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An Exit and Entry Study of Renewable Power Producers: A Real Options Approach

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In recent years, there has been a substantial increase in renewable power production sites. However, such sites operated in the 1980s were often abandoned in the 1990s and their remnants are still visible. Hence, it is highly desirable to understand the exit and entry decisions of such sites. Toward this goal, we formulate and analyze models for such decisions regarding a single site from a real options perspective when the operation and maintenance costs follow a geometric Brownian motion and derive policy implications. An extensive numerical example for a wind farm illustrates some of the key features of this study.

Introduction

In recent years, across the United States as well as several other regions such as Western Europe and East Asia, there has been a massive increase in construction and operation of renewable power production sites (in this article *renewable site*). By *renewable power*, from an economic perspective, we mean electric power generated by energy sources that are without input fuel costs. Currently, such power is primarily wind and solar power. However, it also includes geothermal, tidal, and hydroelectric power (when the purpose of the dam is primarily for power generation).

For example, in 2009, the total capacity of wind power alone was 34,296 MW compared to 24,651 MW in 2008 (U.S. Energy Information Administration [EIA] 2010a) in the United States. The main drivers for this phenomenal growth have not only been the economic efficiency and technology breakthroughs in renewable power production but also the favorable government support due to environmental concerns as well as higher oil and natural gas prices. We note that currently such government support typically is in the form of subsidies and incentives that are front-loaded in the construction and early operating years (Wiser et al. 2007).

This massive increase, however, leads to a few critical questions that are both old and new. One major recurring question is regarding the exit of such renewable sites in a massive scale. That is, in the 1990s there were numerous cases of abandonment of renewable sites that had been in operation since the 1970s and 1980s where the renewable site decision

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makers simply walked away (WebEcoist.com 2011) as the economic and noneconomic conditions deteriorated. For example, there are thousands of abandoned wind turbines still littering the landscape in the areas of Altamont Pass, Tehachapi, and San Gorgonio in California alone (Wilkinson 2010). The situation for solar farms is quite similar, and certainly no better (The Center for Land Use Interpretation 2011).

At this point in time, the "rebirth" of renewable sites is of an order of magnitude larger than the previous manifestation in the 1970s and 1980s, and a priori there is no basis on which one can assume that abandonment on a truly massive scale will not occur in the near future. In fact, currently, in numerous regions, if a renewable site decision maker walks away (i.e., abandons the renewable site), there are few, if any, consequences or penalties (WebEcoist.com 2011). Specifically, we observe that, as the operation and maintenance (O&M) costs increase with respect to time (Wiser and Bolinger 2008), there is a substantial incentive to walk away even when there remains some "physical" life in the renewable site (see, e.g., Myers and Majd 1983). Furthermore, the exact amount of the O&M costs at a given future time is typically stochastic not only because numerous repairs are unpredictable but also because the current physical age is often quite young relative to the physical life estimated by the site builders (see. e.g., Martínez et al. 2009). This implies that the availability of the relevant empirical data is rather limited at this point in time.

More recently, we note that, in a few places, there has been some consideration or implementation of a simple and straightforward exit fee as insurance to enable the proper disposal of the renewable site in case of abandonment (New Hampshire Office of Energy and Planning 2008). This observation has provided us with the motivation for this study.

Under these circumstances, it is highly desirable to understand the economically rational decisions on exits from and entries to renewable sites, of which understanding includes timely policy implications especially in the areas of government's incentives and fees. As a first step toward this goal, in this article, we (1) formulate and analyze mathematical models of exit and entry decisions for a single renewable site from a real options perspective when the O&M costs follow a geometric Brownian motion (GBM) process (see, e.g., Ye 1990) while the selling power price is fixed by a contract (see, e.g., Roques et al. 2008), (2) show the managerial insights from sensitivity analyses surrounding the exit and entry decisions that are made by renewable site decision makers, and (3) derive policy implications that are theoretically interesting and practically timely. We hope to provide specific insights for relevant decisions and policies and stimulate the critical discussion among renewable site decision makers, government regulators, legislative policy makers, as well as academics.

What distinguishes this article from the extensive literature on renewable site economics and finance is that (1) this is the first quantitative study that, to our knowledge, explains the economically rational decisions of the exit from and entry to renewable sites under the assumption of GBM-based O&M costs, (2) the decision and policy insights are provided concerning critical threshold values such as the optimal O&M costs to exit and the maximum initial O&M costs to enter with respect to various parameters such as the electricity price and the renewable site capacity, and (3) the policy implications of government subsidies and penalties on the decisions are provided with respect to some key parameters such as the initial investment and exit fee.

Several groups of literature are relevant this study. First, in the area of financial and economic applications in the electric power industry, Wang and Min (2006) applied a real options approach to a case of interrelated generation projects. Also, in Wang and Min (2008), a financial portfolio consisting of electric power commodities was managed, and in Wang and Min (2010), financial hedging techniques were developed for electric power producers.

We also note that, in the context of nuclear power plants (which is not renewable power), Takashima et al. (2007) investigated decommissioning by applying a real options approach. That paper differs from our approach because the primary driver of the exit was claimed to be the stochastic electricity price, not the cost components, which were assumed to be deterministic. We caution that there are other reasons for an exit such as competitive or regulatory concerns, which are beyond the scope of this article. As for solar power, Lorenz et al. (2008) advocated phasing economic incentives out, but only prudently and gradually.

For wind power management, Fleten and Maribu (2004) addressed the investment timing and capacity choice of wind power under uncertainty, and Fleten et al. (2007) examined the investment strategies when renewable power is generated in a decentralized manner. These papers are different from our approach because they would not be able to explain the exit behavior that can be theoretically predicted or empirically observed. In addition, the stochastic components in these papers are not the O&M costs (e.g., electricity price).

As for the stochastic O&M costs, Khoub Bakht et al. (2008) investigated statistical repair and maintenance cost models for tractors, and Leung and Lai (2003) statistically studied the quality and reliability aspects of bus engines. Almansour and Insley (2011) applied the stochastic costs model for oil sands in Canada. In addition, in Costa Lima and Suslick (2006), both the price and operating costs were modeled as GBM processes for mining projects.

In the area of equipment replacement, there are numerous examples of more conventional (i.e., non-real options approaches) papers (see, e.g., Hartman and Murphy 2006). In Ye (1990), the replacement strategy was derived under the assumption of a GBM O&M costs, and Zambujal-Oliveira and Duque (2011) considered both O&M costs and salvage value as GBM processes.

