# Optimal Bidding Strategy for GENCO with Green Power in Day-ahead Electricity Market

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Abstract—The electricity market has evolved from a regulated monopoly to a more liberalized competitive market, which allows a generating company (GENCO) to bid to provide energy. The two-period structure of the electricity market (day-ahead and real-time market) introduces a mechanism for determining the GENCO's optimal bidding strategy. The difference between clearing prices for each period adds uncertainty to GENCO's determination of its optimal bid. In addition, the fast growth of renewable energy sources (wind and solar power) and their increasing penetration to the power system adds uncertainty over how much energy the GENCO can actually produce in the realtime market. Based on the two-period market structure, we develop an optimization model for a single GENCO with green power to derive an optimal strategy to bid a price and quantity for the day-ahead market with the objective of maximizing its expected profit. Furthermore, we apply the optimization model with risk-aversion attitude to reduce chance of negative profits for GENCO. We fit probability distributions to historical data to reflect the uncertainties, and Monte-Carlo simulation allows us to solve the stochastic optimization problem. The optimization model and corresponding algorithm are verified in Southern California Edison, a GENCO in California ISO.

*Index Terms*—GENCO, bidding strategy, green power, electricity market, risk-aversion, utility function, stochastic optimization

# NOMENCLATURE

**Decision** variables

G <sub>BID</sub>	Bid quantity for day-ahead market (MW)
$\rho_{RID}$	Bid price for day-ahead market (\$/MWh)

#### Exogenous random variables

$G_i$	Actual generated quantity for real-time market from
	generation technology $i$ (MW), $i = 1, 2, 3, 4$ , 1-
	wind, 2-solar, 3-coal, 4-gas
-	

- **G** Total actual generated quantity for real-time market (MW),  $G = \sum_{i=1}^{4} G_i$
- $\rho_{DA}, \rho_{RT}$ , Clearing price for day-head (DA) or real-time (RT) market (\$/MWh), i.e., DA-price or RT-price

## Parameters

$c_i$	Production costs for generation technolog
	<i>i</i> (\$/MWh), <i>i</i> = 1, 2, 3, 4
$c_{CT,i}$	Curtailment costs for variable generation technolog
	i (\$/MWh), $i = 1, 2$
Pj	The probability of scenario $j = 1, 2,, 7$
F.	The profit made by scenario <b>i</b>

 $F_j$  The profit made by scenario j

# I. INTRODUCTION

Since the 1990s, the electricity market has evolved from a vertically integrated structure to a competitive deregulated market design. The reconstructed market is similar to an imperfect competition or oligopoly market due to the special characteristics, such as a limited number of suppliers, long construction periods of power plants and large capital investment sizes [1]. The current market structure is consisted of two separate financial settlements: day-ahead market and realtime market. The day-ahead market is a forward market, which is settled 24 hours before the operating day and allows market participants to commit selling offer or buying biddings for the next operating day. The real-time market is settled every five minutes in the specific operating day, which balances the difference between day-ahead commitments and the real-time actual production and demand. A typical energy market timeline is shown in Figure 1, which illustrates the well-organized structure of the two-period settlements in ISO-New England (Independent System Operator) [2].



Fig. 1. Electricity market timeline

Under such a market setting, the supplier (generation company (GENCO), or virtual bidder) can maximize its profits by strategic bidding, and the buyers (load-serving entities) hope to pay a reasonable amount to satisfy their demand requirements. Thus, the development of optimal bidding strategy for GENCO is crucial.

With the fast growth of renewable energy, especially wind and solar power, a GENCO's optimal bidding strategy becomes more complicated and challenging. In addition to dealing with the possible load deviations, the output uncertainty from solar and wind power should also be considered when a GENCO

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develops a bidding strategy. In particular, the GENCO should pay for energy production deviations resulted from the prediction error [3]. Thus, researching the optimal bidding strategy for GENCO with green power in day-ahead electricity market is of interest.

Prior research has modeled the optimal bidding strategy for GENCOs ([1], [4]). The models fall into four categories: 1) a single GENCO optimization model [5-7]; 2) game theory with multiple GENCOs and buyers [8-10]; 3) agent-based models [11,12] and 4) hybrid models [13,14]. Most of them focus on the bidding strategy based on the day-ahead market, and only a few consider the background of the real-time market [15]. Some literature addresses the bidding strategy when relying on renewable energy [16-18]. A mixed-integer nonlinear programming model determines the optimal hydro scheduling and offering strategies in the Portugal energy market [16]. A stochastic Cournot model based on generated scenarios from the Auto Regressive Moving Average is proposed to realize strategic bidding for wind-dominated GENCOs [17]. Angarita et al. [18] introduce a combined bidding strategy for wind farm and hydro generating units, which can reduce the loss brought by the fact that variable energy has uncertain output.

