	5.29	3.45	5.86	0.75	1.04
C7	0.25	2.04	2.72	4.99	6.57	0.56	6.44
C8	1.06	1.07	6.09	4.23	2.9	6.20	0.37
C9	1.77	3.01	0.41	6.75	5.83	5.19	2.55
C10	2.71	2.54	6.83	6.39	6.56	2.86	3.81

This example were solved in three different approaches introduced in this paper. First, it was solved by considering a separated modeling approach in which direct shipment is not allowed. The optimal solution of using separated problem is presented in Table 2.5. As shown in Table 2.5 separated problem results in establishing two distribution centers leading to enhancement of objective function value. The Second approach is using integrated problem with single-product delivery and direct shipment. Table 2.6 reports optimal solution obtained by employing this formulation. Integration of problems results in opening one distribution center and decreasing facility location cost. This achievement leads to less overall cost compared to separated problem. However, due to small number of delivery vehicles available in each distribution center, two customers' demand

Table 2.3: Transportation cost from all nodes to customer nodes

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
DC1	1.47	2.51	0.16	1.78	2.45	2.89	0.77	1.30	0.90	1.89
DC2	2.25	2.54	2.29	2.82	1.41	0.65	3.08	1.20	1.34	2.04
S1	0.15	0.39	6.41	4.94	1.59	5.29	2.72	6.09	0.41	6.83
S2	0.79	2.94	5.46	3.05	5.78	3.45	4.99	4.23	6.75	6.39
S3	4.12	1.83	2.29	0.20	1.07	5.86	6.57	2.90	5.83	6.56
S4	1.56	2.32	6.41	3.99	6.24	0.75	0.56	6.20	5.19	2.86
S5	0.98	0.59	5.21	0.77	6.15	1.04	6.44	0.37	2.55	3.81
C1	0	0.96	2.90	2.87	2.42	0.65	1.96	1.30	2.72	1.54
C2	0.14	0	0.24	2.15	2.58	1.61	2.59	2.48	1.10	1.65
C3	1.65	0.45	0	1.76	0.32	0.41	2.86	1.82	1.64	0.95
C4	1.14	1.94	2.74	0	1.76	0.42	1.00	0.32	0.18	1.05
C5	1.94	3.02	1.42	0.72	0	2.75	1.57	0.4	2.22	1.19
C6	1.01	0.82	2.17	3.01	2.48	0	1.82	1.57	2.85	2.98
C7	1.14	2.40	1.12	2.97	1.63	2.29	0	0.77	2.97	0.71
C8	1.89	2.42	1.43	0.14	0.44	0.42	1.32	0	2.82	2.99
C9	2.42	1.67	1.31	0.65	1.30	2.55	0.89	0.59	0	1.61
C10	0.70	1.32	1.11	1.19	2.27	0.43	0.35	1.07	0.77	0

Table 2.4: Parameter values in example problem

	DC1	DC2	S1	S2	S3	S4	S5	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
FC_j	825	937															
CC_j	336	352															
CS_p			241	294	250	264	183										
a_{p1}			0	1	1	1	1										
a_{p2}			1	0	0	0	0										
d_i								8	14	14	25	25	28	19	26	17	16
b_{i1}								1	1	0	0	1	1	0	1	1	1
b_{i2}								0	0	1	1	0	0	1	0	0	0

were met by direct shipment. The Third approach is to solve the example with integrated problem with multi-product delivery and direct shipment. Optimal solution of this model is shown in Table 2.7. As discussed for second approach, this method also decides to establish one distribution center. Since in this approach multi-product delivery is allowed, the two available delivery vehicles in distribution center were able to meet all customers' demand. Establishing less number of distribution centers and using less number of vehicles to deliver products to customers resulted in better objective function value in third approach.

Table 2.5: Optimal solution for separated problem

Opened DC(s)	DC1, DC2
Pickup tour(s)	DC1→S1→DC1
	DC1→S2→DC1
	DC2→S3→DC2
Delivery tour(s)	DC1→C3→C4→C7→DC1
	DC1→C9→C8→C5→C10→DC1
	DC2→C6→C2→C1→DC2
OFV	3057.58

Table 2.6: Optimal solution for single-product delivery integrated problem

Opened DC(s)	DC1
Pickup tour(s)	DC1→S1→DC1
	DC1→S2→DC1
	DC2→S3→DC2
Delivery tour(s)	DC1→C3→C4→C7→DC1
	DC1→C9→C8→C5→C10→C1→DC1
Direct shipment	S4→C6
	S5→C2
OFV	2290.70

2.5.2 Integrated versus separated models (without direct shipment)

The first analysis compares the integrated problem (IP) with separated problem (SP) in terms of objective function value (OFV) and CPU run-time in small size instances. The main goal of this analysis is to validate that integrated problem outperforms the separated problems even without direct shipment feature. The integrated problem is the formulation introduced in previous section but without direct shipment variables. The separated problem divides integrated problem into two mixed integer linear models. The first model solves a location routing problem (LRP) formulated by distribution center and delivery nodes. The mathematical formulation of the LRP is shown as following:

Table 2.7: Optimal solution for multi-product delivery integrated problem

Opened DC(s)	DC1
Pickup tour(s)	DC1→S1→DC1
	DC1→S2→DC1
Delivery tour(s)	DC1→C7→C10→C6→C2→C3→C1→DC1
	DC1→C9→C4→C8→C5→DC1
Direct shipment	
OFV	1894.40

$$\text{Min } f = \sum_{j \in N_C} FC_j w_j + \sum_{i \in N_C} \sum_{j \in N_{CD}} \sum_{v \in V_D} FDV x_{ijv} + \sum_{i \in N_{CD}} \sum_{j \in N_{CD}} \sum_{v \in V_D} cd_{i,j} u_{ijv} \tag{2.38}$$

Subject to:

Constraints (2.2)-(2.14),(2.29)-(2.30),(2.35)-(2.36) and

$$\sum_{j \in N_C} z_{ij} = 1 \quad i \in N_D \tag{2.39}$$

The second model is obtained by assigning values of variables in solved LRP model to the corresponding variables in the integrated model. Solving the second model represents the final solution for the separated problem.

The computational results have been reported in Table 2.8. Two important observations can be made. On one hand, the results indicate that compared to integrated problem, CPU run-time for solving the separated problem is 12.34 times faster on average.

Table 2.8: Integrated model versus separated model, small size instances

	Separated problem		Integrated Problem		
Instance	OFV	t(s)	OFV	t(s)	OFV improvement %
1	2972.70	0.74	1966.68	21.81	33.84
2	2567.01	0.50	2043.84	0.59	20.38
3	2771.71	0.42	1965.13	0.63	29.10
4	2502.73	0.44	1834.35	1.50	26.71
5	1672.10	0.27	1672.10	3.98	0.00
6	3057.58	0.47	2290.70	10.50	25.08
7	1613.03	0.25	1613.03	0.38	0.00
8	2419.97	0.77	1577.95	0.79	34.79
9	2805.71	0.52	2052.48	17.12	26.85
10	3169.03	0.34	2219.39	0.71	29.97
Avg.	2555.16	0.47	1923.56	5.80	24.72

It makes sense, because separated problem can be considered as a relaxation of the integrated problem. On the other hand, solving the integrated problem reduces the total system costs by 24.72 % on average. As shown in Figure 2.3, the integrated problem improves total cost in 8 out of 10 small instances.

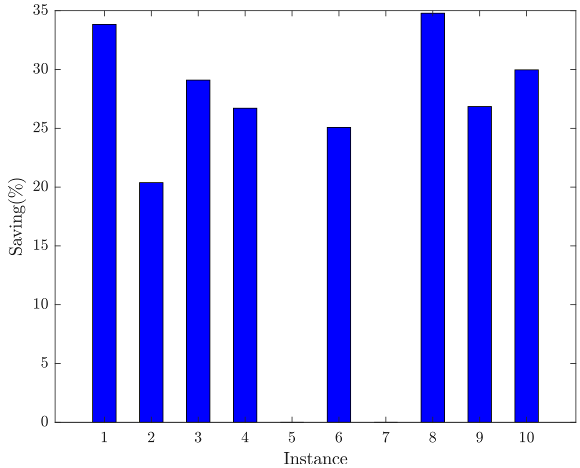


Figure 2.3: Total cost improvement in changing from SP model to IP model in small size problems

2.5.3 Single-product delivery versus multi-product delivery

This section evaluates the effect of loading strategy in delivery routes. Two scenarios have been considered: First, delivery vehicles are allowed to deliver various type of products so called multi-product delivery. Second, each delivery vehicle is allowed to load same type of products, namely single-product delivery. The formulated mathematical model in Section 2.4 with constraints (2.2)-(2.36) follows multi-product delivery scenario and adding constraints (2.37) results in single-product delivery scenario.

Table 2.9 includes the results of implementing multi-product and single-product delivery on small instances. As expected, CPU run-time for solving model with multi-product delivery scenario is longer than model with single-product scenario in all instances due to larger feasible solution space to explore. On average, it can be observed that the consumed CPU time in multi-product delivery scenario is 11.35 times greater than single-product delivery scenario. Regarding the objective function value, single-product delivery model is expected to have higher total costs because the system is expected to use more delivery vehicles than in single-product delivery scenario. In the case there is not enough number of delivery vehicles, the system uses direct shipment, inevitably. Both cases would increase the total costs of the distribution system. As indicated in Figure 2.4, allowing the multi-product delivery increases the total cost saving by 7% to 31.27%.

Table 2.9: Single-product delivery versus multi-product delivery, small size instances

Instance	Single-product OFV	t(s)	Multi-product OFV	t(s)	OFV improvement %
1	1966.68	21.81	1778.80	332.88	10.56
2	2043.84	0.59	1556.94	0.54	31.27
3	1965.13	0.63	1736.29	22.37	13.18
4	1834.35	1.50	1506.89	1.00	21.73
5	1672.10	3.98	1593.97	72.21	4.90
6	2290.70	10.50	1894.40	183.51	20.92
7	1613.03	0.38	1500.17	0.49	7.52
8	1577.95	0.79	1282.20	1.53	23.07
9	2052.48	17.12	1731.66	32.32	18.53
10	2219.39	0.71	2074.19	11.57	7.00
Avg.	1923.56	5.80	1665.55	65.84	15.49

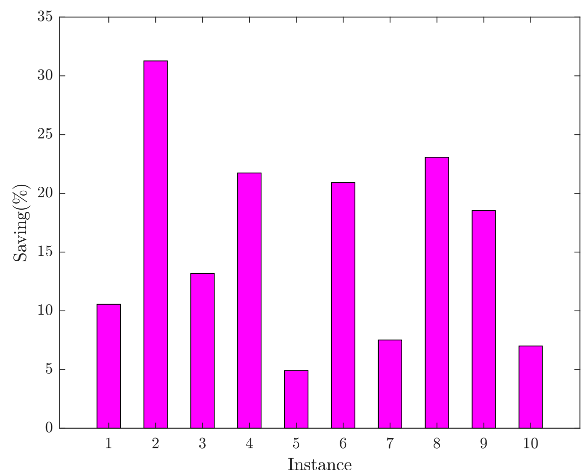


Figure 2.4: Total cost saving in multi-product delivery scenario (small size problems)

2.5.4 Pickup and delivery with location-routing and direct shipment: single-product delivery scenario

As discussed before, there are two delivery scenarios and the experiment results showed that multi-product delivery outperforms single-product delivery in terms of objective function value. But usually using single-product delivery is inevitable due to the delivery products nature and unloading difficulties. This section provides comprehensive analysis of computational results of solving introduced problem with single-product delivery scenario (constraints (2.2)-(2.37)) using three solution methods on small, medium and large instances.

Table 2.10 shows the comparison of deterministic mode, opportunistic mode and benders decomposition algorithm. It reports best objective function value, consumed CPU time, and gap in small size instances. For opportunistic mode each instance runs for 10 times and best

Table 2.10: Numerical results for single-product delivery scenario, small size instances

	Deterministic			Opportunistic			Benders		
Instance	OFV	t(s)	Gap	OFV	t(s)	Gap	OFV	t(s)	Gap
1	1966.68	52.52	0.00	1966.68	36.02	0.00	1966.68	3049.05	0.00
2	2043.84	0.58	0.00	2043.84	0.38	0.00	2043.84	48.19	0.00
3	1965.13	0.60	0.00	1965.13	0.48	0.00	1965.13	23.02	0.00
4	1834.35	1.25	0.00	1834.35	1.07	0.00	1834.35	5.95	0.00
5	1672.10	3.38	0.00	1672.10	2.69	0.00	1672.10	126.69	0.00
6	2290.70	10.82	0.00	2290.70	7.36	0.00	2290.70	358.40	0.00
7	1613.03	0.43	0.00	1613.03	0.30	0.00	1613.03	5.01	0.00
8	1577.95	0.82	0.00	1577.95	0.51	0.00	1577.95	10.65	0.00
9	2052.48	15.56	0.00	2052.48	3.77	0.00	2052.48	145.14	0.00
10	2219.39	0.70	0.00	2219.39	0.55	0.00	2219.39	36.38	0.00
Avg.	1923.57	8.67	0.00	1923.57	5.31	0.00	1923.57	380.85	0.00

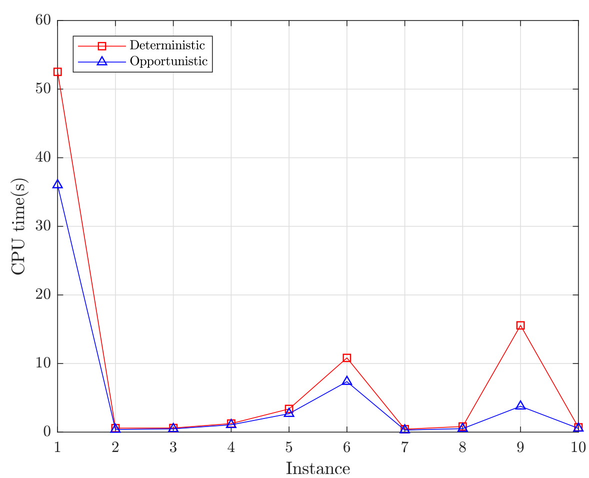


Figure 2.5: Comparison of computational time between deterministic and opportunistic modes

results based on objective function value have been reported.

Reported results in Table 2.10 indicate that all three methods can find optimal objective function value in small size problems. In terms of CPU run-time, benders decomposition algorithm has the worst performance and as shown in Figure 2.5, opportunistic mode outperforms deterministic mode in all small instances.