The rest of the article is organized as follows. We first model the exit decision of a currently operating renewable site under the assumption of a GBM-based O&M costs. This model leads to the derivation of the threshold O&M costs, above which the renewable site exits the market, as well as to the derivation of the expected remaining life of the renewable site. Next, we conduct sensitivity analysis of the exit threshold O&M costs and the expected remaining life with respect to various critical parameters such as the exit fee. We then consider the entry decision of a new renewable site and derive the threshold O&M costs below which such a renewable site will enter the market, as well as the expected life of the renewable site. This is followed by a sensitivity analysis of the entry threshold O&M costs and the expected life. We next present some of the features of our model through an illustrative numerical example based on a wind farm and discuss the policy implications and the various assumptions defining the scope of our study. Finally, we make concluding remarks and comment on future research.

Modeling and Analysis of Renewable Site Exit Decision

In this section, let us consider a firm consisting of a single renewable site currently in operation. For the decision maker of this firm, we assume that there exists an option to abandon the renewable site and to exit the electricity market. Furthermore, we make the following critical assumptions that enable us to model and analyze the exit decision, which is economically rational as well as tractable. The relaxation of the assumptions that would enhance the realism of the models in this article will be addressed in the Discussion section.

Assumption 1: There already exists a power purchase contract between the renewable site and a utility company with the appropriate transmission connection at a fixed selling price of P (\$/MWh) at any time point (see, e.g., Roques et al. 2008). We assume that the upper bound of the renewable power purchase quantity of the contract will not be reached at this site.

Assumption 2: In making the exit decision, the decision maker will rely on the fixed (expected) production quantity per unit time, which is equal to the quantity demanded by the utility (e.g., via a contract). By this assumption, we are not implying that the actual production and demand quantities remain constant over time, but, for planning purposes, we are simplifying the dynamic aspects of renewable energy source such as wind speed, daylight availability, etc. (see, e.g., Fleten and Maribu 2004). Specifically, we utilize the fixed annual production quantity, K (MWh/year). We further assume that this production quantity is equal to the nameplate capacity times the capacity factor, which is a standard procedure for generation planning in the electric power industry, alternative procedures notwithstanding (see, e.g., Wiser and Bolinger 2007).

Assumption 3: The O&M costs (\$/MWh) at any time point, C, follows the GBM process. Specifically,

$$dC = \alpha_C C dt + \sigma_C C dz \tag{1}$$

where α_C is the instantaneous growth rate of the O&M costs (% per year), σ_C is the instantaneous volatility of C (% per square root of year), and dz is the increment of a standard Wiener process z ($dz = \varepsilon_t \sqrt{dt}$ where $\varepsilon_t \sim N(0, 1)$).

Typically, the O&M costs include costs associated with repairs, spare parts, maintenance, and consumables necessary for O&M (see, e.g., European Wind Energy Association [EWEA] 2004). Also, we note that modeling the O&M costs as a GBM process is not new, as explained in the Introduction.

Assumption 4: There exists a fixed exit fee of W ($W \geq 0$) when the option to abandon is exercised, and the decision maker is aware of this a priori. Currently, in the United States, few local, state, and federal rules and regulations exist that require the removal and disposal of renewable sites when the option to abandon is exercised, and some authors have claimed that the renewable site decision makers have strong incentive to abandon their renewable sites even if they are physically viable for operation (Wilkinson 2010). Our model will allow this current situation as a special case of $W = 0$.

Assumption 5: The remaining cost components are irrelevant to the decision maker regarding the option to abandon; that is, the cost components such as nonspecific overheads, taxes, etc., are negligible for our planning purposes. We also assume that there is no salvage value at the time point of exit, and our model does not explicitly factor in specific government programs such as the production tax credit (see, e.g., Union of Concerned Scientists 2011).

Assumption 6: More sophisticated options such as a partial shut-down, mothballing, etc., are not available for the renewable site. These options are beyond the scope of this article and will not be considered.

Under these assumptions, our problem can be interpreted as an optimal stopping problem. We observe that the higher the O&M costs, the stronger the incentive to abandon. Therefore, intuitively, the range of the O&M costs where the optimal decision is to abandon may be characterized by a single scalar threshold of C^*. That is, if C is between $[C^*, \infty)$, then the optimal decision is to abandon. Otherwise, the optimal decision is to continue operation. It can be shown (see, e.g., Dixit and Pindyck 1994) that indeed there will be only one threshold C^* for this problem.

As long as it is not optimal to abandon the renewable site (i.e., C is in the continuation region), the value of the renewable site project, $V(C)$, must satisfy the following differential equation, which results from Bellman's principle of optimality (Dixit and Pindyck 1994):

$$\rho V dt = (P - C)K dt + \mathrm{E}[dV] \tag{2}$$

where ρ is the annual discount rate (% per year), which is often called the expected rate of return (Dixit and Pindyck 1994). The left-hand side of Equation (2) is the total return on the value of the renewable site. The first term on the right-hand side is the immediate profit flow from keeping the renewable site in operation, and the last term is the expected capital appreciation of the renewable site value function.

By Ito's lemma (see, e.g., Dixit and Pindyck 1994), it can be verified that dV is given by

$$dV = \frac{\partial V}{\partial C}(\alpha_C C dt + \sigma_C C dz) + \frac{1}{2}\frac{\partial^2 V}{\partial C^2}\sigma_C^2 C^2 dt \tag{3}$$

Substituting Equation (3) into Equation (2), and after rearranging and simplifying terms, we have

$$\frac{1}{2}\frac{\partial^2 V}{\partial C^2}\sigma_C^2 C^2 + \frac{\partial V}{\partial C}\alpha_C C - \rho V + (P - C)K = 0 \tag{4}$$

We note that Equation (4) is a differential equation. To guarantee its convergence, we impose that $\alpha_C < \rho$ (see, e.g., Costa Lima and Suslick 2006). We also note that the following boundary conditions between the operation and abandonment states are needed to obtain the optimal threshold C^*.

$$V(C^*) = -W \tag{5}$$

$$V'(C^*) = 0 \tag{6}$$

The first is the value-matching condition and the second is the smooth-pasting condition. The value-matching condition (Equation (5)) requires that at the exit threshold, the value of the renewable site project equals the value of exit. The smooth-pasting condition (Equation (6)) assures that C^* is the optimal exercise point by defining the continuance and smoothness of $V(C)$ at C^*.

