Analyzing Different Decision-Making Methods for Situations with Deep Uncertainty

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Motivation

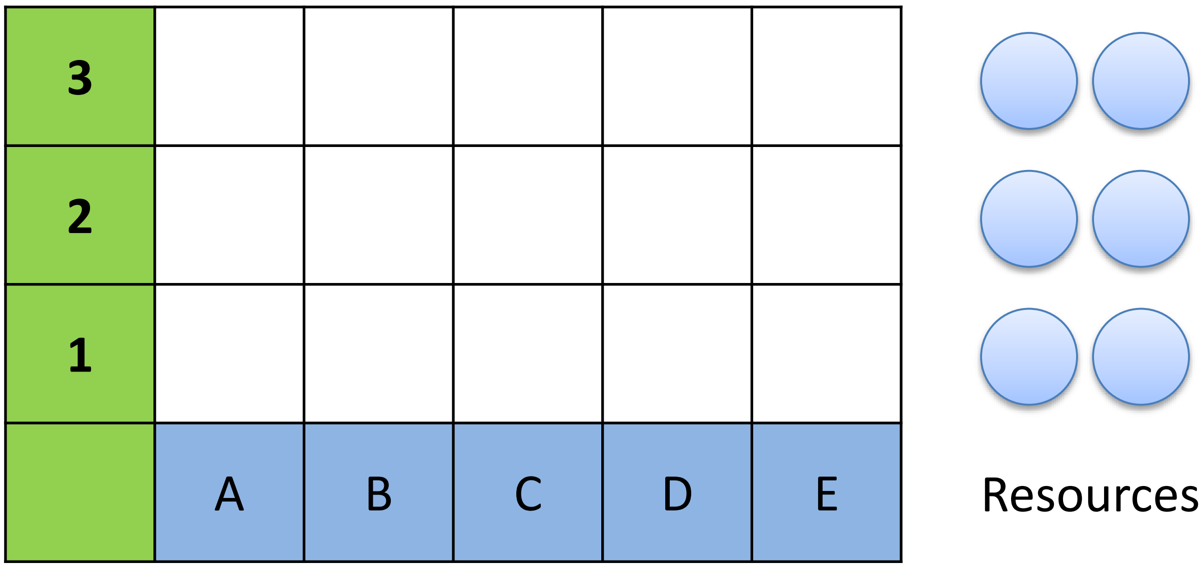
Security & Defense
- Airport
- Terrorism
- Military

Limit Resource
- Equipment
- Guards
- Budget

Deep Uncertainty
- Uncertain probability
- Hard to estimate

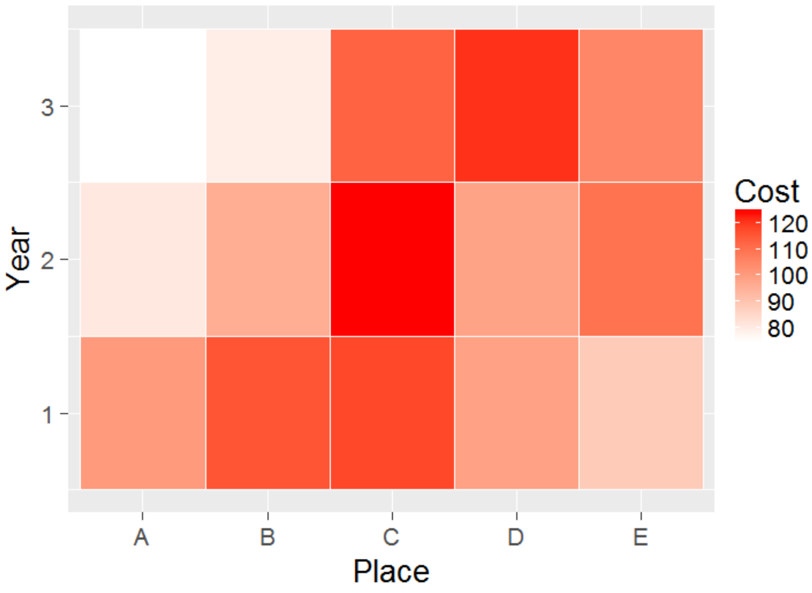
Decision Making
- severe consequence
- Long-term

