

# **Optimal Resource Allocation for Recovery of Interdependent Systems: Case Study of the *Deepwater Horizon* Oil Spill**

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## **Abstract**

Investing in infrastructure and industry sectors can lessen the direct impacts of disruptive events, and a risk-based input-output model can demonstrate how those direct impacts propagate to other economic sectors. We develop and compare two different decision models to determine the optimal resource allocation to assist impacted sectors to recover. The first decision model minimizes direct impacts from a disruption, and the second model minimizes direct and indirect impacts, or total production losses. We solve for the optimal allocation in each model as a function of model parameters, and we compare the two models' solutions. We deploy these models to a data-driven case study analyzing the economic impacts of the *Deepwater Horizon* oil spill, which adversely impacted several industries in the region such as tourism, fishing, and real estate. These models can be applied to different homeland security situations to help governments and organizations determine proper resource allocation during and after a disruption.

## **Keywords**

Resource allocation, optimization, input-output, oil spill, homeland security

## **1. Introduction**

On April 20, 2010, an explosion on the *Deepwater Horizon* offshore oil drilling rig claimed 11 lives, injured 16 other employees, and led to nearly 5 billions of crude oil spilling into the Gulf of Mexico over a span of three months. The loss of human life, the damage to the environment and wildlife, the loss of business to several Gulf industries, and the technical and engineering challenges of stopping the oil leak combined to make this incident the largest marine oil spill and perhaps the most devastating [1].

The *Deepwater Horizon* oil spill embodies the type of a large-scale disruptive event that concerns homeland security officials in federal agencies, state and local governments, and foreign governments. The magnitude of the disruption, the complex interdependencies in the impacted ecosystem and economy, and the uncertainty involved hinder efforts to contain and recover from such a disruption. Determining where to devote resources and the necessary tasks for recovery presents a challenge to incident-response officials.

This paper addresses resource allocation for regional economic recovery, focusing on the interdependent economic impacts among the homeland security concerns discussed above. We develop an optimization problem to allocate resources to specific industries to effectively reduce the adverse impact of a disruptive event. Section 2 reviews previous optimal resource allocation models and outlines the unique contributions of this paper. Section 3 develops and provides solutions for two optimization models: (i) a model of direct impacts from a disruption, and (ii) a model of both direct and the indirect impacts from a disruption. Section 4 applies these two models to the *Deepwater Horizon* oil spill, and we perform sensitivity analysis on key parameters. Concluding remarks appear in Section 5.

## **2. Literature Review**

A resource allocation model generally addresses the fundamental economic question of how to satisfy unlimited wants with limited resources in a specific domain. Such resource allocation models that attempt to effectively divide a fixed budget have been developed and deployed in numerous domains, a small subset of which are reviewed here.

Most resource allocation models are formulated as static optimization problems with a resource budget serving as the primary constraint. Estimating parameters to accurately measure the objective function in these optimization problems can pose a challenge for modelers. For example, optimally allocating efforts among quality control tasks in order to

minimize software defects requires a careful assessment of a cost function and a large amount of data for parameter estimation [2]. Resource allocation models in the medical field also involve several parameters, including the type of disease, treatment options and effectiveness, and patient characteristics. Clinical studies often provide data to enable accurate parameter estimation [3]. The models presented in this paper encounter a similar challenge of estimating parameters.

Homeland security resource allocation models often employ game theory to protect citizens and infrastructure against possible terrorist attacks. Zhuang and Bier [4] identify equilibrium strategies for a defender choosing to allocate investments to protect several targets from a strategic attacker (e.g., a terrorist) and a non-strategic actor (e.g., a natural disaster). Because assuming that an attacker is perfectly rational or strategic may not be realistic, another model [5] determines the optimal resource allocation where a chance exists that the terrorist or attacker is not strategic.

Specifically for oil spills, Psaraftis and Ziogas ([6]) develop a resource allocation model to determine the appropriate type of equipment needed to clean up a spill. The decision maker's objective is to minimize a weighted combination of the damage costs from the spill and the costs of responding to the spill (i.e., the resource budget).

