

# Probabilistic methods for long-term demand forecasting for aviation production planning

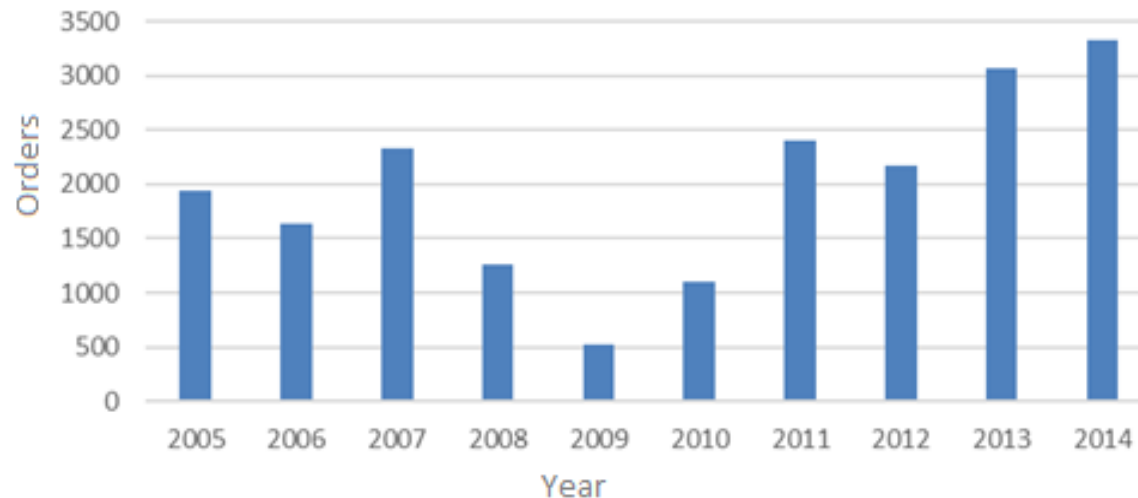
Minxiang Zhang, Cameron A. MacKenzie, Caroline  
Krejci, John Jackman, Guiping Hu

Industrial & Manufacturing Systems Engineering  
Iowa State University

Charles Y. Hu, Gabriel A. Burnett, Adam A. Graunke  
Boeing Research & Technology

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# Motivation



Historical order of global commercial airplanes

- Is painting capacity expansion necessary?
- How many hangars need to be built for Boeing?
- When to build?