Security & Defense Problem

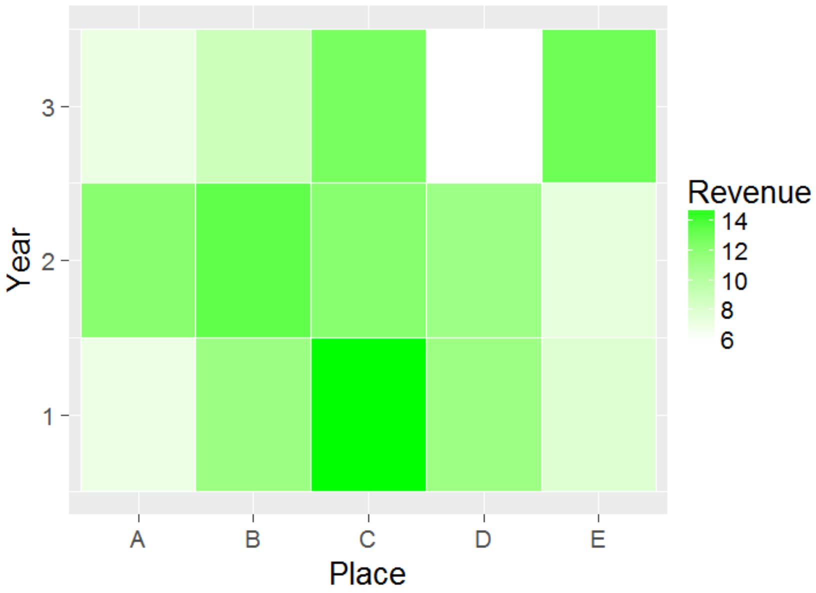


- Revenue\Gain
- Cost
- Probability of Disruption
- Risk Attitude

Security & Defense Problem

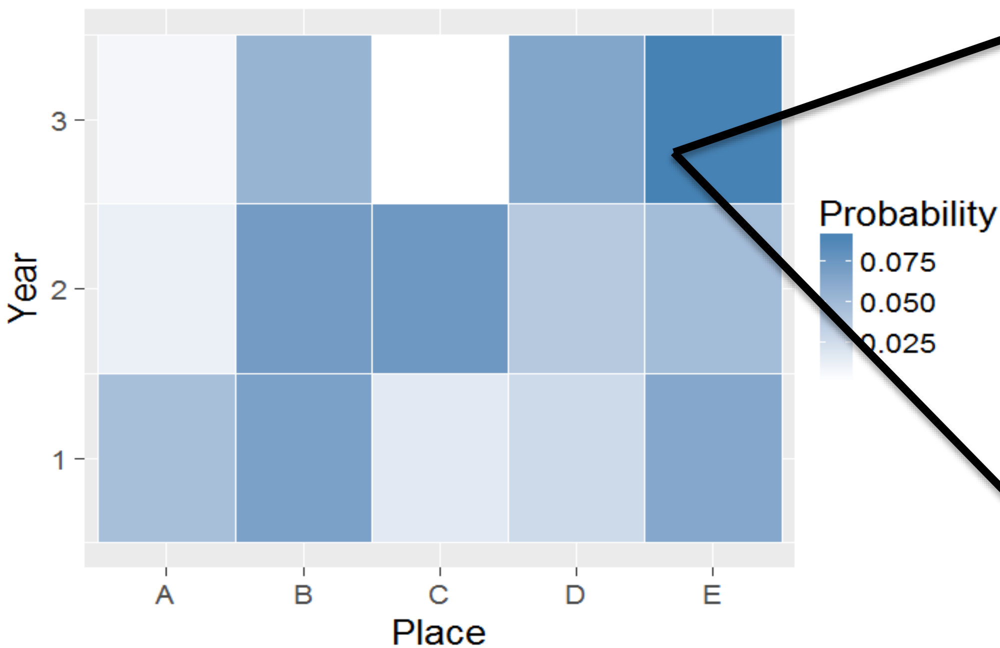


Cost Matrix

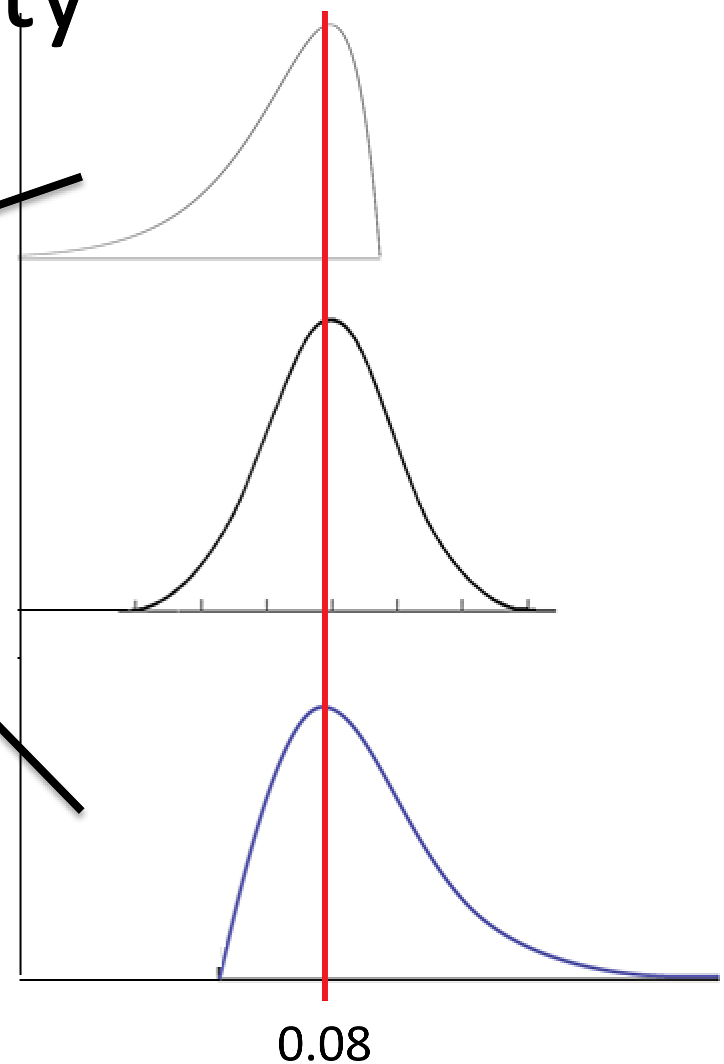