Unlike many other homeland security resource allocation models, the modeling approach in this paper focuses on post-disruption decision making in order to limit the impacts and enhance recovery. Because preparing for every possible type of disruption is practically impossible, empowering decision makers to make good decisions following a disruption is of importance in homeland security. Our models seek to minimize the economic impact caused by a disruptive event, and similar to other studies, a resource budget serves as the primary constraint. Applying the model to an oil spill disruption also requires estimating several parameters, a task we accomplish by relying on media articles, scholarly work, think-tank reports, and government data. Although the specific application and motivation for our model is an oil spill recovery, the models developed in this paper can be applied to a wide variety of disruptive events.

Disruptions can have indirect impacts as well as direct impacts, the former being a result of the interdependent nature of an economy, infrastructure system, or ecosystem. Perhaps the most important contribution of this paper is the development of an optimal resource allocation model that includes both direct and indirect impacts, and we compare the allocations between such a model and another model that only incorporates direct impacts. Our modeling approach also allows us to compare the benefits of allocating resources that can help multiple industries recover simultaneously with the benefits of targeting individual industries for recovery.

### 3. Resource Allocation Models

We present two static allocation models, both of which measure the economic consequences from a disruption. The first model minimizes the direct economic impacts from a disruption. The second model minimizes the total production losses from a disruption, which include both direct and indirect impacts. Both models assume that a disruption directly impacts  $m$  industries in an economy with  $n$  industries, where  $m \leq n$ . Each subsection presents the model and necessary conditions for optimal allocation. Table 1 defines the variables used in the two models.

#### 3.1 Model 1: Direct Impacts

For the first model, a decision maker wishes to effectively allocate resources to minimize  $D$ , the direct impacts caused by the disruption.  $D$  is the scalar product of two vectors of length  $m$ :  $\tilde{\mathbf{x}}$  describes each industry's as-planned production in dollars and  $\mathbf{c}$  measures the direct impact to each industry if recovery resources are allocated. Vector  $\mathbf{c}$  quantifies each industry's inoperability, or the extent to which the sector is not productive, in proportional terms, as the result of a disruptive event. Equation (1) models the decision maker's problem as an optimization problem. The total budget,  $Z$ , is divided into resources allocated to each industry,  $z_1, \dots, z_m$ , and to all industries simultaneously,  $z_0$ . These  $z_i$  and  $z_0$ , which serve as the decision variables in the optimization problem, are investments to promote recovery following a disruptive event.

$$\begin{aligned}
 & \text{minimize} && D = \tilde{\mathbf{x}}^T \mathbf{c} \\
 & \text{subject to} && c_i = \hat{c}_i \exp(-k_i z_i - k_0 z_0^2) \quad i = 1, \dots, m \\
 & && z_0 + \sum_{i=1}^m z_i \leq Z \\
 & && z_0, z_i \geq 0 \quad i = 1, \dots, m
 \end{aligned} \tag{1}$$

Because the first constraint describing the impacts on each industry can be substituted directly into the objective function, the problem has one principal constraint, the resource budget  $Z$ , which cannot be exceeded.

Table 1: Definitions for model variables

$n$	Total number of industries or sectors in economy
$m$	Number of industries directly impacted by disruption
$D$	Economic impact due to directly impacted industries (Model 1)
$\tilde{\mathbf{x}}$	Vector of length $m$ where $\tilde{x}_i$ is industry $i$ 's production in dollars before disruption
$\mathbf{c}$	Vector of length $m$ describing direct impacts on each industry in proportional terms
$c_i$	Direct impact on industry $i$ after resources are allocated
$\hat{c}_i$	Direct impact on industry $i$ if no resources are allocated
$z_i$	Decision variable of amount of resources allocated to industry $i$
$z_0$	Decision variable of amount of resources allocated that simultaneously benefits all directly impacted industries
$k_i$	Effectiveness of allocating resources to industry $i$
$k_0$	Effectiveness of allocating resources simultaneously for all industries
$Z$	Total resource budget
$Q$	Total production losses from both direct and indirect impacts (Model 2)
$\tilde{\mathbf{B}}$	$n \times m$ matrix translating direct impacts into direct and indirect impacts
$\mathbf{x}$	Vector of length $n$ of each industry's production in dollars before disruption

The impact on each industry  $c_i$  is a function of the direct impact if no resources are allocated,  $\hat{c}_i$ , the allocation amounts, and the effectiveness of the resource allocation,  $k_i$  and  $k_0$ . We assume that allocating resources reduces the impacts exponentially.

