Value of a Generator Construction Option in a Transmission Network under Demand Uncertainty

Abstract— **In this paper, we investigate how to value an option to construct a power plant when electricity demand fluctuates over time. Towards this aim, we first construct a transmission network, and obtain locational marginal prices for the network buses utilizing optimal power flow. Next, we construct a lattice model under the assumption that the demand fluctuation over time is represented by a geometric Brownian motion. Based on this demand lattice, we derive the economic consequences of costs to a bus with and without a power plant in a risk neutral world. These in turn will lead to the computation of the value of an option to construct a power plant. This value of the option will be useful for the electric power planning. For example, the bus with a higher value of this option indicates that the community in this bus is demonstrating a higher degree of potential need for such a power plant.**

I. INTRODUCTION

Whether to build a new power plant at a community or transmit from another community to meet its demand is a significant decision for generation and transmission planners as such a decision has a significant consequence on labor and capital requirements as well as the entire transmission network. This paper aims to address this issue by showing how to value an option to build such a power plant for the transmission network when demand is uncertain.

To achieve our aim, in this paper, we will first utilize an optimal power flow (OPF) framework leading to a locational marginal price (LMP) at each bus with demand [1]. Next, under demand uncertainty, based on the real options framework of geometric Brownian motion (GBM; continuous) and binomial lattice (discretized) [2], we will show how to compute the said value of a generator construction option in a transmission network.

II. BACKGROUND

We note that electric power operations are uncertain and often volatile. For example, the recent power outage in Texas led to a residential bill that is as high as $17,000 for a few days of electricity [3]. The locational marginal price for some communities remained near $9,000/MWh for several days [4]. With this backdrop, in our paper, we model uncertain power demand as GBM [5]. Moreover, as in Kucuksayacigil and Min [6], the continuous GBM process is discretized as a lattice (see e.g., Cox et al. [7]). The real options analysis for transmission planning has been used previously. For example, Abadie and Chamorro [8] worked with a binodal transmission network where the decision was adding a power line between two buses.

III. PROPOSED IDEA

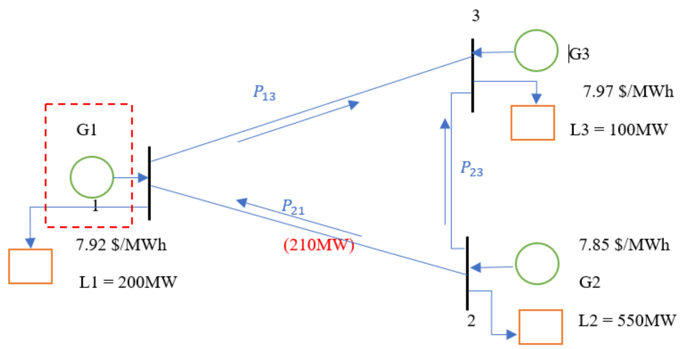

Figure: 1 3- bus network (case 1)

We present a 3-bus network as above, which can be considered as a simplified version of an example in Chapter 8 of Wood et al. [1]. We note that the textbook configuration as well as this paper's modification are of similar complexity as a simplified IEEE benchmark 9 bus system [9]. We note that there are three generators over bus 1, 2 and 3. The marginal cost of generator 1, 2 and 3 is ($7.92/MWh when it is built), $7.85/MWh and $7.97/MWh, respectively. The physical transmission limit of transmission line P_{12} is 210MW. There are consumption centers at all the buses, and the total demand load is 850 MW.

Per unit system

We use a per unit (PU) system as in [1]. For instance, let us consider a generator, which has a rating of 150 MWh and the system base power as 100 MVA. If the generator produces 120 MWh, then it is equivalent to 1.2 PU in per unit system.

We now consider the first case, where Bus 1 has no generator and the demand at this bus is satisfied by Generators 2 and/or 3. In the second case, we will add a generator at Bus 1 and the total demand will be met by the combination of all three generators. The resulting power flow and total generation cost of both cases will be compared.

To obtain the LMPs at different buses, we will utilize a DC-OPF approach [1]. Specifically,

MC_i = marginal cost of bus *i*
G_i = generation at bus *i*
θ_i = phase angle for bus *i*
P_{ij} = Power-flow in line *i-j*
$P_{load\,i}$ = demand load at bus *i*

For Case 1,
Objective Function: minimize $(MC_2 \times G_2 + MC_3 \times G_3)$
Decision variables are G_1, G_2, G_3 and $\theta_1, \theta_2, \theta_3,$ and $P_{12},\ P_{13}, P_{23}$ and the constraints are:

Nodal power balance constraints:
$100 \times [B_x]\underline{\theta} = \underline{P_{gen}} - \underline{P_{load}}$

Where, $\underline{\theta}$ is in radians and $[B_x]$ is in per unit.
We multiply $[B_x]\underline{\theta}$ or simply the values in $[B_x]$ matrix by 100 to keep the values of $\underline{P_{gen}} - \underline{P_{load}}$ in MW which means we convert the power from per unit to MW with an MVA system base of 100 MVA.