Revenue Matrix

Uncertainty



Estimated Probability Matrix

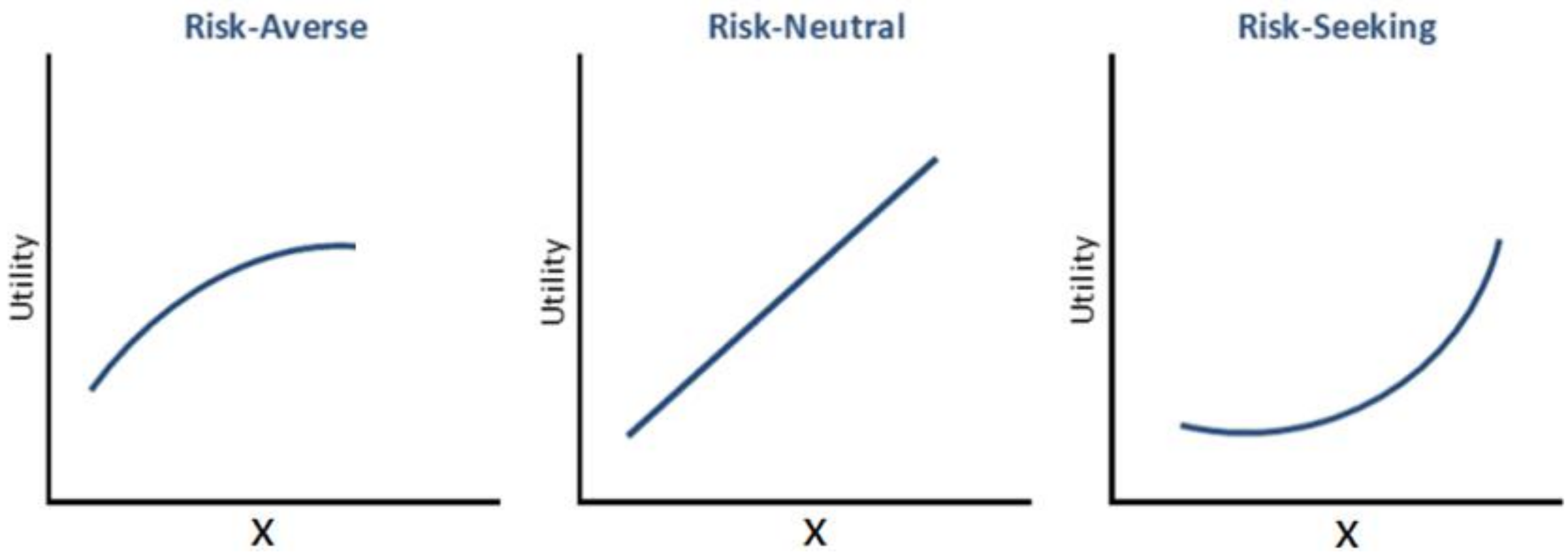


Decision Making Methods

- Expected Utility
- Information Gap
- Robust Decision Making

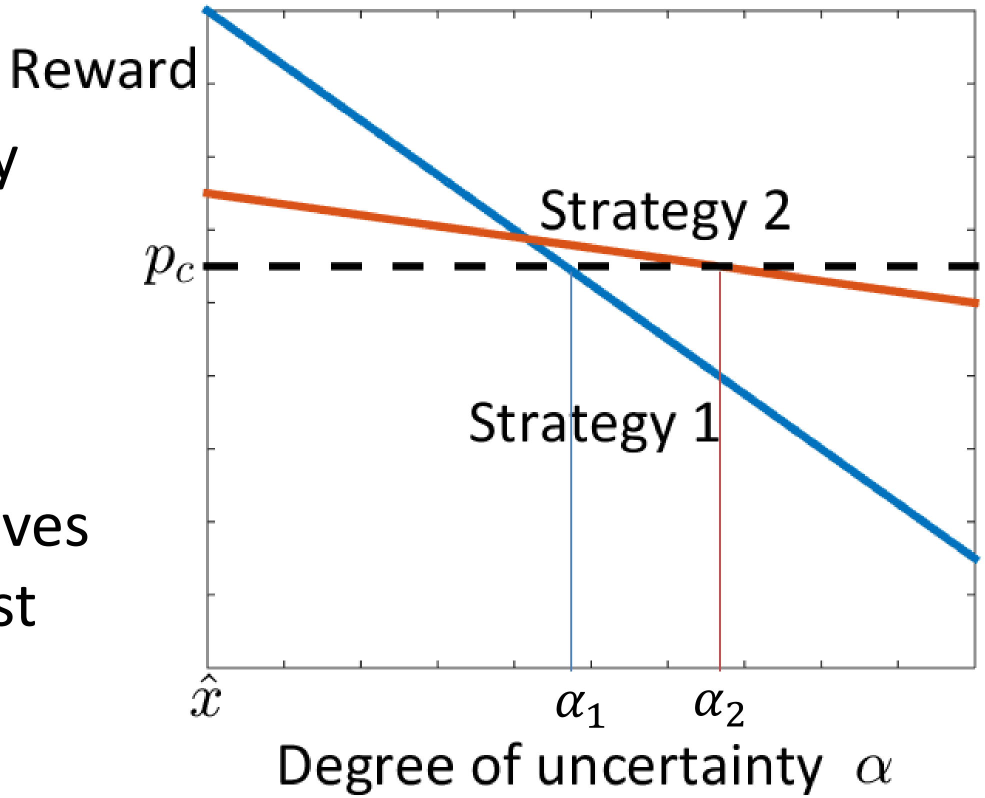
Expected Utility

Utility Function U(x)