Allocating resources to simultaneously benefit all industries could include activities such as cleaning the area and removing debris after the disruption, repairing infrastructure that all the other industries require (e.g., electric power, transportation), and engaging in risk communication efforts to inform the public that a region is safe. The model squares this allocation amount  $z_0$  because we assume that if a major disruption occurs, allocating resources for these types of activities will not enhance recovery unless a significant amount of resources is allocated. Mathematically,  $k_0 < 1$  and squaring  $z_0$  reduces the impact of allocating  $z_0$  if  $\sqrt{k_0 z_0} < 1$ . If  $\sqrt{k_0 z_0} > 1$ , squaring the term enhances the effect of this allocation.

A solution to the optimization problem can be found by forming the Lagrangian and applying the Karush-Kuhn-Tucker conditions for optimality. Equations (2) - (4) depict the necessary conditions, where  $\lambda$ ,  $\lambda_i$ , and  $\lambda_0$  are the Lagrange multipliers for the budget constraint, the nonnegative constraints for  $z_i$ , and the nonnegative constraint for  $z_0$ , respectively.

$$\lambda = \left[ \prod_{i:z_i>0} (\tilde{x}_i \hat{c}_i k_i)^{1/k_i} \right]^{(\sum_{i:z_i>0} 1/k_i)^{-1}} \exp \left( Z - z_0 + \sum_{i:z_i>0} \frac{k_0 z_0^2}{k_i} \right)^{-(\sum_{i:z_i>0} 1/k_i)^{-1}} \quad (2)$$

$$z_i = \frac{1}{k_i} \log \left( \frac{\tilde{x}_i \hat{c}_i k_i}{\lambda - \lambda_i} \right) - \frac{k_0 z_0^2}{k_i} \quad \lambda_i z_i = 0 \quad (3)$$

$$-2k_0 z_0 \sum_{i=1}^m \tilde{x}_i \hat{c}_i \exp(-k_i z_i - k_0 z_0^2) + \lambda - \lambda_0 = 0 \quad \lambda_0 z_0 = 0 \quad (4)$$

Because the optimization problem is non-convex in  $z_0$ , the above conditions represent necessary but not sufficient conditions. However, if  $z_0$  is known, the equations for  $\lambda$  and  $z_i$  represent both necessary and sufficient conditions. Equation (4) has at most two real roots, and solving this equation generates at most two potential optimal allocations. Comparing the values of the objective function at these local minima enables us to determine the optimal allocation of resources.

As long as some resources are allocated to industry  $i$ , the optimal allocation for that industry,  $z_i$ , monotonically increases with  $\tilde{x}_i$  and  $\hat{c}_i$ . If an industry produces more or if the direct impacts are larger, homeland security officials

should devote more resources to reducing losses in that industry. The optimal allocation to industry  $i$  increases as  $k_i$  increases for smaller values of  $k_i$  but decreases for larger values of  $k_i$ . If allocating resources to an industry becomes more effective, the industry requires fewer resources, leaving more resources available for other industries.

### 3.2 Model 2: Direct and Indirect Impacts

Because of the complexity and connectedness of the modern economy, direct impacts in some industries lead to indirect impacts in all industries. Measuring those indirect impacts is important to accurately quantify the losses from a disruption and may influence decision making during the recovery phase. The direct impacts on the  $m$  industries have interdependent impacts on the rest of the economy, which is composed of all  $n$  industries. Economic input-output models, as first devised by Leontief [7], can measure these interdependent impacts. We deploy a risk-based extension of the Leontief input-output model, the Inoperability Input-Output Model [8]. We generate the normalized interdependency matrix  $\mathbf{A}^*$  for the Inoperability Input-Output Model (see [9]), and  $\mathbf{B} \equiv (\mathbf{I} - \mathbf{A}^*)^{-1}$  is a square matrix of size  $n$  that translates direct impacts into direct and indirect impacts in all  $n$  industries. Because the disruption directly impacts  $m$  industries,  $\tilde{\mathbf{B}}$  is a  $n \times m$  matrix whose columns correspond to the directly impacted industries from  $\mathbf{B}$ . The vector  $\mathbf{x}$  is a vector of length  $n$  representing normal economic output in all  $n$  industries in the economy.