Table 2.11: Numerical results for single-product delivery scenario, medium size instances

	Deterministic			Opportunistic			Benders		
Instance	OFV	t(s)	Gap	OFV	t(s)	Gap	OFV	t(s)	Gap
11	4315.89	18000.00	1.21	4315.89	3415.97	0.00	4586.47	18000.00	35.65
12	3929.98	7458.53	2.65	3929.98	18000.00	2.65	4122.76	18000.00	21.25
13	4667.10	10058.76	3.26	4667.10	1165.85	0.00	4684.76	18000.00	13.71
14	3255.68	18000.00	8.19	3257.46	18000.00	6.10	3339.57	18000.00	23.74
15	4131.84	18000.00	6.90	4124.23	18000.00	5.90	4271.46	18000.00	24.70
16	4325.03	6172.48	12.83	4265.10	6111.41	11.52	4421.89	18000.00	24.65
17	5434.17	3064.25	7.24	5434.17	6757.82	4.64	5434.16	18000.00	13.81
18	4739.08	18000.00	2.11	4739.01	18000.00	1.65	4758.15	18000.00	14.07
19	4437.37	18000.00	5.26	4437.37	9007.92	4.49	4667.07	18000.00	28.15
20	3632.40	9632.87	17.53	3532.92	5628.36	15.30	3870.17	18000.00	28.77
Avg.	4286.85	12638.69	6.72	4270.32	10408.73	5.22	4415.65	18000.00	22.85

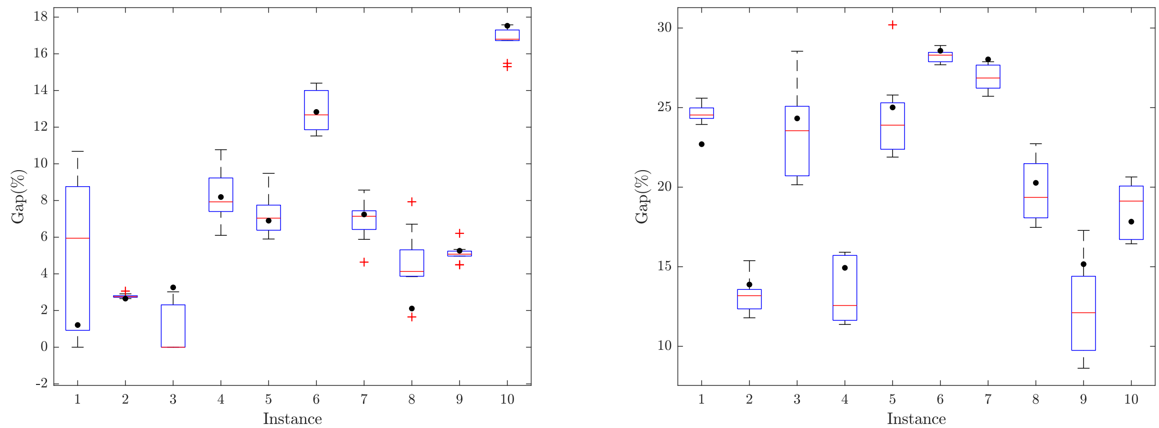
Table 2.12: Numerical results for single-product delivery scenario, large size instances

	Deterministic			Opportunistic			Benders		
Instance	OFV	t(s)	Gap	OFV	t(s)	Gap	OFV	t(s)	Gap
21	4738.99	18000.00	22.70	4782.99	18000.00	23.94	5389.16	18000.00	36.54
22	4329.26	18000.00	13.88	4311.79	18000.00	11.79	5256.76	18000.00	35.36
23	4264.58	18000.00	24.32	4059.86	18000.00	20.15	6023.53	18000.00	50.07
24	4296.91	18000.00	14.93	4245.63	18000.00	11.37	5210.58	18000.00	36.37
25	4553.97	18000.00	25.01	4546.24	18000.00	21.89	3340.84	18000.00	36.52
26	4455.27	18000.00	28.57	4442.45	18000.00	27.69	5363.21	18000.00	44.72
27	4272.78	18000.00	28.03	4153.80	18000.00	25.71	5151.77	18000.00	42.73
28	4604.58	18000.00	20.27	4604.57	18000.00	17.47	5259.81	18000.00	32.82
29	5220.48	18000.00	15.16	5082.84	18000.00	8.62	6304.50	18000.00	38.05
30	4725.72	18000.00	17.83	4711.04	18000.00	16.44	5521.63	18000.00	34.78
Avg.	4546.25	18000.00	21.07	4494.12	18000.00	18.51	5282.18	18000.00	38.80

Detailed results of medium and large instances have been listed in Table 2.11 and Table 2.12, respectively. Due to size of these problem sets, it may be computationally infeasible to find the optimal solution by CPLEX. Therefore, a time limit of 18000 seconds was imposed for the medium and large instances. Similar to small instances in opportunistic mode, each instance runs 10 times and best results based on objective value are reported. The cases with gaps greater than zero and CPU-time less than 18000 seconds indicate out of memory status. Reported results in Table 2.11 show that out of memory status happens for 6 out of 10 instances in deterministic mode and 7 out of 10 instances in opportunistic mode.

Based on Tables 2.11 and 2.12, it can be concluded that benders decomposition fails to find solutions with lower gap, compared to the other methods.

Figure 2.6 illustrates box plot of the gap obtained by opportunistic mode combined with scatter plot of the gap reported by deterministic mode for medium and large size instances, respectively.



(a) Medium size instances

(b) Large size instances

Figure 2.6: Solution gap (opportunistic mode versus deterministic mode)

Figure 2.6(a) indicates that running opportunistic mode for 10 times for each medium instance is 100% successful to find solution with equal or lower gap in comparison to the deterministic mode. Compared to deterministic mode, 80% of instances opportunistic mode provides the lower 25 percentile of gap. In addition, it can be seen from Figure 2.6(a) that the median value of gap obtained by opportunistic mode is better than gap value reported by deterministic mode in 60% of cases.

Figure 2.6(b) repeats same plots for large size instances. As shown in Figure 2.6(b), opportunistic mode results in better solutions with lower gaps, lower 25 percentile of gaps, and lower median of gaps in 90%, 90% and 80% of the large size instances, respectively.

Figure 2.7 traces the trend of gap in small, medium and large size problem sets in opportunistic, deterministic and benders decomposition algorithm. The gap for opportunistic mode is the best with running each instance for 10 times. As depicted in Figure 2.7, the gap increases as the number of nodes in instances increase. Deterministic and opportunistic modes follows each other closely, but benders decomposition performance shows an inefficient solution method for the introduced problem. Based on the results in Table 2.11 and Table 2.12, compared to average gap in medium size instances, the average gap for large size instances enhances 14.35%, 13.29%, and 15.95% for

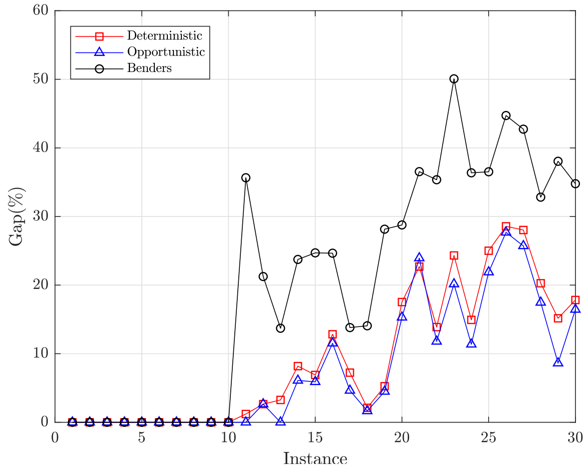


Figure 2.7: Solution gap by different solving methods

deterministic, opportunistic and benders algorithm, respectively. Obviously opportunistic outperforms two the other methods in these instances.

2.6 Conclusions

This paper presents a new supply chain design model in which the location of distribution centers, pickup and delivery process and direct shipment are considered as an integrated model. A mathematical formulation has been developed and two different scenarios have been analyzed for delivery routes. The first scenario refers to status in which delivery vehicles are allowed to load various type of products, called multi-product delivery, and in the second scenario each delivery vehicle is restricted to load just one type of product, called single-product delivery. To evaluate the formulated models, numerical analysis were conducted for 30 instances. The first part of computational results shows that integrated model proposed in this paper outperforms the separated models. The second part of experiments provide a comparison of multi-product delivery and single-product delivery scenarios and shows multi-product delivery reduces total cost of the system. Considering real life cases sometimes using single-product delivery is inevitable, so the third part of computational experiments devoted to single-product delivery. The instances have been tested in CPLEX using deterministic mode, opportunistic mode, and benders decomposition algorithm. Based on the numerical analysis, opportunistic mode outperforms other solution methods in terms of objective function value and the computational time.

For future studies, proposed models can be extended considering uncertainty in parameters like customer demand, capacity of distribution centers and supply capacity. The other direction can be devoted to develop better exact or heuristic solution methods. Considering objective functions other than cost function such as customer satisfaction and environmental emission effect can also serve as future research direction.

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CHAPTER 3. A two-stage stochastic programming model for multi-period reverse logistics network design with lot-sizing

A paper accepted by *Computers & Industrial Engineering*

3.1 Abstract

This paper proposes an integrated model for a multi-period reverse logistics (RL) network design problem under return and demand uncertainty. The reverse logistics network is modeled as a two-stage stochastic programming model to make strategic and tactical decisions. The strategic decisions are the first stage decisions in establishing network's facilities and tactical decisions are the second stage decisions on material flow, inventory, backorder, shortage, and outsourcing. The uncertainties considered in this study are the primary market return and secondary market demand. The model aims to determine optimal numbers of sorting centers and warehouses, optimal lot sizes, and transportation plan that minimize the expected total system cost over the planning horizon. A case study was conducted to validate the proposed model. Numerical results indicate that the stochastic model solution outperforms result of expected value solution.

3.2 Introduction

Reverse logistics has been gaining popularity in the supply chain design (Agrawal et al. (2015)). The term reverse logistics refers to "the process of planning, and managing the flow of raw materials, in-process inventory, and finished goods from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal" (Rogers et al. (1999b)). Nowadays, manufacturing industry and related stakeholders have recognized that reverse logistics is critical for their success in current competitive market environment. Major companies such as Dell, General Motors, Canon, and Hewlett-Packard have taken advantage of reverse logistics (Jayaraman and Luo (2007)). Hence, reverse logistics network planning is crucial for sustainable competitiveness.

One of the most challenging supply chain problems is the network design for a reverse logistics system (Melo et al. (2009)). It involves locating multiple types of facilities, such as sorting centers, warehouses, disposal centers, and recycling centers, and decisions on material flow between facilities. The designing of reverse logistics network is more complicated compared to the traditional forward logistics network planning due to two reasons (Yu and Solvang (2018)). First, more activities are involved in reverse logistics, such as collection, sorting, stocking, distribution, remanufacturing, recycling, and disposal. Therefore, the structure of network in reverse logistics is more complicated. Second, there are more uncertainties both in terms of quality and quantity in reverse logistics networks.

To cope with these challenges, researchers have developed decision-making models and solution techniques for reverse logistics problems over the past decades. In terms of mathematical modeling, existing literature commonly ignore some real world characteristics of reverse logistics such as backorder and shortage for secondary markets and outsourcing. Regarding the solution method, most of the studies do not include appropriate scenario generation and scenario reduction methods to approximate underlying distributions of uncertain parameters. This paper aims to overcome these drawbacks with a two-stage stochastic programming model for multi-period reverse logistics which includes lot-sizing (allowing backorder and shortage) and outsourcing. Moment matching method has been used to generate scenarios and fast forward selection method is used as a reduction method to select a proper subset of generated scenarios as the most representative scenarios. A case study was conducted to illustrate and validate the model and solution method. The computational results have been provided to evaluate the stochastic programming model's performance.

3.3 Literature review

The earlier models have focused on the decision-making in deterministic environments (Govindan et al. (2015)). However, it is essential to consider uncertainties in reverse logistics system design (Govindan et al. (2017)). Demand quantity is among the common uncertain parameters considered in the literature. Aghezzaf (2005) presented a robust optimization model for warehouse capacity and location problem under demand uncertainty. The author developed a Lagrangian relaxation algorithm to solve the problem. Lee and Dong (2009) formulated a dynamic location and allocation model for reverse logistic network design problem. They formulated the problem as a two-stage stochastic programming model and developed a solution method based on sampling and a simulated annealing (SA) algorithm. The other common uncertainty to consider in reverse logistics network is return amount and quality. Ayvaz et al. (2015) studied a reverse logistics network under return, quality and transportation cost uncertainties. They proposed a two-stage stochastic programming model for multi-echelon, muti-product and capacitated reverse logistics network for an electrical and electronic equipment company. They used sample average approximation method to solve the model in order to maximize the total profits. Khatami et al. (2015) designed a reverse logistics network and incorporated it to an existing supply chain network under demand and return uncertainties. The authors generated scenarios based on Cholesky's factorization method and decreased the number of scenarios by k-means clustering algorithm. They used an Epsilon-constraint method to find the solution. Salema et al. (2007) proposed a generic multi-product capacitated reverse logistics under demand and return uncertainty. They formulated the problem as a mixed integer problem and solved it by standard branch and bound solution method. Ene and Öztürk (2015) formulated a linear programming model for reverse logistics in a vehicles' recovery network to minimize pollution and maximize revenue in end-of-life product operations. Trochu et al. (2018) studied a reverse logistics under environmental policies in a wood industry. They conducted a scenario-based analysis to evaluate the influence of uncertainties on the reverse logistics network design. Yu and Solvang (2017) presented a stochastic programming model for a sustainable multi-product multi-echelon carbon-constrained reverse logistics network under uncertainty. The proposed model considers both optimal value expectation and its reliability in decision making.

Closed-loop supply chain networks include both forward and reverse logistics networks as an integrated system. Soleimani et al. (2016) studied a multi-product multi-period closed-supply chain problem with stochastic demand and price. They developed a multi-criteria scenario based solution method to find the optimal solution. A case study of an Indian manufacturer was conducted to validate model and solution approach. Ameknassi et al. (2016) developed a stochastic programming model for a multi-objective closed-loop supply chain under demand, capacity of facilities, quantity and quality of returns, and the transportation, warehousing and reprocessing costs uncertainties. Özceylan et al. (2017) studied a closed-loop supply chain network based on a case study for the end-of-life vehicles (ELV) in Turkey. They formulated problem as a linear programming model and analyzed the results under a variety of scenarios. Kuşakcı et al. (2019) developed a fuzzy mixed integer programming model for reverse logistics network of ELVs under supply uncertainty. A case study was conducted to validate the proposed model. Pishvaee et al. (2009) developed a scenario-based stochastic programming model for a closed-loop logistic network under uncertainty. El-Sayed et al. (2010) addressed multi-period multi-echelon closed-loop logistic problem under uncertain demand. The authors formulated problem as a multi-stage stochastic program to maximize the total expected profit. Pishvaee et al. (2011) studied a closed-loop supply chain network under uncertainty and proposed a robust optimization model to handle the uncertainties.

To better summarize the literature, we constructed two tables to compare the studies from literature to our proposed study. Table 3.1 focuses on the model assumptions and formulation settings and Table 3.2 focuses on the solution methods or techniques.

