Iowa State University
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Comparing Decisions with Intervals and Probability Distributions

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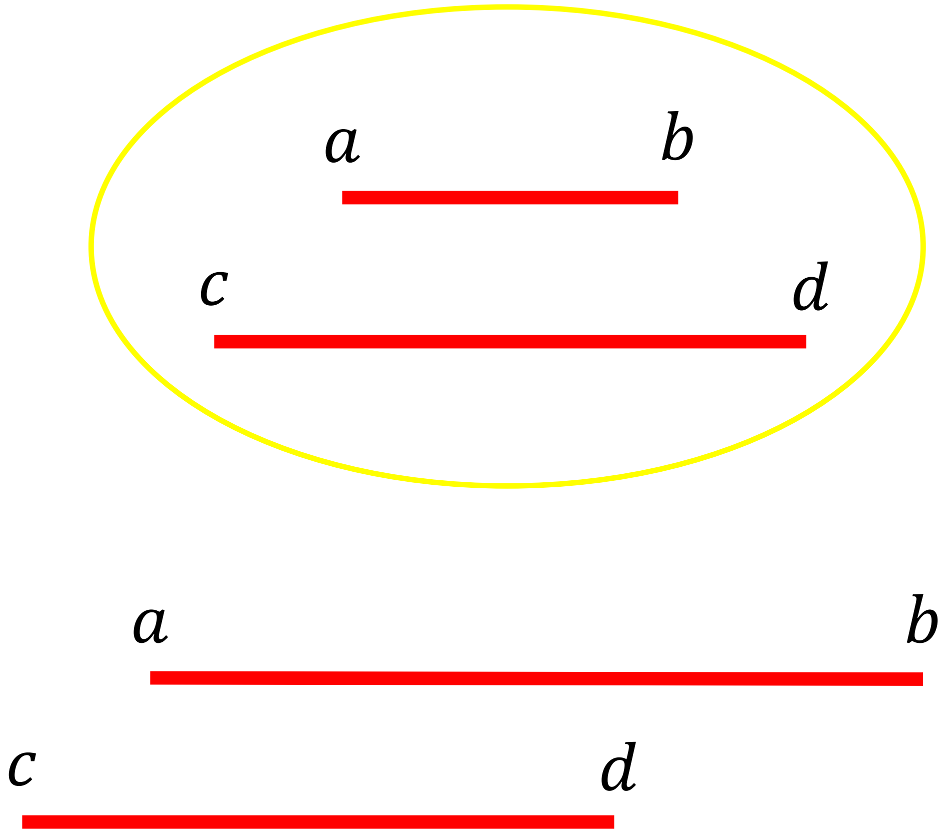
Decision making with intervals

1. Worst-case rule, $a > c$
2. Best-case rule, $b > d$
3. Laplace rule, $a + b > c + d$
4. Minimum regret rule,
   $$d - a < b - c$$
5. Hurwicz rule,
   $$\alpha a + (1 - \alpha)b > \alpha c + (1 - \alpha)d$$
   where $\alpha \in [0,1]$

Barker, K and Wilson, KJ (2012) Decision trees with single and multiple interval-valued objectives. *Decision Analysis* 9(4):348-358.

Assume $c < a < b < d$



Hurwicz rule

1. Worst-case rule, $a > c$
2. Best-case rule, $b > d$
3. Laplace rule, $a + b > c + d$
4. Minimum regret rule, $d - a < b - c$

Hurwicz rule, $\alpha a + (1-\alpha)b > \alpha c + (1-\alpha)d$ where $\alpha \in [0,1]$

Cao, Y (2014) Reducing interval-valued decision trees: Comments on decision trees with single and multiple interval-valued objectives. *Decision Analysis* 11(3): 204-212.

Research questions

- What is the relationship between Hurwicz decision rule and subjective expected utility decision making?

- What is the relationship between decision making with intervals and decision making with probabilities and utilities?