Under this model, the decision maker's goal is to minimize the total impacts or total production losses in a region, as represented by  $Q$ . The optimization problem in Equation (5) is equivalent to Equation (1) except for the objective function.

$$\begin{aligned} & \text{minimize} && Q = \mathbf{x}^T \tilde{\mathbf{B}} \mathbf{c} \\ & \text{subject to} && c_i = \hat{c}_i \exp(-k_i z_i - k_0 z_0^2) \quad i = 1, \dots, m \\ & && z_0 + \sum_{i=1}^m z_i \leq Z \\ & && z_0, z_i \geq 0 \quad i = 1, \dots, m \end{aligned} \quad (5)$$

The necessary conditions for optimality are identical to the optimality conditions given in Equations (2) - (4) except that the scalar product  $\mathbf{x}^T \mathbf{b}_{*i}$  replaces  $\tilde{x}_i$ , where  $\mathbf{b}_{*i}$  is the  $i$ th column of the matrix  $\tilde{\mathbf{B}}$ . Thus, the optimal allocation of resources depends upon the interdependent impacts from the disruption of industry  $i$  rather than industry  $i$ 's own production as in Model 1.

Equation (6) compares the optimal allocations to industry  $i$  if the decision maker only considers direct impacts and if he or she considers both direct and indirect impacts. We let  $\hat{z}_i^*$  and  $\hat{z}_0^*$  be the optimal allocations to industry  $i$  and to all industries, respectively, from Model 2, and we let  $z_i^*$  and  $z_0^*$  be the optimal allocations from Model 1. Equation (6) is only true if  $\hat{z}_i^* > 0$  and  $z_i^* > 0$ .

$$\hat{z}_i^* - z_i^* = \frac{1}{k_i} \log \left( \frac{\mathbf{x}^T \mathbf{b}_{*i}}{x_i} \right) \left( 1 - \frac{1}{k_i \sum_{j=1}^m 1/k_j} \right) - \frac{1}{k_i \sum_{j=1}^m 1/k_j} \sum_{j \neq i} \frac{1}{k_j} \log \left( \frac{\mathbf{x}^T \mathbf{b}_{*j}}{x_j} \right) - \frac{1}{k_i \sum_{j=1}^m 1/k_j} (\hat{z}_0^* - z_0^*) \quad (6)$$

The  $i$ th element of  $\mathbf{b}_{*i}$  always exceeds 1, and  $\mathbf{x}^T \mathbf{b}_{*i} > x_i$  for all  $i$ . Numerical studies reveal that the quotient  $\mathbf{x}^T \mathbf{b}_{*i} / x_i$  ranges between 1 and 3 for any industry  $i$ . Because Equation (6) relies on the natural logarithm of this quotient, the difference between an industry's interdependencies and the industry's own production produces relatively little change in the optimal allocation.

Although the effectiveness of allocating to industry  $i$  is equivalent in both models, the value of  $k_i$  impacts the difference in allocations between the two models. If  $\sum_{j=1}^m 1/k_j > 1$ , larger  $k_i$  values produce smaller changes in the optimal allocations. Smaller  $k_i$  values produce greater differences. If  $\sum_{j=1}^m 1/k_j < 1$ , larger values of  $k_i$  produce greater changes in the optimal allocations. If allocating resources is effective for several industries, accounting for economic interdependencies could lead to allocating more resources to those industries whose allocation is most effective.

## 4. Case Study: Recovery from *Deepwater Horizon* Oil Spill

We apply these resource allocation models to a case study examining the economic impacts of the *Deepwater Horizon* oil spill. As a result of the April 20 explosion on the *Deepwater Horizon* oil rig, almost 5 billion barrels of crude oil spewed into the Gulf of Mexico until the leak was finally capped on July 15. The operator of *Deepwater Horizon*, BP, agreed to establish a \$20 billion fund to pay for damage to the Gulf ecosystem, reimburse state and local governments

for the cost of responding to the spill, and compensate individuals for lost business. The U.S. government imposed a six-month moratorium on deepwater drilling in the Gulf of Mexico, and it did not issue new leases for oil exploration in the Gulf until December 2011 [10].

We quantify the economic impacts of this disaster by focusing on the spill's direct impacts on five different industries. We estimate parameters for the models using publicly available economic data, think-tank and government reports, journal articles, and news stories. We analyze and compare the two models and perform sensitivity analysis on key parameters.