Table 3.1: Literature review (model assumptions)

					Cost elements							
Reference	Network	# Layers	SP	MP	L	T	I	B	S	E	O	CS
Ayvaz et al. (2015)	RL	5	✓		✓	✓						✓
Salema et al. (2007)	RL	4	✓		✓	✓			✓			
Lee and Dong (2009)	CL	3		✓	✓	✓						
Trochu et al. (2018)	RL	4	✓		✓	✓						✓
Soleimani et al. (2016)	CL	6		✓	✓	✓	✓		✓			✓
Ameknassi et al. (2016)	CL	5		✓	✓	✓				✓		✓
Pishvaee et al. (2011)	CL	4	✓		✓	✓						
Miranda and Garrido (2004)	RL	3	✓		✓	✓	✓					
Listeş and Dekker (2005)	RL	4	✓		✓	✓						✓
Ramezani et al. (2013)	CL	5	✓		✓	✓						
Our study	RL	6		✓	✓	✓	✓	✓	✓		✓	✓

RL=Reverse Logistics, CL=Closed-Loop, SP=Single-Period, MP=Multi-Period, L=Location/Allocation, T=Transportation, I=Inventory, B=Backorder, S=Shortage, E=Environmental, O=Outsourcing, CS=Case Study

As shown in Table 3.1, some of the existing mathematical models did not incorporate all of the characteristics of reverse logistics such as backorder and shortage for secondary markets and outsourcing. Table 3.2 illustrates stochastic programming properties such as stochastic parameters and solution method's elements. Scenario generation and scenario reduction are two important components of a stochastic programming model. The two most common methods for scenario generation are by sampling and by statistical methods (Mitra and Domenica (2010)). As shown in Table 3.2, most of the proposed solution methods do not provide appropriate scenario genera-

Table 3.2: Literature review (solution methods)

Reference	# Stages	Stochastic parameters		Solution method elements						
		Return	Demand	SG	SR	SAA	H	EC	RO	EF
Ayvaz et al. (2015)	2	✓				✓				
Salema et al. (2007)	2	✓	✓							✓
Lee and Dong (2009)	2	✓	✓				✓			
Trochu et al. (2018)	2	✓								
Soleimani et al. (2016)	2		✓							
Ameknassi et al. (2016)	2	✓	✓					✓		
Pishvaee et al. (2011)	2	✓	✓						✓	
Miranda and Garrido (2004)	2		✓				✓			
Listeş and Dekker (2005)	3		✓							✓
Ramezani et al. (2013)	2	✓	✓							✓
Our study	2	✓	✓	✓	✓					✓

SG=Scenario Generation, SR=Scenario Reduction, SAA=Sample Average Approximation, H=Heuristic, EC=Epsilon Constraint, RO= Robust Optimizaion EF=Extensive Form

tion and scenario reduction methods to approximate underlying distributions. Some studies focus on the sample average approximation method. However, a key drawback of using sampling-based scenario generation methods is that the obtained scenario tree may have completely different statistical properties from the original distribution if the size of scenario set is small. To solve this problem, the number of scenarios should be increased which may be computationally expensive. The advantage of statistical methods like moment matching is that it can generate better scenarios when the size of generated scenario tree is small (Mitra and Domenica (2010)). Arpón et al. (2018) and Römisch (2010) reviewed comprehensively scenario generation methods. In order to fill the mentioned gaps of mathematical modelling and solution methods, this paper addresses a two-stage stochastic programming model for multi-period reverse logistics which includes lot-sizing and outsourcing. Moment matching and fast forward selection are used as scenario generation and scenario reduction methods, respectively.

The reminder of this paper is organized as follows: Section 3.4 describes the two-stage stochastic programming model. Section 3.5 discusses the computational results and stochastic programming performance. Lastly, conclusions and scope of future research are included in Section 3.6.

3.4 Two-stage stochastic programming model

This section provides a two-stage stochastic programming model for a reverse logistics network with lot-sizing under uncertainty. Return and demand uncertainties are the common uncertain factors for reverse logistics problems thus they are considered in this paper. The first-stage decisions of this study are related with facilities (sorting centers, warehouses, recycling centers and disposal centers) location in strategic level, and the second stage decisions represent the transported and stocked products, unmet demand as backorder, shortage, and outsourcing quantities in tactical level. This paper considers return and demand with continuous distributions as the stochastic factors to be investigated. As problems with uncertain parameters represented with continuous distribution are computationally challenging (Escudero and Kamesam (1995)), scenarios are typically used to approximate the underlying continuous distributions. Each scenario is a set of discrete values showing returns from the primary markets and demands in the secondary markets.

3.4.1 Problem statement

The study addresses multi-echelon and multi-period supply chain design problem for reverse logistics network including a set of distributed primary and secondary markets with locations known and fixed, and a set of facilities to be located among candidate locations. If a product provided by manufacturer does not meet primary market's demand, the product will be returned to sorting center for quality assessment. After collecting products, they are inspected in sorting center and based on quality assessment three outcomes are possible: (1) The quality level of product is acceptable. Therefore, it will be sent to the warehouses where prepare the products to secondary markets; (2) the product is recyclable and needs to be transported to recycling center for re-processing; (3) the product is scrapped and needs to be transported to the disposal centers where it is disposed in proper manner. The quantity of primary markets' returns and secondary markets' demands are assumed to be stochastic. Because of these stochastic factors, the problem is formulated as a stochastic programming model.

The objective is to minimize overall system cost including facility capital investment, transportation cost, inventory cost, backorder cost, shortage cost, and outsourcing cost. The proposed stochastic programming includes following decisions:

- The locations of facilities such as sorting; centers, warehouses, recycling centers, and disposal centers.

- The material flow between various facilities;
- Inventory level in each warehouse;
- Backorder level for each secondary market;
- Shortage level for each secondary market;
- Outsourcing quantity.

3.4.2 Mathematical Formulation

This section introduces the proposed two-stage stochastic programming model. Assumptions are listed as follows:

- Inventory in sorting centers, recycling centers and disposal centers are not allowed.
- Initial inventory is not allowed in warehouses.
- End of each period is set to measure inventory level of warehouses.
- Fulfilling of secondary markets' demand can be delayed or ignored since backorders and shortages are allowed.
- Transportation between the same kind of facilities are not allowed (e.g. transportation between warehouses is prohibited).

The notations of the model formulation are as following.

3.4.2.1 Sets

$\mathcal{PM}$ Set of primary markets, indexed by m

$\mathcal{SC}$ Set of possible facility locations for sorting centers, indexed by c

$\mathcal{W}$ Set of possible facility locations for warehouses, indexed by w and $w\prime$

$\mathcal{SM}$ Set of secondary markets, indexed by i and j

$\mathcal{WS}$ Set of warehouses and secondary markets, $\mathcal{WS} = \mathcal{W} \cup \mathcal{SM}$, indexed by i and j

$\mathcal{R}$ Set of recycling centers, indexed by r

$\mathcal{D}$ Set of disposal centers, indexed by d

$\mathcal{T}$ Set of periods, indexed by t and t'

$\mathcal{S}$ Set of scenarios, indexed by s

3.4.2.2 Parameters

p_s Probability associated with scenario s

es_c Fixed cost for establishing sorting center in location c

ew_w Fixed cost for establishing warehouse in location i

ez_r Fixed cost for establishing recycling center in location r

ey_d Fixed cost for establishing disposal center in location r

q_m^{ts} Return of primary market m under scenario s in period t

tps_{mc} Transportation cost per product unit from primary market m to sorting center c

tsw_{cw} Transportation cost per product unit from sorting center c to warehouse w

tsr_{cr} Transportation cost per product unit from sorting center c to recycling center r

tsd_{cd} Transportation cost per product unit from sorting center c to disposal center d

tws_{ij} Transportation cost per product unit from node $i \in \mathcal{WS}$ to node $j \in \mathcal{WS}$

h_w^t Inventory cost per product unit in warehouse w in period t

b_i^t Backorder cost per product unit for secondary market i in period t

sc_i Shortage cost of one unit of unmet demand of secondary market i

OC Outsourcing cost per product unit

d_i^{ts} Demand of secondary market i under scenario s in period t

dr^t Disposal ratio in period t

rr^t Recycling ratio in period t

CS_c Capacity of sorting center c

CW_w Capacity of warehouse w

CR_r Capacity of recycling center r

CD_d Capacity of disposal center d

3.4.2.3 Binary first stage decision variables

l_c 1 if sorting center c established, 0 otherwise

g_w 1 if warehouse w is established, 0 otherwise

y_d 1 if disposal center d is established, 0 otherwise

z_r 1 if recycling center r is established, 0 otherwise

3.4.2.4 Nonnegative second stage decision variables

α_{mc}^{ts} Amount of product transported from primary market m to sorting center under scenario s in period t

β_{cw}^{ts} Amount of product transported from sorting center c to warehouse w under scenario s in period t

θ_{cr}^{ts} Amount of product transported from sorting center c to recycling center r under scenario s in period t

λ_{cd}^{ts} Amount of product transported from sorting center c to disposal center d under scenario s in period t

μ_{wi}^{ts} Amount of product transported from warehouse w to secondary market i under scenario s in period t

I_w^{ts} Inventory level of product in warehouse w under scenario s in period t

B_i^{ts} Backordered demand for secondary market i under scenario s in period t

so_{mc}^{ts} Outsourced product of shipment from primary market m to sorting center c (because of capacity exceeding in sorting center c) under scenario s in period t

ro_{cr}^{ts} Outsourced product of shipment from sorting center c to recycling center r (because of capacity exceeding in recycling center r) under scenario s in period t

do_{cd}^{ts} Outsourced product of shipment from sorting center c to disposal center d (because of capacity exceeding in disposal center d) under scenario s in period t

wo_{cw}^{ts} Outsourced product of shipment from sorting center c to warehouse w (because of capacity exceeding in warehouse w) under scenario s in period t

$d_i^{[t]s}$ Cumulative total demand of secondary market i over t periods under scenario s, $\left(d_i^{[t]s} = \sum_{t'=1}^{t'=t} d_i^{t's}\right)$

$\mu_{wi}^{[t]s}$ Cumulative total shipment transported from warehouse w to secondary market i over t periods under scenario s, $\left(\mu_{wi}^{[t]s} = \sum_{t'=1}^{t'=t} \mu_{wi}^{t's}\right)$

$B_i^{[t]s}$ Cumulative total backorder for secondary market i over t periods under scenario s, $\left(B_i^{[t]s} = \sum_{t'=1}^{t'=t} B_i^{t's}\right)$

$\beta_{cw}^{[t]s}$ Cumulative total shipment transported from sorting center c to warehouse w over t periods under scenario s, $\left(\beta_{cw}^{[t]s} = \sum_{t'=1}^{t'=t} \beta_{cw}^{t's}\right)$

3.4.2.5 Objective function

The objective function minimizes the total expected costs of network including location costs ($\mathcal{Z}_1$), transportation costs ($\mathcal{Z}_2$), inventory costs ($\mathcal{Z}_3$), backorder costs ($\mathcal{Z}_4$), shortage costs ($\mathcal{Z}_5$) and outsourcing costs ($\mathcal{Z}_6$) over the determined planning horizon. Equalities (3.1)-(3.7) present the objective function and its elements:

$$\text{Min } \mathcal{F} = \mathcal{Z}_1 + \mathcal{Z}_2 + \mathcal{Z}_3 + \mathcal{Z}_4 + \mathcal{Z}_5 + \mathcal{Z}_6 \tag{3.1}$$

$$\mathcal{Z}_1 = \sum_{c \in \mathcal{SC}} es_c l_c + \sum_{w \in \mathcal{W}} ew_w g_w + \sum_{r \in \mathcal{R}} er_r z_r + \sum_{d \in \mathcal{D}} ed_d y_d \tag{3.2}$$

$$\mathcal{Z}_2 = \sum_{s \in \mathcal{S}} p_s \Bigg(\sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{PM}} \sum_{c \in \mathcal{SC}} tps_{mc} \alpha_{mc}^{ts} + \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{SC}} \sum_{w \in \mathcal{W}} tsw_{cw} \beta_{cw}^{ts} + \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{SC}} \sum_{r \in \mathcal{R}} tsr_{cr} \theta_{cr}^{ts} + \tag{3.3}$$

$$\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{SC}} \sum_{d \in \mathcal{D}} tsd_{cd} \lambda_{cd}^{ts} + \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{W}} \sum_{j \in \mathcal{W}} tws_{ij} \mu_{ij}^{ts} \Bigg)$$

$$\mathcal{Z}_3 = \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} p_s h_w^t I_w^{ts} \tag{3.4}$$

$$\mathcal{Z}_4 = \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T} \setminus T} \sum_{i \in \mathcal{SM}} p_s b_i^t B_i^{ts} \tag{3.5}$$

$$\mathcal{Z}_5 = \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{SM}} p_s sc_i^p B_i^{Ts} \tag{3.6}$$

$$\mathcal{Z}_6 = \sum_{s \in \mathcal{S}} p_s \left(OC \left(\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{SC}} so_{mc}^{ts} + \sum_{t \in \mathcal{T}} \sum_{w \in \mathcal{W}} wo_{cw}^{ts} + \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}} ro_{cr}^{ts} + \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} do_{cd}^{ts} \right) \right) \tag{3.7}$$

In Eq.(3.2), the four terms include all first stage decision variables and represent location cost for sorting centers, warehouses, recycling centers and disposal centers, respectively. Eq.(3.3), with five terms, calculates transportation costs of primary markets to sorting center, sorting centers to warehouses, sorting centers to recycling centers, sorting centers to disposal centers, and warehouses to secondary markets, respectively. Eq.(3.7) includes outsourcing costs for sorting centers, warehouses, recycling centers and disposal centers.

3.4.2.6 Constraints

$$\sum_{m \in \mathcal{PM}} \alpha_{mc}^{ts} \leq CS_c l_c \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.8}$$

$$\sum_{c \in \mathcal{SC}} \theta_{cr}^{ts} \leq CR_r z_r \quad r \in \mathcal{R}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.9}$$

$$\sum_{c \in \mathcal{SC}} \lambda_{cd}^{ts} \leq CD_d y_d \quad d \in \mathcal{D}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.10}$$

$$\sum_{c \in \mathcal{SC}} \beta_{cw}^{ts} \leq CW_w g_w \quad w \in \mathcal{W}, t = 1, s \in \mathcal{S} \tag{3.11}$$

$$\sum_{c \in \mathcal{SC}} \beta_{cw}^{ts} = CW_w g_w - I_w^{(t-1)s} \quad w \in \mathcal{W}, t \in \mathcal{T} \setminus 1, s \in \mathcal{S} \tag{3.12}$$

Constraints (3.8)-(3.12) are for the capacity of facilities and transportation amount the facilities. These constraints prohibit product flow between facilities that are not established. Meanwhile, they prevent capacity exceeding in facilities by transported products.

$$\sum_{c \in \mathcal{SC}} (\alpha_{mc}^{ts} + so_{mc}^{ts}) = q_m^{ts} \quad m \in \mathcal{PM}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.13}$$

Constraints (3.13) state that the amount of return products from primary markets include transported products to sorting centers and the portion of return which exceeds sorting centers' capacity.

$$\sum_{m\in\mathcal{PM}} rr^t \alpha_{mc}^{ts} = \sum_{r\in\mathcal{R}} (\theta_{cr}^{ts} + ro_{cr}^{ts}) \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.14}$$

$$\sum_{m\in\mathcal{PM}} dr^t \alpha_{mc}^{ts} = \sum_{d\in\mathcal{D}} (\lambda_{cd}^{ts} + do_{cd}^{ts}) \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.15}$$

$$\sum_{m\in\mathcal{PM}} (1 - rr^t - dr^t) \alpha_{mc}^{ts} = \sum_{w\in\mathcal{W}} (\beta_{cw}^{ts} + wo_{cw}^{ts}) \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.16}$$

Constraints (3.14) calculate transported amount of products from sorting centers to recycling centers. At the same time, these constraints calculate the amount of products exceeding the capacity of recycling centers. Constraints (3.15) and (3.16) do the same task for disposal centers and warehouses, respectively.

$$\sum_{m\in\mathcal{PM}} so_{mc}^{ts} \leq \Big(\sum_{m\in\mathcal{PM}} q_m^{ts}\Big) l_c \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.17}$$

$$\sum_{c\in\mathcal{SC}} ro_{cr}^{ts} \leq \Big(\sum_{m\in\mathcal{PM}} q_m^{ts}\Big) z_r \quad r \in \mathcal{R}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.18}$$

$$\sum_{c\in\mathcal{SC}} do_{cd}^{ts} \leq \Big(\sum_{m\in\mathcal{PM}} q_m^{ts}\Big) y_d \quad d \in \mathcal{D}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.19}$$

$$\sum_{c\in\mathcal{SC}} wo_{cw}^{ts} \leq \Big(\sum_{m\in\mathcal{PM}} q_m^{ts}\Big) g_w \quad w \in \mathcal{W}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.20}$$

Constraints (3.17)-(3.20) state outsourcing is not allowed from facilities that are not established since capacity exceeding does not occur in this situation.

$$I_w^{ts} \leq \beta_{cw}^{[t]s} \quad w \in \mathcal{W}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.21}$$

$$I_w^{ts} = \sum_{c \in \mathcal{SC}} \beta_{cw}^{ts} - \sum_{i \in \mathcal{SM}} \mu_{wi}^{ts} \quad w \in \mathcal{W}, t = 1, s \in \mathcal{S} \tag{3.22}$$

$$I_w^{ts} = I_w^{(t-1)s} + \sum_{c \in \mathcal{SC}} \beta_{cw}^{ts} - \sum_{i \in \mathcal{SM}} \mu_{wi}^{ts} \quad w \in \mathcal{W}, t \in \mathcal{T} \setminus 1, s \in \mathcal{S} \tag{3.23}$$

Constraints (3.21)-(3.23) determine the inventory level at each warehouse.

$$\sum_{i \in \mathcal{SM}} \mu_{wi}^{ts} \leq \sum_{c \in \mathcal{SC}} \beta_{cw}^{ts} \quad w \in \mathcal{W}, t = 1, s \in \mathcal{S} \tag{3.24}$$

$$\sum_{i \in \mathcal{SM}} \mu_{wi}^{ts} \leq I_w^{(t-1)s} + \sum_{c \in \mathcal{SC}} \beta_{cw}^{ts} \quad w \in \mathcal{W}, t \in \mathcal{T} \setminus 1, s \in \mathcal{S} \tag{3.25}$$

$$\sum_{w \in \mathcal{W}} \mu_{wi}^{ts} \leq d_i^{ts} + B_i^{[t]s} \quad i \in \mathcal{SM}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.26}$$

Constraints (3.24)-(3.26) determine the amount of product transportation to each secondary market.

$$\sum_{w \in \mathcal{W}} \mu_{wi}^{ts} = 0 \quad i \in \mathcal{W}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.27}$$

Constraints (3.27) state transportation between warehouses is not allowed.

$$B_i^{ts} = d_i^{[t]s} - \sum_{w \in \mathcal{W}} \mu_{wi}^{[t]s} \quad i \in \mathcal{SM}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.28}$$

Constraints (3.28) calculate backorder level for each secondary market.