In summary, much of the foregoing research focuses on a single market, which does not address the difference between day-ahead market and real-time market. For most of these cases, only a single generation technology is considered, which cannot satisfy the real-world fact that multiple technologies (especially green power) are available within a single GENCO. The unique contribution of this paper is the development of an optimization model based on a two-period market structure to determine the optimal bidding strategy for a GENCO with green power and multiple generation technology. The electricity prices for both periods and the generation output for the green power are uncertain. This type of model allows the GENCO to produce more electricity from fossil fuel sources if the renewable energy sources generate less electricity than anticipated. The GENCO can determine the best way to meet its requirements based on which generation technology costs less and produces the highest profits. We also incorporate risk attitude into the model to understand how the bidding strategy should change for a riskaverse GENCO. This is another significant contribution of this paper.

Based on a typical electricity market structure and management rules, we develop a stochastic optimization model in order to recommend the optimal bidding strategy for a GENCO who wishes to maximize its expected profit or expected utility under this two-period electricity market.

The remaining of the paper is organized as follows. Section II introduces the market structure and management rules, which is the foundation of our analysis. Section III details and formulates the bidding problem and proposes the stochastic optimization model. Section IV applies the model to Southern California Edison (SCE, a GENCO from California ISO, CAISO). Section V concludes the paper.

## II. MARKET STRUCTURE AND MANAGEMENT RULES

#### A. Market structure

A bidding strategy is based on the market structure and auction rules. The typical market structure includes a wholesale market and retail market, and GENCOs participate in the wholesale market, as shown in Figure 2. The buyers include large energy users and distribution utilities and retailers. A GENCO can sell its electricity production directly to power pools by competing in the electricity energy market, entering into bilateral contracts, or providing ancillary service in the corresponding reserve market. This paper focuses exclusively on the bidding strategy of a GENCO in the power pool. Daily auctions for electricity power exist in the pool-based electricity market, which determines the wholesale electricity price.



Fig. 2. General structure of wholesale electricity market

## B. Market clearing mechanism

The pool-based market is a type of mediated market, which serves as an auction center to which all buyers and sellers submit bids. The bid of a GENCO has two components: a bid price and a bid quantity. After the period of submitting bids closes, the generation bids are cleared or scheduled in price ordered from lower to higher prices in order to eliminate the more expensive bids. An ISO conducts this procedure [1].

As shown in Figure 3, the intersection point between the aggregated hourly supply offer curve and the aggregated hourly demand bid curve is the market clearing price (MCP), which is also called equilibrium point. The MCP is set as either the last accepted offer or the first rejected offer. GENCOs whose bid prices are less than or equal to the MCP are "committed" to sell their bid quantity in the following day. The GENCOs are paid by uniform pricing or pay-as-bid [1]. Uniform pricing means that all cleared GENCOs are paid the same MCP, no matter how much their original bidding price is. Pay-as-bid indicates that the committed GENCO will sell its energy at the price that it bid assuming it is less than the MCP. In order to simplify the calculation and analysis, this paper assumes a uniform pricing market, which means the GENCO sells its energy at the MCP.



Fig. 5. Warket clearing proces

# C. Day-ahead market and real-time market

The day-ahead market develops schedules about unit commitment to satisfy the forecasted load requirement. Buyers purchase a certain amount of electricity in the day-ahead market, but the real-time actual demand may be higher or lower than the equilibrium quantity from the day-ahead market. A real-time market exists in which buyers purchase electricity in real time [2]. Two conditions can happen, detailed as follows:

- When the real-time price is higher than the day-ahead price, GENCOs whose bid prices exceed the day-ahead MCP but are less than the real-time price will be committed in the real-time market.
- When the real-time price is less than the day-ahead price, a committed GENCO has the option to "buy out." A GENCO can decide between 1) producing its bid quantity from the day-ahead action or 2) buying the same quantity from the real-time market. It should make this decision based on the alternative that yields higher profits.

Uncertainty in the relationship between the day-ahead price and real-time price plays a significant role in determining the optimal bidding strategy for GENCOs.