#### 4.1 Assumptions and Parameter Estimates

Both models include five Gulf states (Texas, Louisiana, Mississippi, Alabama, and Florida). Economic data collected by the Bureau of Economic Analysis [11–13] provide information on the production of different industries or sectors in each of those states, the vector  $\mathbf{x}$ , and the interdependencies among sectors,  $\tilde{\mathbf{B}}$ . We combine the five Gulf States into a single economy with a total of  $n = 63$  economic sectors.

Both models focus exclusively on business interruption losses, which are defined as production losses due to inoperable facilities or reduced demand, and we ignore the severe environmental damage. The costs of stopping the oil leak or containing and removing crude oil are modeled to the extent that these activities impact demand and production in the Gulf region. Direct impacts from the oil spill include: (i) demand losses because consumers decide to buy or consume fewer goods and services as a result of the oil spill and (ii) less industry production because facilities are inoperable. Demand losses occurred because people did not travel to the Gulf for vacation or buy fish from the Gulf (and fewer fish were caught). The demand for beachfront property also declined. Firms drilled for less oil in the Gulf because of the moratorium, the lack of new leases and licenses, and the need for enhanced safety measures. The models consider that the oil spill directly impacts the Fishing and Forestry, Real Estate, Amusements, Accommodations, and Oil and Gas industries ( $m = 5$ ).

The decision maker for this case study is a hypothetical entity responsible for limiting economic losses in the five Gulf states. The decision maker controls resources that can be used to increase demand for seafood, tourism, and real estate in the Gulf, implement new safety requirements in the offshore oil platforms, and remove crude oil from the Gulf which benefits all of the impacted industries. Although the U.S. federal government and the Department of Homeland Security have responsibility for many of these areas, in practice, the federal government, state and local entities, and the private sector all control resources that can be used for these types of tasks.

Table 2 displays the parameter estimates for the effectiveness of allocating resources,  $k_i$ , and the direct impacts for each industry,  $\hat{c}_i$ . Allocating resources to one of the industries directly impacted by reduced demand means better communication about the risks, safety, and cleanliness of the products and services produced by these industries. We assume that these resources can be expressed in monetary terms. If people are not consuming fish caught in the Gulf of Mexico, resources can be devoted to testing fish for oil contamination and to a public relations campaign explaining that fish are safe for consumption. A National Resources Defense Council report [14] that found that fishing revenue decreased by \$63 million enables us to estimate direct impacts for the Fishing and Forestry industry ( $i = 1$ ). We derive  $k_1$  from two studies [15, 16] examining the effectiveness of positive media stories following two different food scares.

Table 2: Input values for *Deepwater Horizon* case study

$i$	Industry	$k_i$ (per \$1 million)	$\hat{c}_i$
0	All industries simultaneously	$7.4 * 10^{-9}$	
1	Fishing and Forestry	0.074	0.0084
2	Real Estate	0	0.047
3	Amusements	0.0038	0.21
4	Accommodations	0.0027	0.16
5	Oil and Gas	0.0057	0.079

Tourism to the Gulf can be encouraged by ensuring that the beaches are free of oil and debris and showing potential tourists that the beaches are safe and open. The direct impacts for Amusements ( $i = 3$ ) and Accommodations ( $i = 4$ ) are based on an estimate that tourism declined in Louisiana, Alabama, Mississippi, and Florida by 30 percent although tourism in Texas does not appear to have been impacted [17, 18]. We calculate the effectiveness parameters from an Oxford Economics study [18] claiming a return on investment of 15 to 1 in tourism marketing. For the Real Estate

industry ( $i = 2$ ), we assume that the demand for housing in the four states fell 10 percent and that increasing demand for housing depends entirely on tasks devoted to helping all industries such as stopping the oil leak and cleaning the oil. Hence,  $k_2 = 0$ .

Resources allocated to the Oil and Gas industry ( $i = 5$ ) means implementing new safety measures to reduce the risk of an accident on an offshore oil platform. The federal government may have lifted the moratorium earlier and granted more licenses and leases if the oil industry had demonstrated the safety of deepwater drilling. Direct impacts are based on domestic oil production from the Gulf of Mexico in 2010 [19], and we derive  $k_5$  based on an estimate that the new safety measures cost \$183 million [20].