3.5 Case study

In this section, the applicability of the proposed model is demonstrated with a real case study from Europe. Data used in the case study were adapted from Kalaitzidou et al. (2015). The case study focused on a European consumer goods company and due to confidentiality policy, the parameters have been scaled with a common factor and real currency units have been substituted with relative money units (rmu).

3.5.1 Data sources

The potential reverse logistics network consists of 38 nodes, as shown in Figure 3.1. Five of them including Italy (IT), United Kingdom (UK), Belgium (BE), France (FR), and Norway (NO) concern primary markets. Six of them including United Kingdom (UK), France (FR), Sweden (SE), Spain (ES), Austria (AT), and Belgium (BE) are potential sorting centers. Three of them including Netherlands (NE), Italy (IT), and France (FR) represent potential warehouses. Three of them including Germany (DE), France (FR), and Italy (IT) are potential recycling centers. The potential disposal site are same as recycling centers' candidates. The remaining 18 nodes represent secondary market nodes. The secondary markets are located in Italy (IT), United Kingdom (UK), France (FR), Spain (ES), Ireland (IE), Sweden (SE), Greece (GR), Netherlands (NL), Finland (FI), Denmark (DK), Czech Republic (CH), Belgium (BE), Portugal (PT), Norway (NO), Germany (DE), Austria (AT), Turkey (TR), and Poland (PL).

Establishing cost and capacity of facilities including sorting centers, warehouses, disposal centers and recycling centers are shown in Table 3.3. Furthermore, the unit backorder cost, unit shortage

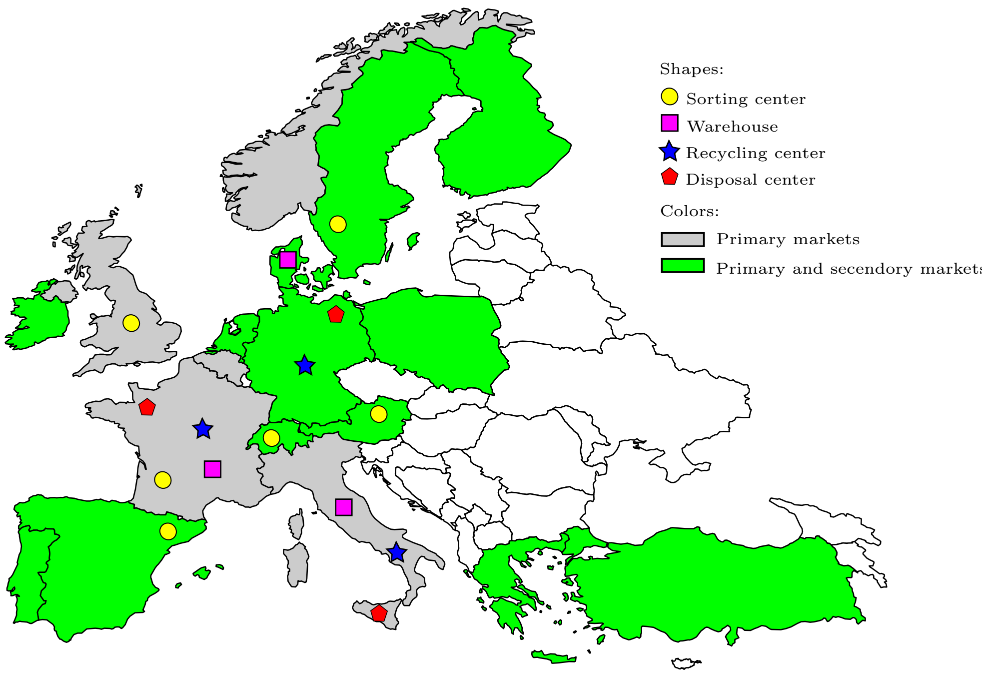

Figure 3.1: Network of facilities, primary markets, and secondary markets

cost, and unit holding cost are included in Table 3.4. The rate of recyclable and disposable products in different periods are listed in Table 3.5.

Quantity of returned products from the primary markets and demand by secondary markets are assumed to follow Normal distributions (Abdallah et al. (2012)). Four moments of return distribution and demand distribution for primary and secondary markets are listed in Tables 3.6 and 3.7, respectively. We assume that returns in different periods are independent from each other. The same assumption holds for demand (i.e. demands in different periods are independent). The planning horizon the model studies is 5 months. Unit outsourcing cost for each facility is considered to be 30 rmu.

3.5.2 Scenario generation

It is computationally challenging to solve stochastic models including parameters with continuous distribution Feng and Ryan (2013). In such a situation, a discretization process so-called scenario generation is used to approximate continuous distribution with a discrete distribution

Table 3.3: Capacity (Cap) and establishing cost (EC) of facilities

	Sorting Center		Warehouse		Recycling center		Disposal center	
Node	Cap	EC	Cap	EC	Cap	EC	Cap	EC
UK	8000	40000						
FR	10000	32500	9500	25000	1500	20000	1500	6000
SE	7000	22500						
ES	9500	20000						
AT	6000	15000						
BE	7500	25000						
NE			6000	12000				
IT			8000	15000	1500	11500	2200	5400
DE					2000	15000	2500	6500

Table 3.4: Holding, backorder and shortage cost

Secondary market	Holding cost					Backorder cost					Shortage cost
	#Period					#Period					
	1	2	3	4	5	1	2	3	4	5	
IT	1.28	1.31	1.32	1.28	1.30	0.46	2.52	1.36	1.16	-	7.46
UK						3.54	1.28	0.22	0.26	-	6.89
FR	0.99	1.03	1.03	0.99	1.00	2.00	2.68	2.40	0.54	-	8.98
ES						0.60	2.94	2.5	2.90	-	4.53
IE						0.76	2.36	1.52	1.40	-	6.12
SE						1.76	3.30	0.54	0.76	-	5.07
GR						0.52	2.22	3.18	3.42	-	7.85
NL	1.13	1.16	1.15	1.13	1.14	2.56	3.36	3.06	3.06	-	7.91
FI						1.04	0.16	0.76	3.58	-	6.24
DK						1.78	3.80	3.20	1.82	-	4.28
CH						3.38	3.96	3.04	3.88	-	9.28
BE						1.04	2.48	2.66	0.46	-	8.96
PT						3.96	2.46	3.30	1.74	-	9.77
NO						0.68	1.94	3.52	1.32	-	6.58
DE						3.64	3.86	1.70	3.54	-	7.20
AT						0.58	3.50	1.88	2.52	-	8.88
TR						0.54	2.76	0.9	0.12	-	5.23
PL						2.84	1.08	2.24	3.52	-	4.42

with limited number of outcomes. In this study the moment matching method is adopted to generate limited number of outcomes to represent each continuous distribution (Høyland and Wallace (2001)). Since statistical properties are able to approximate continuous distributions, we minimize the distance between statistical specifications of continuous distributions and statistical properties of fitted discrete distributions subject to a constraint ensuring summation of branching probabilities to be one. In this study mean, variance, kurtosis and skewness have been used as statistical specifications. The values of these statistical properties for the 5 return distributions and 18 de-

Table 3.5: Recycling and disposal rate

Period	$rr(t)$	$dr(t)$
1	0.24	0.06
2	0.29	0.09
3	0.18	0.03
4	0.16	0.04
5	0.23	0.09

Table 3.6: Return properties in each period

		Properties			
Primary market	PDF	Mean	Variance	Skewness	Kortusis
UK	Normal	2338.27	132986.03	0	3
FR	Normal	2605.81	184908.17	0	3
BE	Normal	2102.58	76003.87	0	3
IT	Normal	2027.70	92385.29	0	3
NO	Normal	1375.71	15104.41	0	3

mand distributions are listed in Tables 3.6 and 3.7, respectively. Following notations are used in moment matching method:

- κ Specifies a statistical property
- K Set of all statistical properties
- w_κ Refers to the importance weight of statistical property κ
- υ Specifies branches (outputs)
- x Vector of realizations for uncertain factors
- π Probability vector for branches (outputs)
- $f_\kappa(x, \pi)$ Mathematical function for representing statistical property κ
- VAL_κ Specified value of statistical property of κ

Scenario generation may be formulated as a nonlinear programming model as follows Høyland and Wallace (2001). Although the optimal solution for this model is not guaranteed but any solution with objective function close to zero can be considered a good solution.

Table 3.7: Demand properties in each period

Secondary market	PDF	Properties			
		Mean	Variance	Skewness	Kortusis
UK	Normal	351.33	2777.24	0	3
ES	Normal	348.00	2724.84	0	3
IT	Normal	317.67	2270.57	0	3
FR	Normal	310.00	2162.25	0	3
SE	Normal	299.67	2020.54	0	3
IE	Normal	233.00	1221.50	0	3
NL	Normal	348.00	2724.84	0	3
GR	Normal	332.00	2480.04	0	3
DK	Normal	317.33	2265.71	0	3
FI	Normal	551.33	6839.20	0	3
PT	Normal	443.00	4415.60	0	3
BE	Normal	462.00	4802.49	0	3
CH	Normal	571.33	7344.41	0	3
NO	Normal	546.33	6715.72	0	3
AT	Normal	529.33	6304.28	0	3
DE	Normal	518.00	6037.29	0	3
PL	Normal	495.00	5513.06	0	3
TR	Normal	461.33	4788.57	0	3

$$\min_{x_\upsilon,\pi_\upsilon} \sum_{\kappa\in\mathcal{K}} w_\kappa(f_\kappa(x,\pi) - VAL_\kappa)^2 \tag{3.29}$$

$$s.t. \quad \sum_{\upsilon\in\Upsilon} \pi_\upsilon = 1 \tag{3.30}$$

$$\pi_\upsilon \geq 0 \quad \upsilon \in \Upsilon \tag{3.31}$$

We assume continuous distribution of uncertain factors are independent from each other. Also, we assume that realization of uncertain factors at a certain point of time during time horizon is independent from previous outcomes. Hence we assume the outcomes in different periods are identical. The minimum number of outcomes can be achieved by formula (32) (Høyland and Wallace (2001)).

$$(D + 1)y - 1 \sim \text{the number of specifications} \tag{3.32}$$

Where D indicates the dimension of scenario in each node, and y represents the number of branches from each node (outcomes). In this paper, D is 115 (product of 23 random variables and 5 periods) and the number of specifications is 460 (product of 23 uncertain factors, 5 periods, and 4 moments). Hence, according to the formula (32) the value of y is 4. To obtain better results we chose 5 as the number of branches of each node in each period. The nonconvex nonlinear program (3.29)-(3.30) was solved by COUENNE solver in GAMS 23.5. Obtained solution with objective function value of zero or close to zero demonstrates that generated outcomes have statistical properties which perfectly match with specified properties of continuous distributions. A full scenario tree for 5 periods with size of 5^5 is created and shown in Figure 3.2.

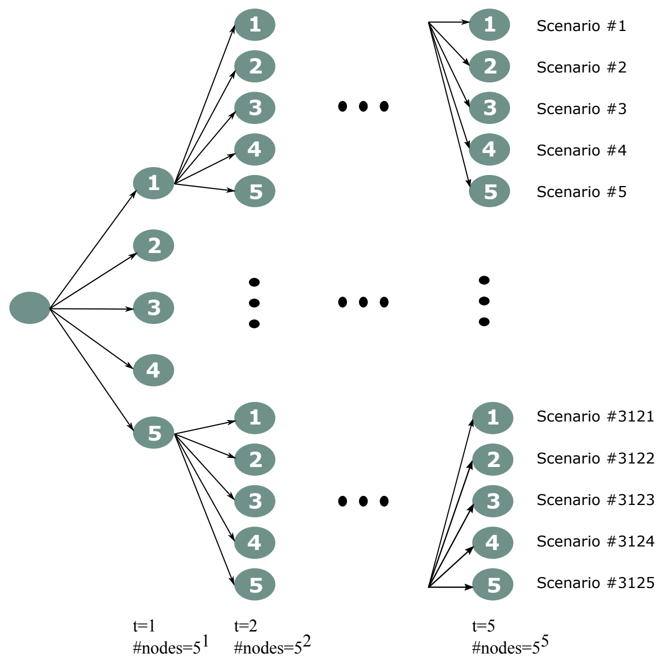

Figure 3.2: Full scenario tree

Since using different importance vectors, w, lead to different optimal solutions, we use different w's to see how uncertainty in return and demand values affects reverse logistics network. Table 3.8 shows 5 importance vectors used to obtain 5 outputs in each node. Vector w_1 considers equal importance to all moments. Vector w_2 gives greater importance to mean and variance. Vector w_3 considers greater weights to skewness and kurtosis. Vectors w_4 and w_5 assign the largest weight to mean and variance, respectively. The results of moment matching method using w_1 are listed in Table 3.9.

Table 3.8: Weight vectors used in moments matching method

Property	w_1	w_2	w_3	w_4	w_5
Mean	0.25	0.35	0.15	0.5	0.2
Variance	0.25	0.35	0.15	0.2	0.5
Skewness	0.25	0.15	0.35	0.15	0.15
Kurtosis	0.25	0.15	0.35	0.15	0.15

3.5.3 Scenario reduction

Increasing the number of uncertain factors and number of time periods leads to increasing the number of scenarios and decreasing the tractability of solution. In such a situation, a strategy is to select a subset of scenarios as representative of whole scenario set which is called scenario reduction. Heitsch and Römisch (2003) introduced forward and backward scenario reduction algorithms. The results of implementation of proposed algorithms show that for small size reduced scenario tree, the fast forward selection (FFS) algorithms is faster and more accurate. Thus, in this paper, the fast forward selection algorithm is employed to reduce the size of scenarios in full size scenario tree. The notations of this algorithm are listed below:

K Scenario set indexed by k and l

β_k Scenario k

γ_k Probability of occurring scenario k

$\eta(.)$ L_2-norm function

$\delta_{k,l}^{[i]}$ Distance value between scenarios k and l at iteration i

$z_l^{[i]}$ Weighted distance of scenario k at iteration i

$J^{[i]}$ Set of scenarios which are not selected up to iteration i

Ω Reduced scenario set

The basic steps of FFS are described as follows:

Step 1: Let $i = 1$, calculate the distance between two scenarios $\delta_{k,l}^{[1]} = \eta(\beta_k, \beta_l), k, l = 1, ..., K$.

