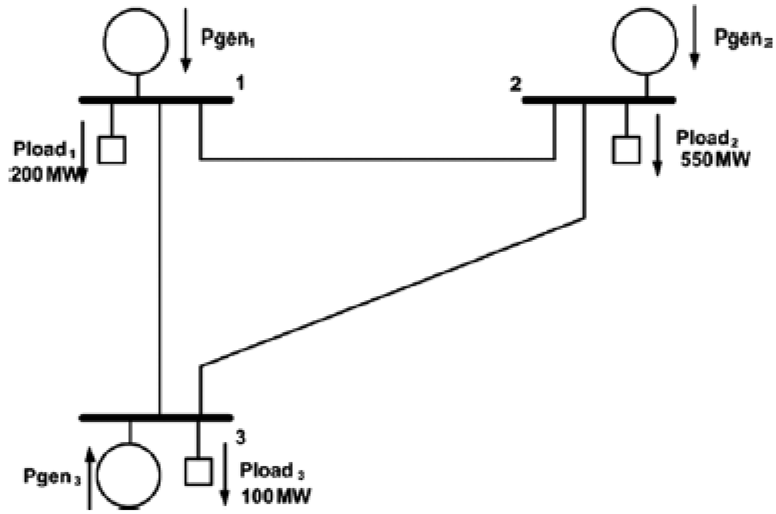
DCOPF with transmission limit imposed

Line 1-2 has transmission limit of 150MW



Objective Function – To minimize cost function.

$\min \sum_{i=1}^{3} F_i\,(P_i)$

The variables are – P1, P2, P3, θ1, θ2 and θ3

Definition of Pij (power flow) is from thetas and admittance, so P1, P2 and P3 and relation of P12, P23 and P13 is not needed.

Subject to –

Generator limit inequality constraint:

$$P_{gen_i}^{min} \leq P_{gen_i} \leq P_{gen_i}^{max},$$

Generator load balance equality constraint:

$$P_{total\ load} - (P_1 + P_2 + P_3) = 0$$

Nodal power balance constraints:

$$100[B_x]\theta = P_i - P_{load}$$

$$1800\theta_1 - 1000\theta_2 - 800\theta_3 = P_1 - 200$$

$$-1000\theta_1 + 1500\theta_2 - 500\theta_3 = P_2 - 550$$

$$-800\theta_1 - 500\theta_2 + 1300\theta_3 = P_3 - 100$$

$$\theta_1 - 0 = 0$$

And since, line 1-2 had limited power supply, we have

$$\frac{100}{0.1}(\theta_1 - \theta_2) \leq 150 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\left(\frac{100}{x_{12}}(\theta_1 - \theta_2) = 150\right)$$

Where, transmission limit on line 1-2 is 150 MW and x_{12} is the line reactance for line 1-2

Actual transmission lines are not limited to flow in one direction but are usually limited in either direction.

This can be achieved by adding constraint that limits the flow to 150MW from 2 to 1, which can be shown as,

$$-\frac{100}{x_{12}}(\theta_1 - \theta_2) \leq 150$$

Adding a slack variable to the above equation,

$$\frac{100}{x_{12}}(\theta_1 - \theta_2) + S_{12} = 150$$

Where, $0 \leq S_{12} \leq 2(P_{flow\,12}^{max})$, $0 \leq S_{12} \leq 300$

Phase angle constraints:

$$-\pi \leq \theta_1 \leq \pi$$

$$-\pi \leq \theta_2 \leq \pi$$

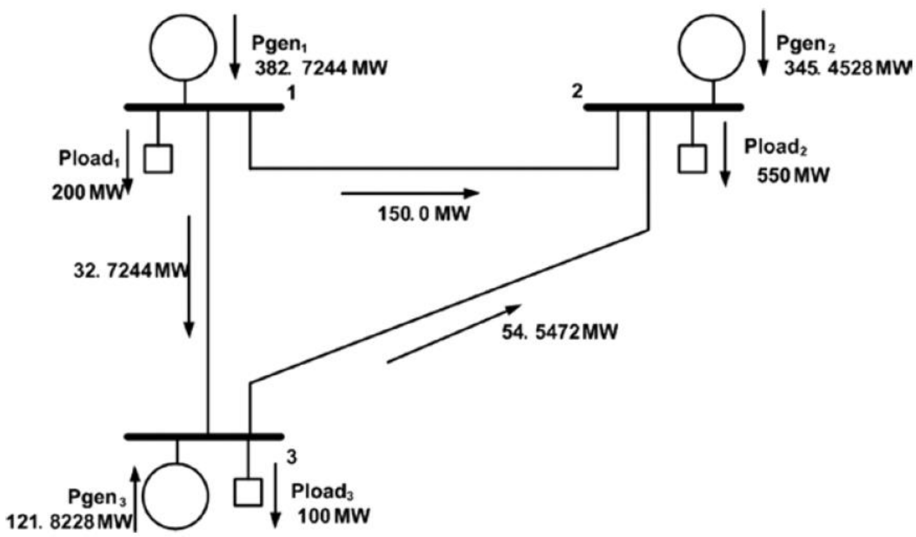
$$-\pi \leq \theta_3 \leq \pi$$

DC Power flow in each branch is given by:

$$P_{ij} = -B_{ij}(\theta_i - \theta_j) = \frac{\theta_i - \theta_j}{x_{ij}} \text{ MW}$$

Locational Marginal Price:

The LMP for every node will be different due to the transmission line constraint. In this formulation, the Lagrangian multiplier (λ) is the LMP.



