SafeLife-X Safe Life Extension management of aged infrastructures networks and industrial plants
This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under Grant Agreement no. 608813

Optimization model to increase resilience, with application to the electric power network

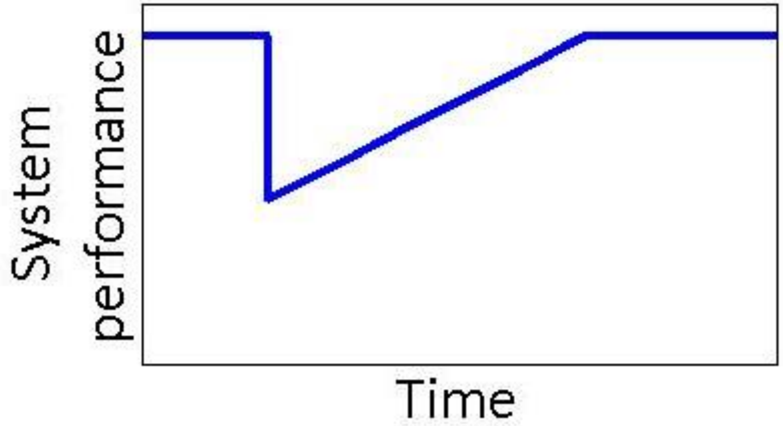
Cameron MacKenzie
Defense Resources Management Institute
Naval Postgraduate School

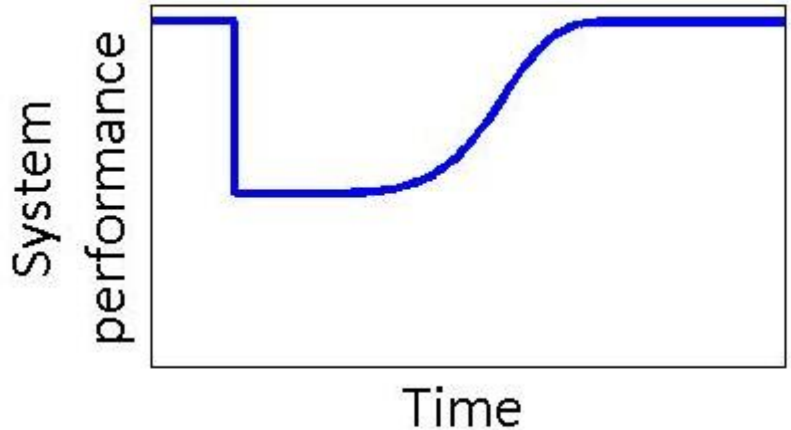
Christopher Zobel
Pamplin College of Business
Virginia Tech

Final conference
June 24-25, 2015 in Brussels

Disaster resilience

- Disaster resilience is the ability to (Bruneau et al. 2003)
  - Reduce the chances of a shock
  - Absorb a shock if it occurs
  - Recover quickly after it occurs
- Nonlinear disaster recovery (Zobel 2014)