Note: For two identical scenarios, the distance is zero.

For each scenario, calculate the total weighted distance $z_l^{[1]} = \sum_{k=1,k\neq u}^{N} \gamma_k \delta_{k,l}^{[1]}$.

Find the scenario l_1 which leads to the smallest $z_l^{[1]}$, $l_1 = \arg\min_{l\in\{1,...K\}} z_l^{[1]}$ and set $J^{[1]} = \{1, ..., K\} \setminus l_1$.

Table 3.9: Moment matching method's results (w_1)

Stochastic parameter	Node	Output				
		1	2	3	4	5
Return	UK	2974.35	1523.25	1992.63	2279.40	2506.73
	FR	2272.81	1594.71	2497.82	2497.82	3234.52
	BE	2590.59	1506.29	1809.50	2120.15	2173.31
	IT	1981.28	1311.83	1821.50	1950.79	2471.86
	NO	1587.29	1095.60	1450.69	1356.83	1276.55
Demand	UK	311.12	227.36	341.59	335.43	428.37
	ES	308.10	225.21	338.15	332.42	424.31
	IT	280.83	205.62	306.82	304.91	387.34
	FR	274.26	200.64	295.94	299.84	377.98
	SE	269.87	193.62	296.18	280.77	365.30
	IE	216.95	150.44	231.31	214.25	284.00
	NL	308.10	225.21	338.15	332.42	424.31
	GR	293.45	214.90	318.84	319.93	404.81
	DK	286.58	204.99	314.01	296.69	386.82
	FI	487.68	356.84	534.18	527.90	672.23
	PT	391.85	286.73	423.31	428.24	540.15
	BE	409.06	298.98	449.00	441.25	563.31
	CH	505.83	369.74	555.16	545.74	696.61
	NO	483.67	353.56	530.79	521.93	666.13
	AT	467.94	342.63	507.59	510.58	645.42
	DE	458.56	335.23	503.18	494.94	631.59
	PL	437.62	320.40	474.36	477.66	603.56
	TR	408.47	298.55	448.35	440.61	562.49
Probability		0.150	0.058	0.216	0.314	0.262

Step 2: Let $i = i + 1$, compute $\delta_{k,l}^{[i]} = \eta(\beta_k^{[i-1]}, \beta_l^{[i-1]}), k, l \in J^{[i-1]}$.

Calculate the total weighted distance $z_l^{[i]} = \sum_{k \in J^{[i-1]} \setminus l} \gamma_k \delta_{k,l}^{[i]}, l \in J^{[i-1]}$.

Find the scenario l_i which leads to the smallest $z_l^{[i]}$, $l_i = \arg\min_{l \in J^{[i-1]}} z_l^{[i]}$ and update $J^{[i]} = J^{[i-1]} \setminus l_i$.

Step 3: If the size of selected scenarios set is equal to the number of scenarios determined by user go to step 4 otherwise return to step 2.

Step 4: Find the set of the closest unselected scenarios to selected scenario l which is called $L(l)$. For this purpose, for each unselected scenario $l(k) = \arg\min_{l \in \Omega} \delta(\beta_k, \beta_l), k \in \{1, ..., K\} \setminus \Omega$ finds the closest scenario; the probability of occuring scenario l is updated by adding its probability before being selected and the probabilities of all unselected scenarios that are close to l, $q_l = p_l + \sum_{k \in L(l)} p_k$ (Hu and Hu (2016)).

We applied FFS for 15, 20, 50, 100, and 200 scenarios on each full scenario tree obtained by importance vectors of Table 3.8. The results of scenario reduction process to obtain 15 scenarios are listed in Table 3.10. This table shows the selected scenarios from whole scenario trees with 3125 scenarios and redistributed probability values. A reduced scenario tree with 15 scenarios from a full scenario tree (obtained from importance vector w_1) is shown in Figure 3.3.

Table 3.10: Scenario reduction results ($|\mathcal{S}| = 15$)

w_1		w_2		w_3		w_4		w_5	
Sc #	Pr	Sc #	Pr	Sc #	Pr	Sc #	Pr	Sc #	Pr
#1744	0.080	#1	0.102	#2225	0.056	#1563	0.109	#1561	0.050
#2324	0.087	#51	0.059	#2350	0.042	#1570	0.083	#1563	0.223
#2334	0.045	#124	0.061	#2450	0.106	#1604	0.087	#1565	0.080
#2342	0.042	#386	0.043	#2469	0.021	#1740	0.061	#1575	0.031
#2344	0.186	#459	0.044	#2869	0.058	#1820	0.056	#1598	0.091
#2345	0.109	#1094	0.046	#2988	0.095	#1844	0.098	#1738	0.083
#2374	0.057	#1719	0.071	#2993	0.062	#1865	0.040	#1740	0.037
#2469	0.083	#1878	0.066	#2994	0.039	#2223	0.092	#1823	0.036
#2470	0.028	#1964	0.052	#2999	0.061	#2225	0.058	#1868	0.041
#2474	0.047	#1975	0.071	#3089	0.050	#2489	0.061	#2438	0.091
#2969	0.084	#2126	0.061	#3090	0.076	#2869	0.054	#2440	0.042
#2970	0.038	#2279	0.044	#3095	0.061	#2875	0.021	#2818	0.082
#2994	0.028	#2344	0.113	#3118	0.084	#2948	0.061	#2824	0.038
#3094	0.028	#2495	0.066	#3124	0.078	#2969	0.072	#2840	0.039
#3120	0.058	#2969	0.101	#3125	0.111	#3021	0.047	#2863	0.036

Sc= Scenario, Pr= Probability

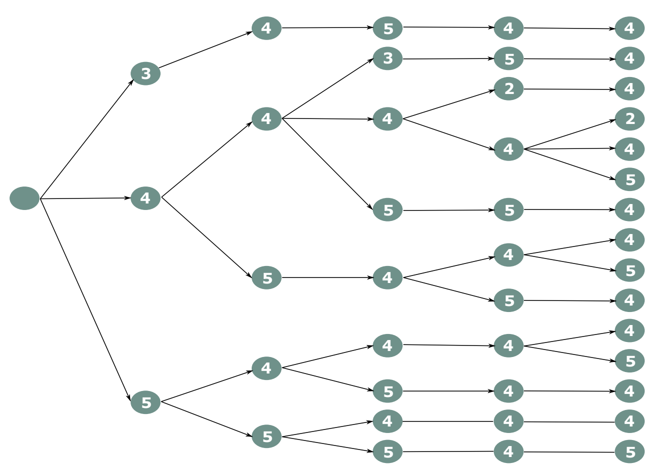

Figure 3.3: Reduced scenario tree (w_1, $|\mathcal{S}| = 15$)

3.5.4 Analysis for stochastic solution

Return and demand are considered as uncertain parameters in the case study. In this section, optimal decisions from deterministic model and stochastic model are compared. Deterministic model, or expected value problem, substitutes uncertain factors of stochastic model with their expected values which are assumed to be known and certain. Using scenario generation algorithm and scenario reduction method described in previous sections provides a smaller set of scenarios which can approximate underlying continuous distribution. However, deciding on the number of the selected scenarios is important to reach acceptable solution quality within a reasonable computational time. Hence, in this study the stochastic problem is solved using scenario sets of different sizes including 15, 20, 50, 100, and 200 obtained from full scenario tree and considering different importance vectors.

Table 3.11 reports the model complexity for both deterministic model and stochastic model. The complexity elements such as the number of variables, the number of constraints and CPU run-time are listed in this table. The deterministic model, as expected, has the least number of variables, constraints and CPU run-time. It can be seen in Table 3.11 that the larger number of scenarios make the problem less tractable.

Table 3.11: Complexity of deterministic and stochastic models

Model	Number of variables	Number of constraints	CPU run-time (s)
Deterministic	1296	491	0.98
RP (w_1, $\lvert\mathcal{S}\rvert = 15$)	12073	4215	30.61
RP (w_1, $\lvert\mathcal{S}\rvert = 20$)	15760	5489	33.12
RP (w_1, $\lvert\mathcal{S}\rvert = 50$)	34408	11957	160.48
RP (w_1, $\lvert\mathcal{S}\rvert = 100$)	65080	22541	669.04
RP (w_1, $\lvert\mathcal{S}\rvert = 200$)	117715	40671	2524.30

To evaluate the performance of stochatic programming model, we use following metrics (Birge and Louveaux (2011)).

A perfect information solution assumes that decision-maker would be able to perfectly predict future uncertainty. This solution method solves the problem for each realization of ξ and determines optimal first stage decisions. Wait-and-see value measures the expected value of solutions.

$$WS = \mathbb{E}_P\Big[\min_{x \in X} f(x;\xi)\Big] \tag{3.33}$$

The expected value of recourse problem is called here-and-now value and is calculated as follows:

$$RP = \min_{x \in X} \mathbb{E}_P f(x;\xi) \tag{3.34}$$

A common measurement to evaluate effect of uncertain parameters in the stochastic model is the expected value of perfect information which is defined as:

$$EVPI = RP - WS \tag{3.35}$$

The expected value problem replaces all random variables by their expected value.

$$EV = \min_{x \in X} f(x;\overline{\xi}) \tag{3.36}$$

Where $\overline{\xi} = \mathbb{E}(\xi)$ indicates the expectation of (ξ). If we denote the optimal solution of expected value problem by $\overline{x}(\overline{\xi})$ then the EEV is defined as:

$$EEV = \mathbb{E}_P f(\overline{x}(\overline{\xi});\xi) \tag{3.37}$$

The quantity, EEV, measures the performance of $\overline{x}(\overline{\xi})$, allowing second stage decision variables to be selected optimally as function of $\overline{x}(\overline{\xi})$ and ξ. The following definition is used to calculate the value of stochastic solution (VSS).

$$VSS = EEV - RP \tag{3.38}$$

VSS refers to the potential cost saving obtained by solving stochastic program instead of solving deterministic model.

Table 3.12: Summary of stochastic programming results

Weight vector	$\lvert\mathcal{S}\rvert$	EV	WS	RP	EEV	EVPI	VSS
w_1	15	194437.56	199682.05	201660.09	203463.67	1978.04	1803.58
	20	194437.56	200066.46	201739.92	204084.13	1673.46	2344.21
	50	194437.56	199933.51	201820.39	205848.39	1886.88	4028.00
	100	194437.56	200782.74	202338.56	209001.07	1555.82	6662.51
	200	194437.56	201326.54	203008.20	212729.05	1681.66	9720.85
w_2	15	196505.58	200428.70	203413.81	206220.69	2985.11	2806.88
	20	196505.58	200162.14	203181.66	206051.58	3019.52	2869.92
	50	196505.58	199978.30	203039.17	205985.19	3060.87	2946.02
	100	196505.58	200146.85	203283.15	206409.73	3136.30	3126.58
	200	196505.58	199993.86	203095.92	206207.89	3102.06	3111.97
w_3	15	184554.10	199064.12	202704.76	253928.04	3640.64	51223.28
	20	184554.10	199345.01	202968.51	255737.66	3623.50	52769.15
	50	184554.10	199489.12	203194.50	258705.41	3705.38	55510.91
	100	184554.10	199307.69	203009.78	259191.77	3702.09	56181.99
	200	184554.10	199154.35	202837.87	258619.83	3683.52	55781.96
w_4	15	185333.82	198640.81	202278.57	256315.44	3637.76	54036.87
	20	185333.82	198786.13	202384.94	256336.13	3598.81	53951.19
	50	185333.82	198883.69	202517.64	258725.07	3633.95	56207.43
	100	185333.82	198829.73	202520.82	258908.37	3691.09	56387.55
	200	185333.82	198913.79	202594.20	259504.86	3680.41	56910.66
w_5	15	193269.97	200190.40	203084.40	244471.89	2894.00	41387.49
	20	193269.97	200443.73	203495.94	248396.53	3052.21	44900.59
	50	193269.97	200310.75	203490.02	253271.99	3179.27	49781.97
	100	193269.97	200073.67	203337.41	253235.21	3263.74	49897.80
	200	193269.97	199884.01	203040.30	252913.72	3156.29	49873.42
Avg		190820.21	199752.73	202801.62	235770.53	3048.89	32968.91
Std		5016.11	689.54	532.69	24585.046	717.20	24482.17
Min		184554.10	198640.81	201660.09	203463.67	1555.82	1803.58
Max		196505.58	201326.54	203495.94	259504.86	3705.38	56910.66

The results of deterministic problem and stochastic program for different number of scenarios are reported in Table 3.12. Figure 3.4 shows EV, WS, RP, and EEV values for different importance vectors obtained by solving stochastic program with 200 scenarios. EVPI and VSS values are illustrated in Figure 3.5.

Table 3.13: Cost components (w_1)

	EV	EEV($\lvert\mathcal{S}\rvert$ =200)	RP($\lvert\mathcal{S}\rvert$ =200)
Total	194437.56	212729.05	203008.20
Location	91900.00 (47.26 %)	91900.00 (43.20%)	111900.00 (55.12%)
Transportation	99787.16 (51.32%)	103819.04 (48.80%)	85019.90 (41.88%)
Inventory	2417.83 (1.25%)	3012.84 (1.42%)	3675.43 (1.81%)
Backorder	332.57 (0.17%)	2336.01 (1.10%)	2327.45 (1.15%)
Shortage	0	134.16 (0.06%)	85.40 (0.04%)
Outsourcing	0	11526.98 (5.42%)	0

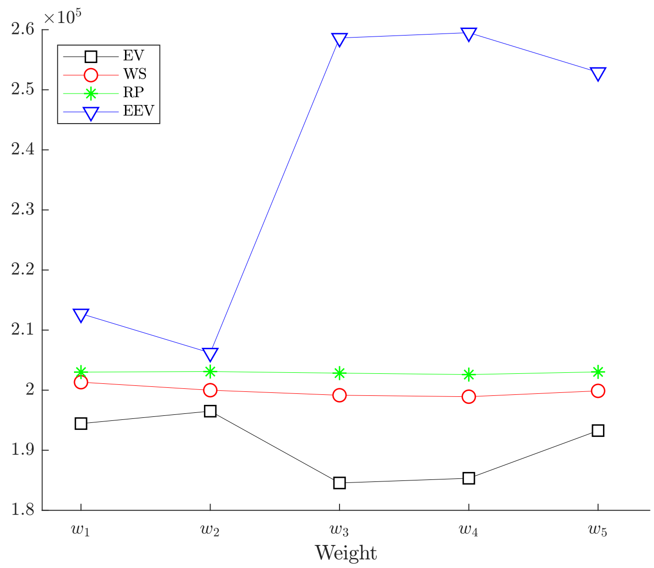

Figure 3.4: Metrics' value ($|\mathcal{S}| = 200$)

As shown in Table 3.12 and Figure 3.5, applying deterministic (EV problem) decisions to stochastic scenarios (EEV problem) results in larger objective function value which means the total cost increases. Considering this point and VSS values in Table 3.12 and Figure 3.5, stochastic program's solution outperforms the EEV solution. Also the EVPI values demonstrates the potential worth of achieving more precise predictions.