## **III. PROBLEM FORMULATION**

The GENCO's optimization problem is to maximize its expected profit (or utility) by bidding a paired price and quantity in the day-ahead market. Four parameters are uncertain in the model: the day-ahead MCP (denoted as day-ahead price), the real-time MCP (denoted as real-time price), the actual real-time output of wind power, and the actual real-time output of solar power. We assume the capacity mix of the GENCO is wind power, solar power, natural gas, and coal. To simplify the analysis, we focus on a bidding strategy for a particular hour.

## A. Uncertainty analysis

# 1) Day-ahead price and real-time price

The day-ahead price and the difference between the dayahead and real-time prices are modeled as independent random variables. There are several studies researching price forecasting [19], including statistical methods and optimization methods. In this paper, we derive probability distributions for the day-ahead price  $\rho_{DA}$  and the difference between the two prices in order to preserve dependence between the day-ahead and real-time prices. The real-time price  $\rho_{RT}$  equals to  $\rho_{DA}$  plus the difference between the two prices. The model assumes that both prices are determined exogenously from the GENCO's bid price.

## 2) Wind output and solar output

As variable and intermittent generation, wind and solar power have uncertain output, especially 24 hours prior to generation. Unlike traditional gas-fired and coal-fired units, which can produce exactly what is bid in the day-ahead market, the output of wind and solar is challenged by over-generation or under-generation. If solar and wind power are over generated, the GENCO curtails the extra power with specific curtailment costs. If solar and wind power are under generated, the GENCO will need to buy additional power from the real-time market. Prior research has studied forecasting the output of variable energy [20-21]. In this paper, we adopt a similar method to the uncertain prices by modeling wind and solar power generation as random variables whose distributions are derived from historical data.

The model assumes that four generation technologies are available to the GENCO. The first two generation technologies are wind (i = 1) and solar (i = 2) and the other two generation technologies are coal (i = 3) and natural gas (i = 4). To calculate the curtailment costs, the generation technology with lower curtailment costs (Technology A) will be chosen to reduce power output first. If reducing all available power from A still cannot solve the problem of over-generation, the curtailment behavior will come to technology B. The corresponding curtailment costs is denoted as  $f(c_{CT}, G - G_{BID})$  where  $c_{CT}$  are the curtailment costs, G is the total amount of energy produced, and  $G_{BID}$  is the GENCO's bid quantity in the day-ahead market.

The GENCO submits a single bid price  $\rho_{BID}$  and a single bid quantity  $G_{BID}$ , regardless of the power sources. The time horizon is one hour, i.e., the decision variables are the GENCO's bidding quantity and price for a particular hour of the next day. We assume that the GENCO can purchase as much energy as they need from the real-time market.

## B. Model formulation with expected profits-Model I

1) Objective function: maximize the expected profits.

$$\max E\left\{F\left(G_{BID}, \rho_{BID}\right)\right\} = \sum_{j=1}^{\prime} P_j F_j$$
(1)

where  $P_j$  is the probability of each scenario, and  $F_j$  is the corresponding profit for each scenario. The scenarios are classified based on the relationships i) between the day-ahead price and real-time price and ii) between the actual output *G* and the bid quantity  $G_{BID}$ . The seven scenarios are detailed as follows:

a)  $\rho_{BID} \leq \rho_{DA} \leq \rho_{RT}$ ,  $G \geq G_{BID}$ : The GENCO is committed in the day-ahead market and does not have the option to "buy out" because the real-time price is more than the bid price. The profit equals the revenue from selling electricity in the day-ahead market minus the production costs and the curtailment costs in which  $G - G_{BID}$  must be curtailed.

$$F_{1} = \rho_{DA}G_{BID} - \sum_{i=1}^{4} c_{i}G_{i} - f(c_{CT}, G - G_{BID})$$
(2)

b)  $\rho_{BID} \leq \rho_{DA} \leq \rho_{RT}$ ,  $G < G_{BID}$ : The GENCO is committed in the day-ahead market and does not have the option to "buy out". The wind and/or solar power does not

generate enough, and the GENCO should buy the additional power from the real-time market. The profit equals the revenue from the day-ahead market minus the production costs and the costs for purchasing additional power in the real-time market.

