The influence of the unit commitment plan on the variance of electric power production cost

by

Jinchi Li

A Creative Component paper submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE WITH CREATIVE COMPONENT

Major: Industrial Engineering

Program of Study Committee:

Sarah M. Ryan, Major Professor

Gary Mirka

Iowa State University

Ames, Iowa

2018

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

In electricity production systems, generating unit failures will result in higher electricity production cost. For a system operator, the variance of production cost is an important factor for choosing the proper commitment plan, because generating unit failures require a much more expensive unit to replace their electricity production task. Unit Commitment (UC) seeks a thermal generator commitment schedule that meets the net electricity demand in the most cost-effective way while satisfying operational constraints on both transmission and generation systems. The unexpected, extremely high operational cost is undesirable for system operators. The production cost variance could be helpful when decision makers try to select a commitment plan with low risk in its operational cost. In this project, we focus on the behavior of production cost variance for different commitment plans under a simplified model of an electric production system. In the simplified system, the warm up and shut down periods of generating units are ignored. First, a simulation model and an approximation method are built to estimate the production cost variance of this simplified system. Then, the influence of the commitment plan is observed based on simulation results. After that, the performance of both estimation approaches is tested using commitment plan data. The contribution of this project is that we incorporate the influence of the commitment plan into the production cost variance calculation and propose a potential approach for commitment plan evaluation. In the proposed approach, the decision makers could use the approximation method to select some good commitment plans from a large set of commitment plans and use the simulation method to select the best commitment plan from those good commitment plans.

CHAPTER 1. OVERVIEW

1.1 Introduction

The deepening penetration of renewable energy helps society to solve the problems of limited quantities of fossil fuel and carbon dioxide emissions. However, it also brings a new problem, as most of the renewable energy sources are associated with high variability in energy production. For example, the wind power can be very high on some days but very low on the next days, or solar power can be high on a sunny day but low when it is raining. The International Energy Outlook 2017 from U.S Energy Information Administration (EIA) predicts the growth of electricity consumption and the growing share of renewable energy in the next 23 years [1]. To deal with the variability in electricity supply, people could increase the electricity storage units. However, an important property of electricity is that it cannot be stored easily. Large-scale storage of electricity is very expensive. So, electricity system operators should try to meet the current net load as much as they can. Net load is defined as the difference between the electricity demand and the output of renewable energy production. If there is a generating unit failure at the same time as high net load, then the system operator needs to increase production from expensive generators or to use expensive storage units. In those extreme situations, the production cost will skyrocket. Those high costs threaten the daily operation of electricity production systems. This concern can be addressed by considering and evaluating the risk associated with a given unit commitment plan. Unit commitment seeks for generator commitment schedules (unit commitment plans) that meet the net electricity load in the most cost-effective way while satisfying operational constraints on both the transmission and the generation system.

1.2 Problem statement

The risk of a unit commitment plan is evaluated by estimating the production cost of an electricity generation system following this unit commitment plan in unexpected situations. An unexpected situation could be a failure of scheduled generating units in the production system. Different unit commitment plans will react differently in those unexpected situations. Evaluating the performance of some unit commitment plans would be helpful to select one with acceptable operation costs in those situations. To assess the risk numerically, the variance of the production cost for a given unit commitment plan is the focus of this project.

1.3 Literature review

There already exist several numerical measures of risk which are used in UC. Loss of Load Probability (LOLP), Value at Risk (VaR) and Conditional Value at Risk (CVaR) are three different risk measures. LOLP is the most widely used measure for evaluating system-wide risk. However, LOLP also requires significant computational power. On the other hand, those measures like CVaR, which have been introduced more recently, are more computationally efficient. CVaR is the expected loss (cost) for the worst cases with probability lower than a given threshold value. More detail about these different risk measures can be found in Zheng's review of unit commitment [2]. Compared with CVaR, variance is describing production cost from a more general perspective, and variance along with the mean can also be used to calculate the CVaR of a normally distributed random variable. Over a long time period, the production cost is asymptotically normal according to central limit theory.

There are some previous researches about the variance associated with electricity production systems. In 1995, Shih verified the asymptotic behavior of electric power generation cost using simulation [3] and Shih et al. also developed a method to utilize the state space of Markov chain to calculate the asymptotic mean and variance of production cost [4]. In 1997, Ryan presented an approximation method for variance of electric power production cost based on renewal reward theory [5]. In 2000, Valenzuela and Muzumdar further explored the relationship between generating units and electric demand by incorporating time series analysis for electric demand and Monte Carlo simulation of the generating units' availability [6]. Their research suggested that the load variability plays an important role in the variation of production cost. In 2016, Kazemzadeh's research applied the approximation method from Ryan to the estimation of CVaR [7]. The approximation method for CVaR introduced in Kazemzadeh's research can be used when asymptotic assumption is appropriate. However, those previous studies did not examine the influence of the unit commitment plan to the production cost variance.

1.4 Model and power generating system

Assume there are n generators (generating units) in a given electric generation system. For each generation unit j there is a capacity c_j (in MW) and a variable production cost d_j (in \$/MW h) associated with it. The list of units is ordered as $d_1 \leq d_2 ... \leq d_n$. The times between unit state changes are modeled as exponentially distributed random variables with failure rate λ_j and repair rate μ_j. The minimum up and down time of the generating units are ignored in this model, but they are considered in the unit commitment problem. The minimum cost to satisfy a given electricity demand (net load) is obtained by dispatching the least expensive unit available first. Dispatching means to assign total load to generation units.

The variation from electricity demand and the uncertainty of generating unit availability are two important factors in estimating electricity production cost variance. To capture the variation in electricity demand, the estimation of the generation system's net electricity demand (DLE), also called the net load for the system, is used. The net load estimation seeks to predict the future load from historical data. In these predictions, the historical data is not just about the electricity demand. Weather data is also used in many net load estimation models. Net load estimation is a hard task because of the complexity of the electricity market. To capture the variation in the production system, the estimation of the state of a production system (SPE) is developed. In time t, the state of generating unit j reflects whether unit j is available for production at that instant. The state of the system is a big set which includes the state of each generation units. The state estimation of a production system is dealing with the combination of both the deterministic schedule and the stochastic availability of generation units. A vector S is used to represent the state of the production system at time t (hr). A variable S_j in this vector will be zero if either unit j is scheduled to be down or is failed. On the other hand, a vector I is used to represent only the condition of generating units in the system. A variable I_j is zero if unit j is failed and one otherwise.

By combining DLE and SPE, a production cost is estimated. For a given DLE and SPE in a time period, the corresponding part of the production cost is calculated by assigning load to units following the unit selection rule discussed previously in this section. For simplification purposes, the failure of the generator unit can be treated as independent of the unit's operational status. With this simplification, the demand side of system will not influence the failure of a generator unit in our model. Thus DLE and SPE can be done separately. The unit commitment plan will directly influence the state of the production system in a given time period. For some time periods, the old version of cost effective dispatch solutions which do not consider the unit commitment plan will be no longer feasible. In those time periods, the system has to dispatch more electricity production on more expensive units compared with the old version. As a result, this increases the total production cost as well.

1.5 Overview

To estimate the variance of electricity production with unit commitment plans, both the Monte Carlo simulation approach and an approximation approach based on renewal reward processes are considered in this project. Some of the results also appear in [8]. This paper will go over those procedures considered in simulation after the introduction to the input data of simulation in Chapter 2. The approximation method adapted from Ryan's method will be discussed with the derivation process in Chapter 3. The dataset from Kazemzadeh will be used as a test case to compare these two approaches. The result for a test case from both the simulation approach and the approximation approach will be listed in their sections of this paper. Finally, a conclusion will be given in Chapter 4 as well as some potential topics which interest us throughout the research process.

CHAPTER 2. MONTE CARLO SIMULATIONS

2.1 Simulation model

The result from a simulation program built in MATLAB was used to test the influence of unit commitment plan on electricity production cost variance and the hypothesis of normal distribution. The normal distribution is a crucial factor for the connection between variance and CVaR. Since the failure of a generating unit does not happen very often, it is very likely that for many runs there will be no failure of any unit. However, the time generating units is failed is the time that we want to focus on. To deal with this, the time horizon for simulation was extended by repeating the new load patter of a given day multiple times.

The input parameters for the simulation include: the new load profile; unit commitment (UC); time horizon considered (TL); repair rate (μ) and failure rate (λ); the capacity (c) and variable cost (d) of the generating units; and a penalty cost for not satisfying net load. The warm-up and shut-down times for the generating units were ignored in our model, but they have been taken care of in the UC. The program will output the expected value and standard deviation for electricity production cost for the given electricity production system. In addition, the a goodness-of-fit test result will also be provided for the hypothesis that production cost is normally distributed.

The way to decompose the problem discussed in the previous section can be treated as a guideline for the simulation. The initial state of the power generation system is determined by the forced outage rate of each unit p_j. The forced outage rate can be calculated by $\lambda_j/(\lambda_j + \mu_j)$ which represents the long-run proportion of time that unit j is unavailable due to the unexpected failure of the unit itself. Then, the program generates a time point for each generating unit when will change its state from failure rate and repair rate. The state of a generating unit represents whether it is ready to generate electricity power. At the same time, the system state will change as any generating unit state changes. There are system state changes due to failure of generating units (SCF). The unit commitment plan can cause system state changes as well. Different from the changes due to generating units, those system state changes due to the unit commitment plan (SCP) are predetermined. In our simulation process, the SCF is captured by the simulation of generators' failures while the SCP is embedded in the calculation of production cost for each time segment. In our model, the net load has been considered as a value that varies only hourly. This pattern is consistent with those unit commitment plans which schedule the generators (generating units) every hour.

