- The purpose of optimal power flow is to minimize the total cost of electricity generation while maintaining the electrical power system within safe operating limits.
- Economic Dispatch is the calculation that finds the lowest-cost generation dispatch for a set of generators that is constrained within the individual generator limits and results in a total generation that equals the total load plus losses.
- OPF couples the Economic Dispatch calculation with power flow equations.
- DC power flow gives a good approximation to AC power flow and is much faster and easier to set up and solve. This approximation is the DC optimal power flow which can be formulated as follows:

Objective Function:

To Minimize: min $\sum_{i=1}^{3} F_i\,(P_i)$

F_i represents the cost rate of the unit, P_i represents the electrical power generated by that particular unit. Therefore, $F_i P_i$ represents the cost function which is the total cost for supplying the indicated load.

$F_1(P_1)$= 561 + 7.92P_1 + 0.001562P_1^2 \$/h

$F_2(P_2)$ = 310 + 7.85P_2 + 0.00194P_2^2 \$/h

$F_3(P_3)$= 78 + 7.97P_1 + 0.0048P_3^2 \$/h

> Generating units consume fuel (H) at a specific rate (MBtu/h)
>
> Fuel cost(F) times fuel consumption (H) gives \$/h input to the unit for fuel

$$\min\,[F_1(P_1) + F_2(P_2) + F_3(P_3)]$$

$$\min\,(561 + 310 + 78 + 7.92P_1 + 7.85P_2 + 7.97P_1 + 0.001562P_1^2 + 0.00194P_2^2 + 0.0048P_3^2)$$

Subject to:

Generator limit inequality constraint:

The power output of each unit must be greater than or equal to the minimum power permitted and must also be less than or equal to the maximum power permitted on that particular unit.

$$P_{gen_i}^{min} \le P_{gen_i} \le P_{gen_i}^{max},$$

$$150 \le P_1 \le 600 \text{ MW}$$

$$100 \le P_2 \le 400 \text{ MW}$$

$$50 \le P_3 \le 200 \text{ MW}$$

Generator load balance equality constraint:

The sum of power generated by all units must be equal to the total load on the system.

$$P_{total\ load} - (P_1 + P_2 + P_3) = 0$$

Where, $P_{total\ load} = P_{load1} + P_{load2} + P_{load3}$

$$P_{load1} = 200MW, P_{load2} = 550MW, P_{load3} = 100MW$$

Nodal power balance constraints:

$$100[B_x]\theta = P_i - P_{load} \qquad \text{where, } [B_x] = \begin{bmatrix} 18 & -10 & -8 \\ -10 & 15 & -5 \\ -8 & -5 & 13 \end{bmatrix} \qquad \theta_i = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

Therefore,

$$1800\theta_1 - 1000\theta_2 - 800\theta_3 = P_1 - 200$$

$$-1000\theta_1 + 1500\theta_2 - 500\theta_3 = P_2 - 550$$

$$-800\theta_1 - 500\theta_2 + 1300\theta_3 = P_3 - 100$$

Phase angle constraints:

For most typical operating conditions, the difference in angles of the voltage phasors at two buses k and j connected by a circuit, which is θi-θj for buses i and j, is less than 10-15 degrees. It is extremely rare to ever see such angular difference exceed 30 degrees. Thus, we say that the angular difference across any transmission circuit is "small." When the angle is given in radians better approximation is that the sine of a small angle is the angle itself. Therefore, we measure θ_i in radians. (1° x $\pi/180$ = 0.01745rad) Calculations

$$-\pi \le \theta_1 \le \pi$$

$$-\pi \le \theta_2 \le \pi$$

$$-\pi \le \theta_3 \le \pi$$

DC Power flow in each branch is given by:

$$P_{ij} = -B_{ij}(\theta_i - \theta_j) = \frac{\theta_i - \theta_j}{x_{ij}} \text{ MW`}$$

Where, B_{ij} is the susceptance of the branch i to j given by:

$$B_{ij} = -\frac{1}{x_{ij}}$$

Line	Line Reactance (x_{ij}) (ohms)
1 – 2	x_{12} = 0.1
1 – 3	x_{13} = 0.125
2 – 3	x_{23} = 0.2

Refer - page 43 Section 2.5 of "Optimization of Power System Operation by Jizhong Zhu"

In this problem the decision variables are P_1, P_2, P_3, θ_1, θ_2, θ_3

Excel Solver

Step 1 –

Write down the variables in one column and values in the next column. Let the values be 0 initially. Similarly input the values of P_{load} which are already given.