Capping the oil leak, containing the spill, and removing crude oil from the ocean can simultaneously benefit all five directly impacted industries. If less oil spills or if the oil is cleaned up more quickly, people are more likely to eat fish from the Gulf and vacation on its beaches. The Oil and Gas industry can also benefit because lifting the moratorium is less politically sensitive if the consequences of the oil spill are limited. Approximately \$11.6 billion was spent on stopping the oil leak and cleaning up the oil [21], and we estimate  $k_0$  by assuming that  $\sqrt{k_0} * \$11600 = 1$ . This assumption implies that billions of dollars must be allocated in order to reduce substantially the direct impacts on the five industries.

#### 4.2 Model 1 Results

We input these parameters into Model 1, whose objective is to minimize direct impacts. Figure 1 depicts the optimal resource allocation for different budgets ranging from \$0 to \$20 billion, where \$20 billion reflect the amount in BP's fund for reimbursing cleanup costs and lost business.

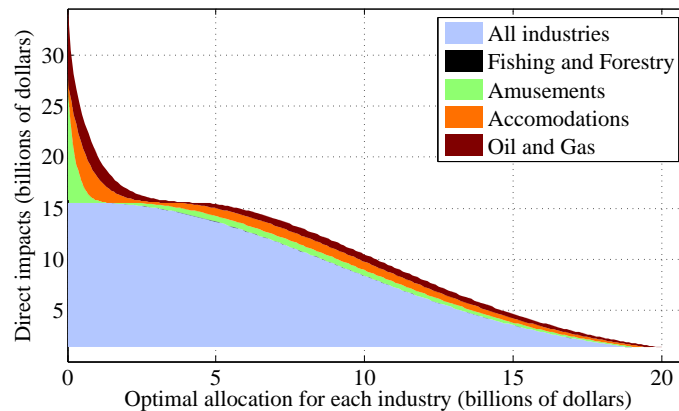


Figure 1: Optimal resource allocation for Model 1 at different budget amounts

Direct impacts total \$34.5 billion if no resources are allocated. Optimally allocating a budget of \$20 billion reduces the direct business losses to \$1.4 billion. If the budget is less than \$4.8 billion, the decision maker should not devote any resources to simultaneously help all industries because these industries do not benefit as much as they do from targeting individual industries. Dividing the budget roughly equally among Amusements, Accommodations, and Oil and Gas is ideal. Because the direct impacts in Fishing and Forestry are less than those in the other industries and because allocating resources to this industry is the most effective, allocating about \$50 million for this industry is optimal if the budget is \$4 billion.

The decision maker should spend \$1.7 billion to benefit all the industries if the budget is \$5 billion. Proportionally more resources should be allocated to help all industries as the budget increases. Almost 95 percent of a \$20 billion budget should be spent on this category. Ensuring that the oil leak is capped and crude oil is removed from the Gulf benefits the economy more than increasing demand by targeting individual industries through a media campaign.

#### 4.3 Model 2 Results

Direct impacts on the five industries lead to indirect impacts in all 63 industries in the regional economy because these five industries that produce less because of the oil spill consequently reduce their demand for intermediate inputs. A

decision maker concerned about the economic vitality of the region may want to consider these interdependent impacts, and Model 2 seeks to minimize total production losses in the Gulf region. Losses increase by about 40 percent if the model includes indirect as well as direct impacts. Total production losses are \$49.1 billion if no resources are allocated and drop to \$2.0 billion if the budget is \$20 billion (Figure 2).

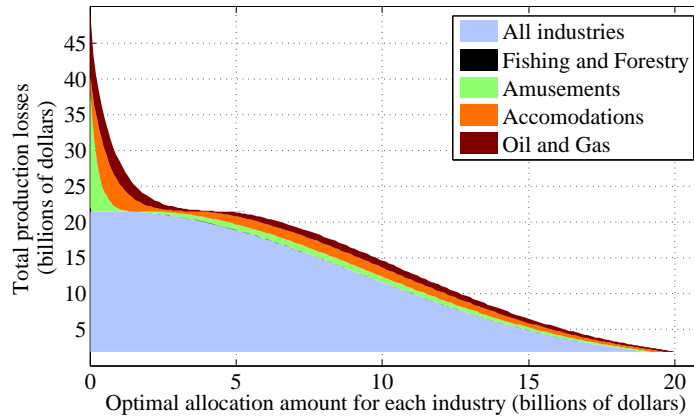


Figure 2: Optimal resource allocation for Model 2 at different budget amounts

Comparing the optimal allocations from Model 1 and Model 2 reveals that including interdependencies does not substantially change the optimal division of a budget (Figure 3). The largest difference between the two models occurs if the budget is \$4.8 billion, which is the smallest budget amount at which it is optimal to allocate resources for all industries. If the model only considers direct impacts, \$1.47 billion should be allocated for all industries, but \$505 million should be allocated for all industries if the model incorporates an interdependent framework. However, if the decision maker follows the optimal allocation suggested by Model 1, total production losses are only \$51 million greater than the production losses if the decision maker follows the optimal allocation suggested by Model 2.