In our simulation model, the time point that a generating unit's failure happens is marked as SCFT. The time points that the system state changes due to the unit commitment plan (SCPT) are at the beginning of every demand hour. Both SCPT and SCFT are considered as the time points in which the system change its state (CT). From a simulation point of view, CT determines when the production cost calculation will be different. The simulation is all about figuring out the next CT and calculating the cost between two CTs. Those steps are also listed in Algorithm 1.

Algorithm 1 Monte Carlo Simulation

```
while n < total number of trials [TS] do                    ▷ Do simulation multiple trials
    Step 1. Initialize two variables: t = 0, Cost = 0
    Step 2. Generate initial value for I according to their force outage rate (p)
    while t ≯ TL do                                          ▷ Simulate until reach end time
        Step 3. Generate a time (t*) for each machine when it changes state (repairing finishes or
        failure happens) according to I.
        Step 4. Set t_next = the closest next time point where a machine state will change or the
        demand will change.                                  ▷ Figure out next CT
        Step 5. Calculate the cost between (t, t_next) and add it to Cost.   ▷ Calculate cost in CT
        Step 6. Change I according to step 4, and update t = t_next
    end while
    Step 7. Record the Cost for this trial
end while
Step 8. Calculate variance and mean for all the Cost and perform K-S test
return the statistic result from Step 8                      ▷ Output from simulation
```

When testing the normality of the simulation result for production cost, both Pearson's chi-square test and the Kolmogorove-Smirnov (K-S) test are commonly used methods. However, the Pearson's chi-square test requires one to pool the results from different trials into several groups.

The number of groups, g, is a crucial factor for the result of this test, but choosing a good g is very hard. Thus, the K-S test is the one used as our criterion to determine the normality of the simulation result. The Kolmogorov-Smirnov test in statistics is used to test the null hypothesis that the empirical Cumulative Distribution Function (CDF) is close to the hypothesized CDF. Its test statistic is based on the maximum absolute difference between the two CDFs.

2.2 Results and comparison

2.2.1 Test simulator functionality

The functionality of the simulator was tested on the data set from Ryan's paper [5]. Since the model in [5] did not incorporate a UC plan, a unit commitment plan which schedules every unit to be available at all time periods was used in this subsection. For each TL value, the simulator simulated and recorded results 1000 times. The mean and standard deviation (square root of variance) for the simulation results are calculated and shown in Table 2.1. In this testing, the standard deviation was used to be consistent with the result format in [5]. As shown in Table 2.1, this simulator gets results close to the results in [5] when using same dataset.

Table 2.1 Comparison of results listed in Ryan (1997) and results from this simulator

	mean ($)		standard deviation ($)	
TL(hr)	Ryan	this simulator	Ryan	this simulator
168	9.300E+06	9.32E+06	2.600E+06	2.501E+06
672	3.720E+07	3.720E+07	6.400E+06	6.490E+06
2016	1.120E+08	1.113E+08	1.200E+07	1.212E+07

After testing simulator functionality, this simulator is ready for the test case from Kazemzadeh's paper [7].

2.2.2 Input data

The numerical experiments are based on Kazemzadeh's calculation for net load in [7]. Kazemzadeh used the load data and units from the modified 24-bus IEEE RTS-96 system with 32 generators and 38 transmission lines [9]. In [7], there are 20 scenarios which are constructed from 365 scenarios using the fast forward selection algorithm as shown in Figure 2.1. The 20 scenarios include one with a probability of 0.58 and three with probabilities of 0.09, 0.11, and 0.13, and the remainder with probability lower than 0.05. They correspond to the High-, Medium- and Low- probability in Figure 2.1.

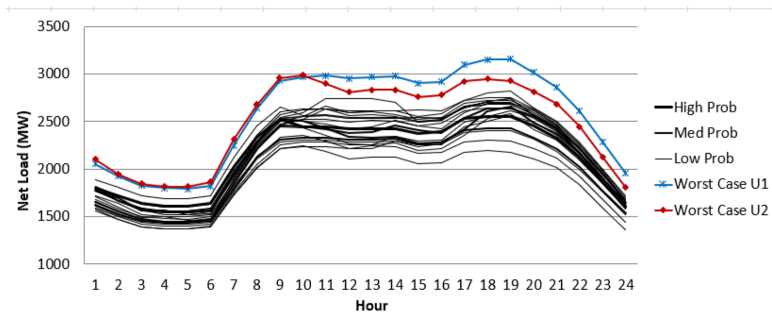

Figure 2.1 Net load cycles (20 scenarios)

For simplification purposes, the numerical testing is focused on the net demand of the High-probability scenario (Figure 2.2). The numerical values for this net load data are provided in Table 2.2.

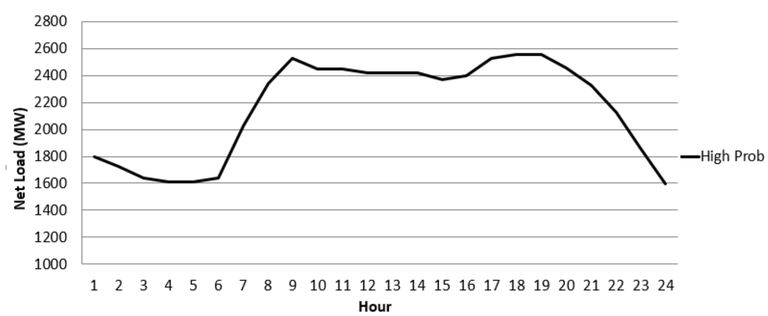

Figure 2.2 Net load cycle (high-probability scenario)

The data for production system is in Table 2.3.

The simulation tests two kinds of different unit commitment plan results from Kazemzadeh's paper. These unit commitment plans are generated from a stochastic program model with CVaR (SP_CVaR in short) and the Robust Optimization with uncertainty set (RO_U1). Kazemzadeh tried three different parameter settings for both methods. In this paper, the SP_CVaR without

Table 2.2 Net load (highest probability)

Period	Net load required (MWh)
1	1800
2	1722
3	1637
4	1608
5	1608
6	1637
7	2028
8	2337
9	2526
10	2446
11	2448
12	2421
13	2422
14	2420
15	2371
16	2396
17	2529
18	2556
19	2558
20	2452
21	2324
22	2123
23	1862
24	1596

specification refers to SP_CVaR with gamma=0.002 (SP_CVaR_002) and RO_U1 refers to RO_U1 with delta=0.05 (RO_U1_05). The section where all six unit commitment plans is discussed will be using the same format to name those unit commitment plans. The case without considering the unit commitment plan is called the default plan. In the default plan, every generating unit is scheduled to be available in every hour. The default plan is considered in order to figure out the influence of introducing the unit commitment plan. The unit commitment plans can be founded in APPENDIX A.

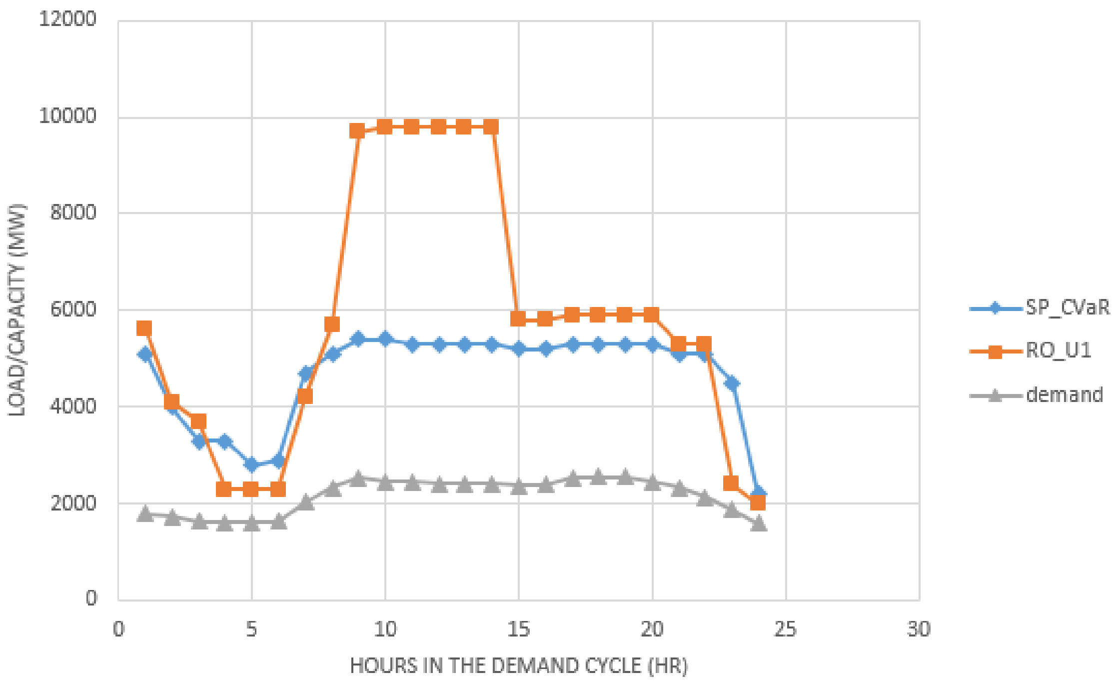

Figure 2.3 Net load and total capacity committed for different commitment plans

The total capacity unit committed for every hour is plotted in Figure 2.3. It is clear that the RO_U1 unit commitment plan schedules generators with much higher total capacity between hours eight and fifteen. This coincides with the property of two unit commitment plans described in Kazemzadeh's paper where the Robust Optimization method tries to guarantee the performance of the system in the worst scenario. The minimal dispatch cost for every unit commitment plans (assuming no failure) is shown in Figure 2.4. From this figure, the total production cost of two unit commitment plans should not have much difference in the long run.