By solving Equation (4) with Equations (5) and (6), we have the following proposition:

Proposition 1. *Given* $0 < \alpha_C < \rho$, *the value of an operating renewable site is*

$$V(C) = A_1 C^{\beta_1} - CK/(\rho - \alpha_C) + PK/\rho \tag{7}$$

$$where\ \beta_1 = \left(1/2\sigma_C^2 - \alpha_C + \sqrt{\left(\alpha_C - 1/2\sigma_C^2\right)^2 + 2\sigma_C^2\rho}\right)\Big/\sigma_C^2 \tag{8}$$

$$C^* = \frac{(PK/\rho + W)(\rho - \alpha_C)\beta_1}{(\beta_1 - 1)K} \tag{9}$$

$$A_1 = \frac{K}{(\rho - \alpha_C)\beta_1 (C^*)^{\beta_1 - 1}} \tag{10}$$

The proof is given in the Appendix. The economically rational decision is as follows: As soon as the O&M costs are equal to or greater than C^*, the firm will pay the exit fee and abandon the renewable site with the corresponding renewable site project value of $-W$. Otherwise, the firm continues to operate the renewable site with the corresponding value of $V(C)$.

We also note that the value function $V(C)$ given by Equation (7) has the following interpretation. Before the decision maker chooses to abandon (i.e., the decision maker is still holding the option to abandon), the value of the renewable site consists of two parts: the value of operating the renewable site and the value of the option to abandon the renewable site. Hence, in Equaiton (7), the first term is the value of the option to abandon and the last two terms represent the expected costs and revenue streams when the initial price and costs are observed as P and C.

Thus far, we have derived the value function of the renewable site project and the threshold value of C^* in terms of aforementioned critical parameters. We now proceed to derive the expected remaining life of the renewable site under the technical assumption of $\alpha_C > 1/2\sigma_C^2$, as shown in the relevant literature (see, e.g., Mauer and Ott 1995). Let $F(C) = \ln C$. Then, $dF(C)$ can be expanded by Ito's lemma as $dF(C) = (\alpha_C - 1/2\sigma_C^2)dt + \sigma_C dz$ and therefore, for any finite time period T, the change in $F(C)$ is distributed with mean $(\alpha_C - 1/2\sigma_C^2)T$ and variance $\sigma_C^2 T$. Hence, the expected first passage time of C from C_c to C^*, where C_c denotes the current level of O&M costs, can be calculated as $(\ln C^* - \ln C_c)/(\alpha_C - 1/2\sigma_C^2)$.

Hence, the expected remaining operating life is given by

$$T_{EX}^* = (\ln C^* - \ln C_c)/\left(\alpha_c - 1/2\sigma_C^2\right) \tag{11}$$

where C^* is the exit threshold as in Equation (9).

Sensitivity Analysis on the Exit Decision

Given the expression for C^* shown in Equation (9), the sensitivity analysis can be performed in a straightforward manner with respect to the key parameters σ_C, α_C, W, P, and K, and the results are summarized in the following proposition.

Proposition 2. *Given* $0 < \alpha_C < \rho$, $\frac{\partial C^*}{\partial \sigma_C} > 0$, $\frac{\partial C^*}{\partial \alpha_C} < 0$, $\frac{\partial C^*}{\partial W} > 0$, $\frac{\partial C^*}{\partial P} > 0$, *and* $\frac{\partial C^*}{\partial K} < 0$.

The outline of the proof is given in the Appendix, and the economic interpretation of the results is as follows: $\frac{\partial C^*}{\partial \sigma_C} > 0$ indicates that an increase in the volatility leads to an increase

in the threshold value. This is because with increased volatility there is a greater chance of a greater reduction in the O&M costs in the near future, and it is beneficial to wait a little longer (hence, C^* becomes higher). As for $\frac{\partial C^*}{\partial \alpha_C} < 0$, we note that as the rate of growth of O&M costs increases, it is beneficial to exit earlier (hence, C^* becomes lower). $\frac{\partial C^*}{\partial W} > 0$ indicates that an increase in the exit fee leads to an increase in the threshold value. This should be clear, as in the case of $\frac{\partial C^*}{\partial \sigma_C} > 0$. Now, as for $\frac{\partial C^*}{\partial P} > 0$, if the contract power price is increasing, we note that the revenue and the value of the renewable site project should increase, and the decision maker has an incentive to continue to operate longer (hence, C^* becomes higher). Finally, $\frac{\partial C^*}{\partial K} < 0$ indicates that as the production quantity increases, the threshold value decreases. This is because the total O&M costs increase even more as such costs are proportional to the production quantity. Therefore, it is beneficial to exit earlier (hence, C^* becomes lower). We note that, due to Assumption 2 (this production quantity is equal to the nameplate capacity times the capacity factor), this sensitivity analysis can be applied to the capacity in place of production quantity interchangeably.

If we assume that preventing a premature exit (relative to the physical life) is environmentally desirable (so that we can delay exploiting new resources), increases in W and P are desirable, whereas an increase in K is not. Hence, any government policies through incentives and fees to increase the contract power price or the exit fee are desirable, whereas increasing capacity is not. For example, if a community is recruiting a single renewable site, the smaller capacity is more desirable than the larger capacity, assuming that preventing a premature exit is the primary criterion. We caution that this interpretation is strictly focusing on the exit decision regarding the existing renewable sites under the aforementioned criterion and will be revisited numerous times in the succeeding sections of this article.

Finally, we note that the sign of $\frac{\partial C^*}{\partial \rho}$ is ambiguous because some components of C^* increase whereas others decrease in a way such that the total effect is unwieldy to interpret. We also note that, with the expected remaining operating life of Equation (11), we have $\frac{\partial T^*_{EX}}{\partial C_c} < 0$, $\frac{\partial T^*_{EX}}{\partial \sigma_C} > 0$, $\frac{\partial T^*_{EX}}{\partial \alpha_C} < 0$, $\frac{\partial T^*_{EX}}{\partial W} > 0$, $\frac{\partial T^*_{EX}}{\partial P} > 0$, and $\frac{\partial T^*_{EX}}{\partial K} < 0$. The interpretation of these results is analogous to the case with respect to C^*.

Modeling and Analysis of Renewable Site Entry Decision

In this section, we extend the previous model by considering the entry decision for the renewable site. For this extension, we assume that the aforementioned power purchase contract is available for a new renewable site. Two additional assumptions are made as follows.

> Assumption 7: The construction period of the renewable site is assumed to be negligible in our model. This simplifying assumption is made to focus on the entry decision without diluting our attention regarding how best to make economic decisions during the construction period. This type of simplifying assumption can be found in numerous papers (see, e.g., Fleten and Maribu 2004).

> Assumption 8: Once the construction occurs, there is a lump sum investment cost of I (\$), which includes the cost of materials, labor, land, etc. This cost is treated as an irreversible sunk cost, which cannot be recovered later.

We note that the firm makes an entry decision by evaluating the direct net revenue (i.e., revenue minus cost) from the renewable site plus the value of the option to exit. We recall that the option value critically depends on the O&M costs, and we will denote C_0 as the initial O&M costs at a time point at which the renewable site starts to operate. We further note that there are no O&M costs prior to the start of the renewable site.

