Proceedings of the 2012 Industrial and Systems Engineering Research Conference
G. Lim and J.W. Herrmann, eds.

A New Fuzzy Logic Approach to Dynamic Dial-a-Ride Problem

Maher Maalouf
Industrial & Systems Engineering
Khalifa University, Abu Dhabi Campus, P.O. Box 127788, UAE

Cameron A. MacKenzie
School of Industrial Engineering
University of Oklahoma, Norman, OK 73019, USA

Sridhar Radakrishnan
School of Computer Science
University of Oklahoma, Norman, OK 73019, USA

Mary Court
School of Industrial Engineering
University of Oklahoma, Norman, OK 73019, USA

Abstract

Almost all dial-a-ride problems (DARP) described in the literature consist of designing optimal routes and schedules for n customers who specify pick-up and drop-off times. In this article we assume that the customer is mainly concerned with the drop-off time since it is the most important to the customer. Then based on the drop-off time specified by the customer and the customer's location, a pick-up time is calculated and given to the customer by the dispatching office. We base our formulation on dynamic fuzzy logic approach in which a new request is assigned to a vehicle. A new route, or a modified route, and schedule are then designed for the chosen vehicle.

Keywords
Dial-a-Ride problem, fuzzy logic, transportation

1. Introduction

Dial-a-Ride transportation services are employed in various areas, either to serve elderly people or people with disabilities. People call Dial-a-Ride services to request transportation to and from a location, and the Dial-a-Ride company tells the customer when a van or car will come to pick up the customer. The request can be made a few days in advance, or it could be made just a few hours before the customer would like to be picked up. A Dial-a-Ride company must decide which vehicle should pick up a customer and the pick-up and drop-off time for each customer.

Dial-a-Ride problems (DARP) are classified into static DARP and dynamic DARP. Static DARP are concerned with finding optimal routes for vehicles based on requests made in advance by customers, and static DARP makes decisions about vehicles and pick-up and drop-off times with full knowledge about customers' requests for a particular day. Dynamic DARP are concerned with requests made in real time when vehicles already have scheduled pick-ups and drop-offs. Both the static and dynamic types are employed in either a single-vehicle or a multi-vehicle DARP systems [**?**]. For the static multi-vehicle case, Jaw et al. [**?**] propose one of the first insertion heuristic algorithms to deal with multi-vehicle DARP using convex optimization techniques. The algorithm considers two types of customers: desired drop off time (DDT) specified by customers, and desired pick-up time (DPT) specified by customers. Kikuchi [**?**] implements a fuzzy logic approach to DARP by considering the desired time of vehicle arrival and travel time between two nodes as fuzzy numbers.

For the dynamic multi-vehicle case, Madsen et al. [**?**] solve a real-life problem in which customers can specify either a pick-up or drop-off time window. The algorithm is based on that of Jaw et al. [**?**] and dynamically inserts new requests into vehicle routes. Colorni and Righni [**?**] consider solving an objective function consisting of maximizing the number of customers served, optimizing the perceived service level, and minimizing the total distance traveled by the vehicles. Teodorovic and Radivojevic [**?**] use fuzzy logic to minimize the total traveling distance, vehicle waiting times, and passenger ride times by developing two approximating algorithms, one for vehicle selection and the other for the optimal insertion of DDT. Fuzzy numbers express the uncertainty involved in picking up and dropping off customers. Fuzzy logic provides an easy way to determine which vehicle is optimal for picking up a customer that reflects the realities of how dispatchers naturally think about the problem.

In this paper, we present FDARP, a dynamic fuzzy logic algorithm for DARP, similar to the one proposed by Teodorovic and Radivojevic [**?**]. Unlike the one proposed by Teodorovic and Radivojevic [**?**], the FDARP in this paper assumes that customers are mainly concerned with the drop-off time instead of the pick-up time. and hence the system specifies the pick-up time. Section 2 states the problem and Section 3 describes the algorithm. Details of the method and results are presented in Section 4. The conclusion is stated in Section 5.

2. Problem Statement

The dynamic Dial-a-Ride system for which FDARP is designed assumes that customers specify DDT. These customers rely on the Dial-a-Ride company to provide them with the best possible pick-up time. Although the customer-provided DDT may be exact, the FDARP algorithm assumes the customer has preferences such as the earliest drop off time (EDT), desired drop off time (DDT), and latest drop off time (LDT).