$$F_{2} = \rho_{DA}G_{BID} - \sum_{i=1}^{4} c_{i}G_{i} - \rho_{RT} \left(G_{BID} - G\right)$$
(3)

c)  $\rho_{BID} \leq \rho_{DA}$ ,  $\rho_{DA} > \rho_{RT}$ ,  $G \geq G_{BID}$ : The GENCO makes decisions of "produce or buy" based on the production costs  $c_i$  and real-time price (plus curtailment costs for wind and solar power) for each generation technology. As with the first scenario, the surplus power  $G - G_{BID}$  must be curtailed.

$$F_{3} = \rho_{DA}G_{BID} - \sum_{i=1}^{2} \min(c_{i}, \rho_{RT} + c_{CT,i})G_{i} - \sum_{i=3}^{4} \min(c_{i}, \rho_{RT})G_{i} - f(c_{CT}, G - G_{BID})$$
(4)

d)  $\rho_{BID} \leq \rho_{DA}$ ,  $\rho_{DA} > \rho_{RT}$ ,  $G < G_{BID}$ : The GENCO is committed in the day-ahead market and has the option to "buy out", but it also needs to buy additional power from the real-time market.

$$F_{4} = \rho_{DA}G_{BID} - \sum_{i=1}^{2} \min(c_{i}, \rho_{RT} + c_{CT,i})G_{i} - \sum_{i=3}^{4} \min(c_{i}, \rho_{RT})G_{i} - \rho_{RT}(G_{BID} - G)$$
(5)

e)  $\rho_{DA} < \rho_{BID} < \rho_{RT}$ ,  $G < G_{BID}$ : The GENCO is not committed in the day-ahead market, but it will be committed in real-time market.

$$F_{5} = \rho_{RT} G_{BID} - \sum_{i=1}^{4} c_{i} G_{i} - \rho_{RT} \left( G_{BID} - G \right)$$
(6)

*f*)  $\rho_{DA} < \rho_{BID} < \rho_{RT}$ ,  $G \ge G_{BID}$ : The GENCO is committed in the real-time market and needs to curtail extra power produced by the green energy.

$$F_{6} = \rho_{RT}G_{BID} - \sum_{i=1}^{4} c_{i}G_{i} - f(c_{CT}, G - G_{BID})$$
(7)

g) All other situations: The GENCO will not be committed but it will need to pay the curtailment costs for wind and solar power.

$$F_{7} = -\sum_{i=1}^{2} c_{CT,i} G_{i}$$
(8)

2) Constraints:

The optimization model has a few constraints. One constraint is:

$$\rho_{BID\_min} \le \rho_{BID} \le \rho_{BID\_max} \tag{9}$$

where  $\rho_{DA\_min}$  is the minimum day-ahead price and  $\rho_{DA\_max}$  is the maximum day-ahead price. The GENCO should not bid a price less than the minimum day-ahead price (to avoid extreme clearing prices) or bid more than the maximum day-ahead price.

The second constraint is that the total bid quantity should not exceed the maximum available capacity (the capacity factor is included):

$$G_{BID} \le \sum_{i=1}^{4} G_{i_{-}\max} \tag{10}$$

where  $G_{i\_max}$  is the maximum amount of electricity that can be produced by generation technology *i*.

The model assumes the GENCO can purchase as much electricity as it needs in the real-time market (e.g., if the wind or solar power under generates). This is a reasonable assumption for our case study because the GENCO only produces about 20% of the region's electricity. However, if the GENCO is a major player in the market, a constraint could be added that limits how much electricity it can purchase in the real-time market.

# C. Model formulation with risk attitude-Model II

A GENCO may be risk averse, which means it may be willing to sacrifice the possibility of higher profits in order to avoid losing money. In order to integrate a GENCO's risk attitude, we propose Model II, which introduces a utility function to describe the GENCO's risk attitude over the uncertain profits.

## 1) Utility function of GENCO

We assume an exponential utility function to represent the GENCO's preferences, as shown as equation (11).

$$U(x) = 1 - e^{-x/R} \tag{11}$$

where x is the GENCO's profit and R represents its risk tolerance (where both are in millions of dollars). When R > 0, the GENCO is risk averse, and a smaller value of R indicates more risk-aversion. If R < 0, the GENCO is risk seeking and if  $R \rightarrow \infty$ , U(x) becomes risk-neutral [22].