We also note that, under the additional assumption of the contract power price following a GBM process, the option value for waiting to enter can be incorporated. In our model construction, however, such an option is excluded by design because (1) there are numerous fixed price contract approaches available (as mentioned in earlier sections) and (2) parallel GBM processes of the price and cost typically make analytical studies infeasible and often make numerical results difficult to sort out (see Discussion section).

Under our model framework, if the value of the potential renewable site project is greater than or equal to the irreversible investment I, the firm will enter the market. Therefore, the condition under which the firm decides to enter becomes

$$V(C_0) = A_1 C_0^{\beta_1} - C_0 K/(\rho - \alpha_C) + PK/\rho \geq I \tag{12}$$

For the boundary "marginal" firm without a strictly positive net benefit, we define another type of the O&M cost threshold, $\bar{C}_0$. Namely,

$$A_1 \bar{C}_0^{\beta_1} - \bar{C}_0 K/(\rho - \alpha_C) + PK/\rho - I = 0 \tag{13}$$

This $\bar{C}_0$ of Equation (13) can be viewed as the upper bound of the initial O&M costs at which the firm will decide to enter the market.

More formally, even though there is no explicit closed-form solution for $\bar{C}_0$ in Equation (13), we have the following proposition that proves the existence and uniqueness of $\bar{C}_0$ under two fairly undemanding conditions.

Proposition 3. *Let us assume that $\bar{C}_0 < C^*$ and $PK/\rho - I > 0$, then there exists a unique solution for $\bar{C}_0$ in $A_1 \bar{C}_0^{\beta_1} - \bar{C}_0 K/(\rho - \alpha_C) + PK/\rho - I = 0$.*

The proof is given in the Appendix. The condition $\bar{C}_0 < C^*$ is not stringent as, if $\bar{C}_0 \geq C^*$, a firm will enter and exit the market instantaneously, which neither leads to practical use nor makes practical sense (not unlike assuming $\alpha_C \geq \rho$). The condition $PK/\rho - I > 0$ indicates that the cumulative revenue ($\int_0^\infty PKe^{-\rho s} ds = PK/\rho$) is greater than the initial investment I. It is a reasonable assumption that any firm considering the entry has a tangible revenue stream that will at least cover the initial investment.

From Proposition 3 and the monotonicity of $V(C_0)$ shown in the proof, we claim that any firm with $C_0 \leq \bar{C}_0$ will enter the market and any other firm with $C_0 \geq \bar{C}_0$ will not.

Finally, we note that, as in the case of the exit decision, we derive the expected economic life of a new renewable site as

$$T^* = (\ln C^* - \ln C_0)/\left(\alpha_C - 1/2\sigma_C^2\right) \tag{14}$$

and that of a marginal renewable site with $C_0 = \bar{C}_0$ as

$$\bar{T} = (\ln C^* - \ln \bar{C}_0)/\left(\alpha_C - 1/2\sigma_C^2\right) \tag{15}$$

Sensitivity Analysis on the Entry Decision

Given the implicit function for $\bar{C}_0$ in Equation (13), sensitivity analysis can be performed with respect to the key parameters σ_C, α_C, W, P, and I, and the results are summarized in the following proposition.

Proposition 4. *Given* $0 < \alpha_C < \rho$, $\frac{\partial \bar{C}_0}{\partial \sigma_C} > 0$, $\frac{\partial \bar{C}_0}{\partial \alpha_C} < 0$, $\frac{\partial \bar{C}_0}{\partial W} < 0$, $\frac{\partial \bar{C}_0}{\partial P} > 0$, *and* $\frac{\partial \bar{C}_0}{\partial I} < 0$.

The outline of the proof is given in the Appendix. The interpretation for $\frac{\partial \bar{C}_0}{\partial \sigma_C} > 0$, $\frac{\partial \bar{C}_0}{\partial P} > 0$, and $\frac{\partial \bar{C}_0}{\partial \alpha_C} < 0$ is straightforward; that is, as the volatility and the contract power price increase, the entry threshold O&M costs increase; that is, more (marginal) firms will enter the market. On the other hand, as the growth rate of the O&M costs increases, less (marginal) firms will enter the market.

As for $\frac{\partial \bar{C}_0}{\partial W} < 0$, an increase in the exit fee will lead to a decrease in the threshold O&M costs. This implies that fewer firms will enter, resulting in a lower power production quantity from renewable energy. At the same time, this will lead to a lower number of marginal firms, reducing the number of premature exits (relative to the physical life). Therefore, the economic and environmental consequences of a government policy for a higher exit fee on the entry (not exit) of renewable sites are far from simple and straightforward.

As for $\frac{\partial \bar{C}_0}{\partial I} < 0$, the increase in the initial investment will decrease the threshold O&M costs. This implies that any initial subsidy provided by the government will lead to more (marginal) firms entering the market, resulting in a higher power production quantity from the renewable energy. On the other hand, this will lead to a higher number of marginal firms, increasing the number of premature exits (relative to the physical life). Once again, the consequence of a government policy for a higher level of initial subsidy on the entry (not exit) is complex.

In addition, we note that the signs of $\frac{\partial \bar{C}_0}{\partial \rho}$ and $\frac{\partial \bar{C}_0}{\partial K}$ are ambiguous for the reason similar to that for the case of ρ in the exit sensitivity analysis.

Let us now turn our attention to the expected life of a new renewable site, T^*. As in the case of the exit decision, we can obtain $\frac{\partial T^*}{\partial C_0} < 0$, $\frac{\partial T^*}{\partial \sigma_C} > 0$, $\frac{\partial T^*}{\partial \alpha_C} < 0$, $\frac{\partial T^*}{\partial W} > 0$, $\frac{\partial T^*}{\partial P} > 0$, and $\frac{\partial T^*}{\partial K} < 0$ with straightforward and intuitive interpretation.

As for the expected life of a new renewable site from a marginal firm, $\bar{T}$, generally, the signs from the sensitivity analysis are ambiguous and the corresponding interpretation is unwieldy. The only exceptions are $\frac{\partial \bar{T}}{\partial W} > 0$ and $\frac{\partial \bar{T}}{\partial I} > 0$; that is, as the exit fee or the initial investment increases, the corresponding expected life increases, as may be expected.

Numerical Analysis: The Case of a Wind Farm

In this section, we numerically illustrate some of the key features of our models as follows.