# Research Overview

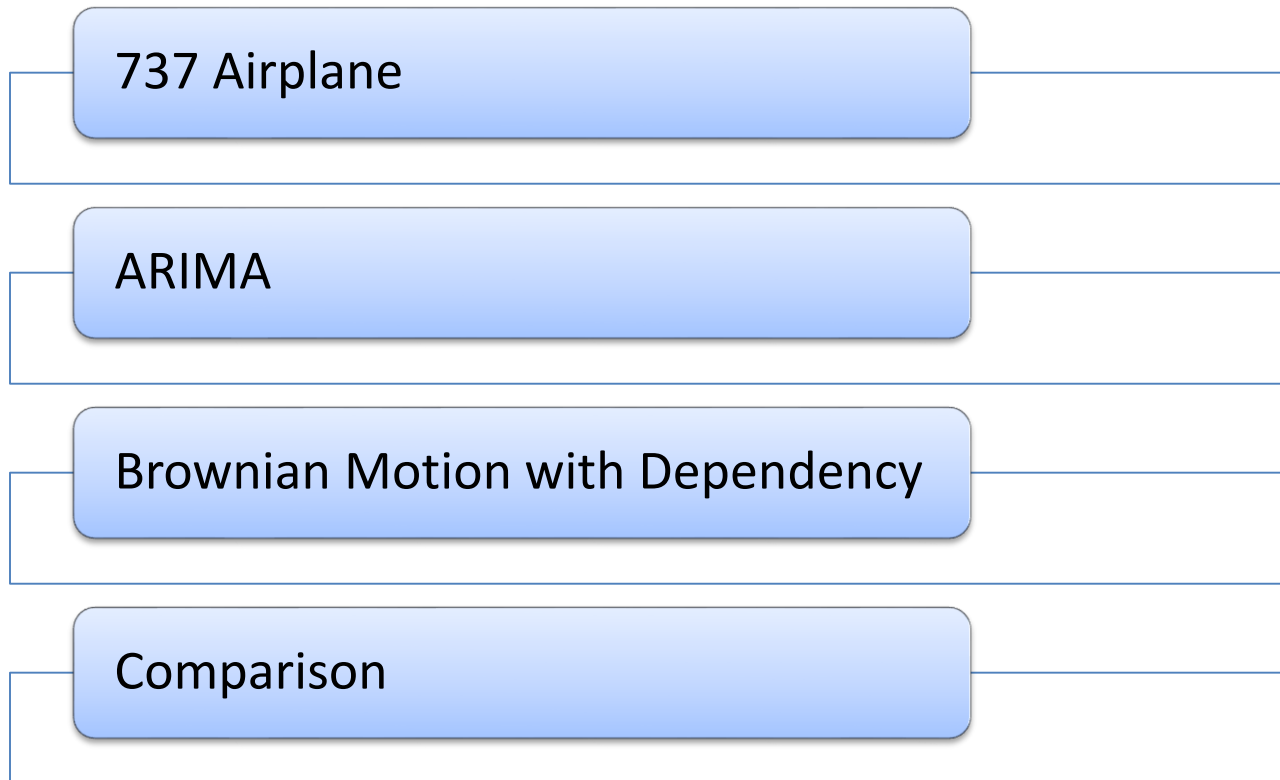
## Forecasting 737 Airplane

- Brownian motion with dependency
- Autoregressive Integrated Moving Average (ARIMA)

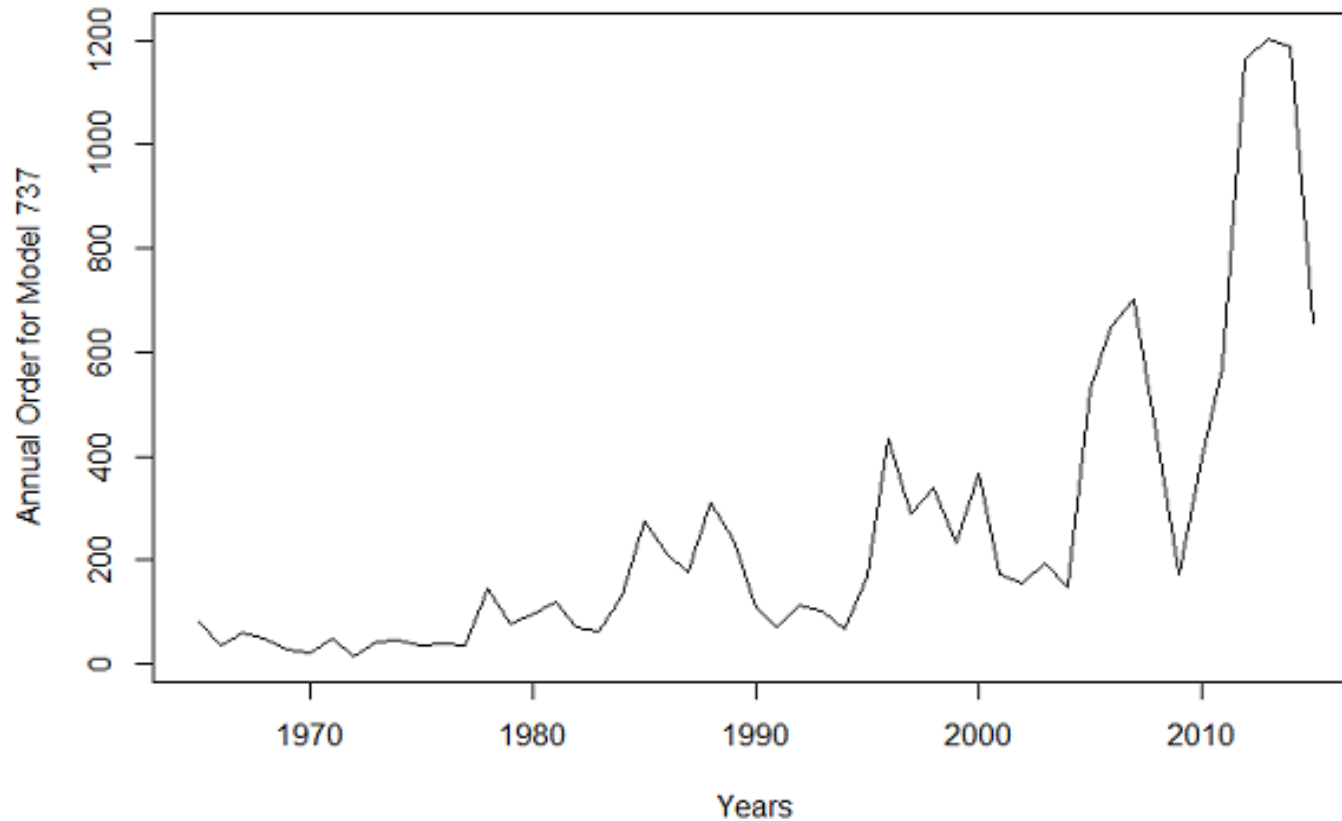
## Forecasting 777 Airplane

- Geometric Brownian motion (GBM)
- Alternative GBM fitting
- Starting point adjustment