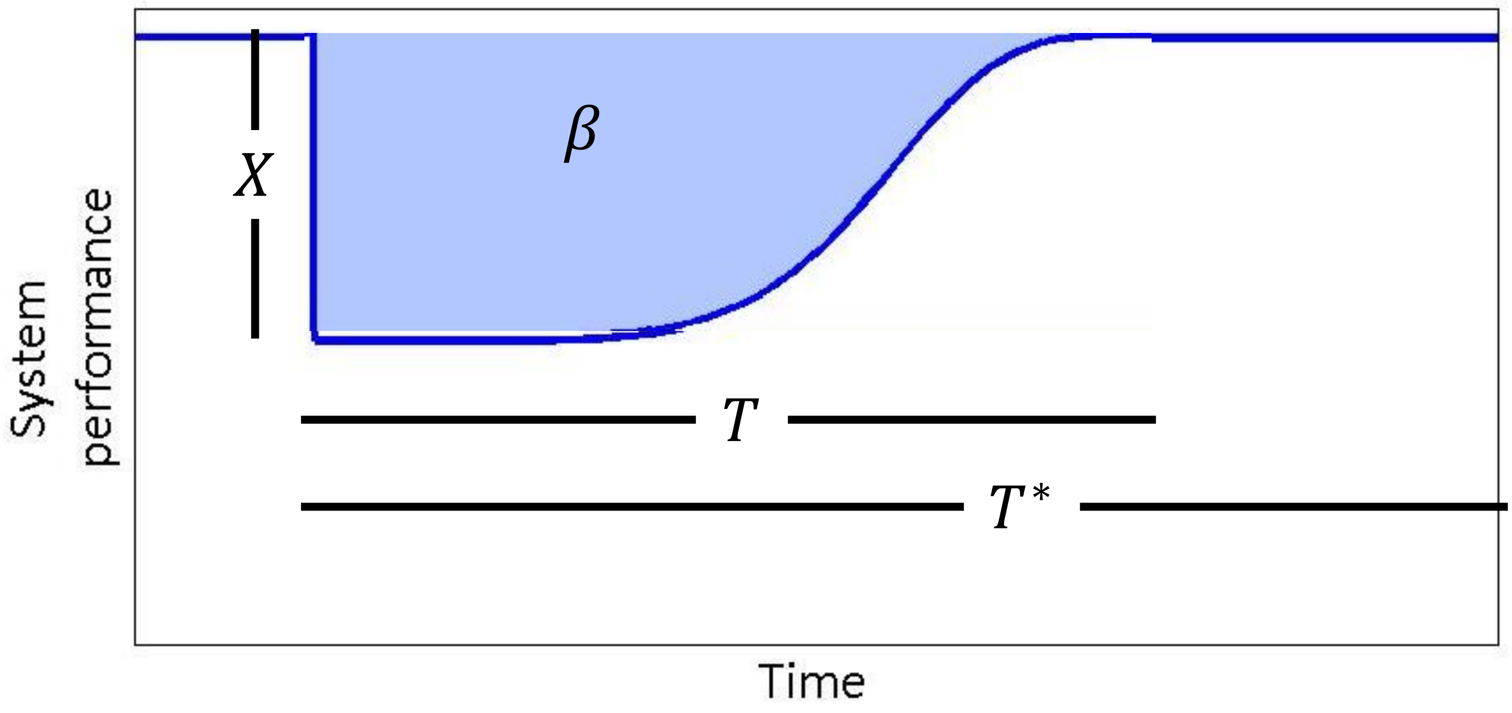


Bruneau, M., Chang, S.E., Eguchi, R.T., Lee, G.C., O'Rourke, T.D., Reinhorn, A.M., Shinozuka, M., Tierney, K., Wallace, W.A., & von Winterfeldt, D. (2003). A framework to quantitatively assess and enhance the seismic resilience of communities. *Earthquake Spectra*, 19(4), 733-752.

Zobel, C.W. (2014). Quantitatively representing nonlinear disaster recovery. *Decision Sciences*, 45(6), 687-710.