1. Parameter values: Let us first present the parameter values used in this section. Even though these values are hypothetical, to be realistic numbers, we have consulted the U.S. Energy Information Administration's *Updated Capital Costs Estimates for Electricity Generation Plants* (EIA 2010b) as well as others (e.g., Kjarland 2007; Takashima et al. 2007). These are summarized in Table 1. As explained in Assumption 2, the linkage between the capacity and production quantity is as follows. Let us assume that the total number of hours of operation per year is 8,760 hours. Then, as the production quantity per year as well as the nameplate

Table 1
Parameters and corresponding values

Parameters	Numerical values
Contract power price P	48 \$/MWh
Nameplate capacity/Ccapacity factor	3 MW/33.33%
Production quantity K	8,760 MWh/year
Investment costs I	\$1,000,000
Exit fee W	\$300,000
Annual discount rate ρ	0.05
Annualized growth rate of O&M costs α_C	0.04
Annualized volatility of O&M costs σ_C	0.10

capacity and capacity factor are assumed to be constant, $K = 3{*}0.3333{*}1{*}8,760 =$ 8,760 (MWh/year).

2. The entry/exit decisions: By applying the parameter values to Equations (7)–(10) and (13), the threshold values of C^* and $\bar{C}_0$ (\$/MWh) as well as the function of $V(C)$ can be calculated. At the same time, with $\bar{C}_0$, we can use Equation (15) to calculate $\bar{T}$ as well. The numerical results are summarized in Table 2. We note that the first term of $V(C)$ is the option value to exit, the second term is the O&M costs, and the third term is the revenue. Also, given a $\bar{C}_0$ value of 30.0549, it can be verified that $V(\bar{C}_0)$ is 1,000,000, which is the initial investment. Furthermore, if a firm is considering entry, given the initial O&M costs, we can calculate the expected life of a new wind farm by Equation (14). Likewise, if a firm is currently operating the wind farm, then given the current O&M costs, we can calculate the remaining life of the existing wind farm using Equation (11). Figure 1 shows the value of the project with respect to the O&M costs when the wind farm is in operation. As indicated in the graph, the value of the project decreases as the O&M costs increase until they reach C^*. Once the cost reaches the exit threshold, the firm will pay the exit fee to abandon the wind farm and thus $V_{EX}(C) = -W$. In Figure 2, the thresholds of the O&M costs C^* and $\bar{C}_0$ are depicted with respect to the volatility of the O&M costs. Both threshold values increase as the O&M costs become more volatile, which indicates that a higher degree of volatility will delay the exit and allow more (O&M cost-wise) marginal firms to enter. It is interesting to note that the slope is much steeper for the exit threshold than that for the entry threshold as

Table 2
Numerical results for the entry/exit decisions

Decision variables	Numerical values
β_1	1.2170
A_1	300,800
C^*	55.7623
$\bar{C}_0$	30.0549
$V(C)$	$300{,}800C^{1.2170} - 876{,}000C + 8{,}409{,}600$
$\bar{T}$	17.6592

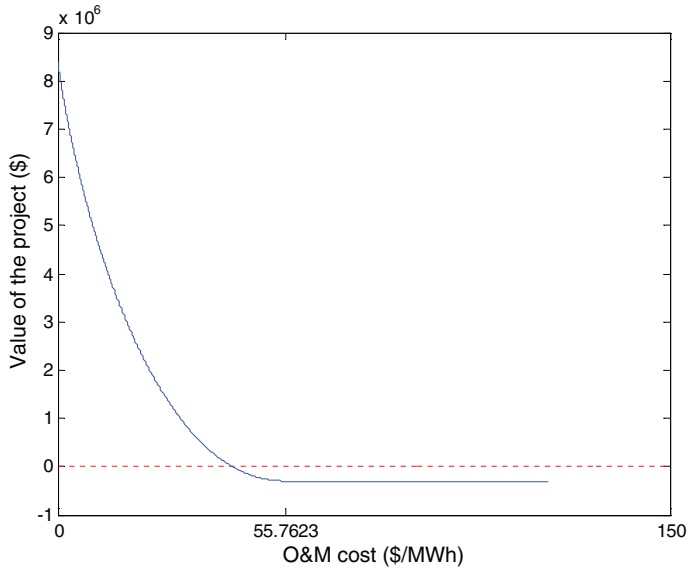

Figure 1. Value of the project vs. the O&M costs (color figure available online).

the volatility increases; that is, it seems that the exit threshold is more sensitive to volatility than the entry threshold is. (Note that the entry cost is at time zero and the exit cost is some time in the future.) In Figure 3, the thresholds of the O&M costs C^* and $\bar{C}_0$ are depicted with respect to the growth rate of the O&M costs. Both threshold values decrease as the growth rate increases. This implies that a higher growth rate of the O&M costs induces an early exit and allows marginal

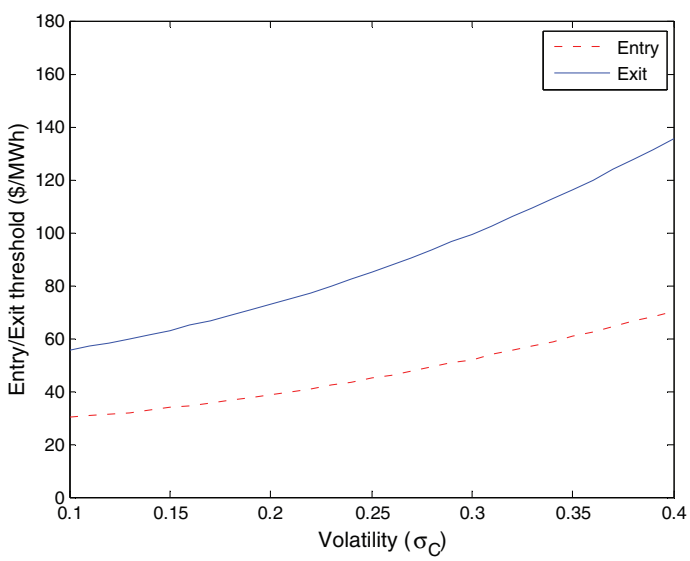

Figure 2. Entry/exit thresholds vs. the volatility of O&M costs (color figure available online).