# Demand Forecasting for 737 Airplane



# 737 Airplane - Historical Annual Order



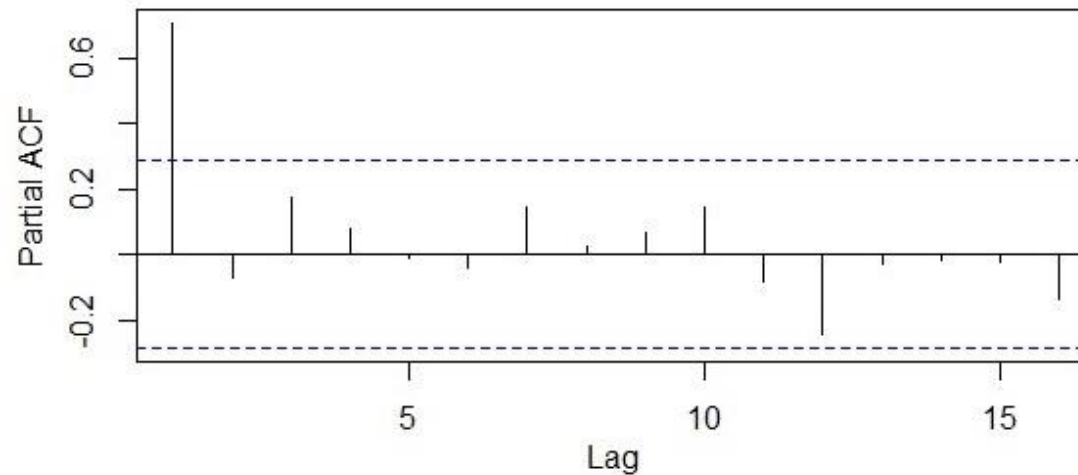
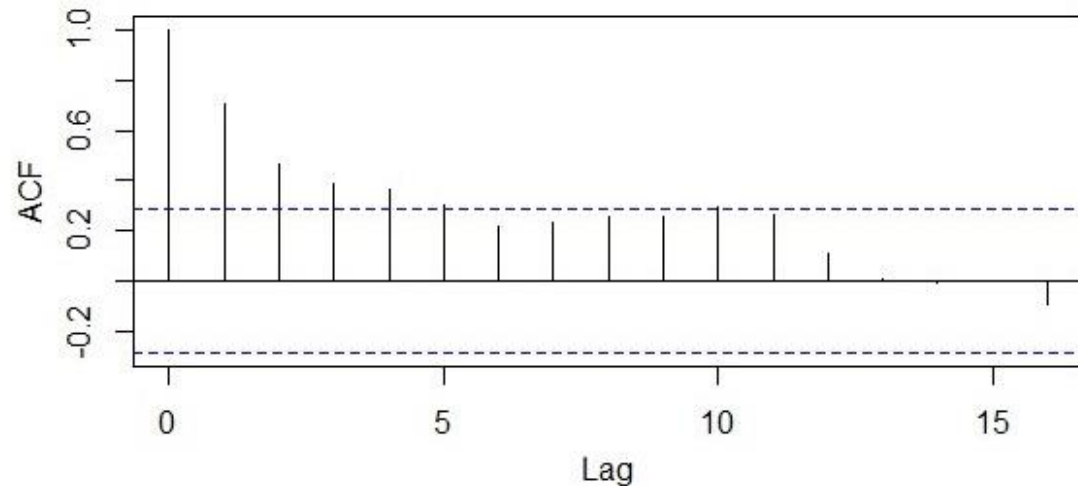
Data source: Boeing Commercial. Available: <http://www.boeing.com/commercial/>, 2015

