Power Research Meeting 6/17/20

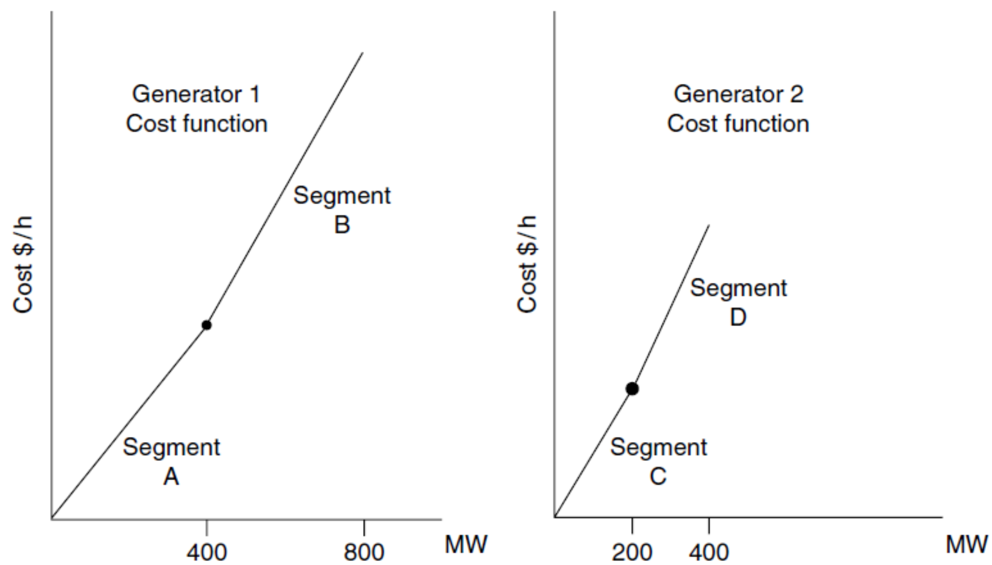
1. Locational Marginal Price

Locational marginal pricing reflects the value of the energy at the specific location and time it is delivered. To understand the practical importance let us look at an example.
Consider a two-bus system as shown in figure below.

The cost function data is as given below.

Generator 1	MW	Marginal Cost ($/MWh)	Generator 2	Bid MW	Marginal Cost ($/MWh)
Segment A	400	5.00	Segment C	200	6.50
Segment B	800	7.50	Segment D	400	8.00

We will use piecewise linear cost functions for the ease of calculation.



When performing economic dispatch, the calculation simply begins increasing the generation on the segment with the lowest marginal price (slope) until a limit is hit or the segment MW limit is hit. Therefore,

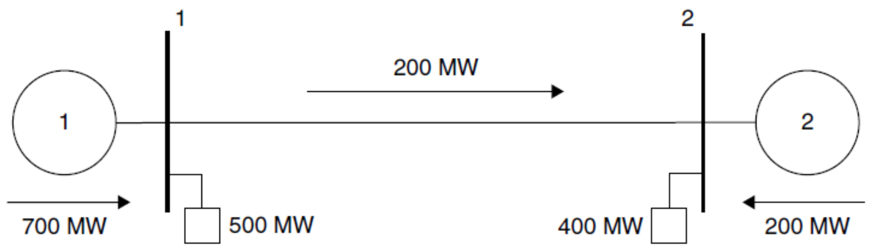
i. We start by segment A of generator 1 with cost of 5.00 and use all 400MW of it generation power.
ii. Next, we will use segment C of generator 2 with cost of 6.50 and use 200MW. At this stage we are generating 600MW and need 300MW more power.
iii. Hence, we will use segment B which next highest cost of 7.50 and only 300MW out of its total 800MW.
iv. The resulting dispatch is -

Segment	MW	Price
A	400	5.00
C	200	6.50
B	300	7.50

v. The final cost of 7.50 is called as the "clearing price" since at that cost all the price is satisfied.

vi. A very important observation at this point is that if we were to add 1 MW to the load at either bus, that additional load would be supplied from segment B at a cost of 7.50 $/MWh. Thus, we can say that the cost of additional load at either bus is 7.50 or that the marginal price of power at either bus is 7.50.

vii. The resulting power flow diagram is as follows-



1.1 LMP with Transmission Limit Imposed

Suppose transmission line 1-2 has a limit imposed of 100MW power flow in either direction then,

i. Again, we start with the lowest cost segment, segment A of generator 1, and bring generator 1 up to 400 MW. We will assume for simplicity that the load at bus 1 is satisfied first, so that at this point the 400 MW of segment A flows into the load at bus 1 leaving another 100 MW to go.

ii. Now, we will use segment C of generator 2 and use all of its 200MW power which flows into load at bus 2.

iii. Next, we will use segment B of generator 1 to generate 100MW to satisfy remaining load at bus 1 and 100MW to flow to load at bus 2 since there is a transmission limit of 100MW. Hence segment B produced 200MW.

iv. The remaining load at bus 2 is 100MW which is satisfied by segment D from generator 2.

v. The resulting dispatch is,

Segment	MW	Price
A	400	5.00
C	200	6.50
B	200	7.50
D	100	8.00

vi. If we add 1 MW of load to Bus 1, it must come from segment B at a cost of 7.50, hence LMP is 7.50 at bus 1.

vii. If we add 1 MW of load to Bus 2, it must come from segment D at a cost of 8. Hence LMP is 8 at bus 2

Conclusion - When all LMP values are the same, none of the transmission lines are at maximum flow. When the LMP values at different buses differ, the cause is a limiting transmission line.

2. **DCOPF with transmission limit imposed.**

   Line 1-2 has transmission limit of 150MW

Objective Function – To minimize cost function.