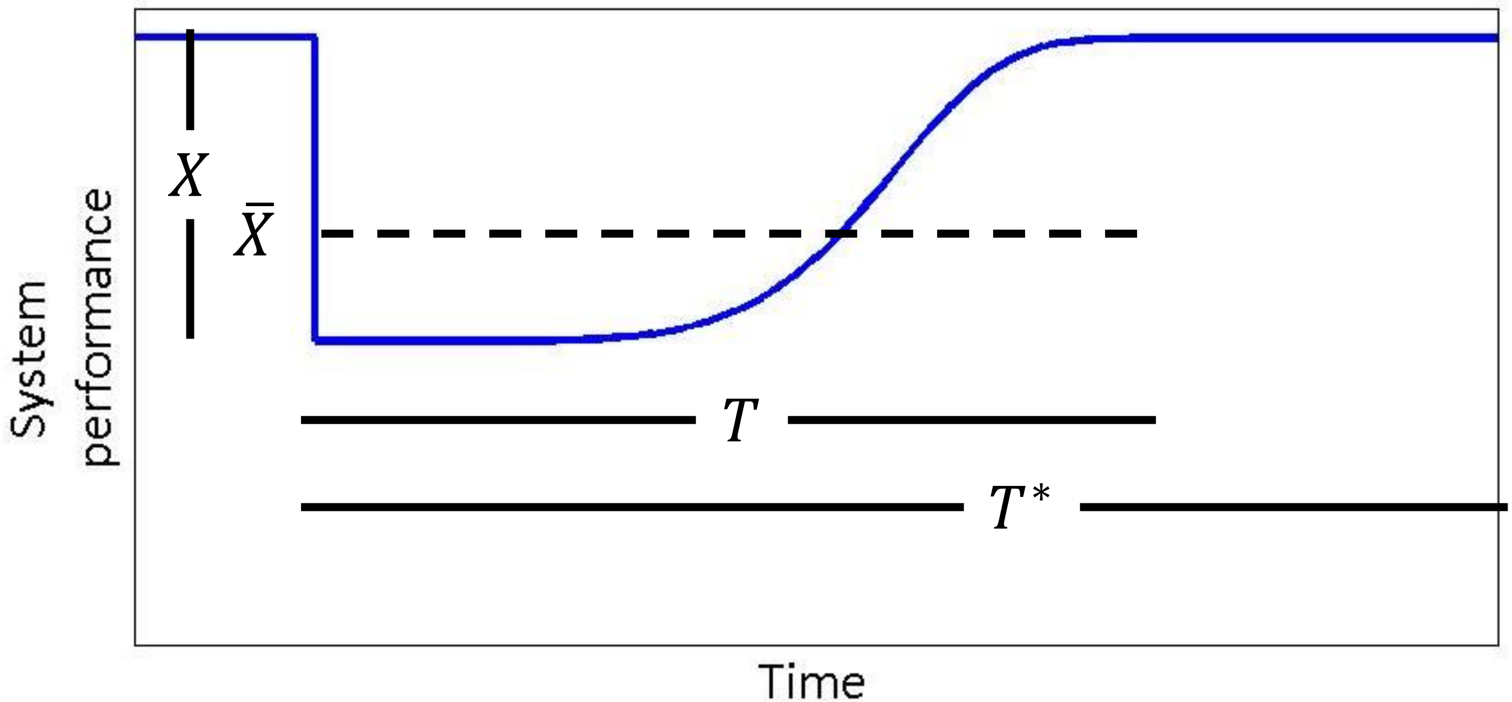
Quantifying disaster resilience

$$R_*(\beta, X, T) = 1 - \frac{\beta X T}{T^*}$$



Quantifying disaster resilience

$$R_*(\bar{X}, T) = 1 - \frac{\bar{X}T}{T^*}$$



Research questions

1. How should a decision maker allocate resources between reducing loss and decreasing time in order to maximize resilience?
2. How should the allocation change based on the assumptions in the allocation functions?
3. Does the optimal decision change when there is uncertainty?
4. How can this theoretical model be applied to a real-world disruption?

Resource allocation model

$$R_*(\bar{X}, T) = 1 - \frac{\bar{X}T}{T^*}$$

Factor as a function of resource allocation decision

maximize $\quad R_*\left(\bar{X}(z_{\bar{X}}), T(z_T)\right)$

minimize $\quad \bar{X}(z_{\bar{X}}) * T(z_T)$

subject to $\quad z_{\bar{X}} + z_T \leq Z$

Budget

$$z_{\bar{X}}, z_T \geq 0$$

Allocation functions

- $\bar{X}(z_{\bar{X}})$ and $T(z_T)$ describe ability to allocate resources to reduce each factor of resilience
- Requirements
  - Factor should decrease as more resources are allocated: $\frac{d\bar{X}}{dz_{\bar{X}}}$ and $\frac{dT}{dz_T}$ are less than 0
  - Constant returns or marginal decreasing improvements as more resources are allocated: $\frac{d^2\bar{X}}{dz_{\bar{X}}^2}$ and $\frac{d^2T}{dz_T^2}$ are greater than or equal to 0

Four allocation functions

1. Linear
2. Exponential
3. Quadratic
4. Logarithmic

Linear allocation function

$$\bar{X}(z_{\bar{X}}) = \hat{X} - a_{\bar{X}} z_{\bar{X}}$$
$$T(z_T) = \hat{T} - a_T z_T$$

- Decision maker should only allocate resources to reduce one resilience factor based on $\max\left\{\frac{a_{\bar{X}}}{\hat{X}}, \frac{a_T}{\hat{T}}\right\}$
- Focuses resources on the factor whose initial parameter is already small and where effectiveness is large