In a dynamic algorithm, vehicles already have assigned routes for picking up and dropping off customers. The fundamental problem is to assign a vehicle for the new request and indicate the pick-up time for the customer. In addition to satisfying the customer's request by dropping the customer off before his or her DDT, the algorithm minimizes the time the customer spends in the vehicle (dwell time) and the distance traveled by a vehicle. We assume that all vehicles originate from one depot and that all vehicles have the same capacity and are of the same type. Once a request is assigned, it is never modified.

Given a list of N customers, each specifying EDT_i, DDT_i, and LDT_i ($i = 1,2\ldots N$), the following relations should hold when determining the actual drop-off time, ADT_i, and the actual pick-up time, APT_i:

$$EDT_i < ADT_i < LDT_i \tag{1}$$

$$EPT_i < APT_i < LPT_i \tag{2}$$

$$ADT_i - DRT_i \geq APT_i \tag{3}$$

$$EDT_i - DRT_i \leq EPT_i \tag{4}$$

$$LDT_i - DRT_i \geq LPT_i \tag{5}$$

$$LDT_i - EPT_i \leq MRT_i \tag{6}$$

Relation (1) indicates that the actual drop-off time falls between the earliest drop-off time and the latest drop-off time. Similarly, relation (2) indicates that the actual pick-up time (APT_i) falls between the earliest pickup time (EPT_i) and the latest pick-up time (LPT_i). Relation (3) indicates that the actual drop-off time minus the direct ride time (DRT_i) is greater than or equal to the actual pick-up time. The direct ride time is the time it takes to go directly from picking up the customer to dropping him or her off at the desired location. Relation (4) requires that the earliest drop-off time minus the direct ride time cannot be greater than the earliest pick up time. This ensures that the customer will not arrive before his or her earliest drop-off time. Relation (5) requires that the latest drop-off time minus the direct ride time cannot be less than the latest pick up time to ensure that the customer does not arrive after his or her latest drop-off time. Finally, relation (6) states that the latest drop-off time minus the earliest pick-up time (EPT_i) must be no greater than the maximum ride time (MRT_i). The maximum ride time is the maximum amount of time a customer will stay in a vehicle, which could be another preference given by the customer or a fixed constraint in the FDARP algorithm. Figure 1 depicts graphically what the above relations show mathematically.

MAHER – YOU MIGHT NOT LIKE WHAT I DID TO THESE RELATIONS. YOU CAN PUT THEM BACK TO WHAT YOU HAD ALTHOUGH I WOULD SUGGEST THAT THE NEW RELATION 3 REMAINS. IF YOU DO LIKE THE CHANGES, FIGURE 1 SHOULD BE CHANGED TO REFLECT IT OR DELETED.

Figure 1: Time windows for the i^{th} passenger.

3. Proposed Solution

As mentioned earlier, Teodorovic and Radivojevic [**?**] solve a dynamic DARP using fuzzy logic with two approximate reasoning algorithms, one for vehicle selection, and the other for the optimal insertion of DDT. Their approximating algorithms are aimed at customers with a specified pick-up time, DPT. Unlike their algorithm, the proposed algorithm for this paper is designed for customers with a specified drop-off time, DDT. While Teodorovic and Radivojevic [**?**] choose fuzzy logic due to the uncertain nature of the problem, we believe more emphasis should be given to DDT. Customers are usually more concerned with the drop-off time than with the pick-up time.

The methodology in this paper consists of two parts. A request for a Dial-a-Ride is made. In the first part, an algorithm determines the best pick-up and drop-off time for each vehicle in the fleet. Certain operational constraints must be satisfied, including vehicle capacity, the maximum ride time, and the customer's drop-off time. After a pick-up and drop-off time for the request is determined for each vehicle, an optimization algorithm in the second part based on fuzzy logic determines the best vehicle to satisfy the customer's request. The optimization algorithm seeks to minimize both the amount of time a customer stays in the vehicle and the distance traveled by the vehicle.

3.1 Part 1: Pick-up and Drop-off Times for Each Vehicle

The first step determines the best time that each of the M vehicles in the fleet can pick up and drop off customer i, who has specified DDT_i, EDT_i, and LDT_i. Following the example of Teodorovic and Radivojevic [**?**], we model these three numbers as a single triangular fuzzy number, such that $DDT_i = (EDT_i, DDT_i, LDT_i)$ as shown in Figure 2.

