IE 361 Module 30

Control Charts for Counts (Attributes Data) Part 2

Reading: Section 3.3 *Statistical Methods for Quality Assurance*

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The scenario under which a u chart or (c chart) is potentially appropriate is one where periodically k inspection units of product or production from a process are looked at and

$$X = \text{the total number of "nonconformities" found across those } k \text{ units}$$

is observed. This is illustrated in the following figure, where "x"'s represent nonconformities.

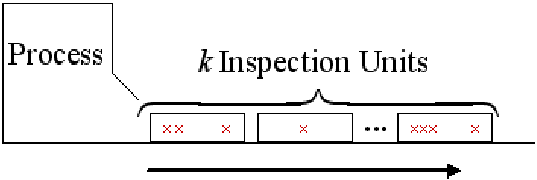

Figure: Cartoon of k Inspection Units of Process Output and X "Nonconformities"

Control Charts for Mean Nonconformities per Unit

In this kind of circumstance, the notation

$$\hat{u} = \frac{X}{k} = \text{the sample rate of nonconformities per unit}$$

is standard, and

- control charts for $\hat{u}$ are called u **charts**
- control charts for the special case where $k = 1$ and thus $\hat{u} = X$ are called c **charts**

Probability Basis for Control Limits

If the process producing items/outcomes is physically stable and produces nonconformities **at a rate of λ per unit**, a reasonable probability model for X (met in Stat 231) is the

$$\text{Poisson } (k\lambda)$$

distribution (the Poisson distribution with mean $\mu_X = k\lambda$).

A Stat 231 fact about the Poisson distribution is that its mean and variance are the same, that is that

$$\sigma_X = \sqrt{\mu_X}$$

So in the present context

$$\mu_X = k\lambda \text{ and } \sigma_X = \sqrt{k\lambda}$$

and in turn that

$$\mu_{\hat{u}} = \lambda \text{ and } \sigma_{\hat{u}} = \sqrt{\frac{\lambda}{k}}$$

Standards Given Control Limits

These probability facts lead to **standards given (u chart) control limits for $\hat{u}$**

$$LCL_{\hat{u}} = \lambda - 3\sqrt{\frac{\lambda}{k}} \text{ and } UCL_{\hat{u}} = \lambda + 3\sqrt{\frac{\lambda}{k}}$$

and for the case of $k = 1$ (so that $\hat{u} = X$) **standards given (c chart) control limits for X**

$$LCL_X = \lambda - 3\sqrt{\lambda} \text{ and } UCL_X = \lambda + 3\sqrt{\lambda}$$

u Chart Example
Example 30-1

Below are some artificially generated (using $\lambda = 2.0$) Poisson data, X. We may consider these to be either counts or rates for a constant ($k = 1$) number of inspection units.

Sample	1	2	3	4	5	6	7	8	9	10
X	2	2	1	2	2	3	4	3	2	0

Sample	11	12	13	14	15	16	17	18	19	20
X	2	0	3	2	1	5	2	2	1	3

Standards given control limits for X would then be

$$CL_X = 2 \text{ and } UCL_X = \lambda + 3\sqrt{\lambda} = 2 + 3\sqrt{2} = 6.24$$

and no lower control limit would be used since $2 - 3\sqrt{2}$ is negative. Clearly, none of the values X in the table would plot above the upper control limit and there is no indication of "lack of control"/deviation from the $\lambda = 2.0$ nonconformities per unit picture of process performance under which the above limits are developed.

To extend this example a bit, suppose that an additional 5 inspection Periods are involved, but now the number of inspection units observed per period is no longer constant (at "$k = 1$"), and in fact that the table below is relevant.

Sample	k	X	$\hat{u} = \frac{X}{k}$	$UCL_{\hat{u}} = 2 + 3\sqrt{\frac{2}{k}}$	$LCL_{\hat{u}}$
21	1.5	2	1.33	5.67	none
22	1	1	1.00	6.24	none
23	.75	2	2.67	6.90	none
24	.5	1	2.00	8.00	none
25	3	5	1.67	5.29	none

u Chart Example

Example 30-1 continued (Shewhart u Chart)

The following figure is a JMP control chart for the whole data set, and there is no reason in these data to doubt that the nonconformity rate is constant at $\lambda = 2.0$ per inspection unit. (Note that JMP in ambiguous fashion indicates that $LCL = 0$. It is better practice and provides clearer intent to say that there is **no lower control limit**.)

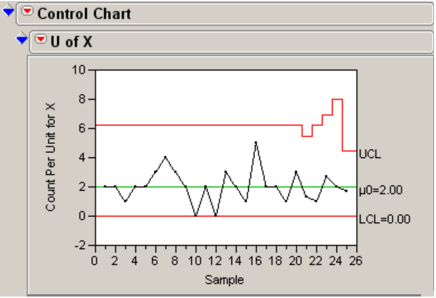

Figure: Standards Given ($\lambda = 2.0$) u Chart for the Artificial Data

Retrospective control limits for $\hat{u}$ are obtained (as always) by making a provisional assumption of process stability and estimating process parameters (here, the value λ). The most sensible estimate obtainable from r values X_i based on respective numbers of inspection units k_i is

$$\hat{\lambda}_{\text{pooled}} = \frac{\text{total nonconformities observed}}{\text{total inspection units}} = \frac{X_1 + X_2 + \cdots + X_r}{k_1 + k_2 + \cdots + k_r}$$

Example 30-1 continued There are a total of 53 nonconforming outcomes indicated among the 26.75 inspection units. So

$$\hat{\lambda}_{\text{pooled}} = \frac{53}{26.75} = 1.98$$

and there is negligible difference between the standards given chart on panel 8 and the retrospective control chart that would be made using this.

p versus u Charts

Students sometimes have a difficult time identifying whether a given application calls for a p chart or for a u chart. Both $\hat{p}$ and $\hat{u}$ are kinds of "rates," but they are intrinsically different. For one thing, it is perfectly possible for $\hat{u}$ to exceed 1.0, while $\hat{p}$ is always between 0 and 1. The figure below lays the two basic scenarios side by side.

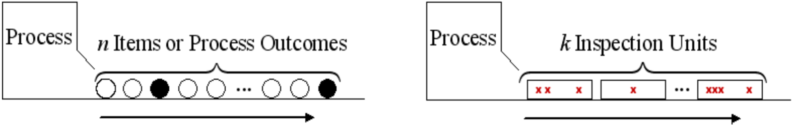

Figure: Comparison of "Fraction Nonconforming" and "Mean Nonconformities per Unit" Scenarios

p versus u Charts

In the p chart/"fraction nonconforming" context, n discrete items or outcomes are individually each either acceptable or nonconforming, and $\hat{p}$ is the sample fraction nonconforming. In the u chart/"mean nonconformities per unit" context, k inspection units of process output potentially suffer nonconformities, and $\hat{u}$ is the sample rate at which these are observed per unit.

It is worth saying again that the table in Section 3.7 of *SMQA* summarizes the control limit formulas for both the "attributes data" of this and the previous module and for the "measurements data" of Modules 26 and 27.