Exponential allocation function

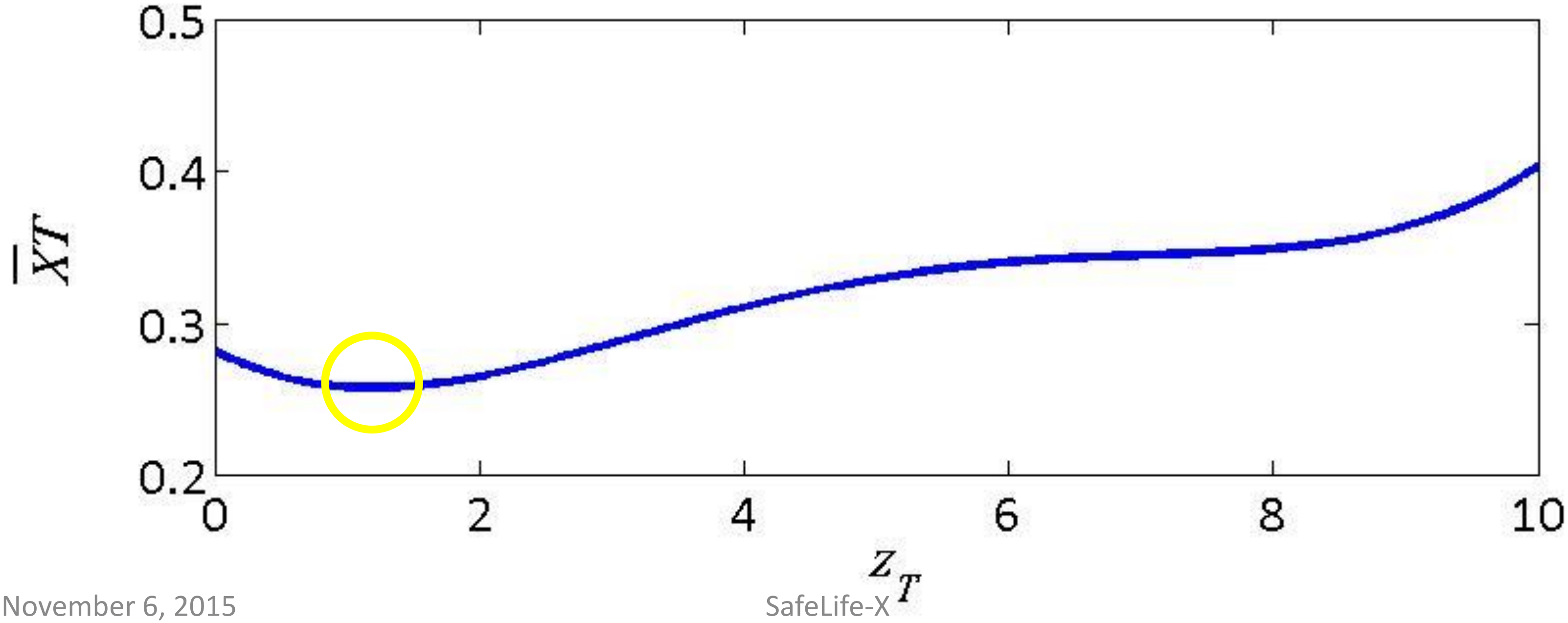
$$\bar{X}(z_{\bar{X}}) = \hat{X}\exp(-a_{\bar{X}} z_{\bar{X}})$$
$$T(z_T) = \hat{T}\exp(-a_T z_T)$$

- Decision maker should only allocate resources to reduce one resilience factor based on $\max\{a_{\bar{X}}, a_T\}$
- Decision depends only the effectiveness and not the initial values

Quadratic allocation function

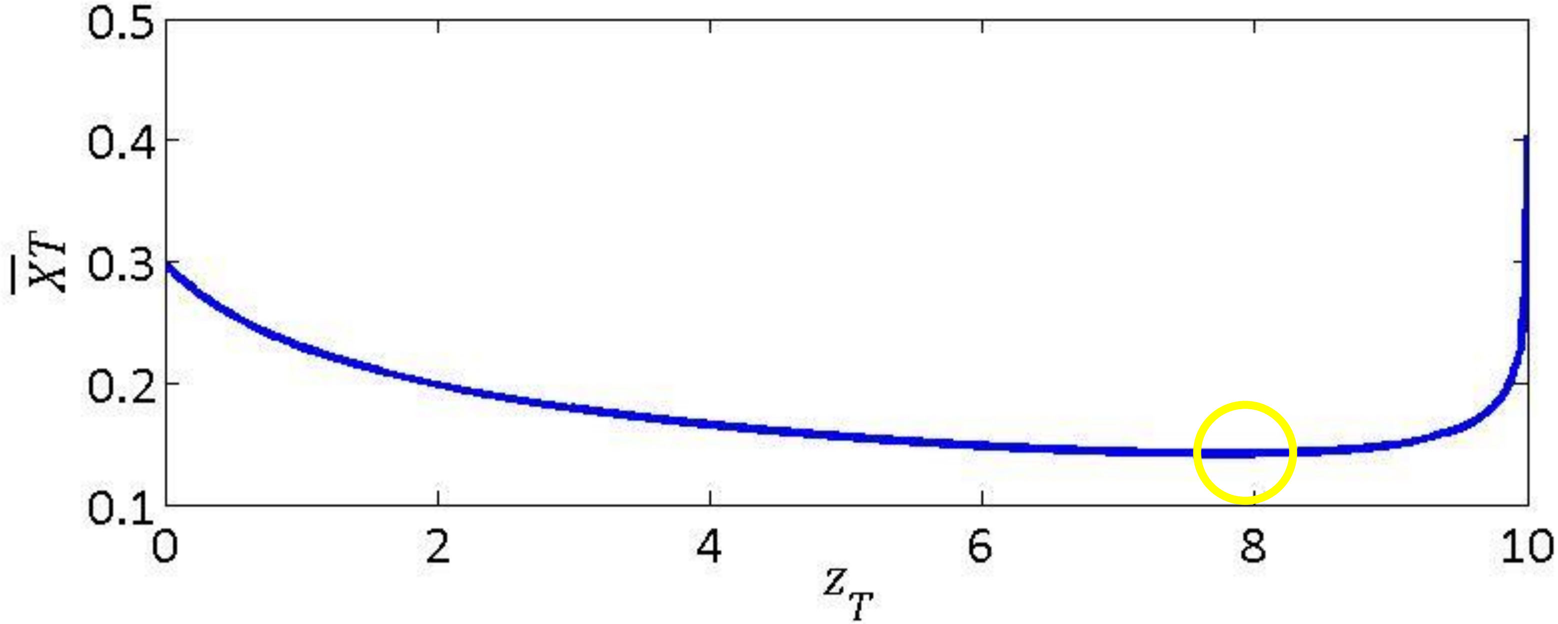
$$\bar{X}(z_{\bar{X}}) = \hat{X} - b_{\bar{X}} z_{\bar{X}} + a_{\bar{X}} z_{\bar{X}}^2$$
$$T(z_T) = \hat{T} - b_T z_T + a_T z_T^2$$