The GENCO's risk tolerance has a direct relation to the GENCO's willingness to accept an uncertain deal between earning \$1 million and losing \$1 million. The value of *R* can be calculated based on the probability *q* for which the GENCO is indifferent between no gain and no loss (\$0) and a *q* probability of earning \$1 million and 1 - q probability of losing \$1 million. Figure 4 depicts that indifference relationship, and equation (12) shows how to calculate *R* given a value for *q*.



## Fig.4. Assessing risk tolerance

2) Model formulation

Given a utility function, Model II calculates the utility for each scenario as depicted in equations (2)-(8) and maximizes the expected utility as given in equation (13). Since higher expected profits may mean more risk, a risk-averse GENCO may choose a bidding strategy that yields lower expected profits than a riskneutral GENCO.

$$\max E\left(U\left\{F\left(G_{BID}, \rho_{BID}\right)\right\}\right) = \sum_{j=1}^{7} P_j \times U\left(F_j\right)$$
(13)

# C. Solving procedure

Monte-Carlo simulation provides an efficient alternative to deal with the four uncertain parameters. The simulation generates thousands of realizations of price and output uncertainties, and an optimization algorithm determines the optimal bidding strategy given these simulated values. The algorithm works identically whether expected profit or expected utility is maximized.

The solving process is detailed as follows:

- Collect historical data for the day-ahead price, real-time price, hourly wind output, and hourly solar output, and fit the corresponding distributions;
- Apply the Monte-Carlo simulation to generate price and variable power output data;
- Apply an optimization tool to solve the model and achieve the optimal bidding strategy for a GENCO.

# IV. NUMERICAL EXAMPLE

We utilize our model and algorithm for a GENCO in CAISO---Southern California Edison (SCE). The generation mix for SCE is shown in Figure 5, where the mix represents the maximum generation that SCE can produce. The production costs for green power is \$0/MWh, \$66.13 for coal-fired units, and \$26.60/MWh for gas-fired units. The curtailment costs for wind and solar power are \$3/MWh and \$0 MWh, correspondingly.



Fig. 5. Modified generation mix of SCE (MW)

## A. Fitting distribution

We collect hourly data of day-ahead price, real-time price, and wind and solar output in 2014<sup>2</sup>. The day-ahead price is subtracted from the real-time price for each data point. These values depend on the season and time of day, and we select a total of 12 conditions: hour 1 (midnight), hour 13 (1 p.m.) and hour 17 (5 p.m.) each for spring (S), summer (Sm), fall (F), and winter (W) seasons. We fit each of the four uncertain parameters to a probability distribution for each of these 12 conditions, as indicated in Appendix Table A-1.

## B. Solving the model

We use Matlab to generate 100,000 cases and use the optimization toolbox Pattern Search in Matlab to solve the model. Pattern search is a direct-search optimization method that does not require calculating the gradient of the objective

<sup>2</sup> Wind and solar output source:

http://www.caiso.com/green/renewableswatch.html;

function. The pattern search algorithm finds a sequence of points that improve the objective function from one sequence to the next [23]. Pattern search has been identified as a successful heuristic for solving non-differentiable optimization problems [24, 25].

We verified each solution for a particular hour with another data set of 100,000 simulated cases, and the optimal bidding strategy for SCE shows little change given the new simulated cases. Thus, 100,000 simulations are reasonable to solve this optimization model with these distributions.

## C. Results analysis

## 1) Bidding strategy analysis

Table 1 indicates the optimal bidding strategy for the GENCO, including its expected profit, the standard deviation in profit, and the probability that the GENCO will lose money. Table 2 shows the results of the simulation, including the average day-ahead and real-time prices, the average output for wind and solar power, and the probability that the day-ahead price exceeds the real-time price. Figure 6 depicts the profit distribution (based on 100,000 new simulations) for each hour given the bidding strategy. (S-spring, Sm-summer, F-fall and W-winter).

For most conditions, the optimal bidding strategy is to bid a price that is less than the expected day-ahead price. This strategy increases the probability that the GENCO will be committed in the day-ahead market. The GENCO should bid more than the expected day-ahead market in hours Sm17 and F17 because the probability that the real-time price will be more than the dayahead price is 0.85 and 1.0, respectively. If the real-time price is more than the day-ahead price, the GENCO will have to buy additional power at the real-time price and sell it for a loss if it does not generate as much electricity as it bids. Thus, the GENCO does not want to be committed in the day-ahead market if the day-ahead price is low. This example demonstrates that a GENCO who fails to consider the full probability distribution of the prices and just bids the expected day-ahead price may not be maximizing its profit.