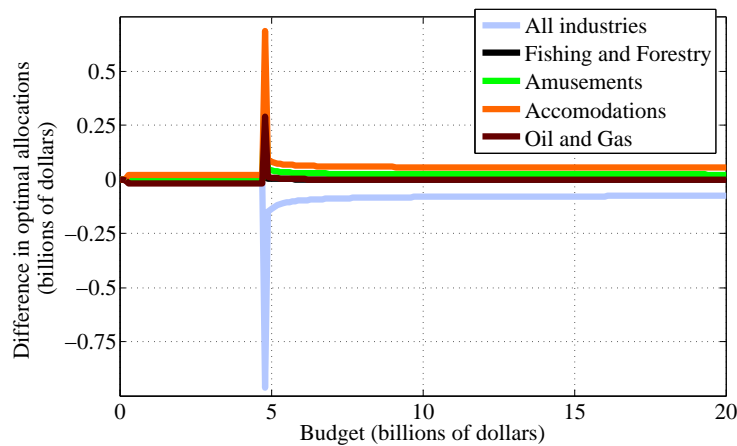


Figure 3: Model 1 optimal allocation amounts subtracted from Model 2

The differences in the two models' solutions are similar for any budget that exceeds \$10 billion. Model 2 recommends allocating \$56 million more to Accommodations than Model 1 because this industry has the least effectiveness (i.e., the smallest  $k_i$  value) and  $\sum -j = 1^m 1/k_j > 1$ . Although Model 2 recommends a similar division of the budget to that of Model 1, the losses in Model 2 are almost 1.5 times greater than those of Model 1. The larger losses may influence a decision maker using Model 2 to spend more, or request a larger budget, than if he or she uses Model 1.

#### 4.4 Sensitivity Analysis

We perform sensitivity analysis on a few key parameters in order to determine how these parameters affect the optimal resource allocation amounts. We focus exclusively on Model 2 results because of the similarity in the results between the two models.

One of the most important parameters in the model is the effectiveness of allocating to all industries,  $k_0$ , which determines the amount that should be allocated to stop the oil spill and clean up the the oil. The proportion of resources allocated to all industries is highly sensitive to small changes in  $k_0$  (Figure 4). Increasing  $k_0$  by  $1 \times 10^{-8}$  can increase the resources allocated to all industries by 20% or more. For large values of  $k_0$ , the entire budget should be allocated to all industries, especially when the budget is greater than \$10 billion. As such, the larger the  $k_0$ , the more effective it is to invest in industry-wide efforts. Because the optimal allocation is highly sensitive to very small changes in  $k_0$ , a careful assessment of this parameter is required for future studies.

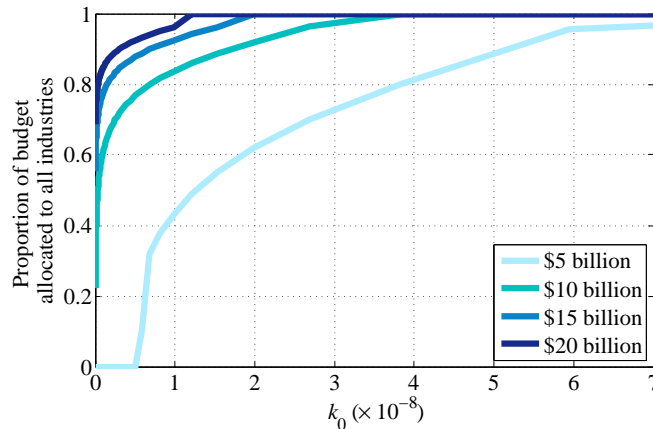


Figure 4: Sensitivity analysis on effectiveness of allocating resources to all industries

Because the base case results recommend allocating less than \$50 million to the Fishing and Forestry industry ( $i = 1$ ), we explore how the optimal allocation changes as the allocation effectiveness,  $k_1$ , and direct impacts,  $\hat{c}_1$ , change (Figure 5). The optimal allocation to this industry increases as  $\hat{c}_1$  increases. This allocation initially increases but then decreases as  $k_1$  increases. If the budget is \$10 billion, the industry should receive almost \$300 billion when  $k_1 = 0.01$  and  $\hat{c}_1 = 0.5$  although this extreme level of direct impacts is very unlikely. The more effective the allocation, the less money needs to be allocated to the industry even if the direct impacts are very large.