Information Gap

- No probability distribution
- Dynamic uncertainty set
  - α: degree of uncertainty
  - $\hat{x}$: most likely state
- Define minimum level of reward p_c
- Select strategy *s* that achieves minimum reward for largest uncertainty



Y. Ben-Haim, 2006

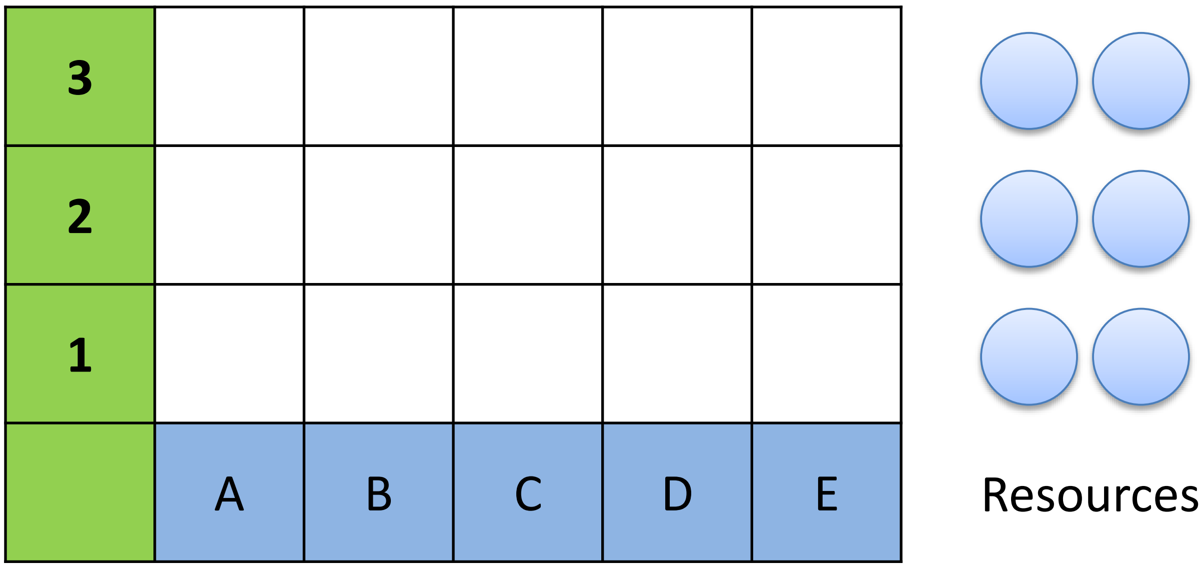
Robust Decision Making

- Set of probability distributions over states x, $\vartheta(x) \in \Theta$
- Reward $P(s, x)$ for each strategy s
- Regret $R(s, x) = \max[P(s', x)] - P(s, x)$
- Expected regret $R\big(s, \vartheta(x)\big)$ for all possible states
  - Best expected regret $R_{best}(s) = \min\limits_{\vartheta(x) \in \Theta} R\big(s, \vartheta(x)\big)$
  - Worst expected regret $R_{worst}(s) = \max\limits_{\vartheta(x) \in \Theta} R\big(s, \vartheta(x)\big)$
- Tradeoff parameter $0 \leq z \leq 1$ between best and worst
- Select s that minimizes $V(s)$

$$V(s) = z * R_{best}(s) + (1 - z) * R_{worst}(s)$$

R. J. Lempert and M. T. Collins, 2007

Security & Defense Problem



- Revenue\Gain
- Cost
- Probability of Disruption
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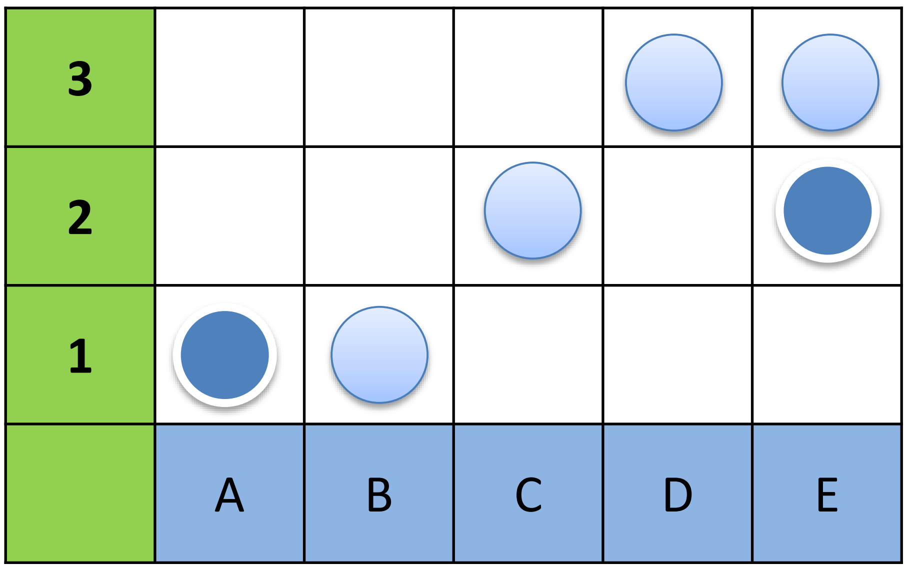
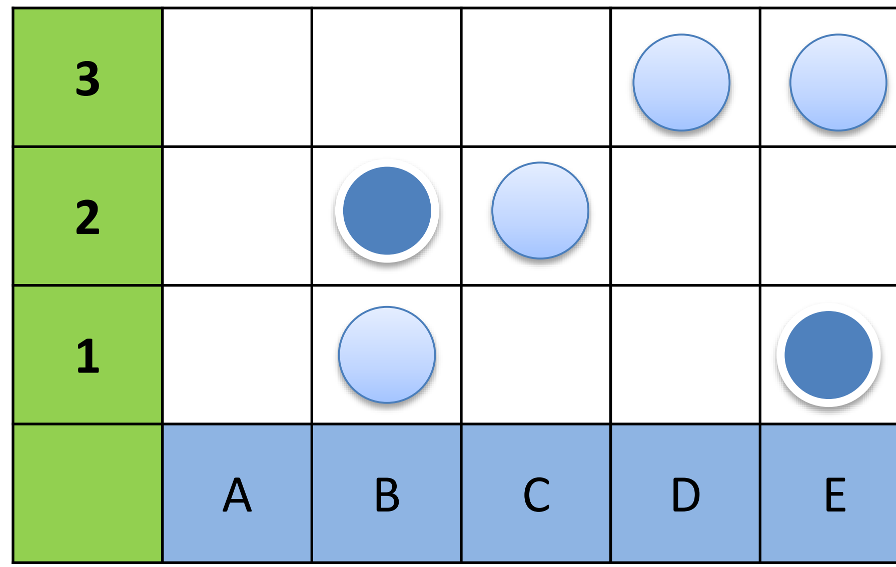
Expected Utility