Main result

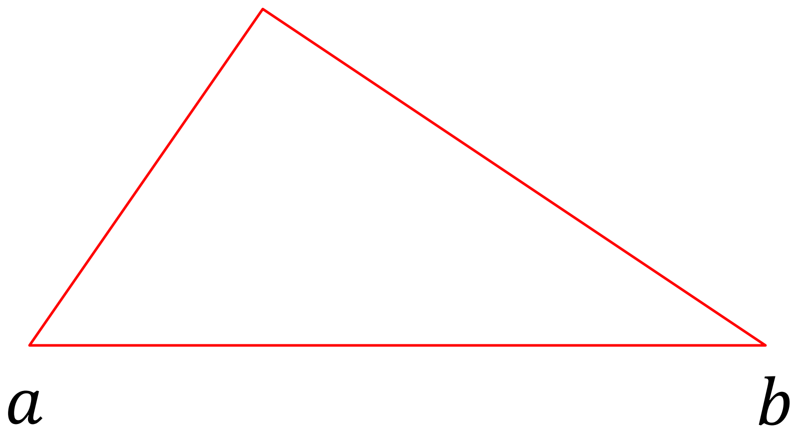
Comparing two intervals using the Hurwicz decision rule results in the same preferred alternative as

1. Expected value with a triangle distribution
2. Expected value with a beta distribution
3. Expected utility with exponential utility and a uniform distribution