By evaluating EV values in Table 3.12 for different importance vector, it can be seen that importance vectors with larger weights for skewness, kurtosis and variance leads to larger objective function value. The reason lies in the fact that larger weights for these properties scatters the selected scenarios in wider interval around mean value in continuous distributions and leads to larger expected value for uncertain variables in selected scenarios which results in larger objective function value for EV problem. Employing w_3 and w_4 leads to smaller objective function values

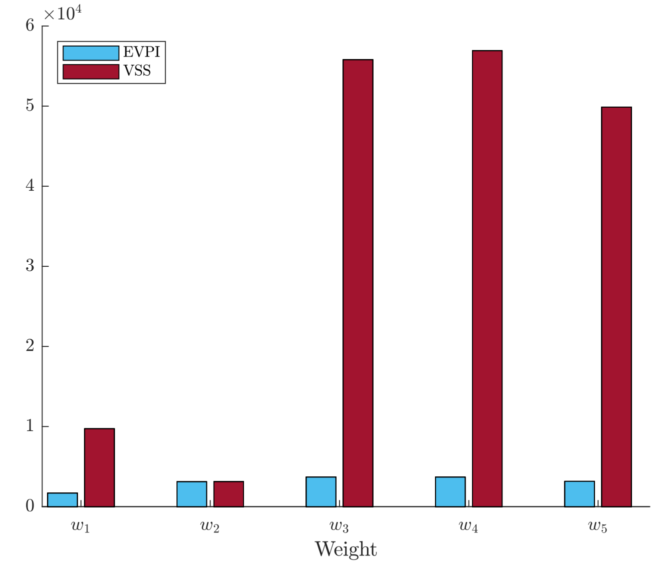

Figure 3.5: EVPI vs. VSS ($|\mathcal{S}| = 200$)

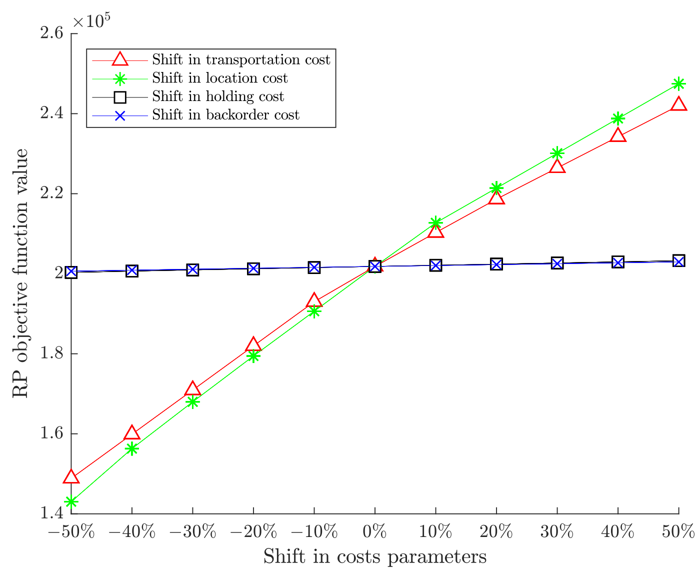

Figure 3.6: Sensitivity analysis ($|\mathcal{S}| = 50$)

compared to other importance vectors with larger wight for variance property. Large standard deviation for EV problem proves discussed analysis. However, using different importance vectors affect WS, RP, and EEV less than EV, because they consider all realizations of uncertain variables in spite of EV problem which replaces uncertain variables with their expected value.

The results of solving deterministic and stochastic models including all components of objective function are shown in Table 3.13. As can be seen, applying optimal decision of EV problem to EEV problem causes an increase in objective function value. Backorder and shortage cost increase because of stochastic nature of EEV problem. Capacity exceeding happens because the number of established facilities with the EV problems are not sufficient to meet the capacity requirements of EEV problem. Results of sensitivity analysis on cost parameters for stochastic programming model are shown in Figure 3.6. As can be seen in this figure, facilities location investing cost and transportation cost are the most critical cost parameters which affect total cost of the system significantly. Since backorder and inventory costs are small portions of RP's total cost, any shift in their cost parameters do not affect the overall cost remarkably.

In terms of managerial insights, the major findings from the case study are given as follows:

- Using stochastic programming approach may lead to cost saving up to 28%.
- Stochastic programming approach eliminates outsourcing costs caused by lack of capacity in facilities. Opening more facilities by stochastic programming model helps to cope uncertainties.
- Based on sensitivity analysis results, investment on facilities location and transportation cost are the two critical cost elements of stochastic programming model.

3.6 Conclusions

This study addresses a two-stage stochastic programming model to consider the return and demand uncertainty in a multi-echelon multi-period reverse logistics network. The first stage decision variables include facility location variables and the second stage decision variables consist of material flow variables, backorder variables, shortage variables and outsourcing variables. The results obtained by solving stochastic program demonstrates the importance of incorporating uncertainty in problem formulation.

A case study for the European consumer goods company with uncertain return and demand is performed. The two-stage stochastic programming approach is employed to study the problem. Scenario generation method and scenario reduction algorithm have been applied to generate a set of scenarios to approximate underlying continuous distributions for return and demand. The numerical results and solution evaluation provide the optimal network structure for reverse logistics system and indicate the usefulness of stochastic programming approaches in an uncertain environment. The results show that the main source of cost enhancement in deterministic model is outsourcing while recourse problem avoids from this cost component by establishing more facilities.

This research is subject to a number of limitations which suggest future research directions. Firstly, the described model assumes that demand and return are independent. However, sometimes in supply chain systems the demand and return in different time periods are dependent to each other. One future research direction is taking account of this assumption and analyzing the correlation between them. Secondly, we considered return and demand as two sources of uncertainty while supply chain systems may include many other uncertain factors such as quality, travel time and facility capacity. Hence, the other future research direction could be considering more uncertain parameters in modeling the problem as a multi-stage stochastic program to represent the system's stochasticity.

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CHAPTER 4. A multi-stage stochastic programming model for the multi-echelon multi-period reverse logistics problem

A paper submitted to *Resources Conservation and Recycling*

4.1 Abstract

Reverse logistics planning plays a crucial role in supply chain management. This paper proposes a multi-stage, multi-period reverse logistics with lot sizing decisions under uncertainties. The main uncertain factors are return and demand quantities, and return quality. Moment matching method was adopted to generate a discrete set of scenarios to represent the original continuous distribution of stochastic parameters. Fast forward selection algorithm was employed to select the most representative scenarios and facilitate computational tractability. A case study was conducted and optimal solution of the recursive problem obtained by solving extensive form. Sensitivity analysis was implemented on different elements of stochastic solution.

4.2 Introduction

Reverse logistics problem is one of the most challenging problems in supply chain management (SCM) which aims to address collecting used, refurbished, or defective products from customers or primary markets and then carrying out some recovery and disposal activities (Govindan et al., 2017). According to American Reverse Logistics Executive Council, reverse logistics is defined as "The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the points of consumption to the point of origin for the purpose of recapturing value or proper disposal". Another report shows 4.7 million tonnes E-waste were generated across world annually (Balde et al. (2015)). The amount of waste generated across the world increases the importance of reverse logistics systems in decreasing waste rate and return the leftover(s) to supply chain.

Reverse logistics has been gaining popularity in recent years. A great portion of reverse logistics literature has been devoted to deterministic reverse logistics problems. John et al. (2018) formulated a multi-stage reverse logistics network for product recovery as a mixed integer linear programming model. The authors validated the model with a used refrigerator recovery network. Min et al. (2006) developed a nonlinear mixed-integer linear programming model for a reverse logistics network which makes decision on the number and locations of centralized return centers. They proposed a genetic algorithm to solve the formulated model. Lee et al. (2009) proposed a mathematical model for a multi-stage, multi-product reverse logistics network. The authors developed a hybrid heuristic based on genetic algorithm for solving the introduced model. Silva et al. (2013) studied a reverse logistics network for a company located in Brazil. They developed a returnable packaging model which decreases material consumption by 18% compared to disposable packaging model. In addition, they concluded that returnable packaging model is the best alternate in terms of environmental concerns since it has less environmental impacts compared to disposable packaging models. Demirel et al. (2016) addressed a reverse logistics network for end-of-live vehicles in Turkey. The authors proposed a mixed-integer linear programming model for the network. Solving the model led to the optimal number of facilities to be located. Alshamsi and Diabat (2017) formulated a reverse logistics network for a case of household appliance in the Gulf Cooperation Council (GCC) region with 68 cities. The authors developed a genetic algorithm with running time reduction up to 38 times compared to GMAS in solving the problem. Ghezavati and Beigi (2016) studied a multi-echelon capcitated reverse logistics network with location-routing, and time window constraints. The authors formulated the problem as a bi-objective mathematical programming model and proposed a non-dominated sorting genetic algorithm II (NSGA II) to obtain Pareto frontier solutions.

Compared to traditional forward logistics, more activities are involved in reverse logistics planning which makes it more challenging. One of the challenges in designing reverse logistics network is the presence of several uncertain factors such as return and demand quantities, and return quality. Therefore considering uncertainty and designing a robust decision making framework are crucial for reverse logistics design. Lieckens and Vandaele (2007) combined queueing models with traditional reverse logistics models to incorporate lead time, inventory positions in an uncertain environment.

This combination led to a mixed integer nonlinear programming model. The authors solved the formulated model by a genetic algorithm with the technique of differential evolution. While most of the studied models in reverse logistics are case based, Salema et al. (2007) proposed a generalized model considering capacity limits, multi-product management, and uncertainty in demand and return quantities. They solved the formulated model by standard branch and bound techniques. Soleimani and Govindan (2014) studied a risk-averse, two-stage stochastic programming approach reverse logistic network design problem. They considered return quantity and price as two sources of uncertainty. Niknejad and Petrovic (2014) focused on a reverse logistics problem with decisions on inventory control and production planning. They considered return and demand quantities as two stochastic parameters and modelled them using fuzzy trapezoidal numbers. The authors developed a two phase fuzzy mixed integer optimization algorithm to solve the formulated model. Ayvaz et al. (2015) studied a reverse logistics network with three uncertainty sources: return quantity, return quality, and transportation cost. They formulated the problem as a two-stage stochastic programming model and validated it by a real world case for waste of electrical and electronic equipment recycling center in Turkey. The authors used sample average approximation method to solve the model. Lee and Dong (2009) proposed a dynamic location and allocation model for reverse logistic problem and formulated it as a two stage stochastic programming model. Also, they developed a heuristic solution method based on sampling to solve the model. Azizi et al. (2020) studied a multi-period reverse logistics network under return and demand uncertainty with lot sizing and formulated it as a two-stage stochastic programming model. They used scenario generation and scenario reduction methods to generate sets of discrete scenarios to approximate underlying probability distributions. The authors used a case of consumer company in Europe to validate the proposed model.

By reviewing body literature of the reverse logistics in deterministic and stochastic environments and review papers (Govindan et al. (2017), Prajapati et al. (2019), and Rachih et al. (2019)) the following gaps are recognized: Firstly, little attention has been paid to multi-echelon, multi-period stochastic reverse logistics with lot sizing. Secondly, multi-stage stochastic programming models for reverse logistics problem has not been investigated. Thirdly, to the best of our knowledge, the solution techniques introduced to solve stochastic reverse logistics problems are not efficient to solve large-scale instances which include large number of scenarios, stages, and decision variables.

This study proposes a multi-stage stochastic program for multi-echelon, multi period reverse logistics program with lot sizing. Scenario generation and scenario reduction methods were employed to generate a representative set of discrete scenarios for underlying distribution of stochastic parameters. Extensive form of problem was used to solve the problem and stochastic solution was evaluated by implementing sensitivity analysis on recursive problem's parameters.

4.3 Problem Statement

In the reverse logistics network considered in this research, returned products flow from primary markets as upstream level to sorting centers. After screening products at sorting centers, the products are sorted to three groups. The products with good quality are transported to warehouses to meet secondary markets' demand. The products with lower quality level that are recyclable will be transported to recycling centers. The rest of the returned products will be transported to disposal centers. During these processes, return and demand quantities, and quality level are the main sources of uncertainty. In this study, we formulate and solve multi-echelon, multi-period reverse logistics problem as a multi-stage stochastic programming model. In fact, this study provides a framework for decision makers to make the optimal decisions on (1) locating facilities such as sorting centers, recycling centers, and disposal centers; (2) the amount of products should be transported between different facilities and from facilities to secondary markets as final customers; (3) inventory, outsourcing, backorder, and shortage levels. Structure of network is illustrated in Figure 4.1.

4.3.1 Model Formulation

This section introduces the proposed multi-stage stochastic programming model.

Assumptions are listed as follows:

- Inventory in sorting centers, recycling centers and disposal centers are not allowed.
- Initial inventory is not allowed in warehouses.
- End of each period is set to measure inventory level of warehouses.

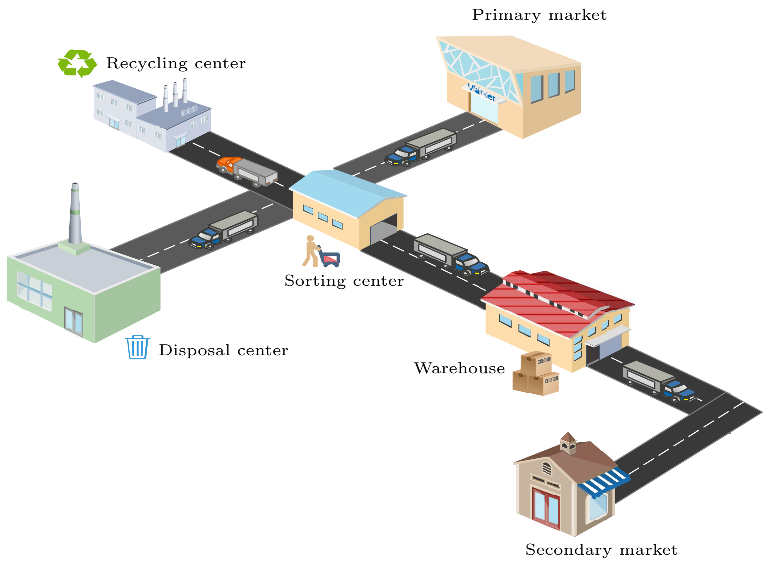

Figure 4.1: Network structure

- Fulfilling of secondary markets' demand can be delayed or ignored since backorders and shortages are allowed.

- Transportation between the same kind of facilities are not allowed (e.g. transportation between warehouses is prohibited).

The notations of the model formulation are as following.