Strong Risk Averse



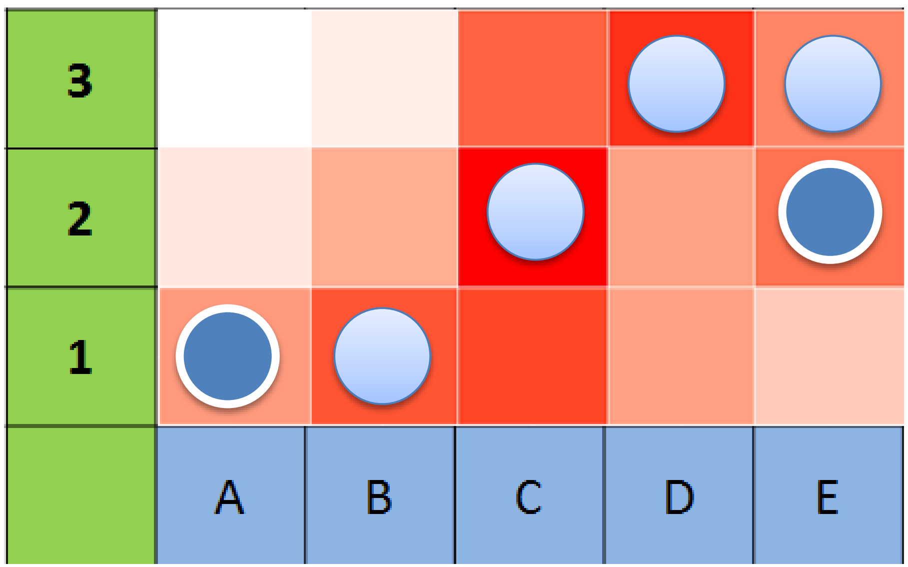
Others



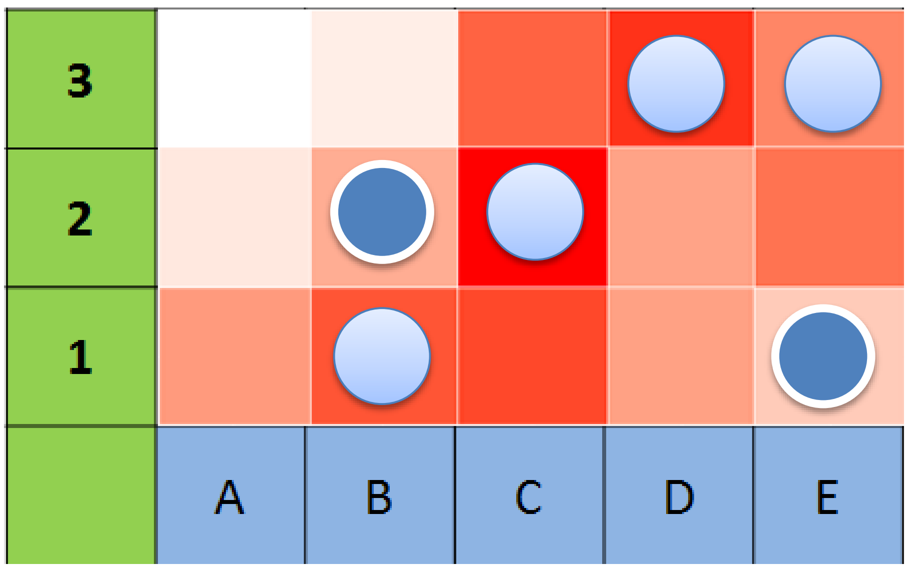
Others include: risk seeking, risk neutral and slight risk averse