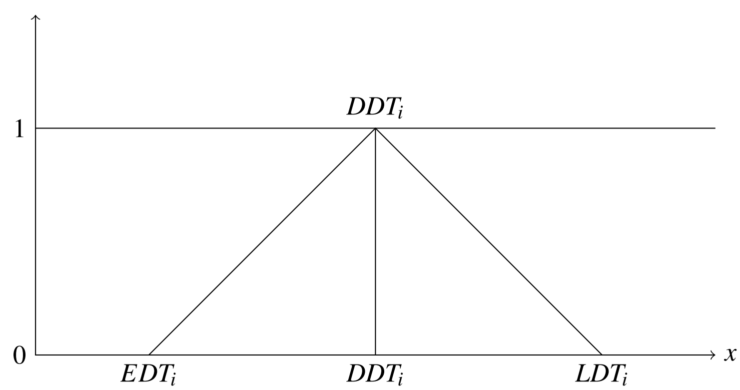

Figure 2: Desired drop-off time represented as a triangular fuzzy number.

FDARP sorts all pick-up or drop-off times for a given vehicle in ascending order. Since DDT_i is the input to FDARP, the insertion of the drop-off node i^- occurs before the insertion of pick-up node i^+. A non-trivial problem emerges when these two nodes must be separated by another node, where each node represents a stop for a vehicle.

The FDARP algorithm for this first part is outlined in Algorithm 1. The following constraints must be satisfied for the actual drop-off time ADT_i:

$$AA_e + TT_{ei^-} \leq ADT_i, \tag{7}$$
$$ADT_i + TT_{i^-f} \leq AA_f, \tag{8}$$

where AA_e is the actual arrival time at node e (the node that precedes the drop-off node of customer i), TT_{ei^-} is the travel time from node e to node $i-$, TT_{i^-f} is the travel time from node $i-$ to node f (the node that succeeds the drop-off node of customer i), and AA_f is the actual arrival time at node f. These numbers can also be expressed as fuzzy numbers, and we use the fuzzy arithmetic methods described by Kaufmann and Gupta [**?**] to ensure that these operational constraints are satisfied.

Algorithm 1: Algorithm to determine pick-up and drop-off time for customer i

---

Data: M, DDT_i
Result: **dwdis**
begin
- **for** $j \leftarrow 1$ **to** M **do**
  - node e = stop immediately preceding i^-
  - node f = stop immediately succeeding i^-
  - **while** *drop-off time*$_j$ *unassigned* **do**
    - **if** $TT_{ei^-} + TT_{i^-f} < AA_f - AA_e$ **then**
      - drop-off time$_j = \min(DDT_i, AA_f - TT_{i^-f})$
      - **if** $TT_{ei^+} + TT_{i^+i^-} + TT_{i^-f} < AA_f - AA_e$ **then**
        - pick-up time$_j$ = drop-off time$_j - TT_{i^+i^-}$
      - **else**
        - node ℓ =node e
        - node k = node preceding ℓ
        - **while** *pick-up time*$_j$ *unassigned* **do**
          - **if** $TT_{ki^+} + TT_{i^+\ell} < AA_\ell - AA_k$ **then**
            - pick-up time$_j = AA_\ell - TT_{i^+\ell}$
          - **else**
            - node ℓ =node k
            - node k = node preceding ℓ
    - **else**
      - node f =node e
      - node e = node preceding f
- max dwell time = max(pick-up time$_j$ for all M vehicles)
- min dwell time = min(pick-up time$_j$ for all M vehicles)
- max distance = maximum additional distance a vehicle travels to pick up and drop off customer i
- min distance = minimum additional distance a vehicle travels to pick up and drop off customer i
- dw_j = function of pick-up time$_j$, min dwell time, and max dwell time; dwell score closer to 1 indicates that passenger's dwell time is small
- dis_j = (additional distance vehicle j travels to pick-up and drop-off customer i - min distance) / (max distance - min distance)

Ideally, $ADT_i = DDT_i$. If $ADT_i = DDT_i$ violates Equation (7) or (8), the operator can change ADT_i to an earlier time. The actual drop-off time may need to be readjusted so that the customer is picked up before the current node e. If this occurs, the current node e becomes node f and the new node e immediately precedes the current node e.

After ADT_i is determined, the next step is the insertion of node i^+, whcih requires the calculation of the actual pick-up time APT_i:

$$AA_k + TT_{ki^+} \leq APT_i, \tag{9}$$

$$APT_i + TT_{i^+\ell} \leq AA_\ell, \tag{10}$$

where AA_k is the actual arrival time at node k (the node preceding the pick-up node of customer i), TT_{ki^+} is the travel time from node k to node i^+, $TT_{i^+\ell}$ is the travel time from node i^+ to node ℓ (the node that succeeds the pick-up node of customer i), and AA_ℓ is the actual arrival time at node ℓ.