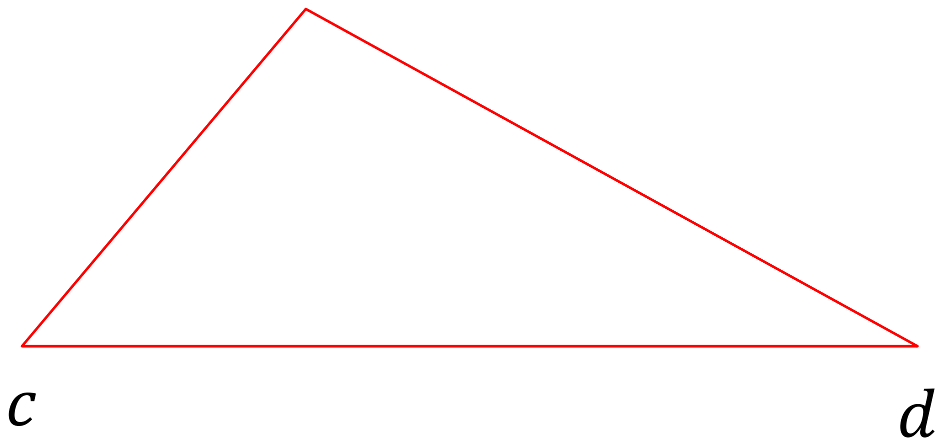
Triangle distribution

If ${}^{1}\!/_{3} \le \alpha \le {}^{2}\!/_{3}$

$\text{mode} = 2b - a - 3\alpha(b - a)$



$\text{mode} = 2c - d - 3\alpha(d - c)$

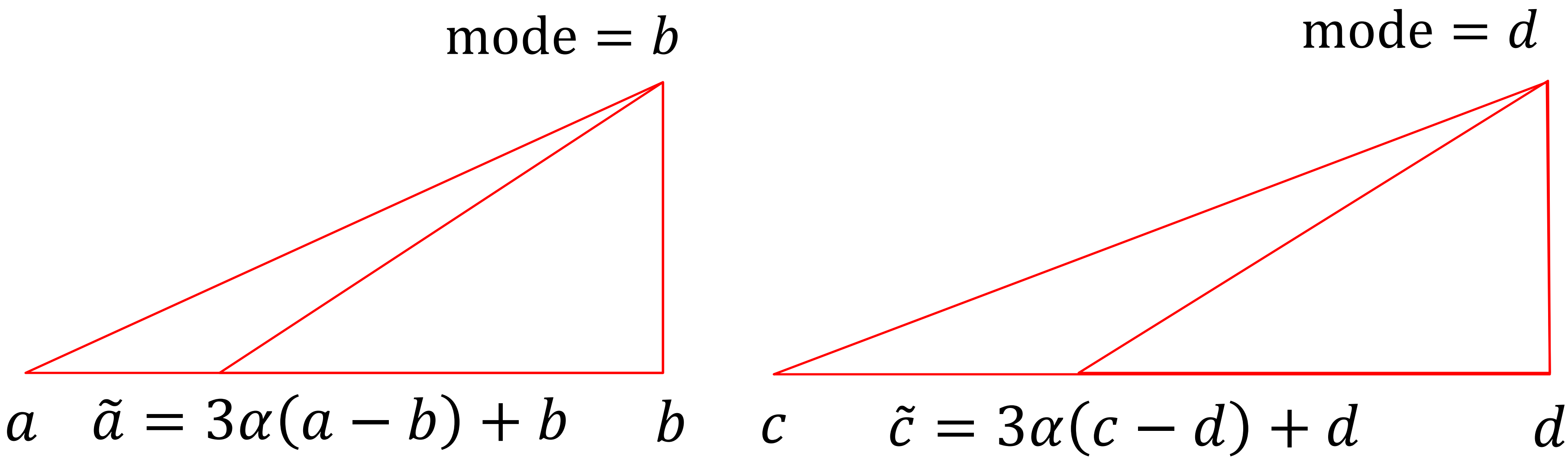


Then expected value of $[a, b]$ triangle greater than expected value of $[c, d]$ triangle if and only if

$\alpha a + (1 - \alpha)b > \alpha c + (1 - \alpha)d$

Triangle distribution

If $\alpha < {}^1/_3$

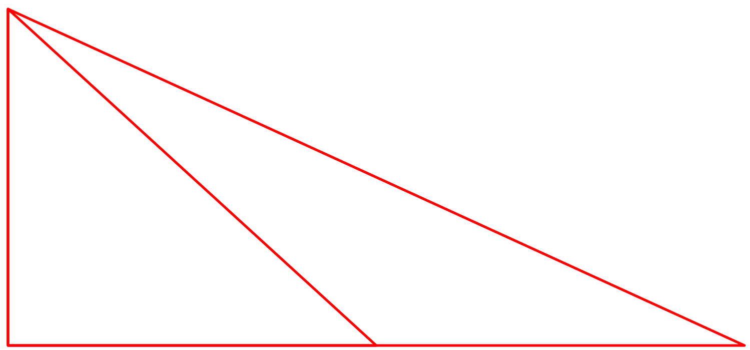


Then expected value of $[\tilde{a}, b]$ triangle greater than expected value of $[\tilde{c}, d]$ triangle if and only if
$\alpha a + (1 - \alpha)b > \alpha c + (1 - \alpha)d$