2.2.3 Influence on total production cost

The expected production cost with a unit commitment plan should be higher than with the default plan. The reduced number of units available for a given hour will potentially require using more expensive units than with the default plan to satisfy net demand. This is the case when the model only captures the production costs and ignores the operational costs required for keeping

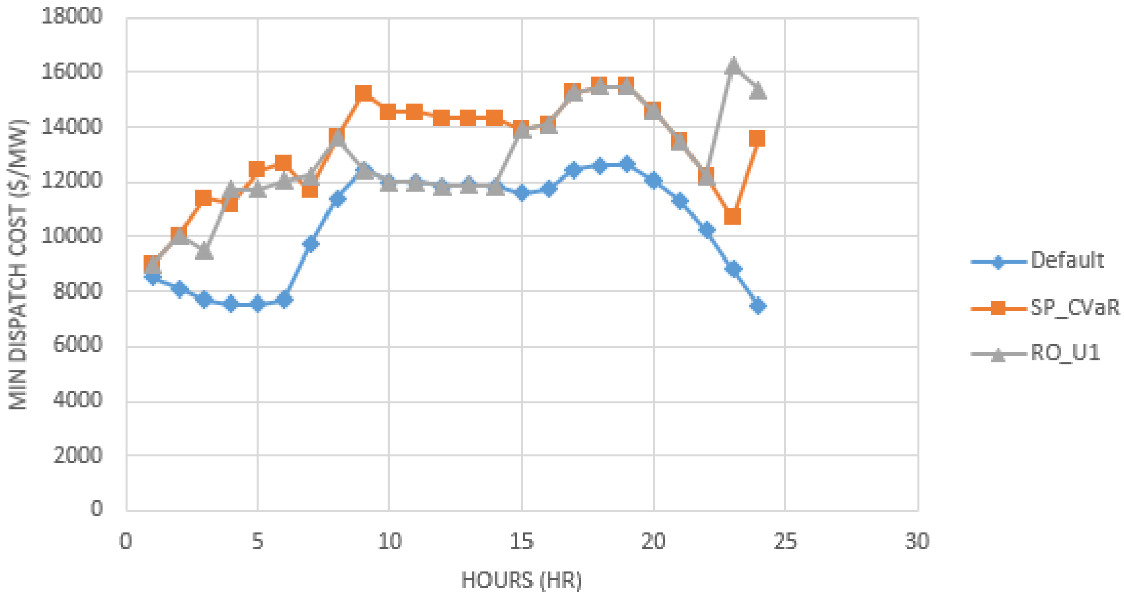

Figure 2.4 The minimal dispatch cost(without failure) from SP_CVaR and RO_U1

generating units ready for production. The cost required for keeping generating units available is deterministic for a given commitment plan. Our goal is to compare the variance of production cost; we choose to ignore those deterministic parts for now.

Figure 2.5 shows the linear relationship between expected cost and time length in consideration. Table 2.4 shows the mean of total production cost that results from the simulation. Just as what was expected, the expected production costs are about the same for SP_CVaR and RO_U1.

2.2.4 Variance of production cost

The simulation results show that the increase in variance tends to have the constant slope over the long run. The slope of variance is estimated by dividing TL into variance. This coincides with the simulation experiment result from Shih [3]. At the same time, both SP_CVaR and RO_U1 commitment plan have greater variance than the default plan. The variance slope of RO_U1 is about six times of the variance slope of SP_CVaR.

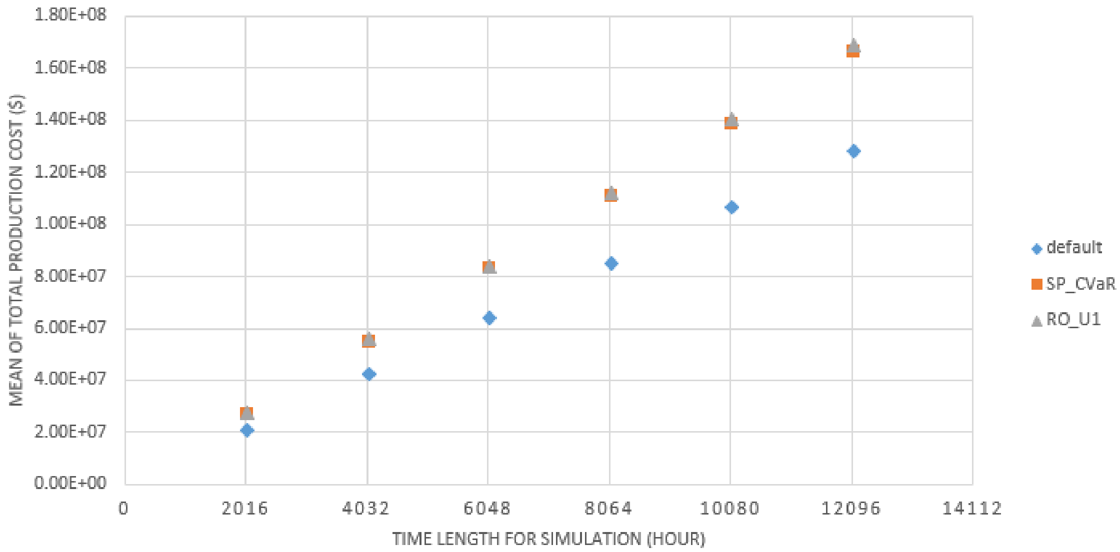

Figure 2.5 Simulation results: mean cost comparison

2.2.5 Normality of simulation results

When applying the simulation method with TS = 1000 and TL = 12096 hrs, running a unit commitment simulation takes more than half an hour. The simulation method would require much more computation power and increase the time to finish computation when the size of the focused system increased. The approximation method could be a good approach to handle those more complex systems. The approximation methods from both Shih [4] and Ryan [5] make the assumption about the asymptotic property of the total cost. Thus, we need to test if this assumption is appropriate for our system. Before running any tests for normality, the histogram of the default unit commitment plan was studied to explore how the shape of the production cost distribution is changing with TL. Figures 2.6, 2.7, 2.8, 2.9 and 2.10 show how the distribution of total production cost approaches a normal distribution as TL increases.

Instead of looking at the histogram of those simulation results one by one, the K-S test is used for testing the normality for all the simulation results. With a significant value of 5%, the hypothesis test result for the K-S test from simulation shows that those results with the default unit commitment plan pass the test when TL reaches 12096 hrs; those results with SP_CVaR pass

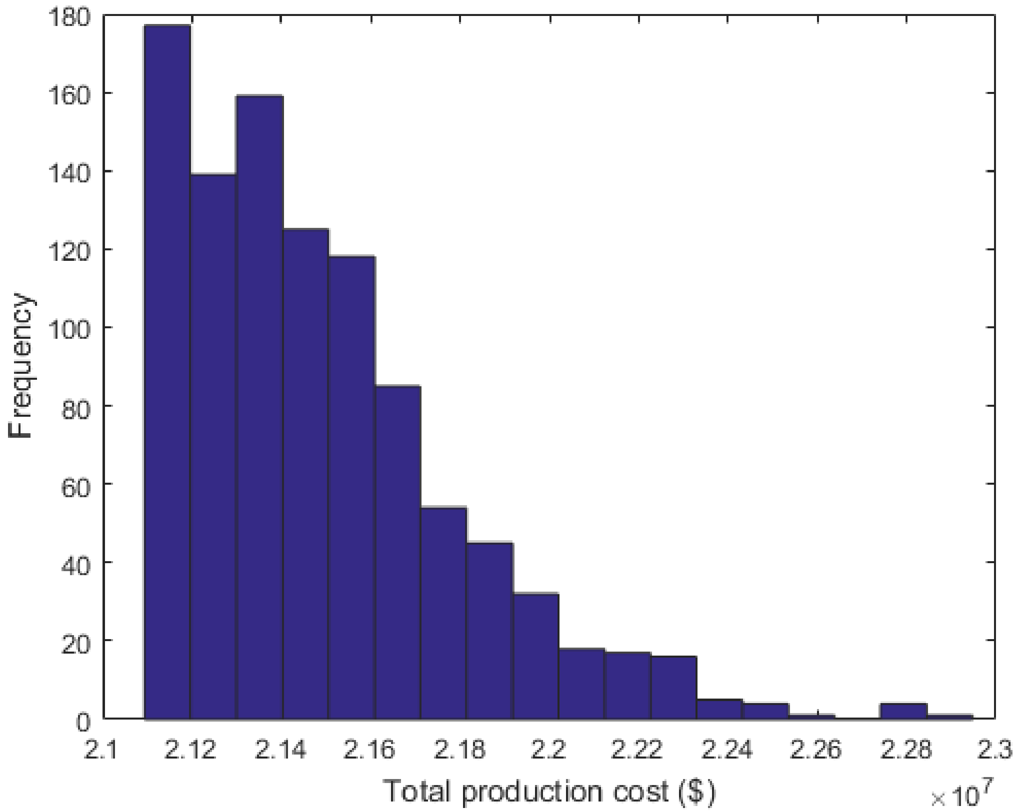

Figure 2.6 Simulation result: histogram of production cost (2016 hr)

the test when the time horizon reaches 4032 hr, but those results with RO_U1 fail to pass the K-S test even when TL reaches 12096 hr. Thus, those unit commitment plans can influence the time required for total production cost to approach a normal distribution.