Where, $[B_x] = \begin{pmatrix} (1/x_{12}+1/x_{13}) & -1/x_{12} & -1/x_{13} \\ -1/x_{12} & (1/x_{12}+1/x_{23}) & -1/x_{23} \\ -1/x_{13} & -1/x_{23} & (1/x_{13}+1/x_{23}) \end{pmatrix}$

$[B_x]$ is the susceptance matrix with x_{ij} components and,

$\theta_i = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$ $\quad \underline{P_{load}} = \begin{bmatrix} P_{load\,1} \\ P_{load\,2} \\ P_{load\,3} \end{bmatrix}$ $\quad \underline{P_{gen}} = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}$

B_{ij} is the susceptance element of ith row and jth column where
$B_{ij} = (-1/x_{ij})$;
$B_{ii} = (1/x_{ij} + 1/x_{ik})$ i.e., the sum of reactance of the lines joining at the bus.
x_{ij} = reactance between line *i* and *j*
Reactance for line 1-2, 1-3 and 2-3 is 0.1 PU, 0.125 PU and 0.2 PU, respectively.

Power-flow in each branch:
$P_{ij} = 100 * B_{ij}(\theta_i - \theta_j) = 100 * (\theta_i - \theta_j)/(-x_{ij})$

Transmission limit constraints:
$100 * (\theta_i - \theta_j)/x_{ij} \leq P_{ijmax}$
$100 * (\theta_j - \theta_i)/x_{ij} \leq P_{ijmax}$

We shall note that there will be additional non-negativity constraints on some variables such as $0 \leq G_1, G_2, G_3$. We also assumed that the transmission losses are negligible, and the line resistances are negligible compared to line reactances. After determining the locational marginal price for 3 buses from the mathematical model, demand growth can be analyzed using a binomial lattice approach.

Demand lattice:

As in [2], the change in some quantity S is determined by multiplication factors "u" and "d". It goes up or down with risk neutral probabilities q and 1-q.

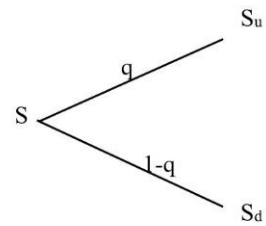

Figure: 2 - Demand Lattice

$u = e^{\sigma\sqrt{\Delta t}}$ $\qquad S_u = u \times S$
$d = e^{-\sigma\sqrt{\Delta t}}$ $\qquad S_d = d \times S$

The values of S_u and S_d can be determined from the above-mentioned equations, where σ is the volatility of the process S, and Δt is the time step in the lattice. As in White [2], we use a risk-free rate and assume a continuous compounding return. Risk-neutral probabilities are probabilities of possible future outcomes, which have been adjusted for risk. Risk neutral approach assumes that the decision maker is indifferent about the risk [2]. In the above example, if the network is congested due to a transmission limit constraint, one viable case is adding one power generator to the network. For this problem, our first case is proceeding without any additional generator, and the second case is adding a generator at the bus with the highest LMP. In what follows, we will focus on the scenario of S_u as the other two scenarios of $S, and\ S_d$ can be verified to result in no changes in LMP's.

IV. DISCUSSION & RESULTS

OPF and LMP:

We use the Excel solver to calculate the values for our DCOPF model. Solving the model using "simplex LP" function and setting the decision variables as generator dispatch, phase angles and subject to the constraints mentioned above, we obtain the DC optimal power flow. The sensitivity report of our model gives us the LMP values as the shadow prices. We can also verify the values from the excel model using the superposition method given by D. S. Kirschen et al. [10].

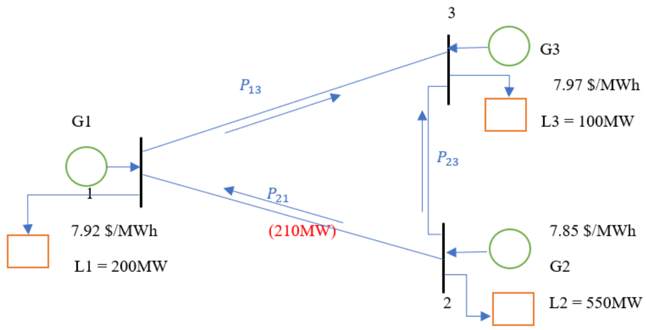

Figure: 3 - bus network (case 2)

In the second case the 3-bus network is modeled and solved for Figure 3. In this, a generator at bus 1 is added with marginal cost of $7.92/MWh. The resulting optimal power flow and LMP of all the buss will be compared with the first case.