- Assume $z_{\bar{X}} \leq \frac{b_{\bar{X}}}{2a_{\bar{X}}}$, $z_T \leq \frac{b_T}{2a_T}$ so that functions are always decreasing



Logarithmic allocation functions

$$\bar{X}(z_{\bar{X}}) = \hat{X} - a_{\bar{X}} \log(1 + b_{\bar{X}} z_{\bar{X}})$$
$$T(z_T) = \hat{T} - a_T \log(1 + b_T z_T)$$

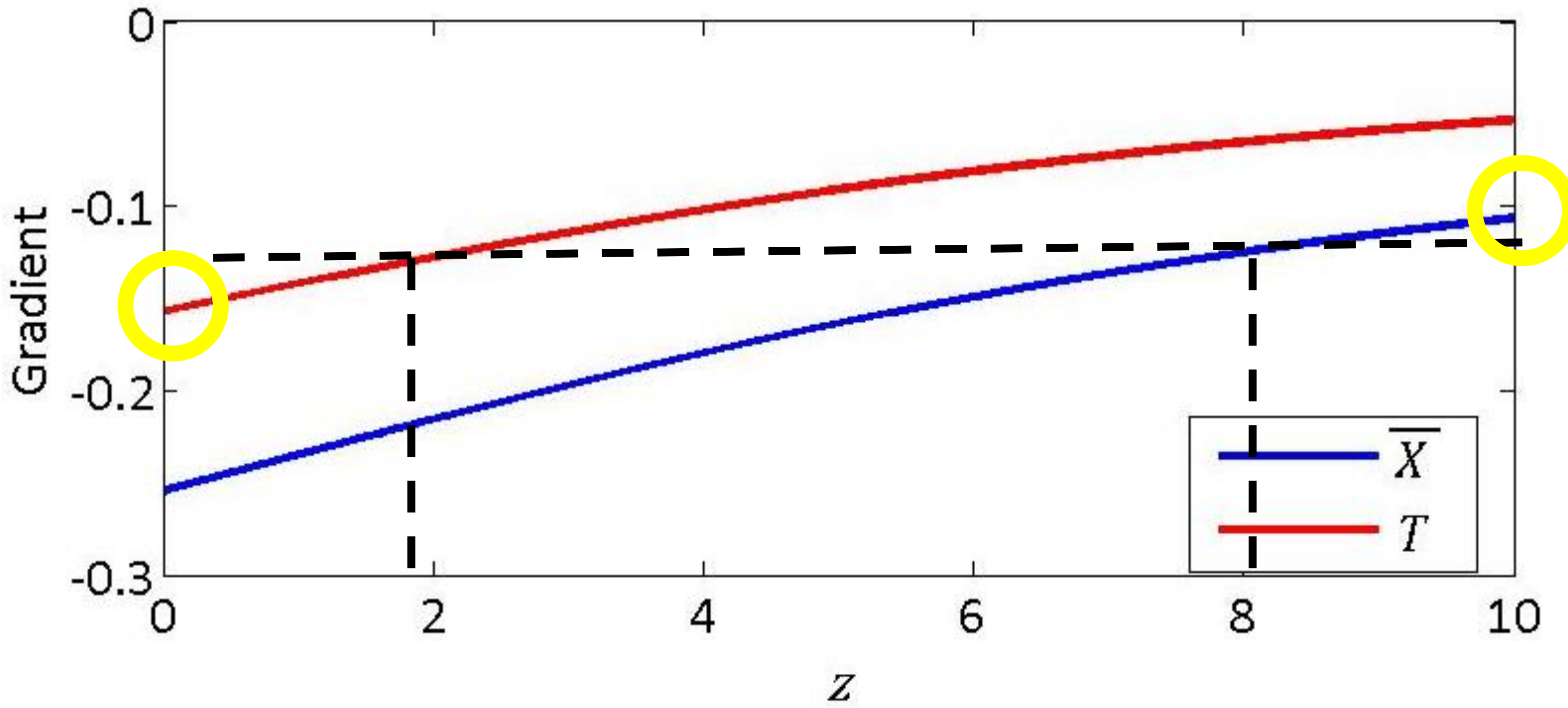


Uncertainty with independence

- Assume $\hat{X}$, $\hat{T}$, a_X, b_X, a_T, b_T have known distributions
- Assume independence
- Maximize expected resilience $E[R_*(\bar{X}, T)]$
- Linear, quadratic, and logarithmic allocation functions: May be optimal to allocate to both factors

Exponential allocation, uncertainty

- Always a convex optimization problem



- $E[(a_T - a_{\bar{X}})\exp([a_T - a_{\bar{X}}]z_{\bar{X}} - a_T Z)] = 0$

Uncertainty without probabilities

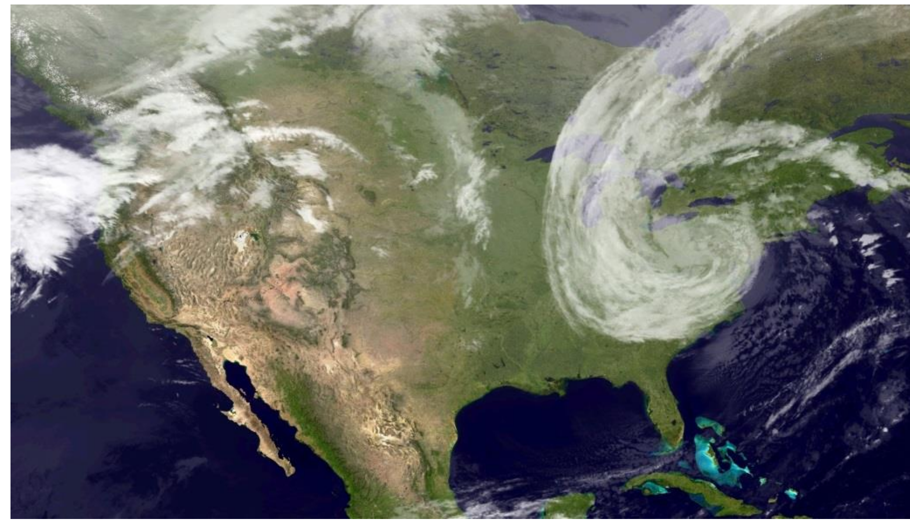
- Each parameter is bounded above and below, i.e. $\underline{\hat{X}} \leq \hat{X} \leq \bar{\hat{X}}$ and $\underline{a}_{\bar{X}} \leq a_{\bar{X}} \leq \bar{a}_{\bar{X}}$
- Maximin approach

$$\text{maximize min } R_*\big(\bar{X}(z_{\bar{X}}), T(z_T)\big)$$

- Same rules as the case with certainty but choose worst-case parameters to determine allocation, i.e. $\bar{\hat{X}}$ and $\underline{a}_{\bar{X}}$

Superstorm Sandy

- October 2012
  - East coast of the U.S.
  - Second costliest hurricane in U.S. history
- ConEdison Electric Utility
  - 670,000 New York city customers without electricity
  - Approximately 1/5 of ConEdison's customers
  - Duration: 13 days



ConEdison's Post-Sandy Plan

- $1 billion over 4 years to increase resilience of electric power network
- Hardening activities (reduce $\bar{X}$)
  - Trimming trees around power lines
  - Building higher flood plains
  - Backup power for substations
- Restoration activities (reduce T)
  - Smart-grid technologies
  - Preemptively shutting down steam plants
  - Deploying advance steams before the storm

Consolidated Edison Co. of New York. (2013). Post-Sandy enhancement plan. Orange and Rockland Utilities.