4.3.1.1 Sets

$\mathcal{PM}$ Primary markets

$\mathcal{SC}$ Candidate locations for sorting centers

$\mathcal{W}$ Candidate locations for warehouses

$\mathcal{SM}$ Secondary markets

$\mathcal{WS}$ Union of warehouses and secondary markets, $\mathcal{WS} = \mathcal{W} \cup \mathcal{SM}$

$\mathcal{R}$ Candidate recycling centers

$\mathcal{D}$ Candidate disposal centers

$\mathcal{T}$ Time periods

$\mathcal{S}$ Set of scenarios

4.3.1.2 Parameters

p_s	Probability of scenario s
es_c	Cost of establishing a sorting center in location c
ew_w	Cost of establishing a warehouse in location i
ez_r	Cost of establishing a recycling center in location r
ey_d	Cost of establishing a disposal center in location r
q_m^{ts}	Return quantity of primary market m in period t under scenario s
tps_{mc}	Cost of transportation for one unit of product from primary market m to sorting center c
tsw_{cw}	Cost of transportation for one unit of product from sorting center c to warehouse w
tsr_{cr}	Cost of transportation for one unit of product from sorting center c to recycling center r
tsd_{cd}	Cost of transportation for one unit of product from sorting center c to disposal center d
tws_{ij}	Cost of transportation for one unit of product from node $i \in \mathcal{WS}$ to node $j \in \mathcal{WS}$
h_w^t	Holding cost for one unit of product in warehouse w in period t
b_i^t	Cost of backorder for one unit of product for secondary market i in period t
sc_i	Cost of shortage for one unit of unmet demand of secondary market i
OC	Cost of outsourcing for one unit of product
d_i^{ts}	Demand of secondary market i in period t under scenario s
dr^{ts}	Ratio of disposal in period t under scenario s
rr^{ts}	Ratio of recycling in period t under scenario s
CS_c	Sorting center c's capacity
CW_w	Warehouse w's capacity
CR_r	Recycling center r's capacity
CD_d	Disposal center d's capacity

4.3.1.3 Decision variables

l_c	1 if sorting center c established, 0 otherwise
g_w	1 if warehouse w is established, 0 otherwise
y_d	1 if disposal center d is established, 0 otherwise
z_r	1 if recycling center r is established, 0 otherwise

α_{mc}^{ts} Amount of products transported from primary market m to sorting center under scenario s in period t

β_{cw}^{ts} Amount of products transported from sorting center c to warehouse w under scenario s in period t

θ_{cr}^{ts} Amount of products transported from sorting center c to recycling center r under scenario s in period t

λ_{cd}^{ts} Amount of products transported from sorting center c to disposal center d under scenario s in period t

μ_{wi}^{ts} Amount of products transported from warehouse w to secondary market i under scenario s in period t

I_{w}^{ts} Inventory level of products in warehouse w under scenario s in period t

B_{i}^{ts} Backordered demand for secondary market i under scenario s in period t

so_{mc}^{ts} Outsourced products of shipment from primary market m to sorting center c (because of capacity exceeding in sorting center c) under scenario s in period t

ro_{cr}^{ts} Outsourced products of shipment from sorting center c to recycling center r (because of capacity exceeding in recycling center r) under scenario s in period t

do_{cd}^{ts} Outsourced products of shipment from sorting center c to disposal center d (because of capacity exceeding in disposal center d) under scenario s in period t

wo_{cw}^{ts} Outsourced products of shipment from sorting center c to warehouse w (because of capacity exceeding in warehouse w) under scenario s in period t

$d_{i}^{[t]s}$ Cumulative total demand of secondary market i over t periods under scenario s, $\left(d_{i}^{[t]s} = \sum_{t'=1}^{t'=t} d_{i}^{t's}\right)$

$\mu_{wi}^{[t]s}$ Cumulative total shipment transported from warehouse w to secondary market i over t periods under scenario s, $\left(\mu_{wi}^{[t]s} = \sum_{t'=1}^{t'=t} \mu_{wi}^{t's}\right)$

$B_{i}^{[t]s}$ Cumulative total backorder for secondary market i over t periods under scenario s, $\left(B_{i}^{[t]s} = \sum_{t'=1}^{t'=t} B_{i}^{t's}\right)$

$\beta_{cw}^{[t]s}$ Cumulative total shipment transported from sorting center c to warehouse w over t periods under scenario s, $\left(\beta_{cw}^{[t]s} = \sum_{t'=1}^{t'=t} \beta_{cw}^{t's}\right)$

4.3.1.4 Objective function

The objective function minimizes the total expected costs of network including establishment costs ($\mathcal{Z}_1$), transportation costs ($\mathcal{Z}_2$), inventory costs ($\mathcal{Z}_3$), backorder costs ($\mathcal{Z}_4$), shortage costs ($\mathcal{Z}_5$), and outsourcing costs ($\mathcal{Z}_6$) over the planning horizon. Equations (4.1)-(4.7) present the objective function and its elements:

$$\text{Min } \mathcal{F} = \mathcal{Z}_1 + \mathcal{Z}_2 + \mathcal{Z}_3 + \mathcal{Z}_4 + \mathcal{Z}_5 + \mathcal{Z}_6 \tag{4.1}$$

$$\mathcal{Z}_1 = \sum_{c\in\mathcal{SC}} es_c l_c + \sum_{w\in\mathcal{W}} ew_w g_w + \sum_{r\in\mathcal{R}} er_r z_r + \sum_{d\in\mathcal{D}} ed_d y_d \tag{4.2}$$

$$\mathcal{Z}_2 = \sum_{s\in\mathcal{S}} p_s \Bigg(\sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{PM}} \sum_{c\in\mathcal{SC}} tps_{mc}\alpha_{mc}^{ts} + \sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{SC}} \sum_{w\in\mathcal{W}} tsw_{cw}\beta_{cw}^{ts} + \sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{SC}} \sum_{r\in\mathcal{R}} tsr_{cr}\theta_{cr}^{ts} + \tag{4.3}$$

$$\sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{SC}} \sum_{d\in\mathcal{D}} tsd_{cd}\lambda_{cd}^{ts} + \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{W}} \sum_{j\in\mathcal{W}} tws_{ij}\mu_{ij}^{ts} \Bigg)$$

$$\mathcal{Z}_3 = \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \sum_{w\in\mathcal{W}} p_s h_w^t I_w^{ts} \tag{4.4}$$

$$\mathcal{Z}_4 = \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}\setminus T} \sum_{i\in\mathcal{SM}} p_s b_i^t B_i^{ts} \tag{4.5}$$

$$\mathcal{Z}_5 = \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{SM}} p_s sc_i^p B_i^{Ts} \tag{4.6}$$

$$\mathcal{Z}_6 = \sum_{s\in\mathcal{S}} p_s \Bigg(OC \Bigg(\sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{SC}} so_{mc}^{ts} + \sum_{t\in\mathcal{T}} \sum_{w\in\mathcal{W}} wo_{cw}^{ts} + \sum_{t\in\mathcal{T}} \sum_{r\in\mathcal{R}} ro_{cr}^{ts} + \sum_{t\in\mathcal{T}} \sum_{d\in\mathcal{D}} do_{cd}^{ts} \Bigg) \Bigg) \tag{4.7}$$

In equation (4.2), the four terms include all first stage decision variables and represent location cost for sorting centers, warehouses, recycling centers and disposal centers, respectively. equation (4.3), with five terms, calculates transportation costs of primary markets to sorting center, sorting centers to warehouses, sorting centers to recycling centers, sorting centers to disposal centers, and warehouses to secondary markets, respectively. Equation (4.7) includes outsourcing costs for sorting centers, warehouses, recycling centers and disposal centers.

4.3.1.5 Constraints

$$\sum_{m\in\mathcal{PM}} \alpha_{mc}^{ts} \leq CS_c l_c \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.8}$$

$$\sum_{c\in\mathcal{SC}} \theta_{cr}^{ts} \leq CR_r z_r \quad r \in \mathcal{R}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.9}$$

$$\sum_{c\in\mathcal{SC}} \lambda_{cd}^{ts} \leq CD_d y_d \quad d \in \mathcal{D}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.10}$$

$$\sum_{c\in\mathcal{SC}} \beta_{cw}^{ts} \leq CW_w g_w \quad w \in \mathcal{W}, t = 1, s \in \mathcal{S} \tag{4.11}$$

$$\sum_{c\in\mathcal{SC}} \beta_{cw}^{ts} = CW_w g_w - I_w^{(t-1)s} \quad w \in \mathcal{W}, t \in \mathcal{T} \setminus 1, s \in \mathcal{S} \tag{4.12}$$

Constraints (4.8)-(4.12) are related to capacities of facilities and transportation amount between the facilities. These constraints prohibit product flows between facilities that are not established. Meanwhile, capacities for the facilities can not be exceeded.

$$\sum_{c\in\mathcal{SC}} (\alpha_{mc}^{ts} + so_{mc}^{ts}) = q_m^{ts} \quad m \in \mathcal{PM}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.13}$$

Constraints (4.13) state that the amount of return products from primary markets include transported products to sorting centers and the amount of returned products which exceed sorting centers' capacity.

$$\sum_{m\in\mathcal{PM}} rr^t \alpha_{mc}^{ts} = \sum_{r\in\mathcal{R}} (\theta_{cr}^{ts} + ro_{cr}^{ts}) \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.14}$$

$$\sum_{m\in\mathcal{PM}} dr^t \alpha_{mc}^{ts} = \sum_{d\in\mathcal{D}} (\lambda_{cd}^{ts} + do_{cd}^{ts}) \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.15}$$

$$\sum_{m\in\mathcal{PM}} (1 - rr^t - dr^t) \alpha_{mc}^{ts} = \sum_{w\in\mathcal{W}} (\beta_{cw}^{ts} + wo_{cw}^{ts}) \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.16}$$

Constraints (4.14), (4.15) and (4.16) calculate the transported amount of products from sorting centers to recycling centers, disposal centers, and warehouses, respectively. At the same time, these constraints calculate the amount of products exceeding the capacities of these facilities.

$$\sum_{m\in\mathcal{PM}} so_{mc}^{ts} \leq (\sum_{m\in\mathcal{PM}} q_m^{ts}) l_c \quad c \in \mathcal{SC}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.17}$$

$$\sum_{c\in\mathcal{SC}} ro_{cr}^{ts} \leq (\sum_{m\in\mathcal{PM}} q_m^{ts}) z_r \quad r \in \mathcal{R}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.18}$$

$$\sum_{c\in\mathcal{SC}} do_{cd}^{ts} \leq (\sum_{m\in\mathcal{PM}} q_m^{ts}) y_d \quad d \in \mathcal{D}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.19}$$

$$\sum_{c\in\mathcal{SC}} wo_{cw}^{ts} \leq (\sum_{m\in\mathcal{PM}} q_m^{ts}) g_w \quad w \in \mathcal{W}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.20}$$

Constraints (4.17)-(4.20) state outsourcing is not allowed from facilities that are not established.

$$I_w^{ts} \leq \beta_{cw}^{[t]s} \quad w \in \mathcal{W}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.21}$$

$$I_w^{ts} = \sum_{c\in\mathcal{SC}} \beta_{cw}^{ts} - \sum_{i\in\mathcal{SM}} \mu_{wi}^{ts} \quad w \in \mathcal{W}, t = 1, s \in \mathcal{S} \tag{4.22}$$

$$I_w^{ts} = I_w^{(t-1)s} + \sum_{c\in\mathcal{SC}} \beta_{cw}^{ts} - \sum_{i\in\mathcal{SM}} \mu_{wi}^{ts} \quad w \in \mathcal{W}, t \in \mathcal{T} \setminus 1, s \in \mathcal{S} \tag{4.23}$$

Constraints (4.21)-(4.23) determine the inventory level at each warehouse.

$$\sum_{i\in\mathcal{SM}} \mu_{wi}^{ts} \leq \sum_{c\in\mathcal{SC}} \beta_{cw}^{ts} \quad w \in \mathcal{W}, t = 1, s \in \mathcal{S} \tag{4.24}$$

$$\sum_{i\in\mathcal{SM}} \mu_{wi}^{ts} \leq I_w^{(t-1)s} + \sum_{c\in\mathcal{SC}} \beta_{cw}^{ts} \quad w \in \mathcal{W}, t \in \mathcal{T} \setminus 1, s \in \mathcal{S} \tag{4.25}$$

$$\sum_{w\in\mathcal{W}} \mu_{wi}^{ts} \leq d_i^{ts} + B_i^{[t]s} \quad i \in \mathcal{SM}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.26}$$

Constraints (4.24)-(4.26) determine the amount of product transported to each secondary market.

$$\sum_{w\in\mathcal{W}} \mu_{wi}^{ts} = 0 \quad i \in \mathcal{W}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.27}$$

Constraints (4.27) state transportation flows between warehouses are not allowed.

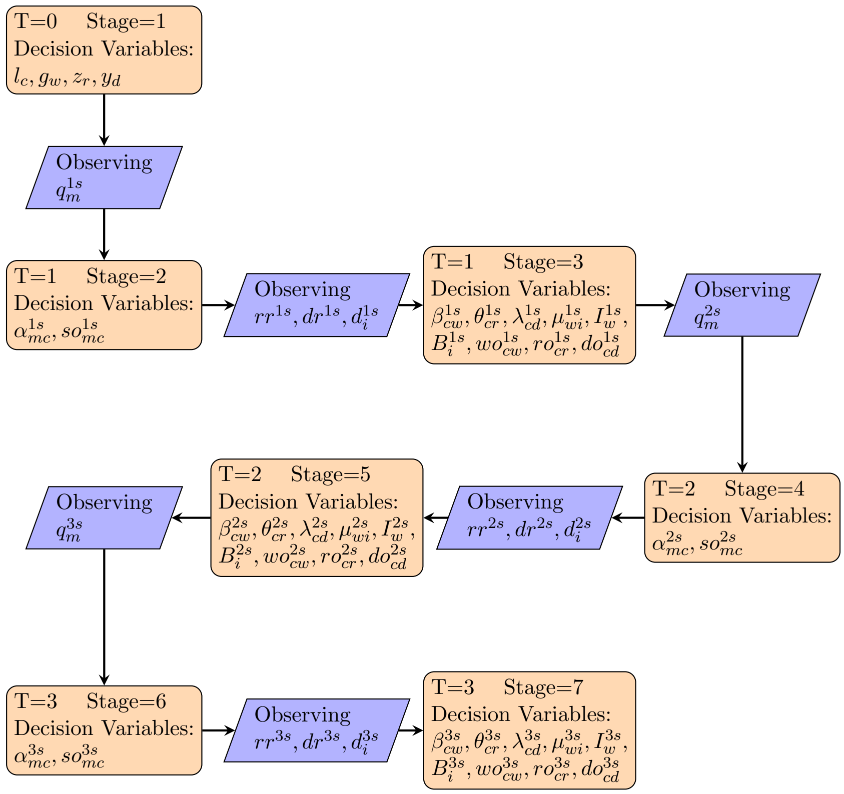

Figure 4.2: Decision variables in different stages

$$B_i^{ts} = d_i^{[t]s} - \sum_{w \in \mathcal{W}} \mu_{wi}^{[t]s} \quad i \in \mathcal{SM}, t \in \mathcal{T}, s \in \mathcal{S} \tag{4.28}$$

Constraints (4.28) calculate backorder level for each secondary market.

Figure 4.2 shows the different decision making stages and their associated decision variables in the problem.

4.4 Case study

To validate the proposed multi-stage stochastic programming model, a case study adapted from Kalaitzidou et al. (2015) was applied. Kalaitzidou et al. (2015) developed a deterministic mixed integer linear program for the case of a European consumer goods company. This study extended their work by proposing multi-echelon, multi-period, and multi-stage stochastic program for reverse logistics network.

The network logistics of this case consists of 38 nodes distributed in different European countries including five primary markets, six potential candidates for sorting centers, three potential candidates for warehouses, three potential candidates for recycling centers, three potential candidates for disposal centers, and eighteen secondary market nodes.

Table 4.1 reports the facilities capacities and establishment costs. Table 4.2 lists the unit holding cost, unit backorder cost, and unit shortage cost. Quantities of returned products from the primary markets and demand of secondary markets are assumed to follow normal distributions (Abdallah et al. (2012)). Four moments of return and demand quantities distributions are reported in Tables 4.3 and 4.4, respectively. Return quantities in different time periods are assumed independent from each other. Demand quantities are also independent from each other in different time periods. The rates of recyclable and disposable products are the other stochastic parameters with 5 possible outputs listed in Table 4.5. The planning horizon for this research problem is considered to be 3 months and outsourcing cost per unit of product for all facilities is assumed to be 30 rmu.

Table 4.1: Establishing cost (EC) and Capacity (Cap)

	Sorting Center		Warehouse		Recycling center		Disposal center	
Node	Cap	EC	Cap	EC	Cap	EC	Cap	EC
UK	8000	40000						
FR	10000	32500	9500	25000	1500	20000	1500	6000
SE	7000	22500						
ES	9500	20000						
AT	6000	15000						
BE	7500	25000						
NE			6000	12000				
IT			8000	15000	1500	11500	2200	5400
DE					2000	15000	2500	6500

The most effective method to solve small to medium size stochastic programs is to generate the deterministic equivalent of the problem so-called extensive form. Extensive form specifies all scenarios in one single mathematical model.