When the load is 200 MW or 148.2 MW at bus 1 (in case 1 and case 2), the LMP at all buses will be 7.85 $/MWh since generator 2 alone is satisfying the total demand. Whereas when the load at bus 1 is 270 MW in case 1, the values of LMP at buses 1, 2 and 3 are 8.045 $/MWh, 7.85 $/MWh and 7.97 $/MWh, respectively. And the values of LMP at buses 1, 2 and 3 in case 2 are 7.92 $/MWh, 7.85 $/MWh and 7.89 $/MWh, respectively.

Demand Lattice:

With the following hypothetical values, a lattice is given as follows (with total construction cost = $100,000).

Drift $(\mu) = 15\%/year$
Risk free discount rate (r_f) = 4.879%
Volatility (σ) = $30\%/year$
Time step (Δt) = $1\ year$
Up-factor (u) = $e^{\sigma\sqrt{\Delta t}} = 1.35$
Down factor (d) = $1/u = 0.741$

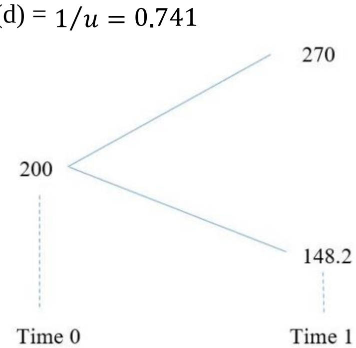

Figure 4: Demand evolution lattice for Bus 1 (in MW)

That is, we have a demand of 200 MW at the beginning of the modelling horizon (time 0) and after one year (time 1), the demand can rise to 270 MW (or drop to 148.2 MW). Assuming continuous compounding, the risk neutral probability is given by,

$q = (e^{r_f} - d)/(u - d) = 0.5074$

Bus based economic consequence:

Case 1:

The first case is to proceed with generator 2 and 3 to fulfill the future demand. Line 2-1 is constrained by a limit of 210 MW. Therefore, the line is congested and locational marginal price at bus 1 will be $8.045/MWh to fulfill the demand of 270 MW.

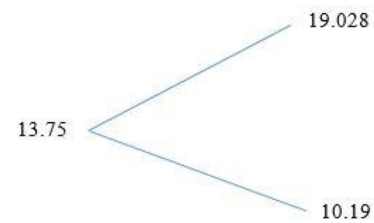

Figure 5: Cost Lattice for Case 1 (in million $)

So, the demand can still be fulfilled. But, due to high locational marginal price ($8.045/MWh), the cost to be paid by the community at bus 1 will be

$$8.045\ X\ 8760\ X\ 270 = \$19{,}028{,}034$$

That means, the community at bus 1 will pay $19.028 million for the whole year to fulfill the demand of 270 MW. The costs at other time points can be calculated similarly.

Case 2:

A generator is installed at Bus 1. If the generator is added, the locational marginal price for bus 1 will be $7.92/MWh after one year to satisfy 270 MW demand. Therefore, if demand goes up, the yearly expense by the community at Bus 1 will be $18.732 million, which is significantly lower than the first case due to the lower locational marginal price.

Net benefit and option value

At the current state, when demand is 200 MW, the locational marginal price will be the same for two cases and therefore the net benefit is zero for this state. It will also be true if the demand goes down (148.2 MW). But, if the demand is up (270 MW) after one year from the starting time, the associated costs for the first and second cases are \$19.028 million and \$18.732 million, respectively. Therefore, the net benefit for that time point is the difference between these costs, which is \$296,000.

We assume that the total construction cost of a power plant at Bus 1 is \$100,000. Therefore, the “net benefit lattice after paying the total construction cost” can be attained by simply subtracting the total construction cost of \$100,000 from net benefit.

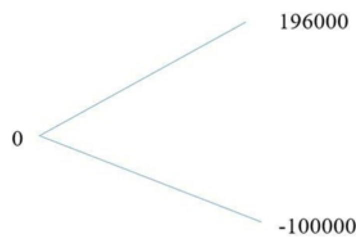

Figure 6: Net benefit Lattice after paying the construction cost (in \$)

The expected net benefit in the risk neutral world after one year from time 1 is

$$\$196000\ X\ 0.5074\ -\ \$100\ 000\ X\ 0.4926\ =\ \$50{,}190.4$$

At time 0, discounted expected net benefit in the risk neutral world is

$$\$50190.4\ X\ e^{-0.04879}\ =\ \$47{,}800.4$$

The numerical figure shows the worth of choosing case 2 (adding one generator at Bus 1) over case 1 (not adding any generator at Bus 1). This indicates that the value of option to build a power plant at Bus 1 is \$47,800.40.