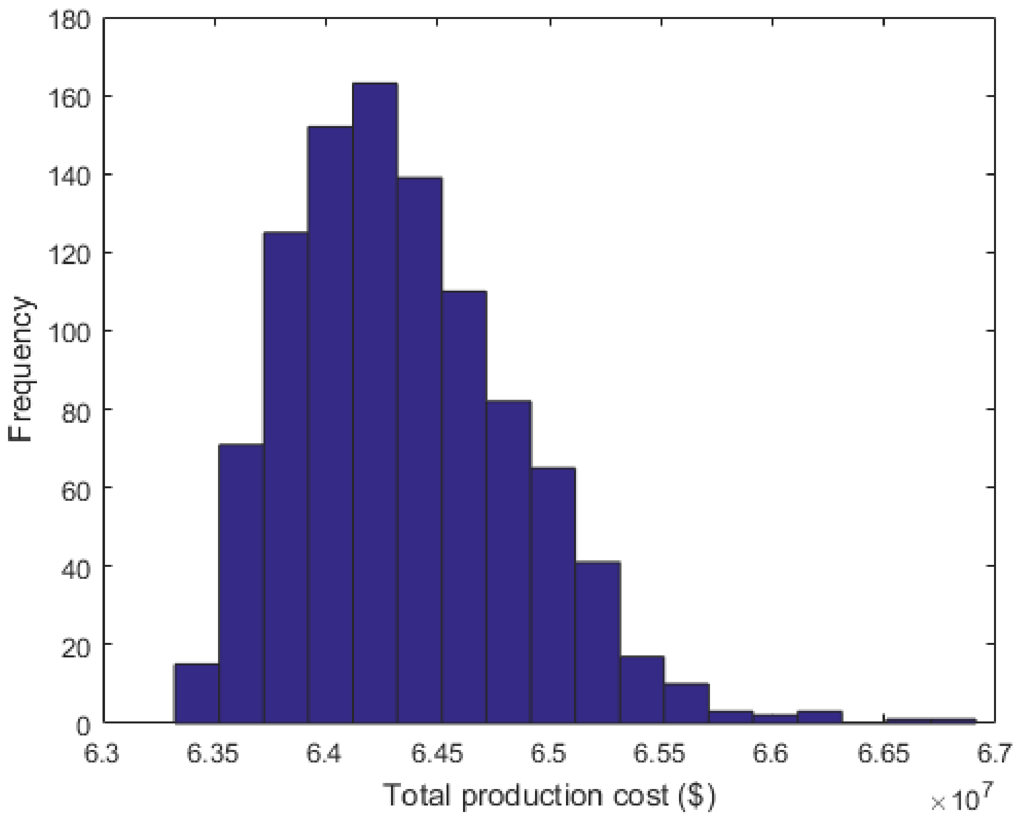

Figure 2.7 Simulation result: histogram of production cost (6048 hr)

Table 2.3 Generating units

unit	capacity (MW)	MTTF ($hours$)	MTTR ($hours$)	d ($\$/MWh$)
1	1000	1440	160	4.5
2	900	1300	150	5
3	700	1200	130	5.5
4	600	1100	110	5.75
5	600	1100	110	5.75
6	500	1150	110	6
7	500	1150	110	6
8	500	1150	110	6
9	400	1100	90	8.5
10	400	1100	90	8.5
11	400	1100	90	8.5
12	400	1100	90	8.5
13	400	1100	90	8.5
14	300	1000	70	10
15	200	850	48	14.5
16	200	850	48	14.5
17	200	850	48	14.5
18	200	850	48	14.5
19	200	850	48	14.5
20	100	600	48	22.5
21	100	600	48	22.5
22	100	600	48	22.5
23	100	600	48	22.5
24	100	600	48	22.5
25	100	600	48	22.5
26	100	600	48	22.5
27	100	350	12	44
28	100	350	12	44
29	100	350	12	44
31	100	350	12	44
32	100	350	12	44

Table 2.4 Mean of total production cost ($) from simulation

TL(hr)	default	SP_CVaR	RO_U1
2016	2.15E+07	2.78E+07	2.83E+07
4032	4.29E+07	5.56E+07	5.64E+07
6048	6.44E+07	8.34E+07	8.46E+07
8064	8.58E+07	1.11E+08	1.13E+08
10080	1.07E+08	1.39E+08	1.41E+08
12096	1.29E+08	1.67E+08	1.69E+08

Table 2.5 Total production cost variance ($\$^2$) from simulation

TL(hr)	default	SP_CVaR	RO_U1
2016	9.55E+10	2.75E+11	1.84E+12
4032	1.61E+11	4.88E+11	3.14E+12
6048	2.58E+11	8.51E+11	5.24E+12
8064	3.41E+11	1.10E+12	6.89E+12
10080	4.25E+11	1.38E+12	8.97E+12
12096	5.31E+11	1.67E+12	9.81E+12

Table 2.6 Production cost variance slopes (σ^2/TL) from simulation

TL(hr)	default	SP_CVaR	RO_U1
2016	4.74E+07	1.36E+08	9.12E+08
4032	4.00E+07	1.21E+08	7.78E+08
6048	4.26E+07	1.41E+08	8.66E+08
8064	4.23E+07	1.37E+08	8.54E+08
10080	4.22E+07	1.36E+08	8.90E+08
12096	4.39E+07	1.38E+08	8.11E+08

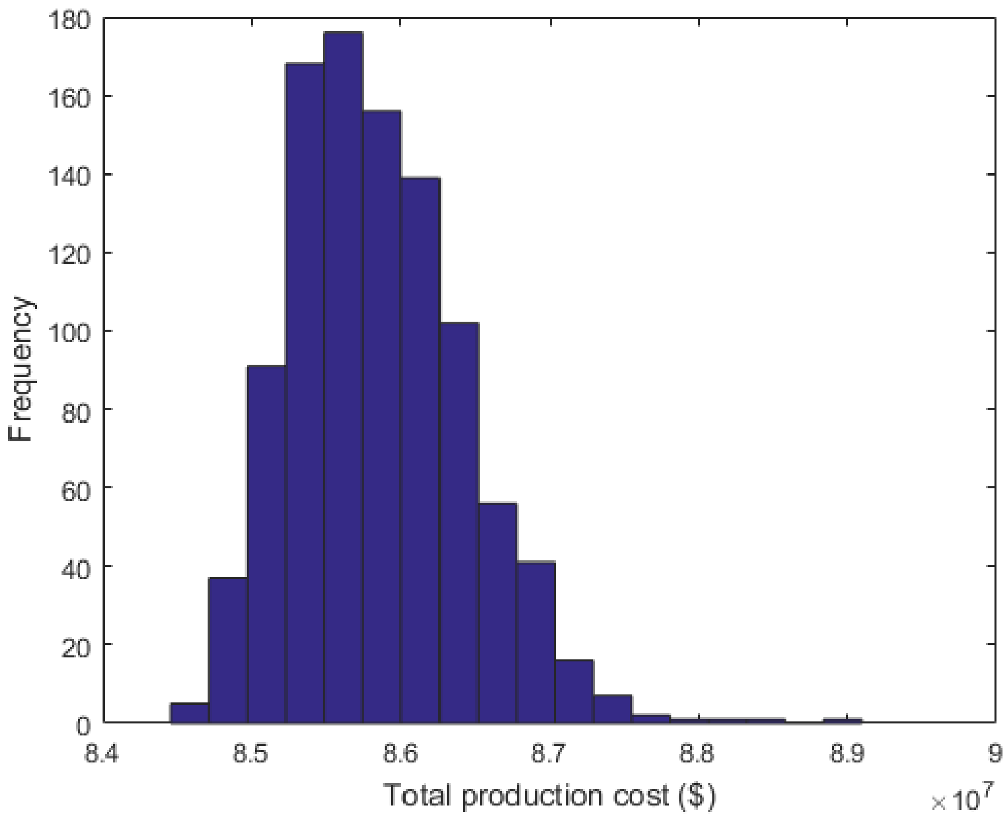

Figure 2.8 Simulation result: histogram of production cost (8064 hr)