# 737 Airplane - Autocorrelation

ACF: decay  
PACF: cut off at lag 1

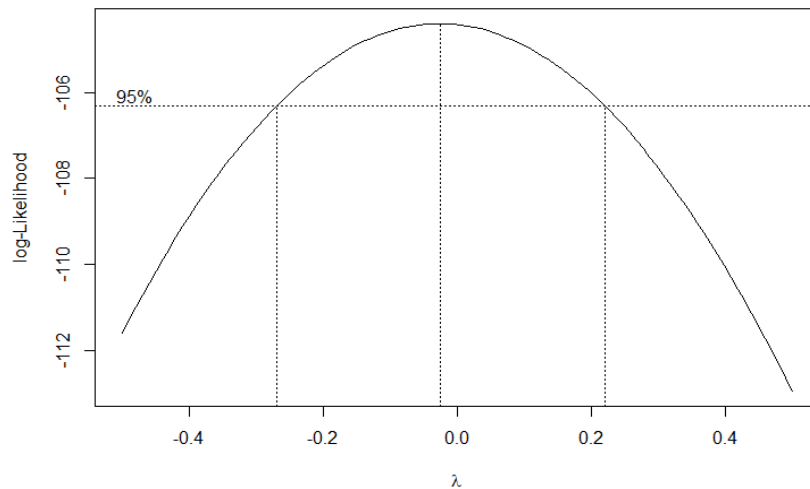
AR(1)?

Non-stationary:  
Increasing variance  
with time

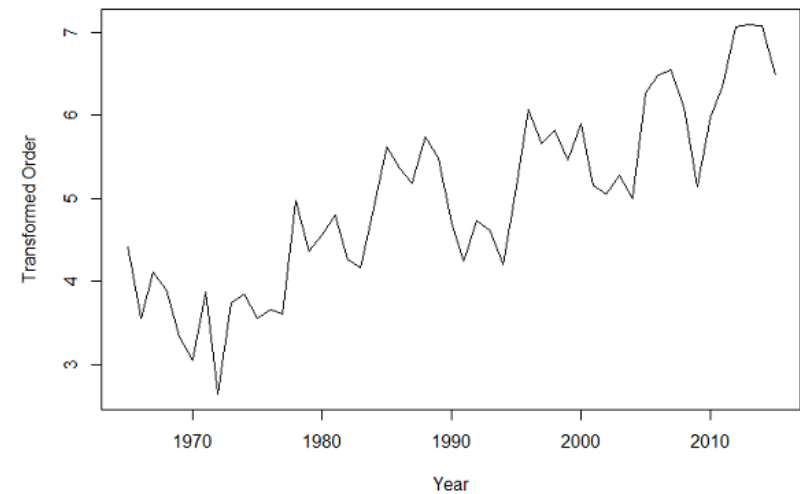


# ARIMA – Transformation

$$S(\lambda) = \begin{cases} \frac{X^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \log(X) & \text{if } \lambda = 0 \end{cases}$$