The FDARP algorithm first assigns $APT_i = ADT_i - DRT_i$. If this assignment violates Equation (9) or (10), FDARP moves node i^+ to an earlier time, up to the maximum ride time MRT_i (Figure 3). Regardless of whether a feasible solution is found for vehicle j, FDARP tries to find a drop-off and pick-up time for customer i for vehicle $j+1$.

FDARP calculates a dwell score and a distance score for each of the M vehicles. Both scores are function of the

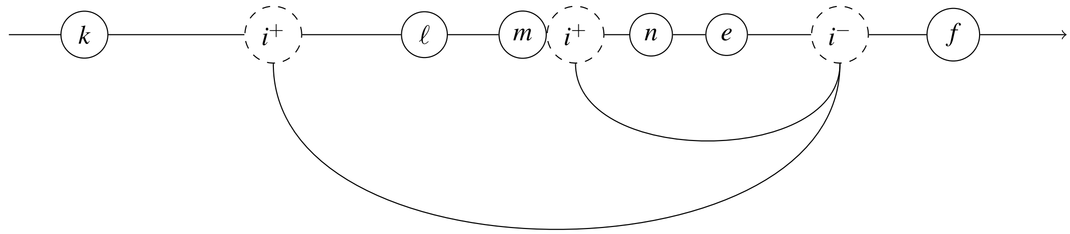

Figure 3: Assignment of new a request i to a route of a vehicle j.

assigned ADT_i and APT_i for each vehicle. The dwell score ranges between 0 to 1 and is based on the length of time the customer spends in vehicle j. If the time spent in vehicle j is close to DRT_i, the dwell score is near 1. The distance score also ranges between 0 and 1 and is based on the distance vehicle j travels. The shorter the distance vehicle j travels, the closer the distance score is to 0. The first part ends by returning two vectors of length M: **dw**, a vector of dwell scores, and **dis**, a vector of distance scores.

3.2 Part 2: Selecting the Best Vehicle

Once feasible insertions for nodes i^- and i^+ are found, FDARP uses a fuzzy optimization approximating algorithm to select the optimal feasible solution. **In addition, instead of using two approximate reasoning algorithms in the optimization step, only one fuzzy approximate reasoning algorithm is needed. In other words, the optimization step determines the choice of the vehicle for which the total time detour of the previously assigned passengers is minimum. Algorithm 2 describes the rules used to achieve this objective.** I'M NOT SURE THIS IS TRUE OR EVEN REALLY WHAT IT MEANS – MAYBE DELETE??

Depending on dis_j, the additional distance traveled by vehicle j (Figure 4), and dw_j, the travel time of the customer (Figure 5), a greater or lesser preference (Figure 6) would be placed on vehicle j to pick up and drop off customer i. According to Algorithm 2, the time to pick up and the dwell time can be considered as fuzzy variables with values ranging between "small", "medium", or "large". Converging scales enable these linguistic terms to be converted into triangular fuzzy numbers, which in turn convert into crisp scores.

Algorithm 2: Fuzzy rules for choosing optimum vehicle

Data: **dwdis**
Result: Optimal vehicle choice
begin
  if *Time to pick-up additional passenger is SMALL and additional passenger's dwell time is SMALL* **then**
    Preference is VERY STRONG
  else
  if *Time to pick-up additional passenger is SMALL and additional passenger's dwell time is MEDIUM* **then**
    Preference is STRONG
  else
  if *Distance to pick-up additional passenger is SMALL and additional passenger's dwell time is BIG* **then**
    Preference is STRONG
  else
  if *Distance to pick-up additional passenger is MEDIUM and additional passenger's dwell time is SMALL* **then**
    Preference is MEDIUM
  else
  if *Distance to pick-up additional passenger is MEDIUM and additional passenger's dwell time is MEDIUM* **then**
    Preference is MEDIUM
  else
  if *Distance to pick-up additional passenger is MEDIUM and additional passenger's dwell time is BIG* **then**
    Preference is WEAK
  else
  if *Distance to pick-up additional passenger is BIG and additional passenger's dwell time is SMALL* **then**
    Preference is WEAK
  else
  if *Distance to pick-up additional passenger is BIG and additional passenger's dwell time is MEDIUM* **then**
    Preference is VERY WEAK
  else
  if *Distance to pick-up additional passenger is BIG and additional passengers' dwell time is BIG* **then**
    Preference is VERY WEAK