Grid based Economic Consequence:

From the LMP calculation, we note how adding the generator affects the performance of the whole grid. If one generator is added to bus 1, it changes the LMP for bus 3, too. So, it can be said that adding an additional generator substantially benefits bus 3 too. In this section, we will discuss the economic consequence of adding the generator at bus 1 on the whole grid.

Case 1:

This is the case where total grid demand fulfilled by Generator 2 and 3. The locational marginal price at bus 1 will be \$8.045/MWh to fulfill the demand of 270 MW and locational marginal price at bus 3 will be \$7.97/MWh in this scenario.

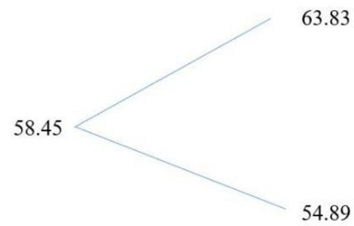

Figure 7: Cost Lattice for Case 1 (Grid based) (in million \$)

The demand can still be fulfilled. But, due to high locational marginal price (\$8.045/MWh at bus 1 and \$7.97/MWh at bus 3), the cost to be paid by the whole community combining 3 buses will be,

$$(8.045 \times 270 + 7.85 \times 550 + 7.97 \times 100) \times 8760$$
$$= \$63831054$$

That means, the total cost paid by 3 communities at 3 busses will be \$63.83 million for the whole year. The costs at other points can be calculated similarly.

Case 2:

If the generator is added, the locational marginal price at Bus 1 will be \$7.92/MWh after one year to satisfy 270 MW demand and the locational marginal price will be \$7.893/MWh at Bus 3. Therefore, if demand goes up, the yearly expense by all the communities will be \$63.46 million, which is significantly lower than the first case due to the lower locational marginal price at bus 1 and 3. Here, costs at other time points are similar to the first case due to the equal LMP.

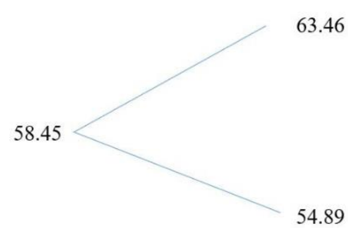

Figure 8: Cost Lattice for Case 2 (Grid based) (in million \$)

Net Benefit and Option Value:

We can build the net benefit lattice similarly here as the bus-based economic consequence evaluation. There is a benefit of using the second case only when the demand at bus 1 goes up at time 1. If the demand goes up (270 MW), the first and second case's associated costs are \$63.83 million and \$63.46 million, respectively. Therefore, the net benefit for that time point is the difference between these costs, which is \$363,102. But, at time 0 and at time 1 (if the demand goes down), there is no benefit of using one case over the other.

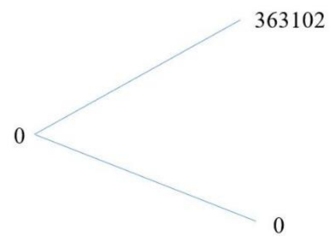

Figure 9: Net Benefit lattice (in $)

We assume that the total construction cost of a power plant at Bus 1 is $100,000. Therefore, the “net benefit lattice after paying the total construction cost” can be attained by simply subtracting the total construction cost of $100,000 from net benefit.

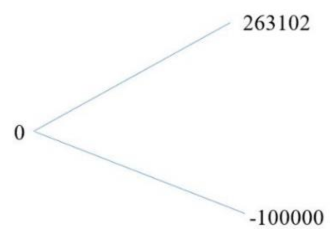

Figure 10: Net Benefit after paying construction cost (in $)

The expected net benefit in the risk neutral world after one year from the starting period is:

$$263102 \times (0.5074) - 100000 \times (0.4926)$$
$$= \$84237.9548$$

At the starting period, discounted expected net benefit in the risk neutral world is:

$$84237.9548 \times e^{-0.04879} = \$80226.64$$

This indicates that the value of the option to build a power plant at bus 1 by considering whole grid’s benefit is $80,226.64. Adding the generator significantly reduced the LMP of bus 3 from $7.97 to $7.893. Due to the added benefit of bus 3 community, the value of the license is increased to $80,226 from $47,800 if we consider the whole grid’s benefit instead of only considering bus 1.

V. Conclusion

In this paper, we have shown how the value of option to build a power plant can be calculated in two different scenarios via a real options approach based on the concepts of optimal power flow and locational marginal price. The first scenario focuses on the net benefit of a single bus while the second scenario focused on the net benefit of the whole grid. Our approach can be expanded to address othercritical option values such as the value of option to add a transmission line. Such a case will lead to interesting research question such as which option is of higher value betweenadding a generator vs. a line.

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