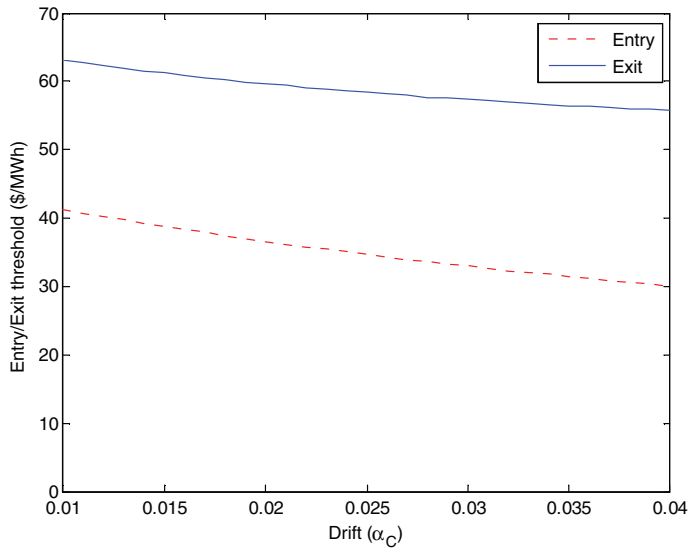

Figure 3. Entry/exit thresholds vs. the growth rate of O&M costs (color figure available online).

firms to enter the market. In Figure 4, the thresholds of the O&M costs C^* and $\bar{C}_0$ are depicted with respect to the exit fee. As we can see from the graph, as the exit fee increases, the exit threshold increases, which leads the existing firm to delay its abandonment. At the same time, it allows marginal firms to enter the market, reducing the total power production from wind energy. We note that similar graphs can be produced for the remaining parameters such as the contract power price P, the power production quantity K, and the initial investment I, among others. We

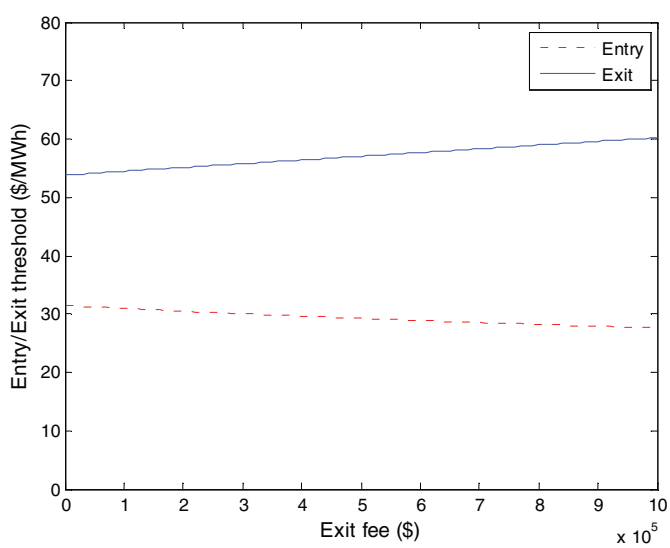

Figure 4. Entry/exit thresholds vs. the exit fee (color figure available online).

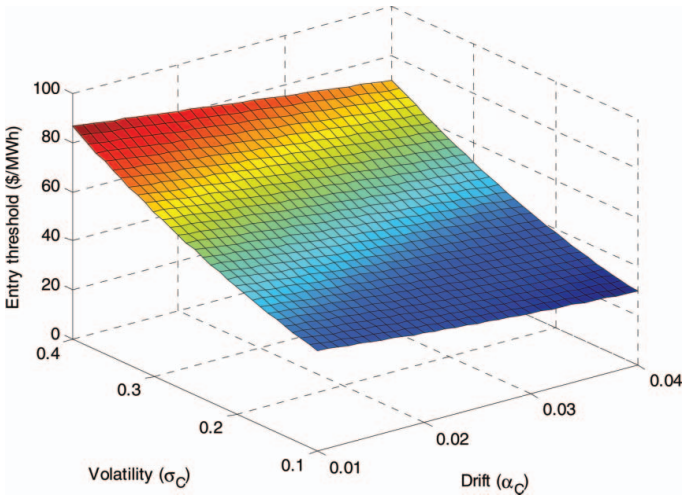

Figure 5. Entry threshold vs. the growth rate and volatility of O&M costs (color figure available online).

now proceed to examine the impact of bidirectional changes on the entry and exit thresholds. First, in Figure 5, we vary the volatility and growth rate and observe that the entry threshold is behaving as predicted. Furthermore, the degree of change seems linear rather than nonlinear. In Figure 6, we vary the volatility and growth rate and observe that the exit threshold is behaving as predicted. Furthermore, the degree of change also seems to be linear rather than nonlinear. We note that this type of bidirectional analyseis can be conducted for the remaining parameters.

3. Monte Carlo simulation: In the previous subsections, the value of a wind farm project, the entry/exit thresholds of the O&M costs, and the expected economic life were calculated based on the fixed hypothetical data. In this section, in contrast,

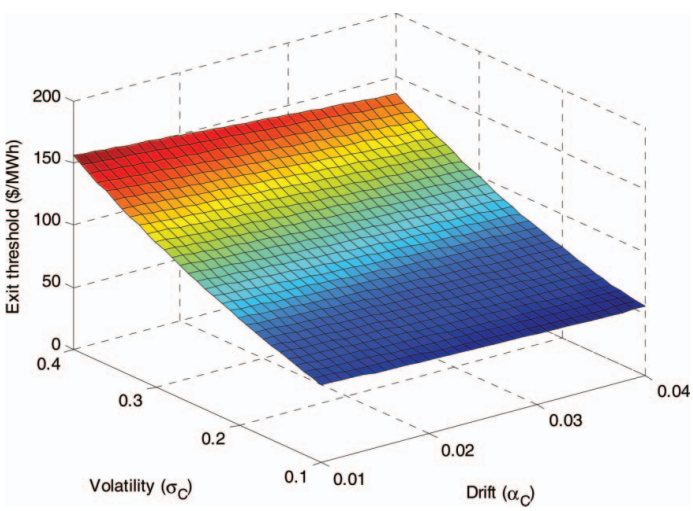

Figure 6. Exit threshold vs. the growth rate and volatility of O&M costs (color figure available online).

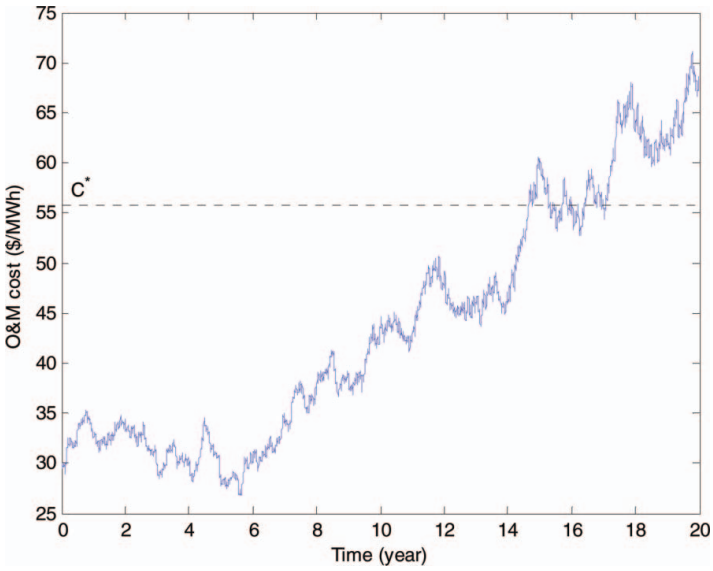

Figure 7. Sample path of a wind farm's O&M costs in 20 years (color figure available online).