Triangle distribution

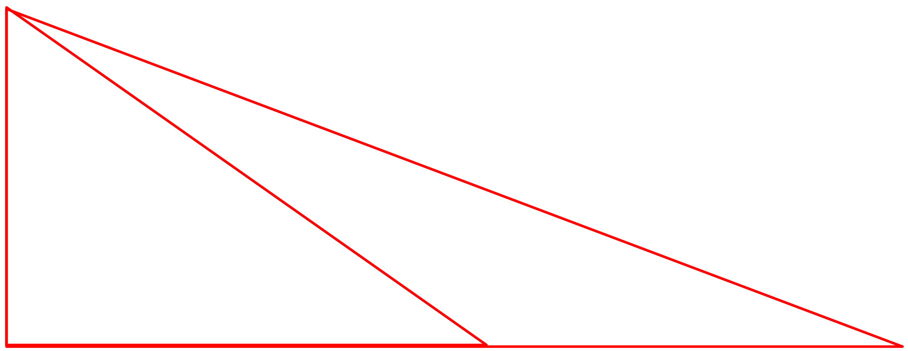
If $\alpha > {}^2/_3$

$\text{mode} = a$



a $\qquad b$

$\text{mode} = c$



c $\qquad d$

$\tilde{b} = (3\alpha - 2)a + 3b(1 - \alpha)$

$\tilde{d} = (3\alpha - 2)c + 3d(1 - \alpha)$

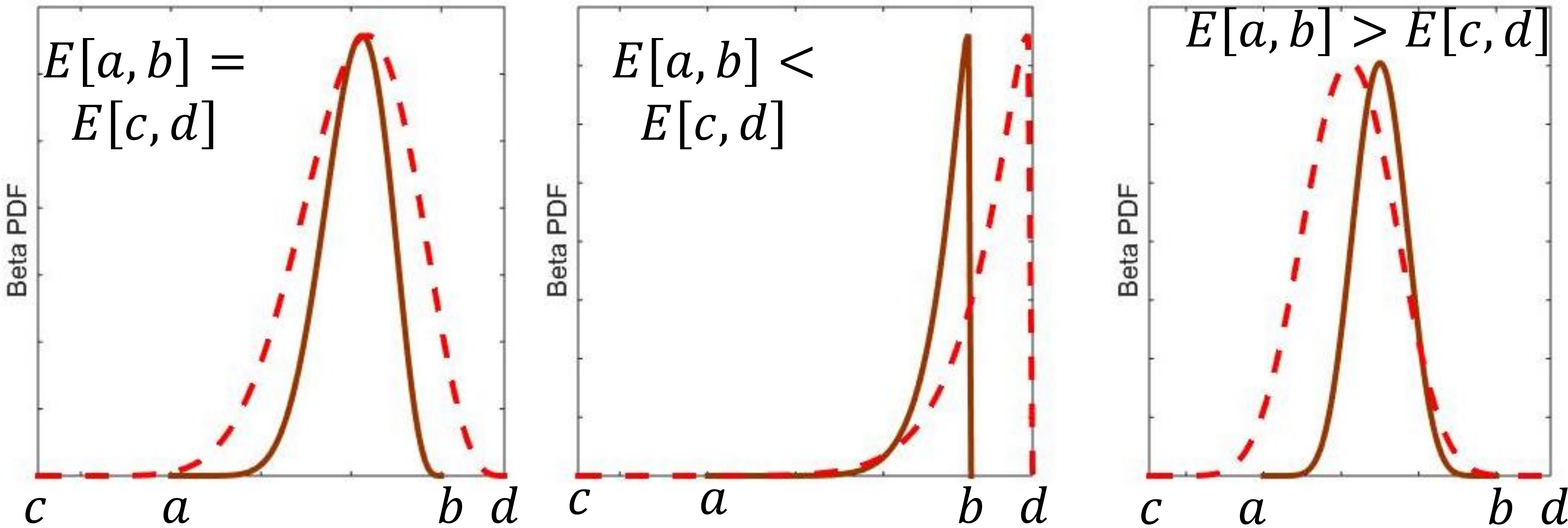
Then expected value of $[a, \tilde{b}]$ triangle greater than expected value of $[c, \tilde{d}]$ triangle if and only if $\alpha a + (1 - \alpha)b > \alpha c + (1 - \alpha)d$

Beta distribution

Four-parameter beta distribution $Beta(\hat{\alpha}, \hat{\beta}, \min, \max)$

$$\frac{\hat{\beta}}{\hat{\alpha}} = \frac{\alpha}{1-\alpha}$$

where $\min = a$ or c and $\max = b$ or d



Exponential utility

$$U(z) = \begin{cases} \dfrac{1 - \exp(-\gamma z)}{\gamma}, & \gamma \neq 0 \\ z, & \gamma = 0 \end{cases}$$

- $[a, b]$ follows a uniform distribution and $[c, d]$ follows a uniform distribution
- $EU([a, b])$ and $EU([c, d])$ are the expected utility for each interval with uniform distribution