$\min \sum_{i=1}^{3} F_i\,(P_i)$

Subject to –

Generator limit inequality constraint:

$P_{gen_i}^{min} \leq P_{gen_i} \leq P_{gen_i}^{max},$

Generator load balance equality constraint:

$P_{total\ load} - (P_1 + P_2 + P_3) = 0$

Nodal power balance constraints:

$100[B_x]\theta = P_i - P_{load}$

$1800\theta_1 - 1000\theta_2 - 800\theta_3 = P_1 - 200$

$-1000\theta_1 + 1500\theta_2 - 500\theta_3 = P_2 - 550$

$-800\theta_1 - 500\theta_2 + 1300\theta_3 = P_3 - 100$

$\theta_1 - 0 = 0$

And since, line 1-2 had limited power supply, we have

$\frac{100}{0.1}(\theta_1 - \theta_2) \leq 150$ ………………….. $\left(\frac{100}{x_{12}}(\theta_1 - \theta_2) = 150\right)$

Where, transmission limit on line 1-2 is 150 MW and x_{12} is the line reactance for line 1-2

Actual transmission lines are not limited to flow in one direction but are usually limited in either direction.

This can be achieved by adding constraint that limits the flow to 150MW from 2 to 1, which can be shown as,

$$-\frac{100}{x_{12}}(\theta_1 - \theta_2) \leq 150$$

Adding a slack variable to the above equation,

$$\frac{100}{x_{12}}(\theta_1 - \theta_2) + S_{12} = 150$$

Where, $0 \leq S_{12} \leq 2(P_{flow\ 12}^{max})$, $0 \leq S_{12} \leq 300$

Phase angle constraints:

$-\pi \leq \theta_1 \leq \pi$

$-\pi \leq \theta_2 \leq \pi$

$-\pi \leq \theta_3 \leq \pi$

DC Power flow in each branch is given by:

$$P_{ij} = -B_{ij}(\theta_i - \theta_j) = \frac{\theta_i - \theta_j}{x_{ij}} \text{ MW`}$$

Locational Marginal Price

As we have seen in the previous sections, the Locational marginal price for each generator is different when the transmission line has a limit imposed. Therefore, all 3 generators will have different LMP.

3. Numerical example

If we impose transmission limit of 150MW for line 1-2, then we obtain the following values based the mathematical formulation mentioned above-

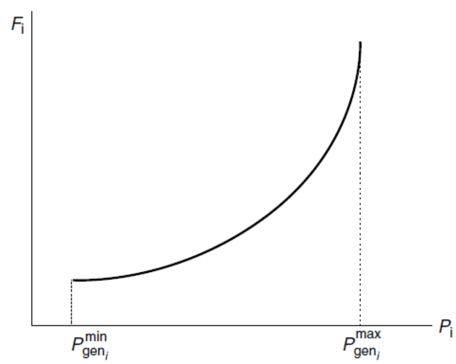
$$P_1 = 382.72\ MW;\ P_2 = 345.45\ MW;\ P_3 = 121.82\ MW$$

$$\lambda_1 = 9.11\ \$/MWh;\ \lambda_2 = 9.19\ \$/MWh;\ \lambda_3 = 9.14\ \$/MWh$$

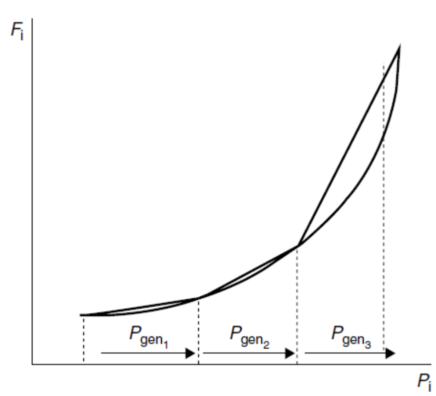
$$\theta_1 = 0;\ \theta_2 = -0.15;\ \theta_3 = -0.04\ radians$$

$$P_{flow1} = 150MW;\ P_{flow2} = 32.72MW;\ P_{flow3} = -54.54MW$$

4. Piecewise Linear Programming



- Consider a typical nonlinear cost curve as shown above.
- We can approximate the nonlinear curve into series of straight-line segments as shown in the figure below



- The three segments for generator i are represented as i1, i2, i3. The variable P_i is replaced by $P_{gen_{i1}}, P_{gen_{i2}} and\ P_{gen_{i3}}$
- Each segment will have a slope designated $s_{i1}, s_{i2} and\ s_{i3}$
- The cost function now becomes sum of the cost at P_i^{min} plus the sum of the linear cost for each segment-

$$F_i(P_{gen_i}) = F_i(P_{gen\ i}^{min}) + s_{i1}P_{gen_{i1}} + s_{i2}P_{gen_{i2}} + s_{i3}P_{gen_{i3}}$$

- Therefore, the objective function is-

Minimize $\sum_{i=1}^{N} F_i(P_{gen\ i}^{min}) + s_{i1}P_{gen_{i1}} + s_{i2}P_{gen_{i2}} + s_{i3}P_{gen_{i3}}$

Subject to-

$$0 \le P_{gen_{ik}} \le P_{gen\ ik}^{max}$$
$$for\ k = 1,2\ and\ 3$$

where

$$P_i = P_i^{min} + P_{gen_{i1}} + P_{gen_{i2}} + P_{gen_{i3}}$$

Power balance constraint

$$100[B_x]\theta = P_{gen} - P_{load}$$

Generator limit constraint

$$P_{gen_i}^{min} \le P_{gen_i} \le P_{gen_i}^{max},$$

Line flow constraint

$$\frac{100}{x_{12}}(\theta_1 - \theta_2) \le P_{ij}^{max}$$