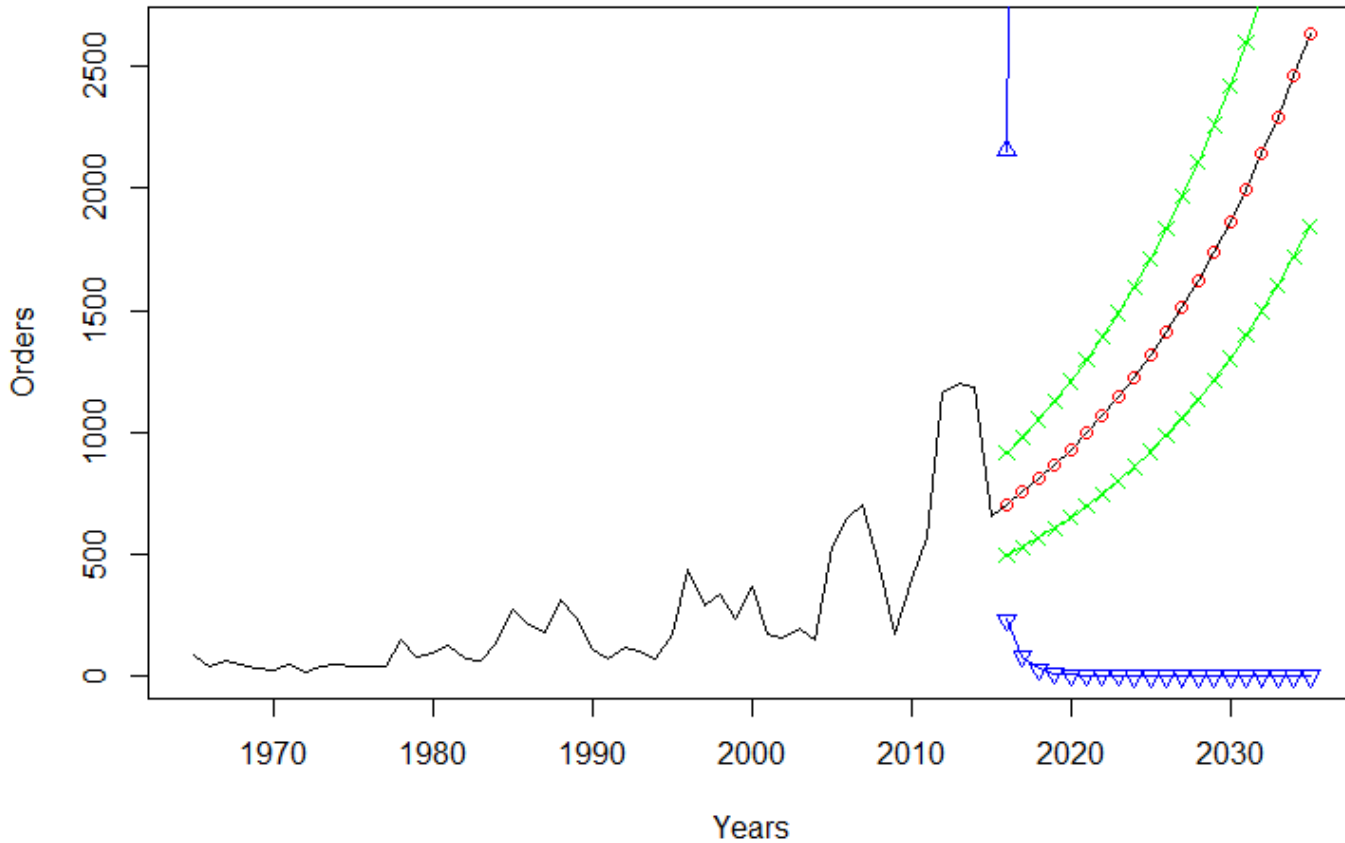
Box-Cox Transformation



After Log Transformation

# ARIMA – Model & Prediction

$$\text{ARIMA}(0,1,1) \quad S_t - S_{t-1} = Z_t - 0.3344Z_{t-1} \quad \{Z(t)\} \sim WN(0, 0.3243)$$