Expected Utility

Strong Risk Averse



Others

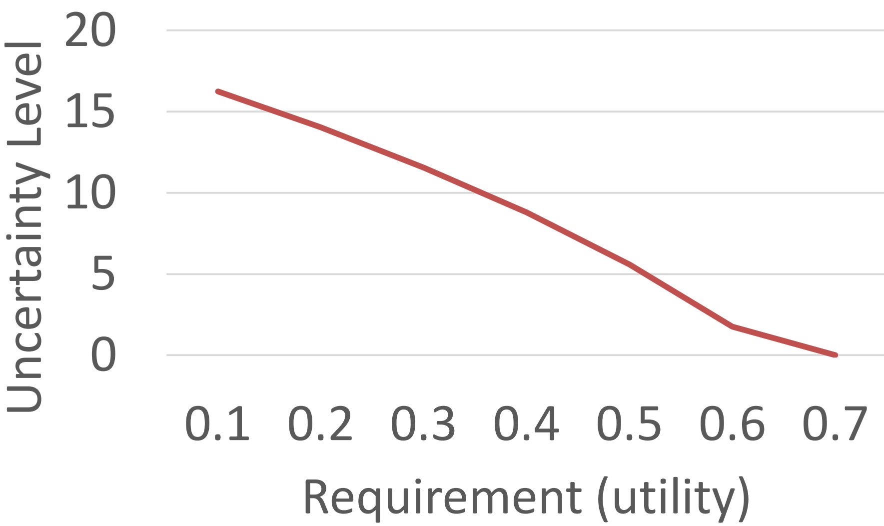


Strong risk averse is more sensitive to high cost

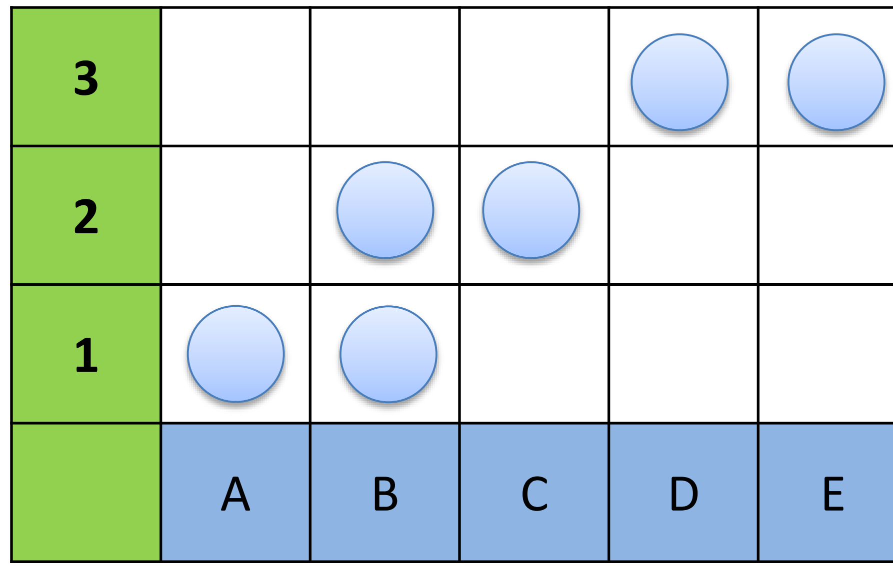
Others include: risk seeking, risk neutral and slight risk averse

Info-Gap

Info-Gap Result



Optimal Decision for Utility >= 0.4

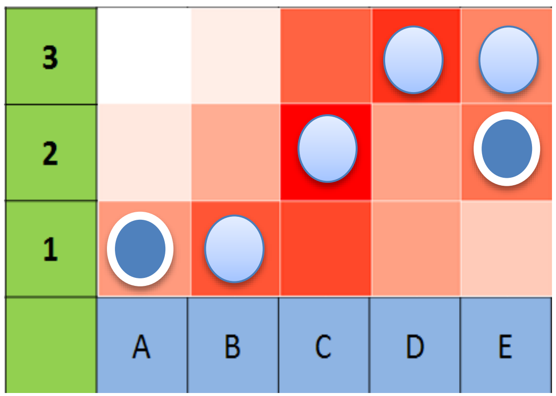


- Uncertainty level: percentage of probability allowed to be changed
- No feasible strategy if utility threshold is 0.7 or greater

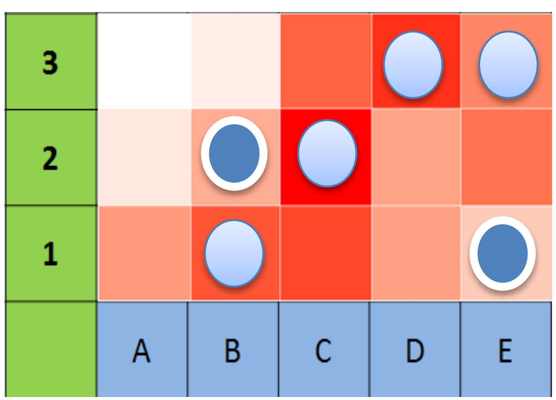
If utility threshold is between 0.4 and 0.6, info-gap recommends strategy