Relationship between α and γ

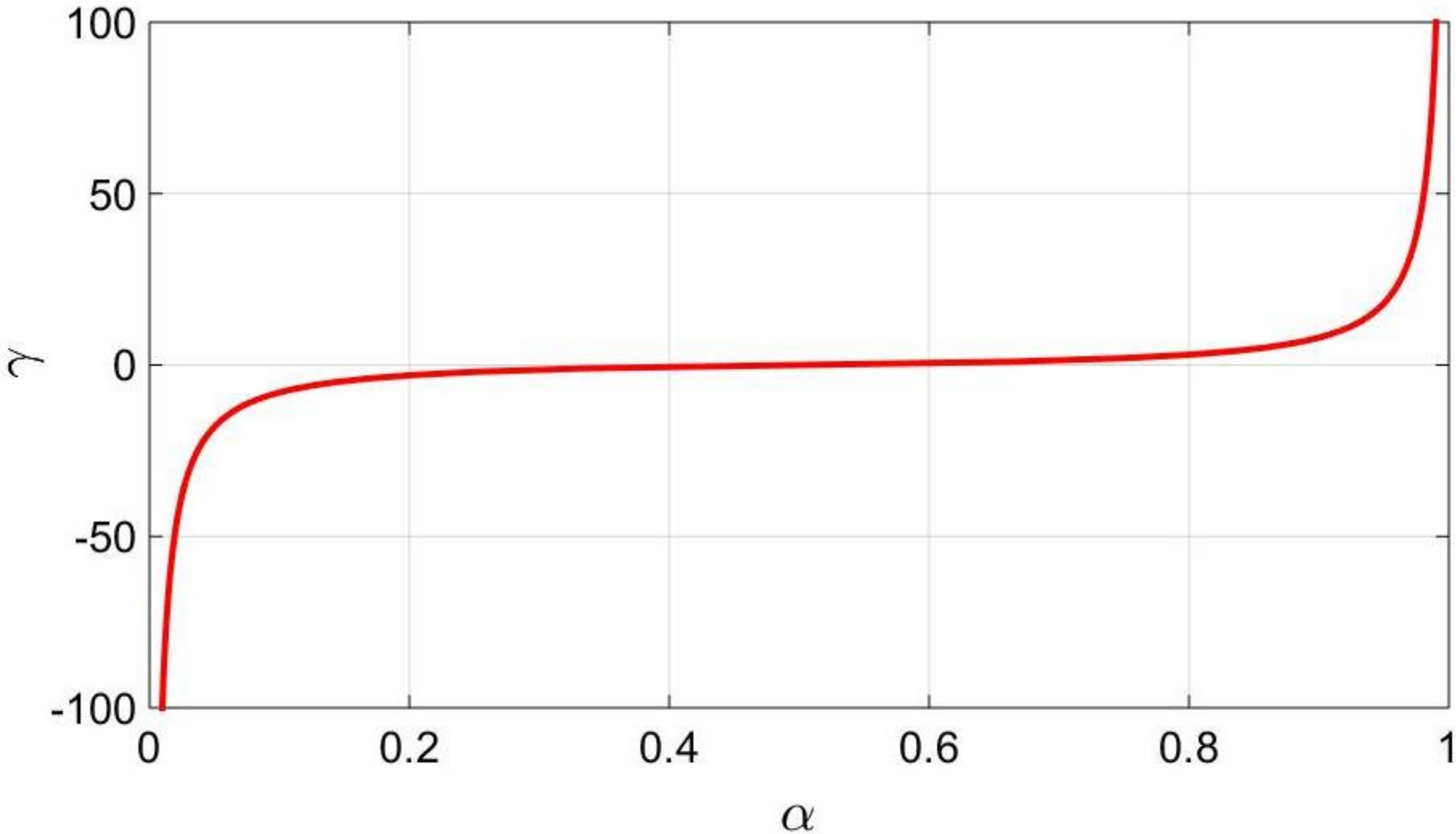
Hurwicz indifferent parameter: α^*

$$\alpha^* a + (1 - \alpha^*) b = \alpha^* c + (1 - \alpha^*) d$$

γ^* such that $EU([a, b]) = EU([c, d])$

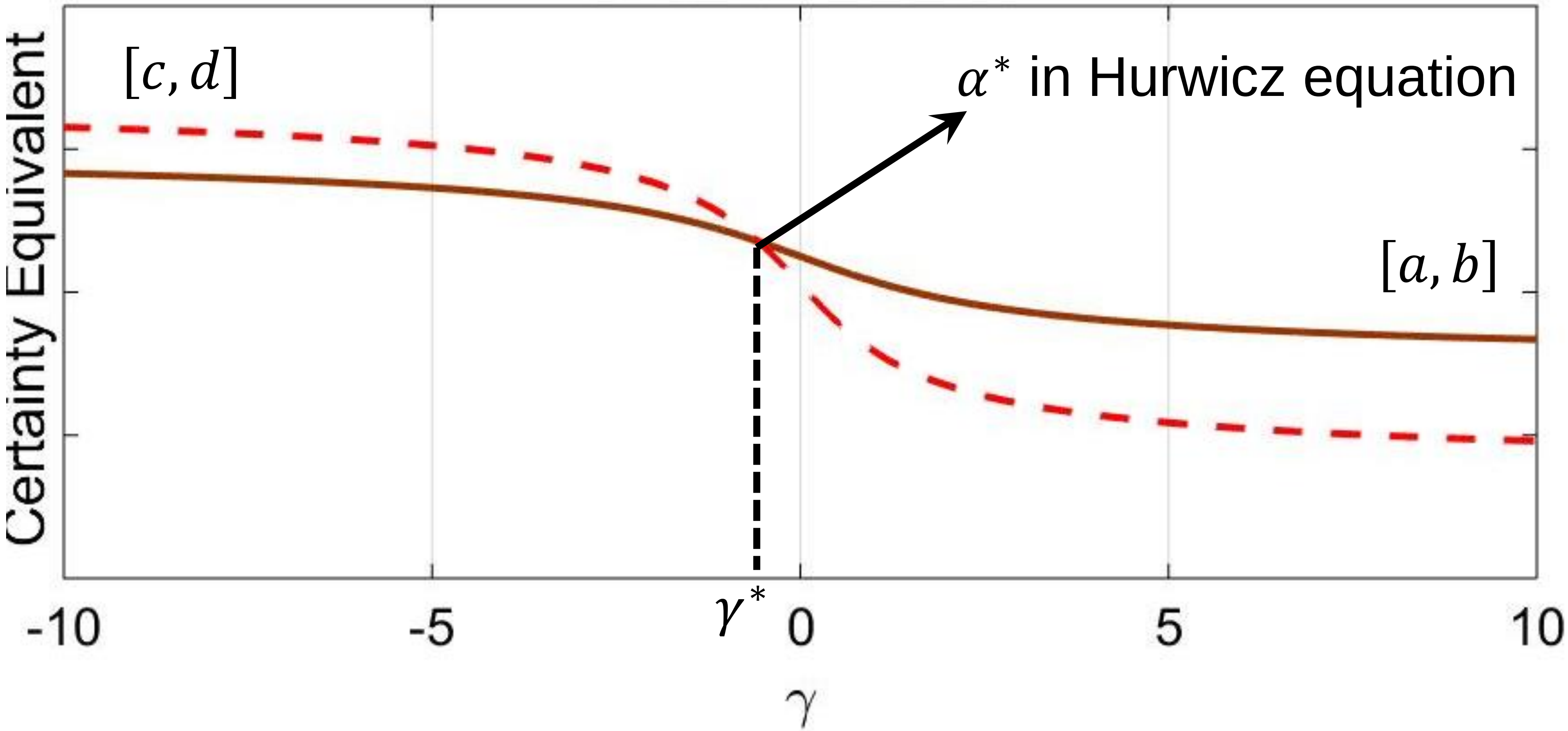
$$\gamma(\alpha) = \begin{cases} \dfrac{\gamma^* \alpha^* (2\alpha - 1)}{\alpha(2\alpha^* - 1)}, & 0 \le \alpha < 0.5 \\ \dfrac{\gamma^* (1 - \alpha^*)(2\alpha - 1)}{(1 - \alpha)(2\alpha^* - 1)}, & 0.5 \le \alpha \le 1 \end{cases}$$

Relationship between α and γ



Expected utility and Hurwicz

If $\gamma(\alpha)$, as defined earlier, then $EU([a,b]) > EU([c,d])$ if and only if $\alpha a + (1-\alpha)b > \alpha c + (1-\alpha)d$



Conclusions

- If making decisions using intervals (i.e., Hurwicz decision rule) results in the same preferred alternative as
  - Expected value with triangle distribution
  - Expected value with beta distribution
  - Expected utility with uniform distribution
- Is interval decision making the same as one of these other rules?
- If a decision maker rejects the triangle / beta / expected utility construct, should he or she reject the interval (or at least the value of α)?

Conclusions

- What is the benefit of intervals for decision making?
- Could be other decision rules for intervals that do not translate to Hurwicz
- Do decision makers prefer making decisions with intervals (i.e., determining a value of α) as opposed to assuming a triangle, beta, and/or utility function?

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