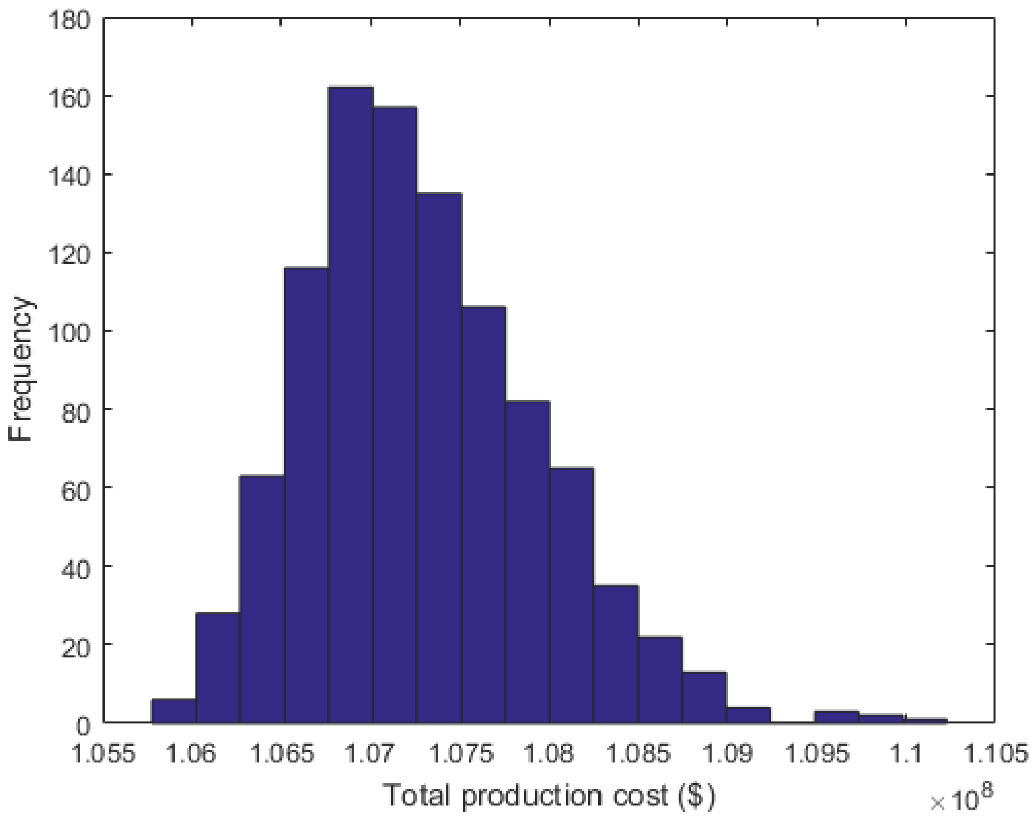

Figure 2.9 Simulation result: histogram of production cost (10080 hr)

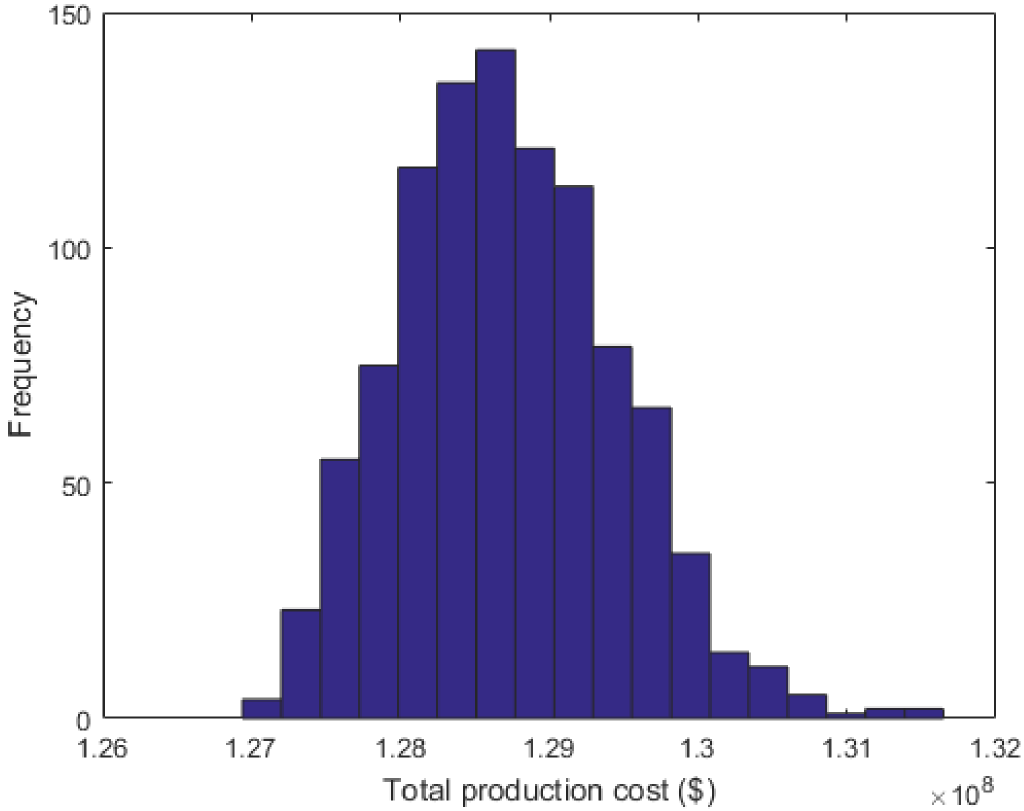

Figure 2.10 Simulation result: histogram of production cost (12096 hr)

CHAPTER 3. APPROXIMATION BASED ON RENEWAL REWARD PROCESS

3.1 Assumption

Different from the system simulated, Ryan's approximation method [5] was built with two more assumptions. The assumptions will be listed below with some explanation.

Assumption one: No more than one scheduled generating unit will fail at one time period. With this assumption, the changing of the generating unit availability state (I) is a renewal process. In probability theory, a counting process with $i.i.d$ times between successive events is called a renewal process.

Assumption two: The failure happens at the beginning of a load cycle.
With this assumption, the calculation of production cost associated with a given time segment is simplified.

3.2 Model

In this approximation method, the total production cost in TL is calculated based on the production cost of renewal cycles. Figure 3.1 shows a single renewal cycle with length T.

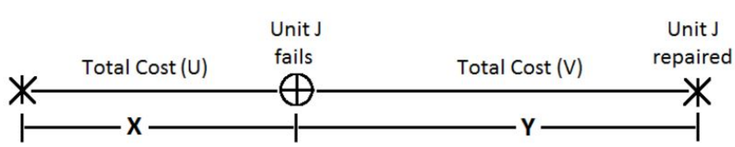

Figure 3.1 A renewal cycle

In time segment X, all the production units scheduled are available and total cost corresponding to this segment is U. For any instant of time t in X, the cost function $u(t)$ is used to represent the cost for that instant t. On the other hand, the time segment Y represents the time in a renewal cycle where one generating unit j is unavailable and the cost for this period is V. In the same manner, the cost function $v_j(t)$ is used to represent the cost for instant t in Y when generating unit j is unavailable. We use Z to represent the total cost associated with a single renewal cycle, where $Z = U + V$.

Based on previous research, the increase of production cost variance is in proportion to the increase of TL. In the long run, how fast the variance increases as TL increases (also called the slope of the variance) will dominate the total variance. To estimate the variance of total production cost, the slope of the variance (a) is our main focus. A study from Brown and Solomon [10] shows that the variance's slope of the total reward can be separated into three parts. The formula shown below is the equation for parameter a written in the form of parameters related to a renewal cycle from [5].

$$a = \frac{E(T^2)E(Z)^2}{E(T)^3} - \frac{2E(ZT)E(Z)}{E(T)^2} + \frac{E(Z^2)}{E(T)}$$

The next few subsections will focus on calculations for all the parameters needed for this formula. Most of these equations used are coming from [5]. In this paper, some extra explanation and deviations are added to make it easier to understand.

3.2.1 The expected renewal cycle length

This section focuses on the expected length of a renewal cycle $E(T)$ and the expected length squared of a renewal cycle $E(T^2)$. Since the length of Y is independent of the length of X, we can write the equations below:

$$E(T) = E(X) + E(Y)$$

$$E(T^2) = E(X^2) + 2E(X)E(Y) + E(Y^2)$$

By the property of exponential distribution, X is exponentially distributed with parameter $\Lambda = \sum_j \lambda_j$ and Y is exponentially distributed with μ_j accordingly. When a generator fails, it happens to be machine j with probability λ_j/Λ. So, the expected values for X and Y can be found by following equations:

$E(X) = 1/\Lambda \qquad E(X^2) = 2/\Lambda^2 \qquad E(Y) = \sum_j \frac{\lambda_j}{\Lambda\mu_j} \qquad E(Y^2) = E_J[E(Y^2|J)] = \sum_j \frac{2\lambda_j}{\mu_j^2\Lambda}$

3.2.2 The expected cost associated with a single renewal cycle

This section focuses on the calculation related to the expected cost when all scheduled generators are available, $E[U]$, and the expected cost when one machine is failed, $E[V]$, in a single renewal cycle. For computational convenience, time segments X and Y are treated as the combination of several demand cycles with length Δ and a remainder part with length R, where $X = R + K\Delta$ and $Y = R_j + K_j\Delta$. Figure 3.2 shows how a time segment is divided into Δ and R .