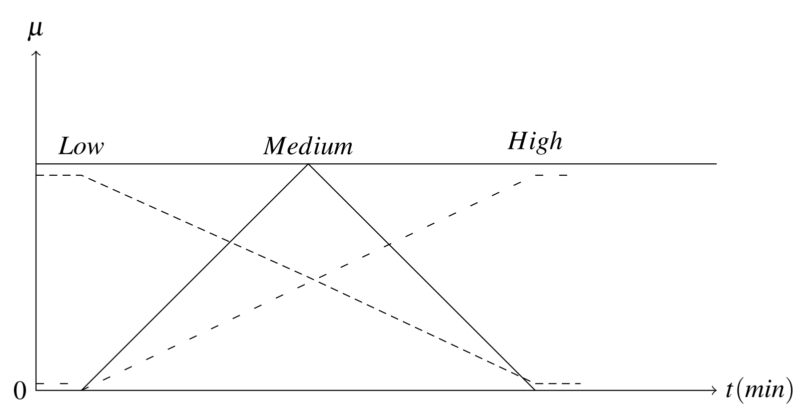

Figure 4: Triangular fuzzy number representing the time to pickup a new passenger.

4. Numerical Example

The developed algorithms above were tested using two different simulations for a single day. Both simulations start with a completely empty system. Customers call one at a time with a DDT_i, EDT_i, and LDT_i and the algorithms

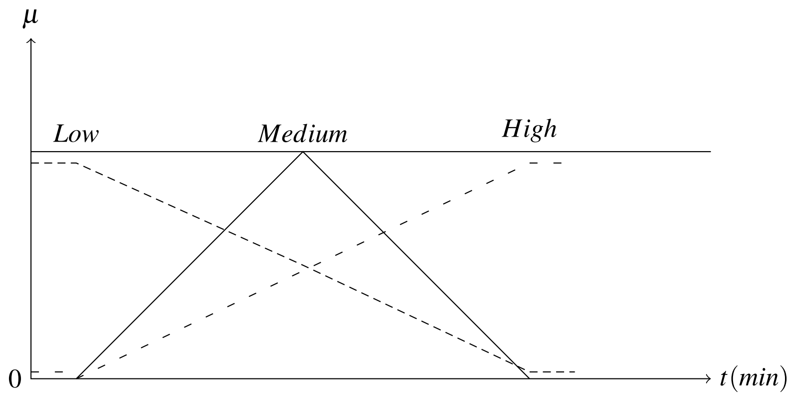

Figure 5: Triangular fuzzy number representing the dwell time of the new passenger.

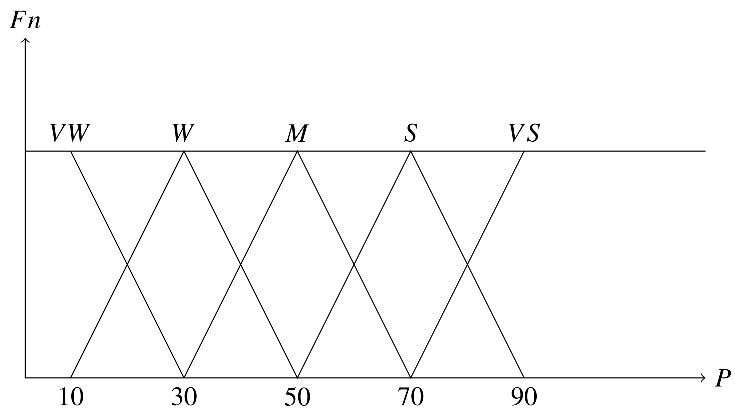

Figure 6: Preference strength to assign a new request to a certain vehicle.

determine the optimal vehicle to pick up and drop off customer i. Each DDT_i is randomly generated from a uniform distribution between 8 AM and 10 PM and $EDT_i = DDT_i - 10\text{min}$ and $LDT_i = DDT_i + 10\text{min}$. Each customer is willing to spend an additional 15 minutes in the vehicle beyond DRT_i, the time it takes to go directly from picking the customer up to dropping him or her off. Each customer is also willing to be dropped off at most 20 minutes before

In the first simulation, the Dial-a-Ride company has a fleet of six vehicles, and 100 customers request a ride. Table 1 displays the results of this simulation.

Table 1: Results for simulation with six vehicles

Customers rejected because constraints not met for all vehicles	100
Average number of minutes customer is early at destination	6
Average time customer spends in vehicle	27.14 min
Average distance traveled by each vehicle	239.93 miles in 14 hours

In the second simulation, the company has a fleet of 30 vehicles, and 900 customers request a ride. Table 2 displays the results of this simulation with a larger fleet of vehicles.

5. Conclusion

6. Acknowledgment

References

Table 2: Results for simulation with six vehicles

Customers rejected because constraints not met for all vehicles	34
Average number of minutes customer is early at destination	3.77
Average time customer spends in vehicle	29.71 min
Average distance traveled by each vehicle	327.67 miles in 14 hours