Monte Carlo simulation is used to simulate sample paths given $C_0 = \bar{C}_0 = 30.0549$ \$/MWh as the starting point and the GBM process as

$$dC = 0.04Cdt + 0.10Cdz$$

A typical sample path is shown in Figure 7 with the horizontal axis as the time (year) and the vertical axis as the O&M costs (\$/MWh). In our study, we used MATLAB (ver. R2010a; The MathWorks, Inc., Natick, MA, USA) on a 64-bit operation system with Intel Pentium D processor (3.00 GHZ) to generate such GBM paths 1,000 times, given all of the parameters and time horizons as we indicated before. The algorithm of computing the expected marginal economic life T^* is simply presented as follows:

Initiation: Set $\text{index}_0 = \emptyset$ and $j = 1$.

Step 1: Generate a GBM process and find the first time it hits C^*as t_j if C reaches C^* within 365×60 days. If C does not reach C^* within 365×60 days, then let $t_j = 60$. $\text{index}_j = \{t_j, \text{index}_{j-1}\}$. Go to step 2.

Step 2: $j = j + 1$. If $j > 1,000$, go to step 3; otherwise go to step 1.

Step 3: Calculate the mean value of all elements in index_{j-1} then STOP.

By executing this simple algorithm, we compute the mean value of the economic life to be 17.4115 years, which is fairly close to the analytical expected value of 17.6592 years that we obtained in the previous subsection. The difference becomes negligible once more simulations are run. Finally, we note that in this algorithm, there is one minor adjustment as follows. We implicitly assumed that the maximum physical life with all of the repair and maintenance would be 60 years. Hence, any simulation run that did not reach C^* in 60 years was terminated and we entered its age as 60 years. By running the Monte Carlo simulation with 60 years 1,000 times, we have the result showing

that the possibility of a wind farm not retiring within 60 years is less than 1%. Hence, we believe that this adjustment is indeed minor.

Discussion on Policy Implications and Assumptions

This section consists of two discussion items: policy implications and assumptions. Let us first address the policy implications. As shown in the previous two sensitivity analysis sections on exit and entry decisions, as the exit fee W increases, the exit threshold C^* increases. This implies that the government could increase the length of the economic life of the renewable site by imposing or increasing the exit fee. At the same time, as W increases, the entry threshold $\bar{C}_0$ decreases. That is, the number of firms that would enter into the renewable power market will decrease as the government imposes or increases the exit fee. Interestingly, the firms that would be able to enter the renewable power market will have a lower level of $\bar{C}_0$, which will increase the economic life of the renewable site.

Hence, the government's exit fee can be viewed as a policy tool to increase the economic life of a renewable site in two different ways. First, it achieves this objective by increasing the exit threshold C^*. Concurrently, it reduces the entry threshold $\bar{C}_0$, contributing positively to the same objective.

Moreover, we recall that, as the initial investment I decreases, the entry threshold $\bar{C}_0$ increases. Hence, if the government provides an initial subsidy, it can be interpreted that more firms will be able to enter. On the other hand, the marginal firms that enter will have a higher level of the initial O&M costs, of which expected value will only increase with respect to time. In this sense, the initial subsidy encourages the premature exit of the (marginal) renewable sites. In view of this observation, let us consider the following scenario.

If the government subsidy is provided as a constant matching fund to C throughout the duration of the renewable site operation, this will directly extend the economic life of many renewable sites through an increase in presubsidy C^*. At the same time, marginal renewable sites will enter through an increase in presubsidy $\bar{C}_0$. In this case, both the expected economic life and the production of electric power from the renewable energy increase.

From our observation, it is certainly worthwhile to investigate the conversion of the initial subsidy to the O&M subsidy. We note that the O&M costs subsidy is different from the production tax credit (PTC) currently in use in the United States because the PTC is front-loaded for early years only and is under the assumption that a sufficient amount of tax has already been paid to the government (see, e.g., Wiser et al. 2007).

Finally, we caution that, for a thorough investigation, the total benefit vs. cost will have to be quantified, which will critically depend on the distribution of firms over key parameters such as $\bar{C}_0$. For example, even in a simpler case of imposing a scalar W, it is recommended that the government strike a careful balance between one environmentally desirable goal of preventing premature exit (relative to the physical life of the renewable site) and another environmentally desirable goal of encouraging the entry of firms to the renewable power market and increase the production of electric power from the renewable energy as the distribution of such firms over $\bar{C}_0$ values may be far from certain.

So far, we have discussed the policy implications. Now we proceed to discuss the assumptions. The most fundamental assumption in our model is that the O&M costs ($/MWh) follow a GBM process and some cases were referred to earlier in this article to support this assumption. We note that, in these cases, the costs were random over a period, and the expected costs as well as the volatility of the costs were increasing.

In the case of the wind farm, we can make similar observations on the O&M costs. In EWEA (2004, p. 101), some of the O&M cost paths over time for wind turbines meet these characteristics. Qualitatively, Wiser and Bolinger (2008) reported that the average O&M costs for a wind turbine ($/MWh) appear to increase with project age; other authors claimed that the operating and maintenance costs escalate over time (Wilkinson 2010).

In practice, we envision that first the raw data on the daily production as well as O&M costs of the renewable site will be recorded over a period. From these data, we will obtain the O&M costs per megawatt hour of data. We note that once enough of these O&M data become available in the future, more rigorous tests to determine the degree of fit can be administered using various statistical tools (e.g., Postali and Picchetti 2006).

In terms of the relaxation of simplifying assumptions, the power production quantity, capacity, and contract power price all seem worthwhile. In the case of power production quantity, a simulation model can accommodate the daily (perhaps hourly) fluctuation of the power production, taking the specific technological characteristics (wind speed, daylight availability, etc.) into account. Such a simulation will add numerical and computational insights to more tactical decisions of temporary contraction or expansion such as partial shut-down due to seasonality.

In the case of capacity, granularity will be a central question. Because the renewable site is subdivided into smaller groups (until an individual renewable generator such as a wind turbine or a solar panel), there will be an explosive number of options for partial shut-downs and gradual entries. We acknowledge that such features will add realism to the models studied here. However, the analytic tractability of any such extension remains to be seen.

Finally, for the contract power price, as mentioned previously, certainly a power price following a separate GBM will allow us to model the option to wait before any entry. With this separate, concurrent GBM process, whether the numerical and computational analyses (analytic solutions are not likely) can exploit the interaction of the price and O&M costs GBMs and yield unambiguous insights will be a significant future challenge.