Compare to EU

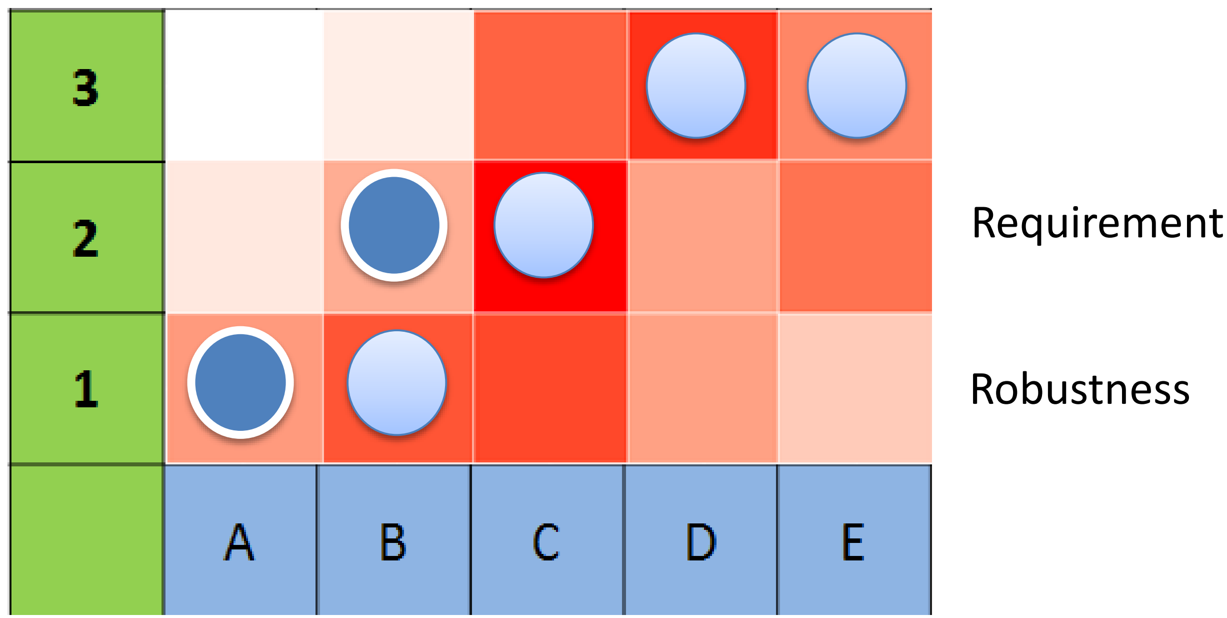
EU - Strong Risk Averse



EU - Others



Info-Gap



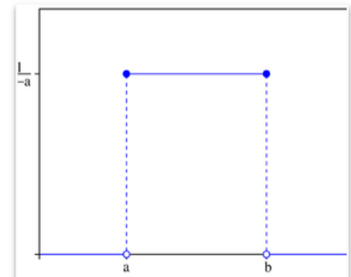
Combination of strong risk averse and risk neutral

RDM – Constant Uncertainty

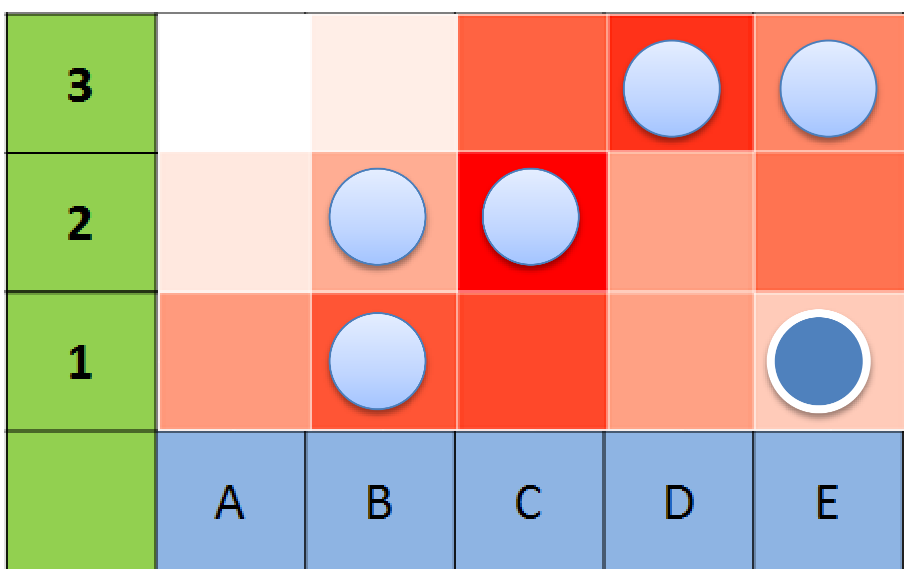
$$V(s) = z * R_{best}(s) + (1 - z) * R_{worst}(s)$$