Section 3 provided extensive form mathematical model of the introduced problem. To solve the problem in extensive form, discrete scenarios are generated by moment matching method and then the total number of scenarios is reduced by fast forward selection algorithm. Scenario generation and scenario reduction were implemented in GAMS 23.5 to create the most representative scenarios for the stochastic problem. The scenarios data files for the problem was generated by Matlab R2020b

Table 4.2: Costs of holding, backorder and shortage per unit of product

Secondary market	Holding cost					Backorder cost					Shortage cost
	#Period					#Period					
	1	2	3	4	5	1	2	3	4	5	
IT	1.28	1.31	1.32	1.28	1.30	0.46	2.52	1.36	1.16	-	7.46
UK						3.54	1.28	0.22	0.26	-	6.89
FR	0.99	1.03	1.03	0.99	1.00	2.00	2.68	2.40	0.54	-	8.98
ES						0.60	2.94	2.5	2.90	-	4.53
IE						0.76	2.36	1.52	1.40	-	6.12
SE						1.76	3.30	0.54	0.76	-	5.07
GR						0.52	2.22	3.18	3.42	-	7.85
NL	1.13	1.16	1.15	1.13	1.14	2.56	3.36	3.06	3.06	-	7.91
FI						1.04	0.16	0.76	3.58	-	6.24
DK						1.78	3.80	3.20	1.82	-	4.28
CH						3.38	3.96	3.04	3.88	-	9.28
BE						1.04	2.48	2.66	0.46	-	8.96
PT						3.96	2.46	3.30	1.74	-	9.77
NO						0.68	1.94	3.52	1.32	-	6.58
DE						3.64	3.86	1.70	3.54	-	7.20
AT						0.58	3.50	1.88	2.52	-	8.88
TR						0.54	2.76	0.9	0.12	-	5.23
PL						2.84	1.08	2.24	3.52	-	4.42

Table 4.3: Return properties

Primary market	PDF	Properties			
		Mean	Variance	Skewness	Kortusis
UK	Normal	2338.27	132986.03	0	3
FR	Normal	2605.81	184908.17	0	3
BE	Normal	2102.58	76003.87	0	3
IT	Normal	2027.70	92385.29	0	3
NO	Normal	1375.71	15104.41	0	3

and stochastic program was coded in Python 3.7 to solve and find the optimal solution. Optimal solution for a relatively small-scale instance with 5 scenario is listed in Tables 4.6 and 4.7. Figure 4.3 shows the objective function value of extensive form for cases with 10 to 500 scenarios. As it can be seen from this figure, the total cost is gradually converging by increasing number of scenarios from 50 up to 500. For analyzing experimental results the case with 300 scenarios is considered. Figure 4.4 shows the run time of the cases with different number of scenarios. As expected, the run time increases by increasing number of scenarios.

To evaluate the quality of stochastic solution, the model is solved for EV (Expected Value), EEV (Expected problem of Expected Value solution) and RP (Recourse Problem) are solved. EV problem assigns fixed values to stochastic parameters. The fixed value is the mean of distribution for each stochastic parameter. In other words, EV problem ignores stochasticity but stochasticity

Table 4.4: Demand properties

Secondary market	PDF	Properties			
		Mean	Variance	Skewness	Kortusis
UK	Normal	351.33	2777.24	0	3
ES	Normal	348.00	2724.84	0	3
IT	Normal	317.67	2270.57	0	3
FR	Normal	310.00	2162.25	0	3
SE	Normal	299.67	2020.54	0	3
IE	Normal	233.00	1221.50	0	3
NL	Normal	348.00	2724.84	0	3
GR	Normal	332.00	2480.04	0	3
DK	Normal	317.33	2265.71	0	3
FI	Normal	551.33	6839.20	0	3
PT	Normal	443.00	4415.60	0	3
BE	Normal	462.00	4802.49	0	3
CH	Normal	571.33	7344.41	0	3
NO	Normal	546.33	6715.72	0	3
AT	Normal	529.33	6304.28	0	3
DE	Normal	518.00	6037.29	0	3
PL	Normal	495.00	5513.06	0	3
TR	Normal	461.33	4788.57	0	3

Table 4.5: Rate of recycling and disposal

Possible output	$rr(t)$	$dr(t)$
1	0.02	0.02
2	0.05	0.05
3	0.10	0.08
4	0.15	0.10
5	0.20	0.13

Table 4.6: First stage decision variables values

Node	Sorting Center		Warehouse		Recycling center		Disposal center	
	l	EC	g	EC	z	EC	y	EC
UK	0	40000						
FR	0	32500	0	25000	0	20000	0	6000
SE	0	22500						
ES	0	20000						
AT	1	15000						
BE	1	25000						
NE			0	12000				
IT			1	15000	1	11500	1	5400
DE					0	15000	0	6500

Table 4.7: Extensive form results ($|\mathcal{S}| = 5$)

Cost	Scenarios				
	#1	#2	#3	#4	#5
Stage 1	71900.00	71900.00	71900.00	71900.00	71900.00
Stage 2	6129.75	6129.75	6129.75	6129.75	7417.03
Stage 3	71256.69	71256.69	71256.69	71256.69	66741.04
Stage 4	6129.75	6129.75	6129.75	7417.03	4115.00
Stage 5	71256.69	71256.69	71256.69	66741.04	90576.36
Stage 6	6129.755	5205.69	7417.03	6129.75	6129.75
Stage 7	71256.69	93332.62	66741.04	71256.69	71256.69
Total cost	304059.34	325211.21	300830.97	300830.97	318135.89
Probability	0.4036	0.0793	0.1705	0.1889	0.1577
Stochastic OFV	306796.27				

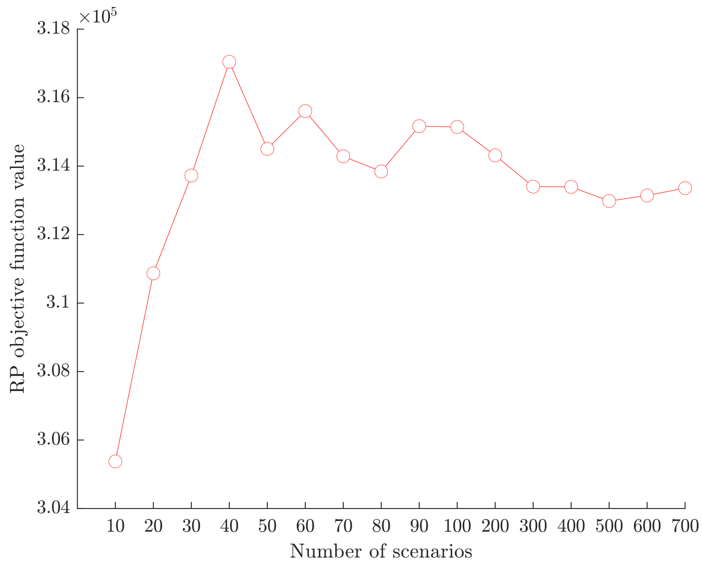

Figure 4.3: Total cost of the system

exists and what will happen in reality (EEV solution) is different from the optimum solution of EV. EEV problem solves the formulation by fixing first stage variables with the solution obtained from EV problem. Table 4.8 shows the solutions obtained by solving EV, EEV, and RP. The value of stochastic solution (VSS) is the difference between EEV and RP which in this case is 12516.7 indicating RP solution outperforms EEV solution.

In the next step we do a sensitivity analysis in outsourcing cost (OC) which is one the important elements in total cost of system. Figure 4.5 shows the change in total cost by decreasing or increasing OC. Linear relationship between OC and objective function value indicates no change in other cost elements of objective function which means optimal solution is not changing by change in OC.

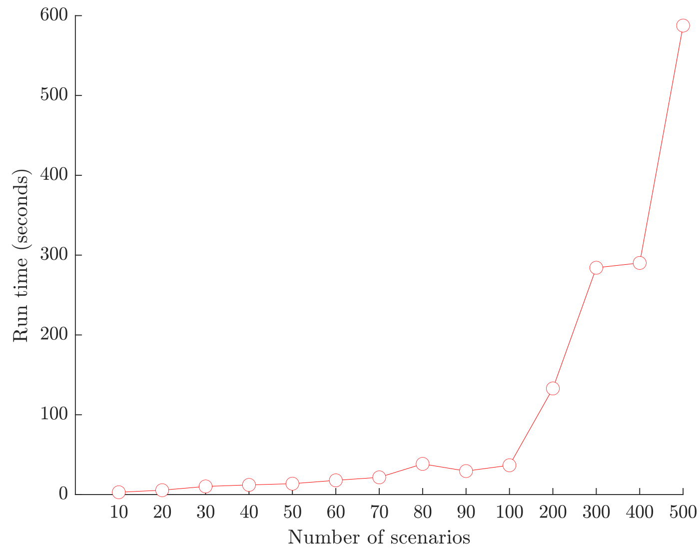

Figure 4.4: Run time for different number of scenarios

Table 4.8: EV, EEV, and RP problems results

	First Stage Variables															
	Sorting Centers						Warehouses			Recycling Centers			Disposal Centers			
Problem	UK	FR	SE	SP	AT	BE	NE	IT	FR	FR	GR	IT	FR	GR	IT	Total Cost
EV	0	0	0	0	0	1	0	1	0	0	0	1	0	0	1	226398.8
EEV	0	0	0	0	0	1	0	1	0	0	0	1	0	0	1	325917.4
RP	0	0	0	0	1	1	0	0	1	0	0	1	0	0	1	313400.7

Since total demand of secondary markets is less than total return of primary markets, final solution always includes outsourced extra returned items.

In terms of managerial insights, the major findings from the case study are given as follows:

- Stochastic solution decides to open two sorting centers. Increasing capacity of one of these sorting centers may result in establishing one sorting center and decreasing total cost of the system.

- Since outsourcing cost is one of the largest portion of total cost of the system, it might be beneficial to decrease this cost or contract with another third party company to handle outsourced products.

4.5 Conclusions

Traditional supply chain design considers product flow from suppliers to customers. However in reverse logistics problem as a supply chain problem, the product flow starts from customers(primary

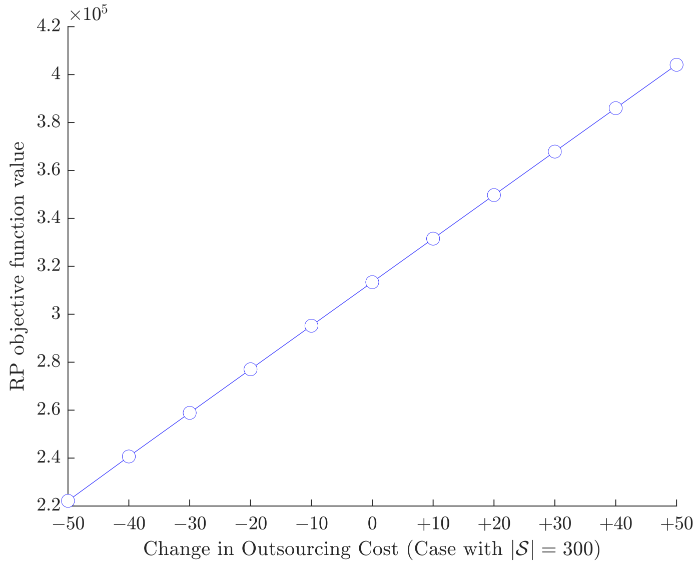

Figure 4.5: Sensitivity Analysis on Outsourcing Cost (OC)

markets) and end at manufacturers(secondary markets). Decision making in such environments face several uncertainty factors. Recently, designing a reverse logistics network involved in stochastic environment has attracted more attention in the literature.

In this paper, we design a reverse logistics network by formulating it as a multistage stochastic programming model. Uncertainty sources in this problem include return quantity in primary markets, demand quantity in secondary markets, recycling rate, and disposal rate. The first two uncertainty sources have normal distribution. Hence, moment matching method was used as a scenario generation approach to create discrete scenarios. Then fast forward selections was applied to decrease the number of scenarios. Finally extensive form of the formulation was solved to find the optimal solution of stochastic problem and sensitivity analysis was implemented to get managerial insights. This study is subject to to a few limitations which suggest some future research directions. First, considering return quality as a continuous variable would be desirable in finding the optimal solution. Second, developing exact and heuristic algorithms to solve the large-scale problems can be appealing. Last but not the least, developing valid inequalities is crucial in decreasing the time complexity of the problem.

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CHAPTER 5. GENERAL CONCLUSIONS

Supply chain management is one one the most critical decision making areas in a wide variety of industries. Designing a robust, cost-effective supply chain network is a crucial factor for manufacturing companies to survive in the current competitive environment. Forward logistics and reverse logistics are two important types of network design in supply chain management field. This dissertation includes three manuscripts aiming to fill some available gaps in the literature of these two problems and contribute on designing supply chain networks in deterministic and stochastic environments.

Forward logistics problem is one of the most studied problems in supply chain management. In this problem product flow starts from suppliers through in-between facilities and end to customers.In the first paper, we studied a multi-product pickup and delivery forward logistics problem with location routing and direct shipment decisions. We proposed a mixed integer linear programming model including two delivery mode for this problem. Two types of delivery mode considered for product delivery process. The first mode allows vehicles load the same type of products. While in the second mode, vehicles can load multiple types of products in each delivery route. Mixed integer programming models were developed for each mode. To solve the models, three solution method were implemented by CPLEX solver: deterministic mode, opportunistic mode, and benders decomposition algorithm. Results show that opportunistic mode outperform other two solution methods. This study is subject to a few limitations, which can be covered by future research directions: strengthening formulation by adding valid inequalities is one potential direction. Exact and heuristic solution methods can be developed to solve the large scale instances of the problem. For future studies, proposed models in this paper can be extended considering uncertainty in parameters like customer demand, capacity of distribution centers and supply capacity. The other direction can be devoted to develop better exact or heuristic solution methods. Considering objective functions other than cost function such as customer satisfaction and environmental emission effect can also serve as future research direction.

Reverse logistics problem is another important network design problem in supply chain management. In reverse logistics problem the product flow starts from customers or primary markets to manufacturers or secondary markets. There are a wide variety of uncertain factors involved in such problems. In the second paper, we designed a reverse logistics network by formulating it as a two-stage stochastic programming model. In this problem primary markets' demand quantity and secondary markets' return quantity, both with normal distribution, were considered as the main sources of uncertainty. Moment matching method and fast forward selection algorithm were used to generate discrete scenarios and reduce the number of scenarios, respectively. A case study was conducted for this problem and stochastic solution analysis showed that recursive problem optimal solution outperforms expected value and wait and see problems' solution.

Recycling and disposal processes are crucial elements of reverse logistics networks. In the third paper, to address uncertainty in these processes, we proposed a multi-stage stochastic programming model for reverse logistics network considering uncertainty in demand and return quantity, and return quality. The stochastic programming was formulated by PySP and solved for a case study. The optimal solution for this problem was obtained for different sets of discrete scenarios and sensitivity analysis was implemented for stochastic solution. In the second and third paper, I studied uncertainty in reverse logistics problems. However, these studies are subject to to a few limitations which suggest some future research directions. First, the described models assumes that demand and return are independent. However, sometimes in supply chain systems the demand and return in different time periods are dependent to each other. One future research direction is taking account of this assumption and analyzing the correlation between them. Second, we considered return and demand quantity and return quality as three sources of uncertainty while supply chain systems may include many other uncertain factors such as travel time and facility capacity. Hence, the other future research direction could be considering more uncertain parameters in modeling the problem. Third, considering return quality as a continuous variable would be desirable in finding the optimal solution. Fourth, developing exact and heuristic algorithms to solve the large-scale problems can be appealing. Last but not the least, developing valid inequalities is crucial in decreasing the time complexity of the problem.