Variables	Values	Load at each node	
p1	0	Pload1	200
p2	0	Pload2	550
p3	0	Pload3	100
θ1	0		
θ2	0		
θ3	0		

Step 2 –

Write the objective function in a different cell.

Objective	$\min \ \sum_{i=1}^{3}(F_i \, P_{gen_i})$								
Minimize	= 561 + 310 + 78 + (7.92*B2) + (7.85*B3) + (7.97*B4) + (0.001562*(B2*B2)) + (0.00194*(B3*B3)) + (0.0048*(B4*B4))								

Step 3 –

Input the constraints to the objective function.

Constraints			
		Equality/Inequality	
p1+p2+p3=850	850	=	850
p1>=150	392.9308019	>=	150
p1<=600	392.9308019	<=	600
p2>=100	334.4112959	>=	100
p2<=400	334.4112959	<=	400
p3>=50	122.6579023	>=	50
p3<=200	122.6579023	<=	200
100*[B]θ=p1-pload1	192.9308019	=	192.9308019
100*[B]θ=p2-pload2	-215.5887041	=	-215.5887041
100*[B]θ=p3-pload3	22.65790227	=	22.65790227
θ1 >= -π	0.158197861	>=	-3.141592654
θ2 >= -π	0	>=	-3.141592654
θ3 >= -π	0.114781686	>=	-3.141592654
θ1 <= π	0.158197861	<=	3.141592654
θ2 <= π	0	<=	3.141592654
θ3 <= π	0.114781686	<=	3.141592654

Step 4 –

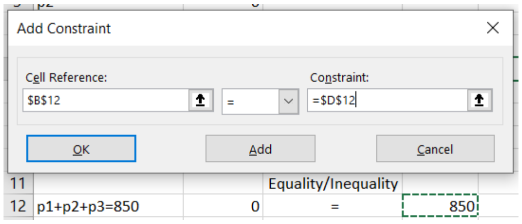
Write down the Line power flow formulation.

Line Power Flow	$P_{ik} = B_{ik}(\theta_i - \theta_k)$	
Pflow12	0	
Pflow13	0	
Pflow23	0	

Step 5 –

Select Data => Solver.

i. **Set Objective** – Select the cell where objective function has been written down.
ii. **To**: Min
iii. **By changing variable cells:** Select the whole column containing values of the variables p1, p2, p3, θ_1, θ_2 and θ_3
iv. **Subject to constraints**: select 'Add'
For Cell reference select the constraint equation cell, select appropriate equality/inequality sign, for Constraint select the cell with constraint value.



v. **Select a solving method** = GRG nonlinear

vi. Select 'solve'

Output

1. The values of decision variables given by the solver are:

Variables	Values
p1	392.9308019
p2	334.4112959
p3	122.6579023
θ1	0.158197861
θ2	0
θ3	0.114781686

2. The Line power flow in each branch is:

Line Power Flow	$P_{ik} = B_{ik}(\theta_i - \theta_k)$
Pflow12	158.1978613
Pflow13	34.73294055
Pflow23	-57.39084282

Summary

1. The values obtained for power generated by each unit are similar to the textbook answers.
2. The values obtained for phase angles are incorrect. In textbook calculations, θ_1 is set equal to 0 as a reference angle whereas the excel solver is showing $\theta_2 = 0$.
   (Generally, the unit with highest power generation is set as the reference bus or slack bus. It provides a voltage reference (typically V = 1.0 p.u. and δ = 0◦) such that the remaining bus voltages are uniquely determined. t is the only bus at which real power is free to vary, the slack bus is required to ensure that the power flow equations have a feasible solution.)
3. The power flow in each branch is also calculated and is similar to textbook values.

Issues pending

1. The important issue to address is the value of phase angle given by the excel solver. The values of θ_1, θ_2 and θ_3 are incorrect. According to the textbook the reference angle should be θ_1 but the excel solver is setting θ_2 equal to 0.

2. Relationship of cost function unit ($/hr) to the unit of power generated (MW)

3. Consider P_{ij} (Power flow in branch *ij*) as one of the constraints.