Model parameters

- From Zobel (2014) and Johnson (2005)

	Most likely	Minimum	Maximum
$\hat{X}$	0.073	0.030	0.22
T	13	3	26

- Assume triangular distribution

Zobel, C.W. (2014). Quantitatively representing nonlinear disaster recovery. *Decision Sciences*, 45(6), 687-710.

Johnson, B.W. (2005). After the disaster: Utility restoration cost recovery. Report prepared for the Edison Electric Institute.

Model parameters

Effectiveness parameters for different allocation functions from

- Brown, R. (2009). Cost-benefit analysis of the deployment of utility infrastructure upgrades and storm hardening programs. Report prepared for the Public Utility Commission of Texas. Quanta Technology.
- Consolidated Edison Co. of New York. (2013). Post-Sandy enhancement plan. Orange and Rockland Utilities.
- Terruso, J., Baxter, C., & Carrom, E. (2012). Sandy recovery becomes national mission as countless workers come to N.J.'s aid. *The Star-Ledger* (Newark). November 11.

Allocation results

Optimal amount (in millions of dollars) to allocate to increase hardness from a $1 billion budget

	Allo-cation function	Amt
Certainty	Linear	0
	Expon	1000
	Quadratic	762
	Logarith	648
Uncertainty with inde-pendence	Linear	0
	Expon	1000
	Quadratic	556
	Logarith	494

	Allo-cation function	Amt
Uncertainty with dependence	Linear	0
	Expon	1000
	Quadratic	840
	Logarith	470
Robust allocation	Linear	0
	Expon	0
	Quadratic	21
	Logarith	286

Resilience results

Resilience given optimal allocation from a $1 billion budget

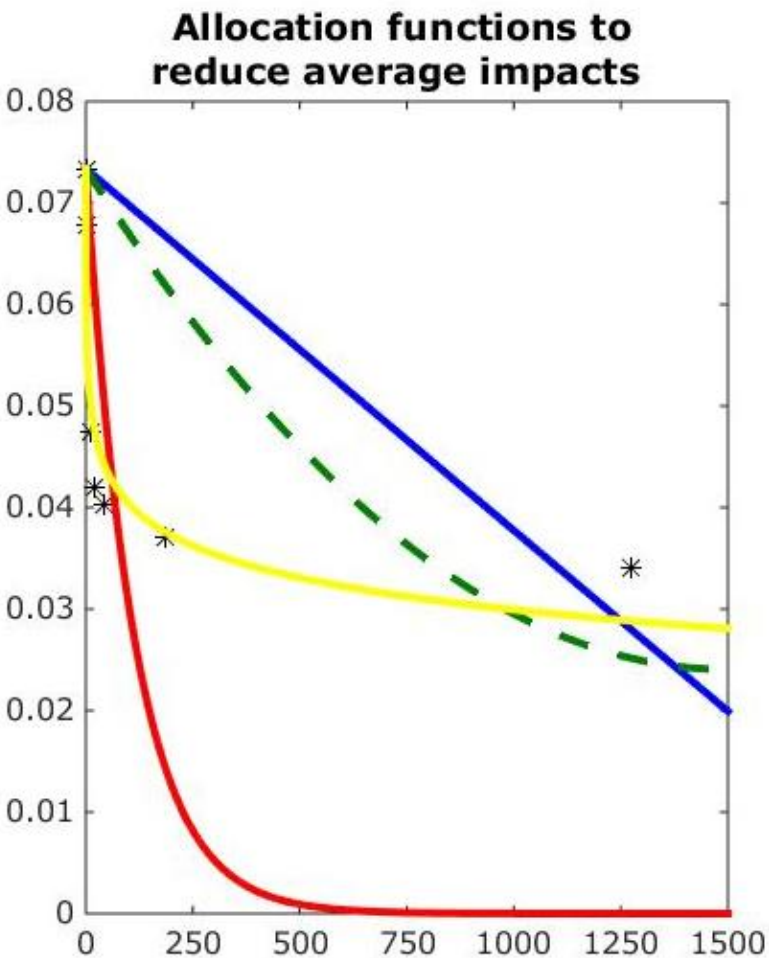
	Allo-cation function	Resi-lience
Certainty	None	0.963
	Linear	0.986
	Expon	1.000
	Quadratic	0.986
	Logarith	0.989
Uncertainty with inde-pendence	None	0.943
	Linear	0.974
	Expon	1.000
	Quadratic	0.985
	Logarith	0.987