	Bid price (\$/MWh)	Bid quantity (MW)	Expected profit(\$)	Profit standard deviation	Probability (profit <=0)		
S1	46.00	11,170	278,240	2.E+05	0.254		
S13	34.26	12,526	262,632	7.E+04	0		
S17	36.95	12,299	365,651	2.E+05	0.069		
Sm1	35.24	11,529	206,154	4.E+04	0		
Sm13	27.80	11,839	329,419	7.E+04	0		
Sm17	71.09	13,305	621,926	9.E+05	0.501		
F1	23.32	10,084	125,303	5.E+04	0.007		
F13	15.98	11,510	216,681	9.E+04	0.011		
F17	60.36	12,300	493,303	3.E+05	0.192		
W1	29.24	10,753	213,219	8.E+04	0.00001		
W13	35.80	11,239	317,355	1.E+05	0		
W17	36.42	11,150	310,821	1.E+05	0.00001		
TABLE 2 Property of uncertain factors							

TABLE 1 Optimal bidding strategy

Day-ahead and real-time price source:

http://www.energyonline.com/Data/GenericData.aspx?DataId=19&C AISO.

	Day- ahead price	Real- time price	Actual wind output	Actual solar output mean (MW) Probability (day-ahead price>real-time price)	Sm13 Sm17 F1	50.12 64.29 36.41	49 74 36	1,478 1,237 513	1,120 818 0	58.52% 15.17% 57.08%	
	mean (\$/MWh)	mean (\$/MWh)	n) (MW)		price>real-time price)	F13 F17	39.53 51.08	35	1,172	769 340	68.74% 0.00%
S1	46.13	55	1221	0	20.87%	W1	45.60	44	705	0	57.31%
S13	41.53	38	743	946	61.63%	W13	49.85	44	536	677	62.39%
S17	51.24	55	1,465	1,759	52.48%	W17	53.95	52	393	241	61.71%
Sm1	41.21	39	747	0	59.89%						



## Fig.6. Distribution of profits for different hours and seasons

Hour S1 is one of the few conditions when the GENCO's optimal bid price is close to the expected day-ahead price. The expected day-ahead price is \$46.13/MWh, and the bid price is \$46.00/MWh. At these prices, the GENCO should not generate coal because its production cost (\$66.13) is \$20 more than the mean day-ahead price. Since it is midnight, solar power is nonexistent. The GENCO will produce natural gas because its production cost (\$22.60) is \$44 less than the mean day-ahead price. The expected output of wind power is 1221 MW. From Table 1, the GENCO should bid to offer 11,170 MW, which is approximately 500 MW more than the expected output from wind (1121 MW) plus the output from natural gas (9472 MW).

The GENCO should bid a higher quantity of output in hours 13 and 17 than in hour 1 because it can depend on solar power. The GENCO tends to generate higher profits in hour 17 because the day-ahead and real-time prices are higher. This reflects more demand for electricity during this hour.

Hour F1 yields the lowest expected profit, which results from the unavailability of solar power, a low expected wind output, and a relatively low day-ahead price.

The GENCO has the highest expected profit in hour Sm17 (\$621,926), and its profit could be as much a \$7 or \$8 million. Both the day-ahead and real-time prices tend to be high, which means that the GENCO should produce from its coal-fired units and it can also sell its electricity at a high price. Both of these conditions lead to a high expected profit.

Hour Sm17 also has the highest probability that the GENCO will lose money during the hour, around 50%. The most it will

lose according to the simulation is \$20,590, which is a lot less than its expected profit. The potential for lost profit is due to: (i) bidding a price greater than the day-ahead and real-time prices so that wind power must be curtailed; (ii) bidding a quantity that exceeds output and needing to buy additional power from the real-time market; and (iii) producing coal to meet its bid quantity when the day-ahead price is less than the cost of producing coal-generated electricity.

## 2) Risk-averse GENCO

As shown in Table 1, the possibility exists that the GENCO could lose money, which suggests the GENCO may want to be risk averse and seek to minimize its losses. Hour Sm17 has the most volatile distribution for profit, and the chance that GENCO may lose money is 50%. If the GENCO is risk averse, it may optimal for a different bidding strategy in order to avoid negative profits.