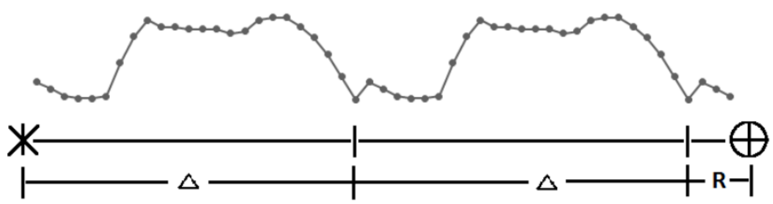

Figure 3.2 A time segment

By some derivations based on the exponential distribution, the expected number of load cycles is $E[K] = \dfrac{e^{-\Lambda\Delta}}{1-e^{-\Lambda\Delta}}$ and the density function of R is $f(r) = \dfrac{\Lambda e^{-\Lambda r}}{1-e^{-\Lambda\Delta}}$ when no generator is failed. $E[U]$ can be written in the formula shown below:

$$E[U] = E[K]E[U|T = \Delta] + E[U|T = R] = \frac{e^{-\Lambda\Delta}}{1-e^{-\Lambda\Delta}} \int_0^\Delta u(t)dt + E\left[\int_0^R u(t)dt\right]$$

The term $E\left[\int_0^R u(t)dt\right]$ can be treated as a cumulative conditional expected value. Using U_i to represent $\int_0^i u(t)dt$, the term $E\left[\int_0^R u(t)dt\right]$ can be written as follows:

$$\begin{aligned} E\left[\int_0^R u(t)dt\right] &= \int_0^\Delta \int_0^r u(t)dt\, f(r)dr \\ &= \sum_0^{\Delta-1} \int_i^{i+1} \int_0^r u(t)dt\, f(r)dr \\ &= \sum_0^{\Delta-1} \int_i^{i+1} (u_{i+1}(r-i) + U_i)\frac{\Lambda e^{-\Lambda r}}{1-e^{-\Lambda\Delta}}dr \end{aligned} \tag{3.1}$$

Similarly, the calculation for $E[V]$ has the same structure:

$$E[V] = P(J = j)E[V|J = j] = \frac{\lambda_j}{\Lambda}\left\{\frac{e^{-\mu_j\Delta}}{1 - e^{-\mu_j\Delta}}\int_0^\Delta v_j(t)dt + E\left[\int_0^{R_j} v_j(t)dt\right]\right\} \tag{3.2}$$

Using $V_{j,i}$ to represent $\int_0^i v_j(t)dt$, the $E\left[\int_0^{R_j} v_j(t)dt\right]$ can be written as follows:

$$\begin{aligned} E\left[\int_0^{R_j} v_j(t)dt\right] &= \int_0^\Delta \int_0^r v_j(t)dt\, f_j(r)dr \\ &= \sum_0^{\Delta-1}\int_i^{i+1}\int_0^r v_j(t)dt\, f_j(r)dr \\ &= \sum_0^{\Delta-1}\int_i^{i+1}(v_{j,i+1}(r-i) + V_{j,i})\frac{\mu_j e^{-\mu_j r}}{1 - e^{-\mu_j\Delta}}dr \end{aligned} \tag{3.3}$$

The expected cost associated with single renewal cycle can be calculated by adding those two expectations together $E(Z) = E[U] + E[V]$.

3.2.3 The second moment of the cycle cost

To compute $E(Z^2)$, the missing part is just $Var(Z)$ according to the equation below:

$$E(Z^2) = Var(Z) + E[Z]^2$$

$$Var(Z) = Var(U) + Var(V)$$

$$Var(U) = Var(K)\left[\int_0^\Delta u(t)dt\right]^2 + Var\left[\int_0^R u(t)dt\right]$$

Since K is geometrically distributed, $Var[K] = \dfrac{e^{-\Lambda\Delta}}{(1 - e^{-\Lambda\Delta})^2}$ and

$$Var\left[\int_0^R u(t)dt\right] = E\left[\int_0^R u(t)dt^2\right] - E\left[\int_0^R u(t)dt\right]^2,$$

where:

$$\begin{aligned} E\left[\int_0^R u(t)dt^2\right] &= \int_0^\Delta \left[\int_0^r u(t)dt\right]^2 f(r)dr \\ &= \sum_0^{\Delta-1}\int_i^{i+1}\int_0^r u(t)dt^2 f(r)dr \\ &= \sum_0^{\Delta-1}\int_i^{i+1}\left[u_{i+1}(r-i) + \sum_{k=0}^{i} u_k\right]^2 \frac{\Lambda e^{-\Lambda r}}{1 - e^{-\Lambda\Delta}}dr \end{aligned} \tag{3.4}$$

The calculation for $Var(V)$ follows the same structure:

$$Var(V) = E_j\left[Var(V|J)\right] + Var_j\left[E(V|J)\right]$$

$$E_j\left[Var(V|J)\right] = \sum_j \frac{\lambda_j}{\Lambda}\left\{Var(K_j)\left[\int_0^{\Delta} v_j(t)dt\right]^2 + Var\left[\int_0^{R_j} v_j(t)dt\right]\right\}, \tag{3.5}$$

where:

$$Var(K_j) = \frac{e^{-\mu_j\Delta}}{(1-e^{-\mu_j\Delta})^2}$$

$$Var\left[\int_0^{R_j} v_j(t)dt\right] = E\left[\int_0^{R_j} v_j^2(t)dt\right] - E\left[\int_0^{R_j} v_j(t)dt\right]^2$$

$$\begin{aligned} E\left[\int_0^{R_j} v_j^2(t)dt\right] &= \int_0^{\Delta}\int_0^{r} v_j(t)^2 dt\, f_j(r)dr \\ &= \sum_0^{\Delta-1}\int_i^{i+1}\int_0^{r} v_j(t)^2 dt\, f_j(r)dr \\ &= \sum_0^{\Delta-1}\int_i^{i+1}(v_{j,i+1}^2(r-i) + \sum_{k=0}^{i} v_{j,k}^2)\frac{\mu_j e^{-\mu_j r}}{1-e^{-\mu_j\Delta}}dr \end{aligned} \tag{3.6}$$

$$Var_j[E(V|J)] = \sum_{j=1}^{n}\frac{\lambda_j}{\Lambda}E(V|J=j)^2 - \left[\sum_{j=1}^{n}\frac{\lambda_j}{\Lambda}E(V|J=j)\right]^2$$

3.2.4 The cross moment of cost and renewal cycle length

The expected product of Z and T is:

$$E(ZT) = E(UX) + E(U)E(Y) + E(VY) + E(U)E(X)$$

$$E(UX) = \left[\Delta E(K^2) + E(K)E(R)\right]\int_0^{\Delta} u(t)dt + \Delta E(K)E\left[\int_0^{R} u(t)dt\right] + E\left[R\int_0^{R} u(t)dt\right],$$

where $E[K^2] = \dfrac{e^{-\Lambda\Delta}(1+e^{-\Lambda\Delta})}{(1-e^{-\Lambda\Delta})^2}$, $E[R] = \dfrac{\left[1-(\Lambda\Delta+1)e^{-\Lambda\Delta}\right]}{\Lambda(1-e^{-\Lambda\Delta})}$ and

$$\begin{aligned} E\left[R\int_0^{R} u(t)dt\right] &= \sum_0^{\Delta-1}\int_i^{i+1}\int_0^{r} ru(t)dt\, f(r)dr \\ &= \sum_0^{\Delta-1}\int_i^{i+1}(u_{i+1}(r^2-ir) + U_i r)\frac{\Lambda e^{-\Lambda r}}{1-e^{-\Lambda\Delta}}dr \end{aligned} \tag{3.7}$$

Similarly, $E(VY)$ can be found, using $E[K_j^2] = \dfrac{e^{-\mu_j\Delta}(1+e^{-\mu_j\Delta})}{(1-e^{-\mu_j\Delta})^2}$ and $E[R_j] = \dfrac{\left[1-(\mu_j\Delta+1)e^{-\mu_j\Delta}\right]}{\mu_j(1-e^{-\mu_j\Delta})}$, as

$$E(VY|j=J) = \left[\Delta E(K_j^2) + E(K_j)E(R_j)\right]\int_0^\Delta v_j(t)dt + \Delta E(K_j)E\left[\int_0^{R_j} v_j(t)dt\right] + E\left[R_j\int_0^{R_j} v_j(t)dt\right]$$

3.3 Results and comparison

3.3.1 Discussion of test case

In this results section, the approximated method will be applied to the same test case as in Section 2.2. Before applying the approximated method, we check if assumption one is appropriate for this case. The forced outage rate p is used to calculate the total proportion of time for one unit failed ($FP1$) or no unit failed ($FP0$). $FP1_j$ is the proportion of time when only unit j is failed. The formulas for $FP1$, $FP1_j$ and $FP0$ are listed below:

$$FP0 = \prod_{j=1}^{n}(1-p_j)\ ;\ FP1 = \sum_{j=1}^{n} p_j \prod_{i\neq j}(1-p_i)\ ;\ FP1_j = p_j\prod_{i\neq j}(1-p_i)$$

In our test case, the $FP0$ value is 0.104494. This means there is at least one unit failed most of the time. The calculation results for $FP1_j$ are shown in Table 3.1. $FP1_j$ is the proportion of time for only unit j is failed. Summing up the proportion of time for each unit gives $FP1 = 0.2455$. Thus, this approximation method seems not well suited for this test case. However, we still apply this approximation method to see the impact of violating this assumption.

Assumption two is discussed in Section 3.3.3.