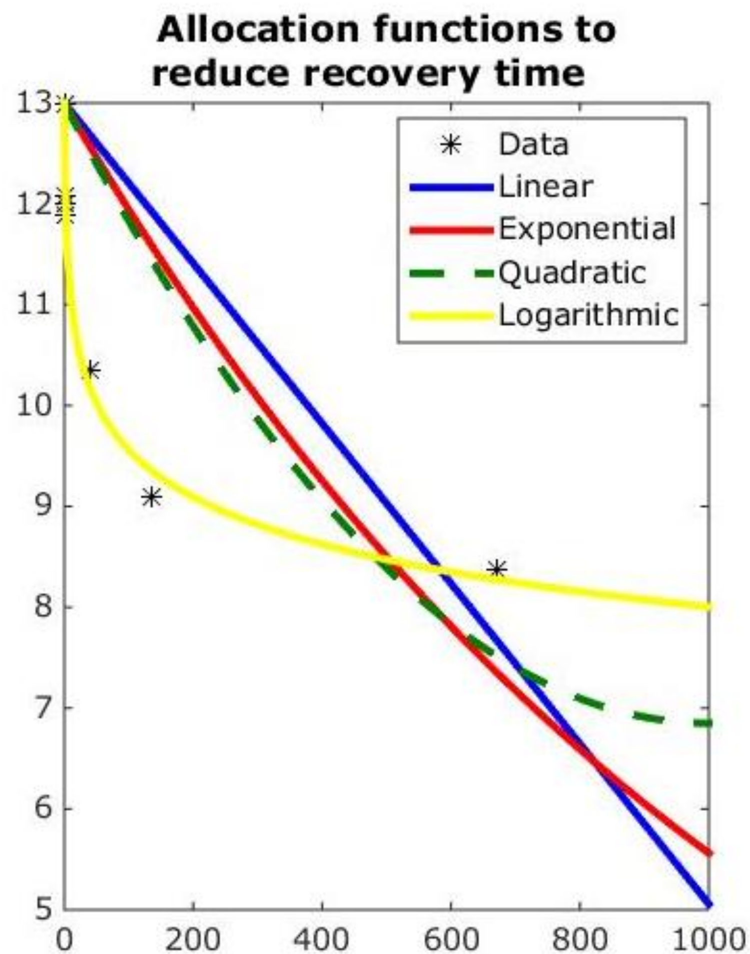
	Allo-cation function	Resi-lience
Uncertainty with dependence	None	0.937
	Linear	0.965
	Expon	0.999
	Quadratic	0.976
	Logarith	0.969
Robust allocation	None	0.784
	Linear	0.788
	Expon	0.786
	Quadratic	0.786
	Logarith	0.832

Resilience in terms of customers and duration

Allocation function	Allo-cation to increase hard-ness ($ million)	Allo-cation to improve reco-very ($ million)	Resi-lience	Average number of custo-mers without power	Du-ration (days)
No efforts			0.963	232,000	13
Linear	0	1000	0.986	232,000	5.1
Exponential	1000	0	1.000	36	13
Quadratic	762	238	0.986	114,000	10.4
Logarithmic	648	352	0.989	101,000	8.7

Best fit for allocation functions





What if logarithmic is wrong?

Allocation function	Given optimal allocation of logarithmic function ($z_{\overline{X}}$ = \$648 and z_T = \$352 mil)			Opti-mal resi-lience	Ave-rage custo-mers with-out power	Du-ration (days)
	Resi-lience	Ave-rage custo-mers with-out power	Du-ration (days)			
Logarith	0.989	101,000	8.7			
Linear	0.980	159,000	10.2	0.986	232,000	5.1
Expon	1.000	36	13	1.000	36	13
Quadratic	0.986	114,000	10.4	0.989	101,000	8.7

Recommendations for ConEdison

- Logarithmic allocation function seems most appropriate
  - Approximates data well
  - Allocation performs well even if another function is correct
- Allocate between 50 and 65% of budget to reduce number of customers who lose power and 35 to 50% to improve recovery

Conclusions

- Assumptions impact optimal allocation
  - Linear or exponential allocation function with certainty → allocate entire budget to reduce one factor
  - Quadratic or logarithmic → may allocate to reduce both factors
- Heuristics
  - Focus resources on small initial value and large effectiveness
  - Uncertainty: divide resources approximately equal manner if marginal benefits decrease rapidly
- Future work
  - Apply allocation model to specific projects
  - Resources can improve both factors simultaneously

MacKenzie, C.A., & Zobel, C.W. (2014). Allocating resources to enhance resilience. Under review. https://faculty.nps.edu/camacken/

Email: camacken@iastate.edu