Where,

$$\lambda_1 = 9.11\ \$/MWh;\ \lambda_2 = 9.19\ \$/MWh;\ \lambda_3 = 9.14\ \$/MWh$$

Are the LMPs of node 1, 2 and 3 respectively.

Excel Input –

Step 1 –

Write down the variables in one column and values in the next column. Let the values be 0 initially. Similarly input the values of P_{load} which are already given.

Variables	Values	Load at each node	
p1	0	Pload1	200
p2	0	Pload2	550
p3	0	Pload3	100
θ1	0		
θ2	0		
θ3	0		

Step 2 –

Write the objective function which is to minimize (561 + 310 + 78 + 7.92P_1 + 7.85P_2 + 7.97P_1 + 0.001562P_1^2 + 0.00194P_2^2 + 0.0048P_3^2) in a different cell.

Objective	min $\sum_{i=1}^{3}(F_i P_{gen_i})$
Minimize	= 561 + 310 + 78 + (7.92*B2) + (7.85*B3) + (7.97*B4) + (0.001562*(B2*B2)) + (0.00194*(B3*B3)) + (0.0048*(B4*B4))

Step 3 –

Write the constraints with appropriate equality signs and values

Constraints			
		Equality/Inequality	
p1+p2+p3=850	0	=	850
p1>=150	0	>=	150
p1<=600	0	<=	600
p2>=100	0	>=	100
p2<=400	0	<=	400
p3>=50	0	>=	50
p3<=200	0	<=	200
100*[B]θ=p2-pload2	0	=	-550
100*[B]θ=p3-pload3	0	=	-100
100/0.1*(θ1-θ2)	0	<=	150
S12	0	<=	300
S12	0	>=	0
θ1-0	0	=	0
θ1 >= -π	0	>=	-3.141592654
θ2 >= -π	0	>=	-3.141592654
θ3 >= -π	0	>=	-3.141592654
θ1 <= π	0	<=	3.141592654
θ2 <= π	0	<=	3.141592654
θ3 <= π	0	<=	3.141592654

For instance, the first constraint is p1+p2+p3=850, hence the formula will be: =

Constraints			
		Equality/Inequality	
p1+p2+p3=850	=B2+B3+B4	=	850

Where B2, B3 and B4 represent the address of cells containing the values of p1,p2 and p3 respectively.

Step 4 –

Write the Line power flow formulation-

Line Power Flow	$P_{ik} = B_{ik}(\theta_i - \theta_k)$
Pflow12	=1000*(B5-B6)
Pflow13	0
Pflow23	0

Step 5 -

Write the LMP formulation-

Lambda1	=7.92+0.003124*C16
Lambda2	9.190356748
Lambda3	9.144371772

where lambda 1 = dF1/dP1

lambda 2 = dF2/dP2

lambda 3 = dF3/dP3

Running Excel Solver -

Select Data (from top bar) => Solver. (Top right corner)

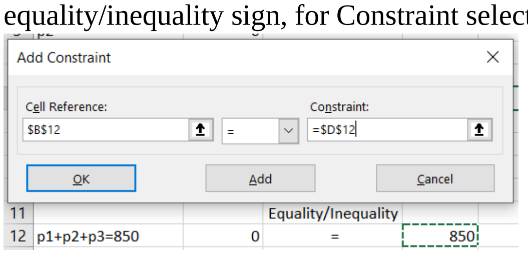
Step 1 - Set Objective – Select the cell where objective function has been written down.

Step 2 - To: Min

Step 3 - By changing variable cells: Select the whole column containing values of the variables p1, p2, p3, θ_1, θ_2 and θ_3

Step 4 - Subject to constraints: select 'Add'

For Cell reference select the constraint equation cell, select appropriate equality/inequality sign, for Constraint select the cell with constraint value.



Step 5 - Do not select the option – **Make unconstrained variables non-negative**

Step 6 - Select a solving method = GRG nonlinear

Step 7 - Select '**Solve**'

Excel Output –

Variables	Values
p1	382.7244314
p2	345.4527697
p3	121.8227989
θ1	0
θ2	-0.15
θ3	-0.04090554

Lambda1	9.115631126
Lambda2	9.190356748
Lambda3	9.144371772