3.3.2 Compare with simulation without two assumptions

The approximation approach has been implemented in MATLAB. The calculation result in Table 3.2 has expected total production which is very close to the mean cost from the simulation approach. Different from the simulation result without those two assumptions, the approximation approach has a ratio between the slope of σ^2 from RO_U1 and SP_CVaR at about 2.4. In other words, the assumptions considered have different impacts on the two unit commitment plans we discussed. This means that the approximation method developed will lose some accuracy compared with the simulation approach, while we want to assess the difference between two unit commitment plans. However, it could be potentially useful for selecting some outstanding unit commitment plan before the implementation of the time consuming simulation method.

3.3.3 Simulation with assumptions

When considering the influence of these two assumptions, a simulation model is developed to test out the two cases where assumptions are included progressively. Table 3.3 shows the results from all cases. When we incorporate only assumption one, the variance from both unit commitment plans have significant changes. The ratio between production cost variance from RO_U1 and SP_CVaR changes from 6 to 2.4. This shows that this assumption could have different influence to production cost with different unit commitment plans. When considering assumption two, the variance of production cost is increased by 34% with both unit commitment plans while the ratio between production cost variance from RO_U1 and SP_CVaR stays the same at around 2.4. An interesting fact is that the simulation method produces a result very close to the approximation method's when simulated with both assumptions.

From the previous discussion and simulation result, the assumption one is the cause for the ratio difference between two approaches. There could be some further research about the influence of relaxing assumption one in approximation method. For example, when can we accept assumption one?

3.3.4 Test on more unit commitment plans

Even though the approximation method does not reflect the simulation result very closely, it can still be useful when there remains some relationship between the unit commitment plans. To check out this idea, both the approximation method and the simulation method are applied to the other four unit commitment plans from Kazemzadeh's research. Figure 3.3 shows the slope of mean vs. the slope of variance from the simulation results. The slope of each unit commitment plan is calculated by applying linear regression on all 6000 simulation results (one thousand for each TL). At the same time, the confidence interval for each slope value is calculated and shown in Figure 3.4. The calculation of the confidence interval for variance is done by the Jackknife re-sampling method [11]. For each data point, our program calculated a ”variance” value without this data point. Then we treat those ”variance” values as our data points to calculate the confidence interval. Unit commitment plans RO_U1_01 and RO_U1_10 are distinguishable from other unit commitment plans. As seen in Figure 3.4, none of those unit commitment plans have both smaller mean and standard deviation than RO_U1_10 and RO_U1_10. Those two interesting unit commitment plans are also distinguishable in Figure 3.5. Following the same rule, the decision maker would choose those two unit commitment plans. If people could choose more unit commitment plans based on Figure 3.5, it would be even less likely that they will miss those two unit commitment plans. This suggests that the approximation method could be useful to select some relatively good unit commitment plans which helps to save some computational power in the simulation stage.

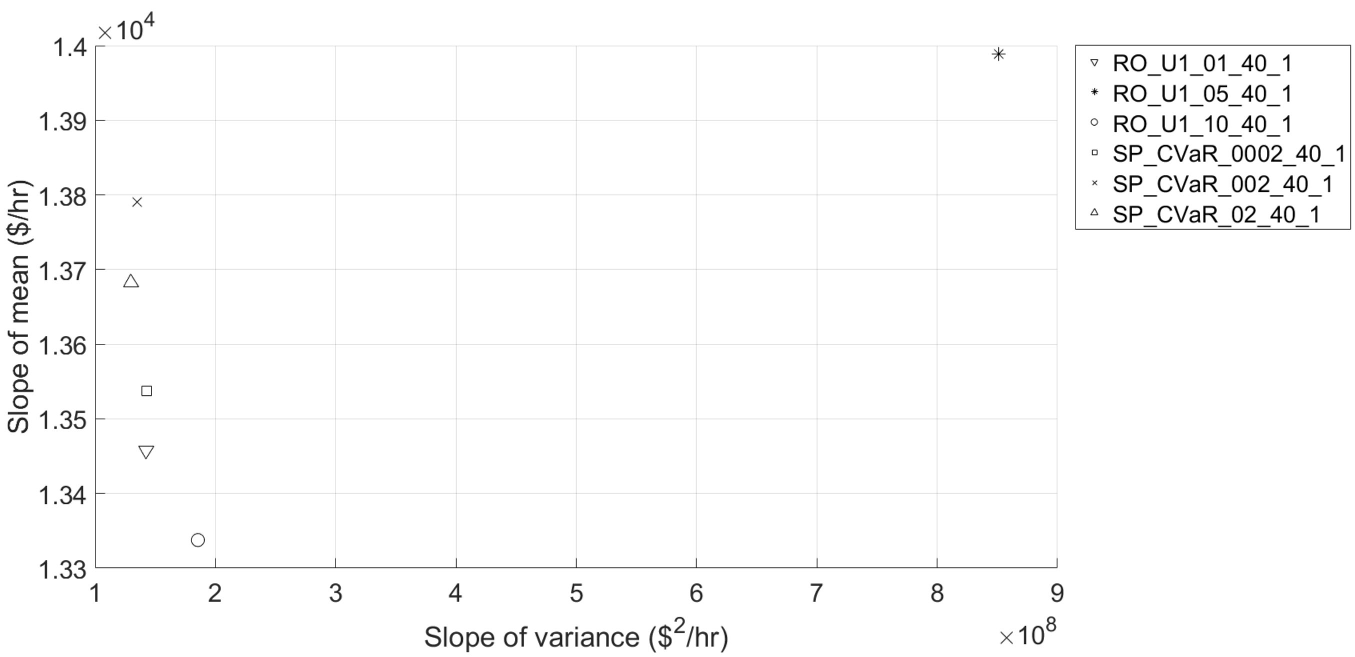

Figure 3.3 Simulation results for other unit commitment plans

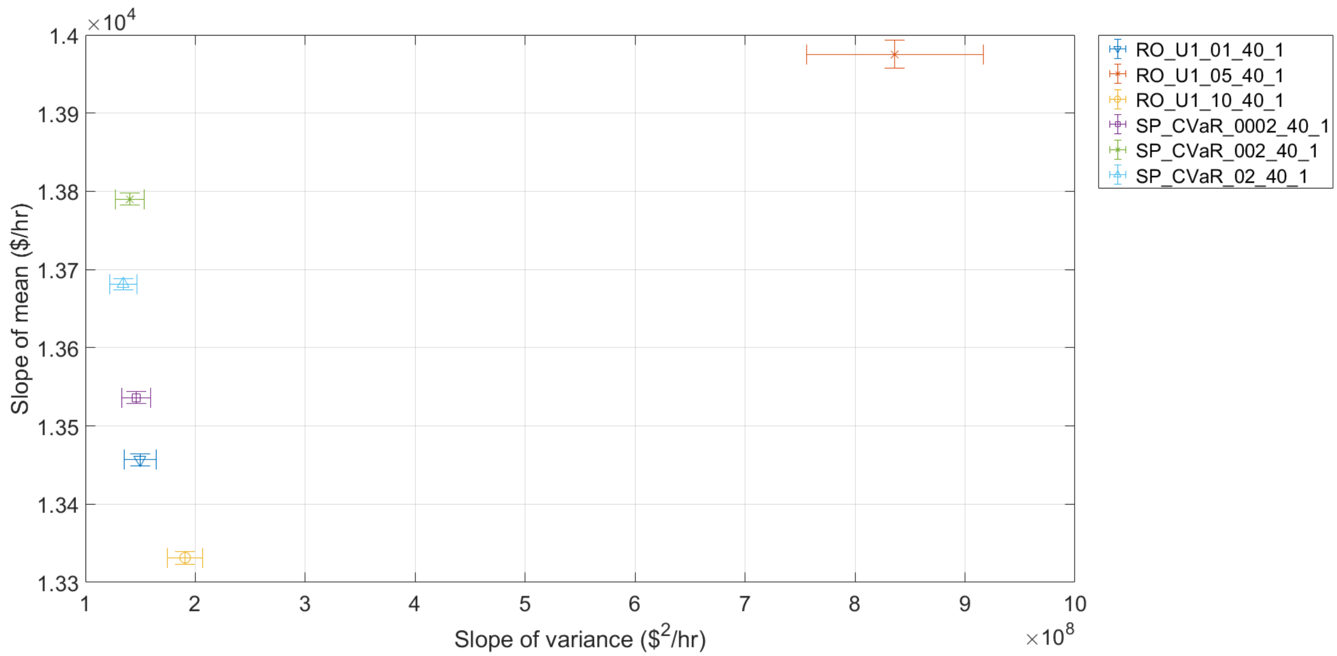

Figure 3.4 Simulation results for other unit commitment plans (with confidence intervals)