We apply Model II with risk attitude to hour Sm17 for several values of q as explained in Section 3.D. Larger values of q indicate more risk aversion. As depicted in Figure 7, the optimal bid price decreases as the GENCO becomes more risk averse. By decreasing its bid price, the GENCO is increasing the chances that it will be committed in the day-ahead market. This suggests that the largest profit losses are due to curtailing wind power or buying additional power from real-time market. Increasing the probability of being committed in the day-ahead market also increases the probability of needing to buy additional power from the real-time market. The optimal bid price continues to fall until it reaches the minimum day-ahead price, approximately \$40 /MWh.



Fig.7. Optimal bid price as a function of risk aversion

The optimal bid quantity stays relatively constant at 13,305 MW as the GENCO becomes more risk averse (Figure 8). The optimal bid quantity does increase by approximately 500 MW as the GENCO becomes more risk averse.



Fig.8. Optimal bid quantity as a function of risk aversion

As displayed in Figures 9 and 10, The GENCO's expected profit decreases with risk aversion, and the standard deviation in the profit and the probability of losing money decreases. The GENCO's expected profit only decreases \$622,000 to about \$590,000 for the most risk-averse GENCO, but the probability of losing money is less than 0.1%. Since the risk-averse GENCO is committed more often in the day-ahead market, it has fewer opportunities to be committed in the real-time market. As shown in Table 2, the probability that the day-ahead price is more than the real-time price is only 15%. A risk-averse GENCO is sacrificing the chance of being able to sell its power for more in the real-time market but is also lessening the chances of not being committed in either market.



Fig.9. Expected profit and standard deviation as a function of risk aversion



Fig.10. Probability of negative profit as a function of risk aversion

Answering what is the proper risk attitude or utility function for the GENCO is beyond the scope of this paper; however, a few remarks are in order. Since the potential losses are on the order of tens of thousands of dollars and the potential gains are on the order of millions of dollars, the GENCO should be able to pay for the losses without too much difficulty. The GENCO will also be bidding for multiple hours and multiple days. If the uncertainties in the model are identically and independently distributed for each day during hour Sm17, the GENCO's average profit should approach the expected profit over the course of a month. Any losses for a particular hour should be compensated for in the next hour or day. Under these assumptions, adopting a risk-neutral attitude and maximizing the expected profit is the best strategy for multiple-hour bidding. However, risk-aversion may be appropriate for single-hour bidding, as shown in this paper.

## V. CONCLUSION

This paper has developed a stochastic optimization model to determine the optimal bidding strategy of a GENCO with green power in the day-ahead market. The uncertain factors—the dayahead and real-time prices and the actual generation output of wind and solar power—are modelled with probability distributions. The paper developed profit equations for the different scenarios based on the relationships among the bid price, the day-ahead price, and the real-time price and between the bid quantity and the actual generation output.

The model is applied to SCE, a GENCO in California, who operates wind, solar, coal, and natural gas power plants. Historical data was used to fit distributions to the four uncertainties for twelve different hour-season combinations. Monte Carlo simulation was used to solve the stochastic optimization problem.

The results indicate that a risk-neutral GENCO should usually bid slightly less than the expected day-ahead price. A risk-averse GENCO should bid a lower selling price in order to increase its chances of being committed in the day-ahead market. If the probability the real-time price exceeds the day-ahead price is greater than 50%, a risk-neutral GENCO can bid more than the day-ahead price because it has a good chance of being committed in the real-time market if its bid price is more than the day-ahead price. The GENCO's optimal bid quantity may or may not include the expensive coal-generated units, depending on how the distribution on the day-ahead price compares with the cost of generating electricity from coal.

Future work can refine the model to account for the possibility that the GENCO may not be able to purchase additional power on the real-time market if actual generation is less than the bid quantity. A GENCO's bid may also influence the market clearing price, which would require the probability distributions to be dependent upon the GENCO's strategy. The effect of including multiple GENCOs could also be included in the model.

To our knowledge, this paper represents the first model that incorporates two periods (day-ahead and real-time markets) with multiple generating technologies including green power. Modeling uncertainties with probability distributions based on historical data allow us to develop realistic results that a GENCO can use to make better decisions in the electricity markets.