## Brownian Motion with dependency

$$X(t) = \sigma B(t) + \mu t + e$$

Where  $B(t) \sim N(0, t)$  is a standard Brownian motion

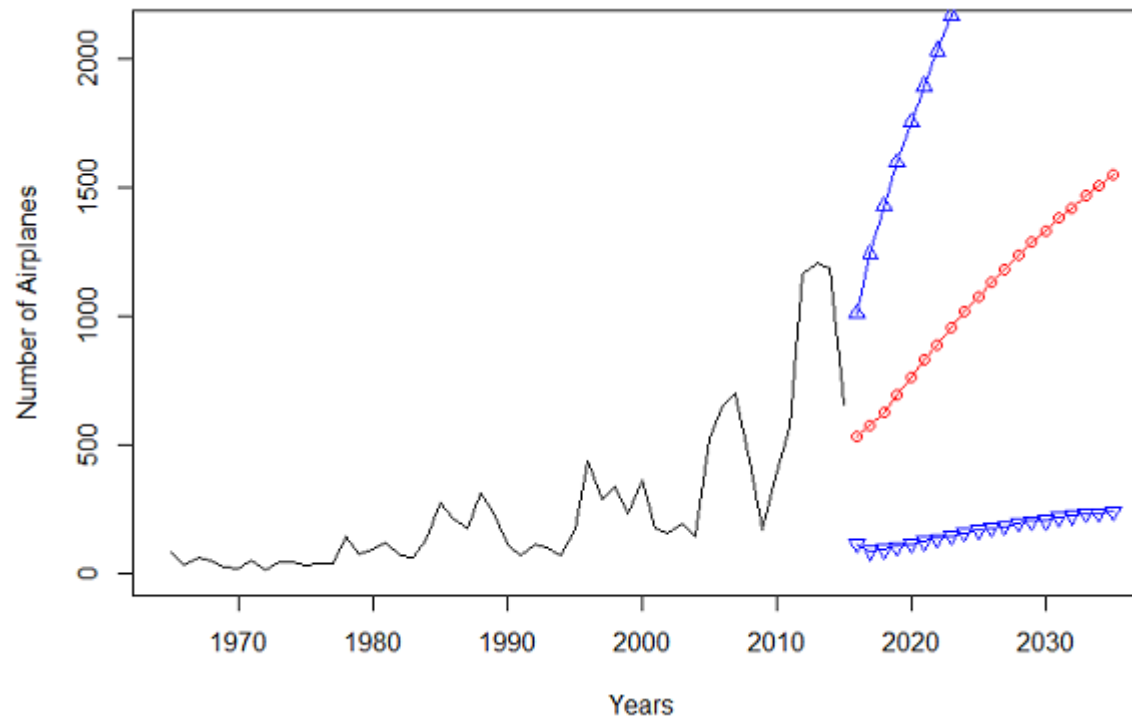
Add correlation  $\rho$  at lag 1

$$N_{cor} = \rho N_1 + \sqrt{1 - \rho^2} N_2$$

correlation between  $X(t)$  and  $X(t + 1)$  equals  $\rho$

# Brownian Motion with dependency

Drift ( $\mu$ )	Sigma ( $\sigma$ )	Baseline ( $b$ )	Correlation ( $\rho$ )
31.7	294.8	483	0.83



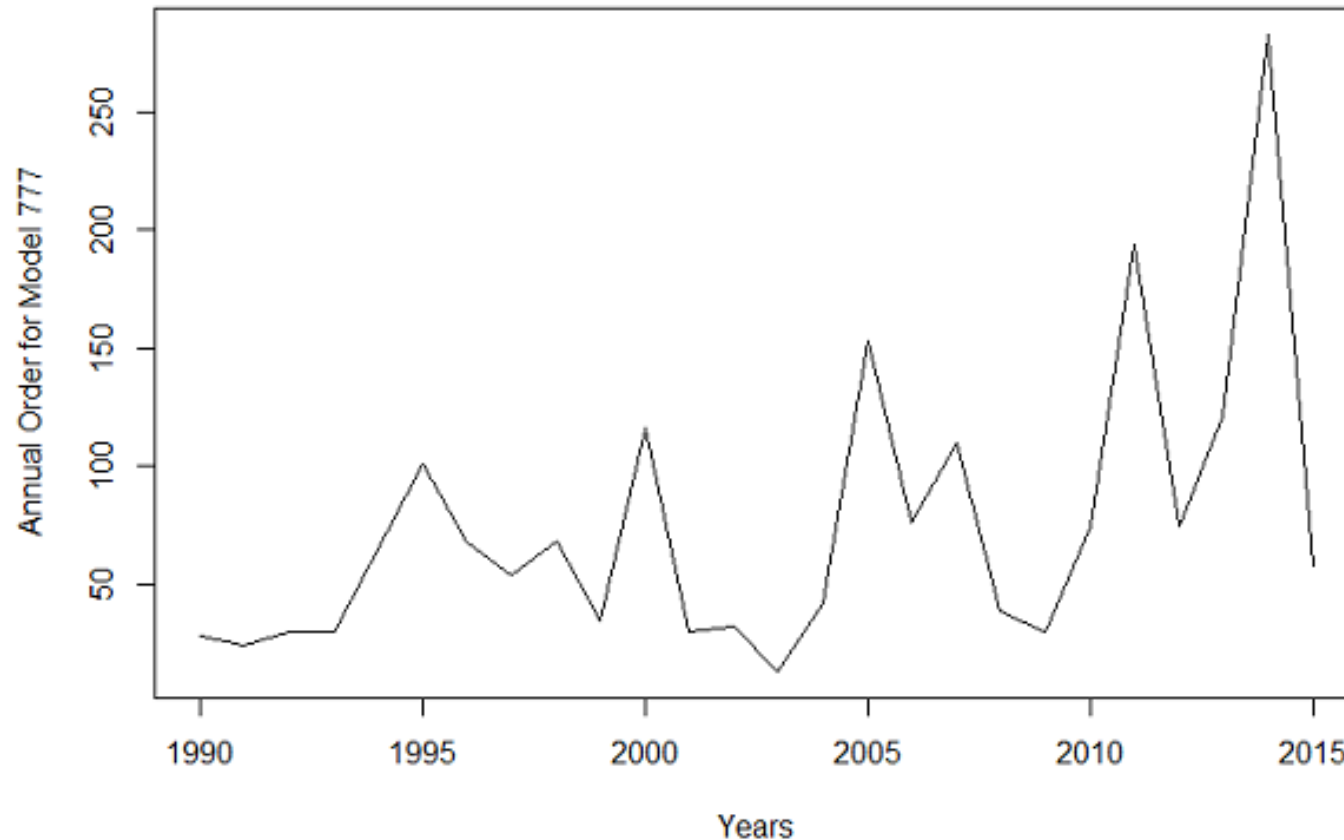
# Comparison

	<b>ARIMA</b>	<b>Brownian Motion with dependency</b>
Prediction trend	Handled by differencing	Defined explicitly
Interval	Confidence interval of mean	Probability interval
Input Data - Stationary	Weak stationary	Non-stationary
Input Data - Correlation	$\text{lag} \geq 1$	$\text{lag} = 1$
Sensitivity	Model parameters	Estimated Trend