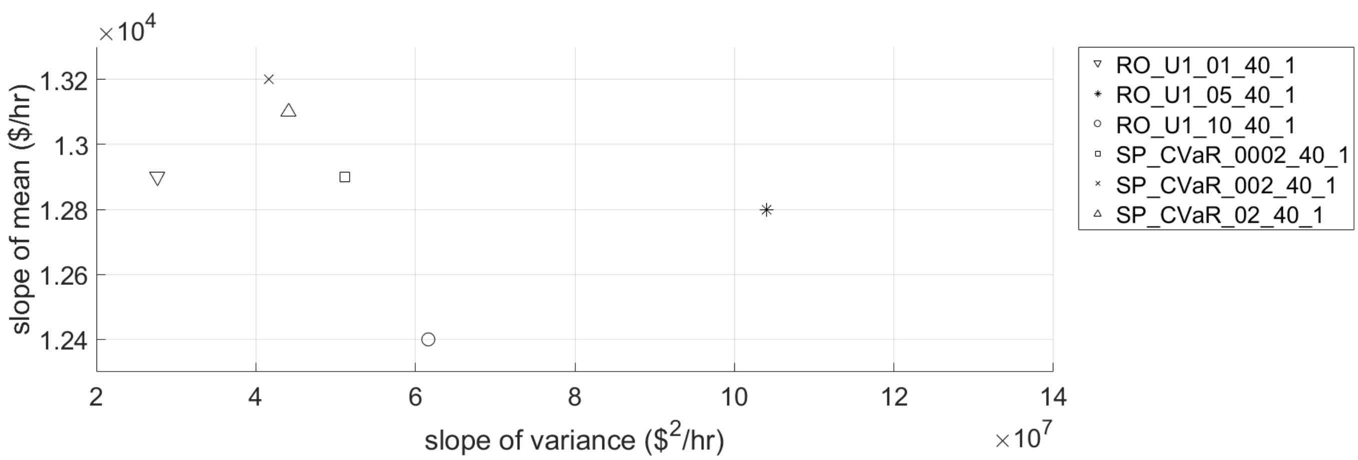

Figure 3.5 Approximation results for other unit commitment plans

Table 3.1 Calculation results for proportion of time that one specific unit failed ($FP1_j$)

unit j	p_j	1-p_j	$FP1_j$
1	0.1000	0.9000	0.0116
2	0.1034	0.8966	0.0121
3	0.0977	0.9023	0.0113
4	0.0909	0.9091	0.0104
5	0.0909	0.9091	0.0104
6	0.0873	0.9127	0.0100
7	0.0873	0.9127	0.0100
8	0.0873	0.9127	0.0100
9	0.0756	0.9244	0.0085
10	0.0756	0.9244	0.0085
11	0.0756	0.9244	0.0085
12	0.0756	0.9244	0.0085
13	0.0756	0.9244	0.0085
14	0.0654	0.9346	0.0073
15	0.0535	0.9465	0.0059
16	0.0535	0.9465	0.0059
17	0.0535	0.9465	0.0059
18	0.0535	0.9465	0.0059
19	0.0535	0.9465	0.0059
20	0.0741	0.9259	0.0084
21	0.0741	0.9259	0.0084
22	0.0741	0.9259	0.0084
23	0.0741	0.9259	0.0084
24	0.0741	0.9259	0.0084
25	0.0741	0.9259	0.0084
26	0.0741	0.9259	0.0084
27	0.0331	0.9669	0.0036
28	0.0331	0.9669	0.0036
29	0.0331	0.9669	0.0036
30	0.0331	0.9669	0.0036
31	0.0331	0.9669	0.0036
32	0.0331	0.9669	0.0036

Table 3.2 Approximation result

UC Plan	Expected cost (\$) at TL=2016	Variance σ^2(\$^{2}) at TL=2016	Slope of $\sigma^2(\$^2/hr)$
SP_CVaR	2.66E+07	8.39E+10	4.16E+07
RO_U1	2.58E+07	2.11e+11	1.04E+08

Table 3.3 Variance estimates from simulation with and without assumptions ($\$^2$)

	With both assumptions		Witn assumption one only		Without assumption	
TL(hr)	RO_U1	SP_CVaR	RO_U1	SP_CVaR	RO_U1	SP_CVaR
2016	2.09E+11	8.77E+10	1.59E+11	6.60E+10	1.84E+12	2.75E+11
4032	3.97E+11	1.66E+11	3.04E+11	1.26E+11	3.14E+12	4.88E+11
6048	5.57E+11	2.27E+11	4.06E+11	1.67E+11	5.24E+12	8.51E+11
8064	8.28E+11	3.09E+11	6.21E+11	2.33E+11	6.89E+12	1.10E+12
10080	1.02E+12	4.40E+11	7.72E+11	3.33E+11	8.97E+12	1.38E+12
12096	1.19E+12	4.83E+11	8.77E+11	3.60E+11	9.81E+12	1.67E+12

CHAPTER 4. CONCLUSION

4.1 The influence of the unit commitment plan on the variance of total cost

Both simulation and approximation approaches were discussed in this paper. When introducing the unit commitment plan in the generating unit schedule, the total cost will increase as well as the variance associated with the total cost as supported by simulation results. In addition, the time needed for the total production cost to approach a normal distribution will be different for different UC plans. When comparing the Stochastic Program with CVaR and Robust Optimization solution from [7], The UC plan from the Stochastic Program with CVaR tends to have a slightly higher mean total production cost than Robust Optimization but lower variance. Compared with the simulation approach, the approximation method introduced is also able to pinpoint the two good unit commitment plans within less computation power required. However, the simulation approach is more appropriate when comparing two unit commitment plans closely, since the different unit commitment plans could react differently as we introduce those assumptions for the approximation method.

4.2 Other factors for future research

4.2.1 The prediction of demand

This research is done on the simplified model described in Chapter 1 in which a simplified net load curve was used. There is no doubt that the net load is a very important factor which influences the production cost and production cost variance. Currently, the short term net load prediction has been considered in the unit commitment plan schedule problem. There could also be a long time trend of net load changes. The research from Wang's team represents a way to capture the tendency and predict the annual load for the next year based on Support Vector Regression (SVR)

[12]. Their research results might be incorporated in the simulation described in this paper to get a result of production cost variance evaluation for a longer period of time. The performance of the unit commitment plan on a longer period of time would be an interesting topic as well.

4.2.2 A generalized approach to the simplified model

When looking at the approximation method in Chapter 3, Assumption one causes the divergence of the approximation method and the simulation method. In Chapter 3, the proportion of time when Assumption one holds is evaluated. Setting a threshold value on this proportion of time could help the decision of how many unit commitment plans should be chosen by this approximation method. There could also be some future work done on relaxing Assumption one or studying the situation when more than one machine can failure at same time. Some network analysis model like Petri Net could be potentially useful to reduce the number of scenarios considered. This direction could possibly produce a method that requires less computation power than the simulation method but more accuracy than the approximation method.

APPENDIX A. COMMITMENT PLAN TABLES

	hour																							
units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
31	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

Table A.1 Commitment plan from SP_CVaR_02

	hour																							
units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
31	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

Table A.2 Commitment plan from SP_CVaR_002

	hour																							
units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
31	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

Table A.3 Commitment plan from SP_CVaR_0002

	hour																							
units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
28	1	0	0	0	0	0	0	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0
29	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
31	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

Table A.4 Commitment plan from RO_U1_01

	hour																							
units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
4	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
8	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
9	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
11	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
25	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
26	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
27	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
28	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
29	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
31	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table A.5   Commitment plan from RO_U1_05

	hour																							
units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
10	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
11	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
25	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
26	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
27	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
28	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
29	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
31	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table A.6 Commitment plan from RO_U1_10

Bibliography

[1] "2017 International Energy Outlook from the EIA Electric Choice." https://www.eia.gov/outlooks/ieo/pdf/0484(2017).pdf/.

[2] Q. P. Zheng, J. Wang, and A. L. Liu, "Stochastic optimization for unit commitment;a review," *IEEE Transactions on Power Systems*, vol. 30, pp. 1913–1924, July 2015.

[3] F. Shih, *A Markov model for evaluation of electric power generation system production costs.* PhD dissertation, University of Pittsburgh, Pittsburgh, Pennsylvania, 1995.

[4] F. R. Shih, M. Mazumdar, and J. A. Bloom, "Asymptotic mean and variance of electric power generation system production costs via recursive computation of the fundamental matrix of a markov chain," *Operations Research*, vol. 47, no. 5, pp. 703–712, 1999.

[5] S. M. RYAN, "A renewal reward approximation for the variance of electric power production costs," *IIE Transactions*, vol. 29, pp. 435–440, Jun 1997.

[6] J. Valenzuela and M. Mazumdar, "Statistical analysis of electric power production costs," *IIE Transactions*, vol. 32, pp. 1139–1148, Dec 2000.

[7] N. Kazemzadeh, S. M. Ryan, and M. Hamzeei, "Robust optimization vs. stochastic programming incorporating risk measures for unit commitment with uncertain variable renewable generation," *Energy Systems*, Dec 2017.

[8] S. Ryan, J. Li, and N. Kazemzadeh, "Asymptotic risk of unit commitment schedule due to generating unit outages," in *Probabilistic Methods Applied to Power Systems (PMAPS)*, 2018.

[9] P. Wong, P. Albrecht, R. Allan, R. Billinton, Q. Chen, C. Fong, S. Haddad, W. Li, R. Mukerji, D. Patton, A. Schneider, M. Shahidehpour, and C. Singh, "The IEEE reliability test system-1996," *Power Systems, IEEE Transactions on*, vol. 14, pp. 1010–1020, Aug 1999.

[10] M. Brown and H. Solomon, "A second-order approximation for the variance of a renewal reward process," *Stochastic Processes and their Applications*, vol. 3, no. 3, pp. 301–314, 1975.

[11] B. Efron, *The jackknife, the bootstrap, and other resampling plans*, vol. 38, ch. 3, pp. 13–18. Siam, 1982.

[12] J. Wang, L. Li, D. Niu, and Z. Tan, "An annual load forecasting model based on support vector regression with differential evolution algorithm," *Applied Energy*, vol. 94, pp. 65 – 70, 2012.