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Fitted distribution		Wind Output (MW)	Solar Output (MW)	Day-ahead Price (\$/MWh)	Real-time price minus Day- ahead price(\$/MWh)	
	Hour 1	Normal	zero	Beta	Log-normal	
Spring		$\sigma = 846.36, \mu = 1739.0$	\	$\alpha_1 = 349.84; \alpha_2 = 105.65; a = -305.56; b = 152.32$	σ = 0.2795, μ = 4.4011, γ = -76.37	
		Normal	Beta	Beta	Log-normal	
	Hour 13	$\sigma = 801.18, \mu = 1152.6$		$\alpha_1 = 1.7445; \alpha_2 = 3.2971;$ a=34.257;b=55.275	$\sigma = 0.2377, \mu = 4.2599, \gamma = -76.08$	
		Normal	Beta	Beta	Log-normal	
	Hour 17	$\sigma = 877.16, \mu = 1629.0$		$\propto_1 = 6.6292; \propto_2 = 9.62E6$ a=18.514;b=4.7455E7	$\sigma = 0.6740, \mu = 3.6265, \gamma = - 43.64$	
		Log-normal	zero	Beta	Beta	
	Hour 1	$\sigma = 0.0435, \mu = 9.6022, \gamma = -$ 13012	١	$\alpha_1 = 2.83; \alpha_2 = 5.16;a=35.24;b=52.14$	$\alpha_1 = 26.45; \alpha_2 = 674.25;,$ a=-51.74;b=1270.5	
		Log-normal	Beta	Beta	Beta	
Summer	Hour 13	$\sigma = 0.7821, \mu = 6.3392, \gamma = -$ 11.94	$\propto_1 = 3.379; \propto_2 = 0.887;$ a=899;b=1602	∝ <sub>1</sub> = 13.46; ∝ <sub>2</sub> = 537.72;a=27.8 ;b=940.9	$\alpha_1 = 1.95; \alpha_2 = 8.304;, a=-34.7; b=141.6$	
		Log-normal	Beta	Beta	Beta	
	Hour 17	$\sigma = 0.2350, \mu = 8.0039, \gamma = -$	$\alpha_1 = 4.4; \alpha_2 =$	$\alpha_1 = 5.232; \alpha_2 =$	$\alpha_1 = 0.4439; \alpha_2 = 3.653; a=-$	
		1670	0.955;a=325.85;b=1276	28024; a=38.81 ;b=1.367E5	50.6 ;b=498.1	
	Hour 1	Log-normal	zero	Beta	Log-normal	
		$\sigma = 1.3962, \mu = 5.8353$	١	$\alpha_1 = 3.37; \alpha_2 = 2.10; a = 21.94; b = 45.47$	$\sigma = 0.2394, \mu = 3.96, \gamma = -54.76$	
	Hour 13	Log-normal	Log-normal	Beta	Log-normal	
Fall		$\sigma=1.2475, \mu=5.7$	$\sigma = 0.3119, \mu = 7.1033$	$\alpha_1 = 5.76; \alpha_2 = 3.86; a=7.95; b=60.7$	σ = 0.4479, μ = 3.505, γ = -41.41	
		Log-normal	Log-normal	Beta	Log-normal	
	Hour 17	$\sigma = 1.4335, \mu = 5.7849$	$\sigma=0.372, \mu=6.617$	$\alpha_1 = 7.11; \alpha_2 =$ 24.38;a=22.44 ;b=149.5	σ = 0.6422, μ = 3.03, γ = -31.78	
	Hour 1	Beta	zero	Beta	Log-normal	
		$(\propto_1 = 0.638; \propto_2 = 1.279;,$ a=7.0;b=2650)	١	$\alpha_1 = 4.54; \alpha_2 = 4.57E6; a=29.24; b=1.65E7$	$\sigma = 0.20468, \mu = 4.1535,$ $\gamma = -66.08$	
		Beta	Log-normal	Beta	Log-normal	
Winter	Hour 13	$(\alpha_1 = 0.591; \alpha_2 =$	$\sigma = 0.21135,$	$\alpha_1 = 1.74; \alpha_2 =$	$\sigma = 0.07759, \mu = 5.785, \gamma = -$	
		2.051;a=23;b=2701)	$\mu = 6.9696$	4.81;a=35.78 ;b=88.61	334.8	
		Beta	Log-normal	Beta	Log-normal	
	Hour 17	$(\propto_1 = 0.556; \propto_2 =$	$\sigma = 0.91126$	$\alpha_1 = 4.52; \alpha_2 =$	$\sigma = 0.4227, \mu = 3.6279, \gamma = -$	
		1.213;a=8;b=2467)	$\mu = 5.6594$	6.49;a=35.29;b=2.68E7	42.65	

Appendix:Table A-1Fitted distribution of uncertain factors