Uniform Distribution

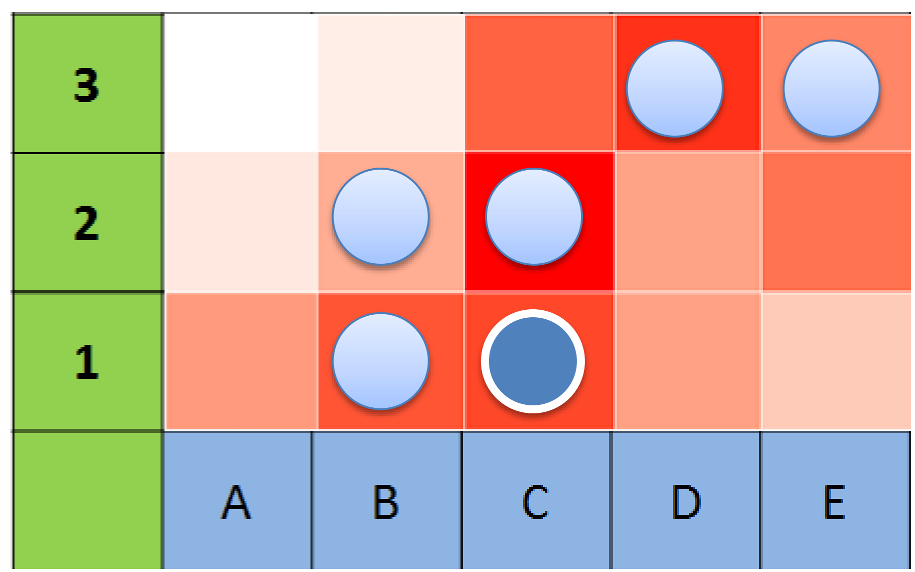
Centralized Distribution



Optimal Strategy when Z = 0.3



Optimal Strategy when Z = 0.8

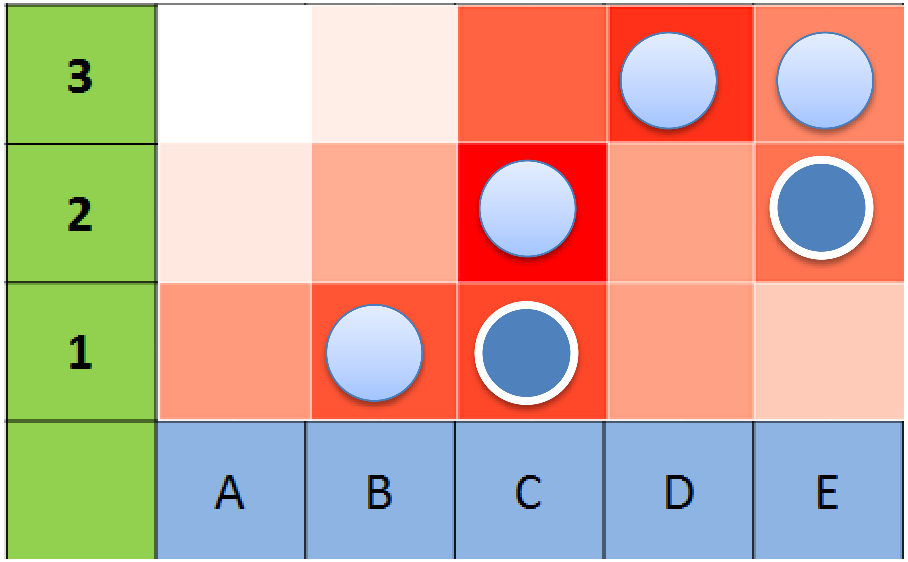


RDM – Increasing Uncertainty

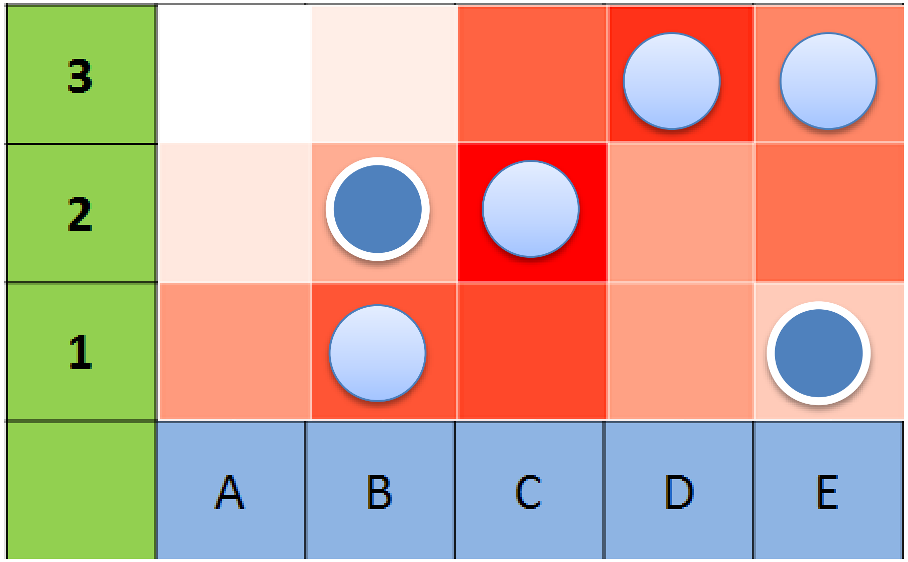
$$V(s) = z * R_{best}(s) + (1 - z) * R_{worst}(s)$$

Centralized Distribution Uniform Distribution

Optimal Strategy when Z = 0.3



Optimal Strategy when Z = 0.8



Conclusions

- Expected Utility
  - High confidence on estimated probability
- Information Gap
  - Balance between robustness & requirement
  - Not looking for optimal performance
  - Good when lack of information
- Robust Decision Making
  - Tradeoff between robustness & optimality (z)
  - Less sensitive to estimated probability
  - Provide flexibility for different risk attitudes
- Future work
  - Flexible strategy

Reference

- R. J. Lempert and M. T. Collins, "Managing the risk of uncertain threshold responses: comparison of robust, optimum, and precautionary approaches," *Risk analysis, vol. 27, pp. 1009-1026, 2007.*
- Y. Ben-Haim, *Info-gap decision theory : decisions under severe uncertainty / Yakov Ben-Haim, 2nd ed.. ed. Oxford: Oxford : Elsevier/Academic, 2006.*
- J. W. Hall, R. J. Lempert, K. Keller, A. Hackbarth, C. Mijere, and D. J. McInerney, "Robust Climate Policies Under Uncertainty: A Comparison of Robust Decision Making and Info-Gap Methods," *Risk Analysis, vol. 32, pp. 1657-1672, 2012.*