# Demand Forecasting for 777 Airplane



# 777 Airplane - Historical Annual Order

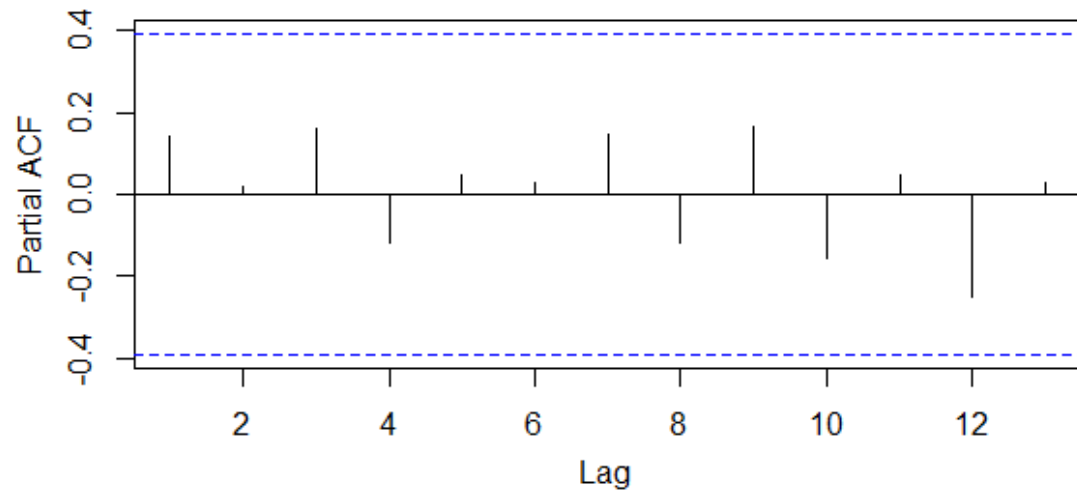
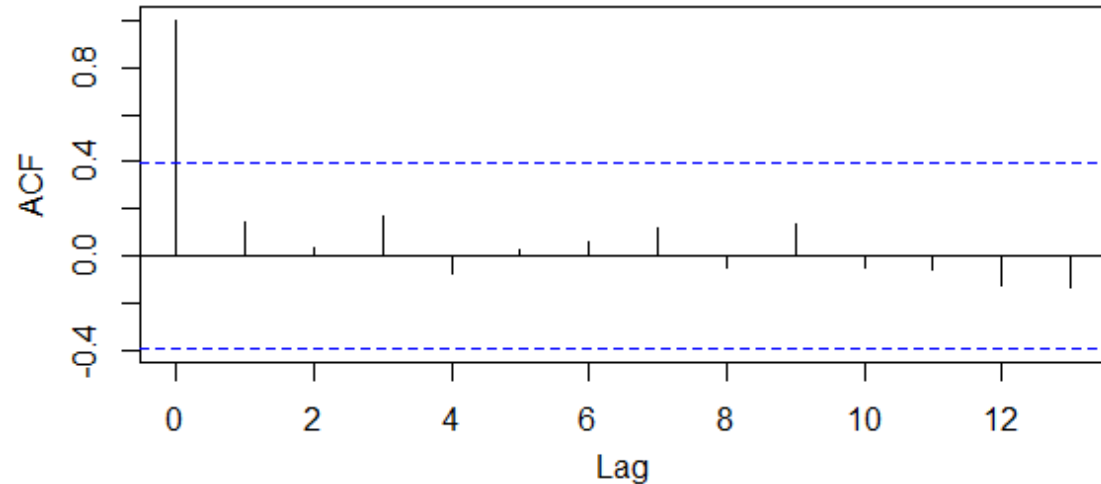


Data source: Boeing Commercial. Available: <http://www.boeing.com/commercial/>, 2015