## 5. Conclusions

The two models presented in this paper can help homeland security officials determine how to allocate resources following a disruption. The first model minimizes direct impacts, and the second model minimizes direct and indirect impacts, or total production losses. The Karush-Kuhn-Tucker conditions for optimality allow us to express the optimal resource allocations as functions of model parameters, such as the initial impact, the effectiveness of allocating resources, and an industry's production or interdependent effects in an economy.

Newspaper accounts, think-tank reports, journal articles, and government data allow us to estimate parameters in order to apply these models to the 2010 *Deepwater Horizon* oil spill. If no money is spent on economic recovery after the Gulf spill, the direct impacts equal \$34.5 billion and total production losses equal \$49.1 billion. Several financial institutions estimated damages from the oil spill between \$10 and \$20 billion [22], and the Oxford Economics study [18] calculated that tourism revenues could decline by as much as \$23 billion over a three-year span. If we assume that total budget for recovering from the spill is \$11.6 billion (the amount that BP spent to stop the spill), the direct impacts and total production losses are \$8.8 billion and \$12.3 billion, respectively, if the decision maker chooses the optimal allocation. These estimates align closely with the other estimates.

The conclusions derived from the models in this paper and the case study can guide federal and state officials in making decisions about recovering from future disruptions. First, considering both direct and indirect impacts may not substantially change the optimal allocation from the allocation if just direct impacts are considered. However, the



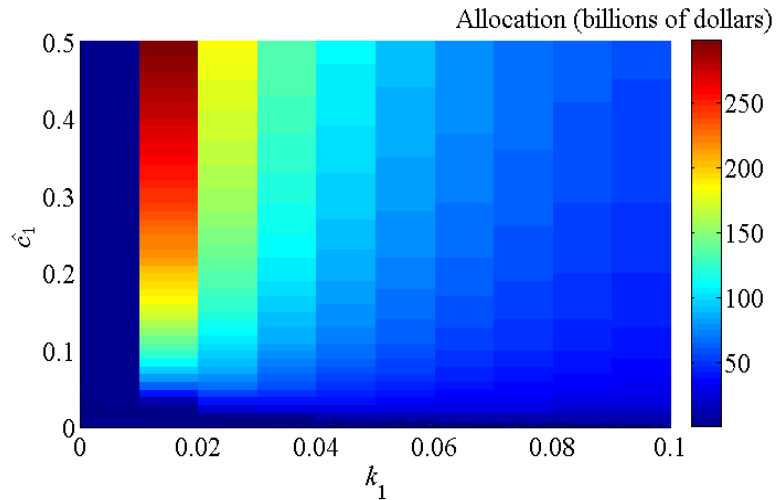


Figure 5: Sensitivity analysis on allocation effectiveness and direct impacts for Fishing and Forestry with a budget of \$10 billion

interdependent effects lead to larger estimates of the economic consequences, and understanding these interdependencies may help determine the total budget that should be available for recovery. Second, the budget for recovering from a disruption should be large enough to repair physical damage and limit environmental damages. These activities can benefit all of the directly impacted industries simultaneously and accomplish more than engaging in a risk communication campaign to help specific industries recover.

Further extensions of this work include developing a dynamic model and modeling preparedness decisions. Disruptions can last a period of time, and recovering from a disruption usually requires allocating resources over time. A dynamic resource allocation model can guide a decision maker in allocating resources at different points in time during a recovery from a disruption. Modeling preparedness decisions would require allocating some resources to prepare for a disruption. This would reduce the likelihood of a disruption, and recovery may require fewer resources if the disruption does occur. Modeling allocation in advance of a disruption would allow a decision maker to trade off investing in preparedness activities with holding resources in reserve to help with recovery.

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