# 777 Airplane - Autocorrelation

No significant autocorrelation

Non-stationary:  
Increasing variance  
with time



## Traditional GBM Fitting

*Brownian motion:*

$$X(t) = \sigma B(t) + \mu t + e$$

*Geometric Brownian motion:*

$$Y(t) = e^{X(t)}$$

$$R(1) = \frac{Y(t+1)}{Y(t)} \sim \text{lognormal}(\mu, \sigma^2)$$

Interested in difference between two adjacent years

## Alternative GBM Fitting

$$R(t) = \frac{Y(t)}{Y(0)} \sim \text{lognormal}(\mu t, \sigma^2 t)$$

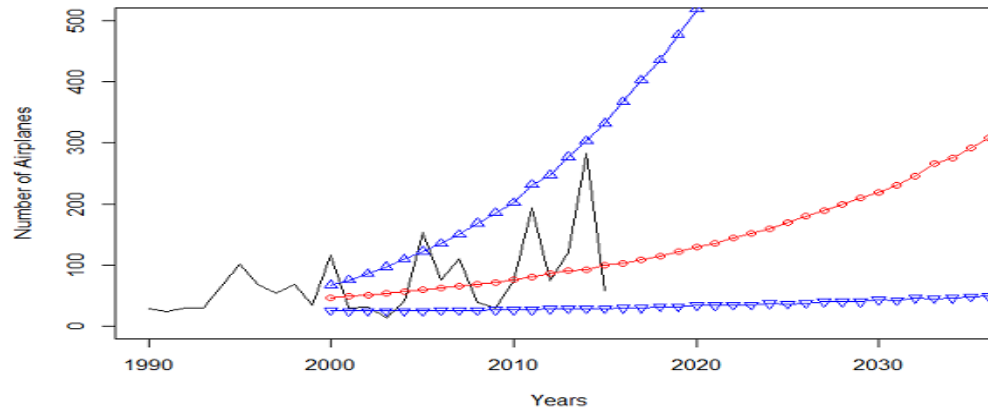
Interested in fitting model over  $t$  years

Method	Time Scale	Drift	Sigma	Baseline
Traditional	Year	0.030	0.847	3.635
Alternative	Year	0.0563	0.1913	3.635

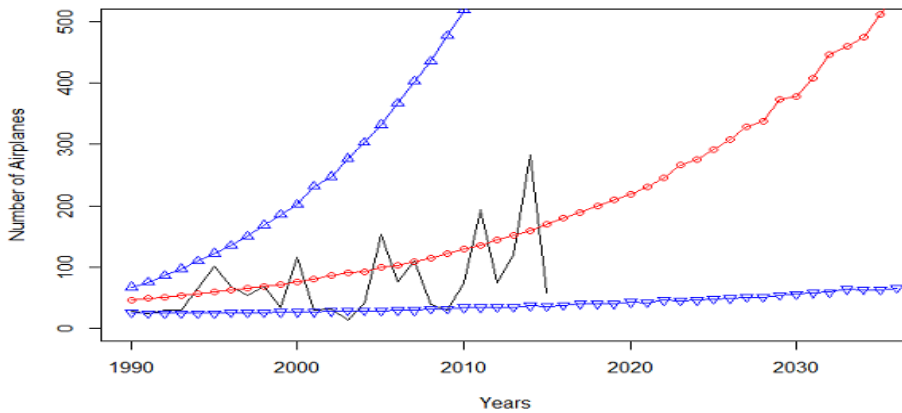
Alternative method reduces variance of estimation significantly



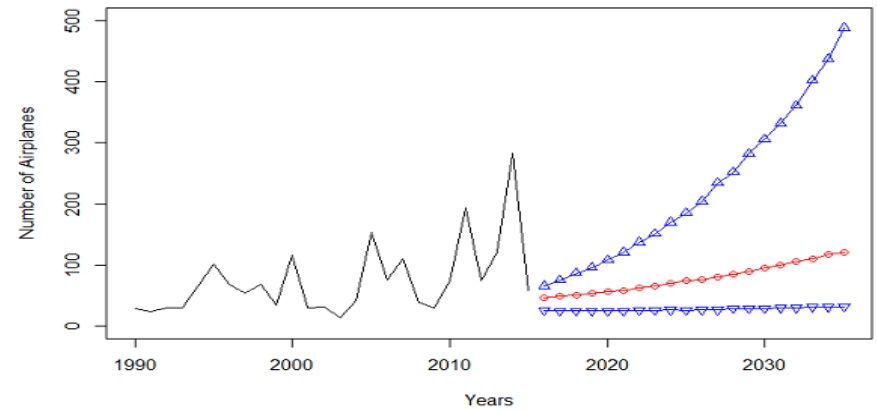
# Start Position Adjustment



Fitted GBM starting at year 2000



Fitted GBM starting at year 1990



Fitted GBM starting at year 2016

## Conclusion

- Incorporate correlation into Brownian motion
- Comparison of probabilistic model and time series model in forecasting
- Geometric Brownian motion at different starting points for increasing variation
- Use probabilistic model in forecasting to capture various scenarios rather than single prediction
- Applied to other airplane models as well
- Future work: Multi-variate forecasting