Concluding Remarks

In this study, we modeled and analyzed how an economically rational decision maker will exit and enter a renewable site when the O&M costs is represented by a GBM process where the renewable site is without any input fuel costs. For such a site, we obtained the threshold level of the O&M costs above which a currently operating renewable site will exit. We also obtained the threshold level of the O&M costs below which a new renewable site will enter.

Based on these two findings, we conducted extensive sensitivity analyses with respect to various critical parameters with major policy implications. For example, the exit fee by the government will help in preventing premature exit relative to the physical life. At the same time, such a fee will prevent O&M cost-wise marginal firms from entering the market, which reduces the total production amount of electric power from the renewable energy. Moreover, the government subsidy for the initial investment allows the O&M cost-wise marginal firms to enter the market. The renewable power sites of such firms, however, do have a shorter expected economic life relative to the physical life. At the same time, such entries will increase the total production amount of electric power from the renewable energy.

As an alternative, it is desirable to investigate diverting the subsidy on the initial investment to the subsidy on the O&M costs because the O&M costs subsidy will extend

the expected economic life. At the same time, more O&M cost-wise marginal firms will enter the market, which will increase the total production amount of electric power from the renewable energy.

We note that for conclusive results over competing policies, the total benefit and cost must be quantified, which is beyond the scope of this article. However, this article has discovered a plausible alternative subsidy to the current government policy of front-loading grants and incentives in the initial and early years of renewable site operations.

Because this article can be seen as an initial exploration, there are numerous worthwhile future studies. Specifically, it may be worthwhile to relax each simplifying assumption and examine the ramifications of such relaxation. For example, by assuming no prior power purchase contract, the electric power price can be modeled as a separate GBM process. This type of endeavor will enhance the realism of our study and widen the applicability of our models.

In addition, as the data on O&M costs across renewable sites accumulate, it is worthwhile to measure the degree of fitness for the GBM assumption. Because such a degree is typically far from being binary, we do anticipate differing degrees of fitness across renewable sites. However, such an examination will enhance our ability to fine-tune the exact GBM process (out of so many GBM-inspired processes) and their corresponding parameter values.

Moreover, one could consider a single firm with multiple sites of the same or different renewable energies whose O&M costs may be positively/negatively correlated. Other expansions could include a competitive model of single-site firms of the same or different renewable energies as well as the management of changing technology with respect to time.

Finally, we note that the massive abandonment of the 1990s has already occurred in the renewable power industries, including wind and solar power producers. Given the current expansion of these industries across the United States and other countries, we believe that the questions raised and addressed (to a varying degree) in this article are timely (if not urgent). Furthermore, we hope that this article contributes positively to the resolution of the upcoming massive-scale problem for the aging and retiring of renewable sites as well as alternative energy facilities of similar economic and environmental characteristics (to a degree, biomass and waste energy).

Acknowledgments

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Appendix

Proof of Proposition 1. The structure of Equation (4)'s solution contains the general solution of the homogeneous part of it as well as a particular solution to the full equation, which is in the form of

$$V(C) = A_1 C^{\beta_1} + A_2 C^{\beta_2} - CK/(\rho - \alpha_C) + PK/\rho$$

where β_1 and β_2 are the roots of the characteristic quadratic equation as follows:

$$\frac{1}{2}\sigma_C^2 \beta^2 + \left(\alpha_C - \frac{1}{2}\sigma_C^2\right)\beta - \rho = 0$$

Solving the quadratic equation we have

$$\beta_1 = \left(1/2\sigma_C^2 - \alpha_C + \sqrt{\left(\alpha_C - 1/2\sigma_C^2\right)^2 + 2\sigma_C^2 \rho}\right) \Big/ \sigma_C^2 > 1$$

$$\beta_2 = \left(1/2\sigma_C^2 - \alpha_C - \sqrt{\left(\alpha_C - 1/2\sigma_C^2\right)^2 + 2\sigma_C^2 \rho}\right) \Big/ \sigma_C^2 < 0$$

We also notice that when $C \to 0$, that is, the O&M costs becomes negligible, the renewable site will not be abandoned, which indicates that the value of the option to abandon approaches zero; therefore, $A_2 = 0$. After eliminating this speculative bubble, the general solution then becomes

$$V(C) = A_1 C^{\beta_1} - CK/(\rho - \alpha_C) + PK/\rho$$

Solving the above solution with boundary conditions (5) and (6) we have the following:

$$C^* = \frac{(PK/\rho + W)(\rho - \alpha_C)\beta_1}{(\beta_1 - 1)K}$$

$$A_1 = \frac{K}{(\rho - \alpha_C)\beta_1 (C^*)^{\beta_1 - 1}}$$

Proof of Proposition 2. To prove $\frac{\partial C^*}{\partial \sigma_C} > 0$, we transform it into an equivalent problem of proving $\frac{\partial C^*}{\partial \sigma_C} = \frac{\partial C^*}{\partial \beta_1} \cdot \frac{\partial \beta_1}{\partial \sigma_C} > 0$ by the chain rule.

For simplicity we denote $\sqrt{\left(\alpha_C - 1/2\sigma_C^2\right)^2 + 2\sigma_C^2\rho}$ as $\sqrt{\Delta}$ and

$$\begin{aligned}\frac{\partial\beta_1}{\partial\sigma_C} &= \left[\left(\sigma_C + \frac{\sigma_C^3 - 2\alpha_C\sigma_C + 4\rho\sigma_C}{2\sqrt{\Delta}}\right)\sigma_C^2 - 2\left(\frac{1}{2}\sigma_C^2 - \alpha_C + \sqrt{\Delta}\right)\sigma_C\right]\Big/\left(\sigma_C^2\right)^2 \\ &= \left(\frac{\sigma_C^4 - 2\alpha_C\sigma_C^2 + 4\rho\sigma_C^2}{2\sqrt{\Delta}} + 2\alpha_C - 2\sqrt{\Delta}\right)\Big/\sigma_C^2 \\ &= \frac{\sigma_C^4 - 2\alpha_C\sigma_C^2 + 4\rho\sigma_C^2 + 4\alpha_C\sqrt{\Delta} - 4\Delta}{2\sigma_C^2\sqrt{\Delta}}\end{aligned}$$

Since the denominator is positive, we only need to check the sign of the numerator to determine the sign of the whole equation. For the numerator, if $4\alpha_C\sqrt{\Delta} > -\sigma_C^4 + 2\alpha_C\sigma_C^2 - 4\rho\sigma_C^2 + 4\Delta$, it is positive. Thus, we square both sides of the inequality and derive the difference between them as

$$\left(4\alpha_C\sqrt{\Delta}\right)^2 - \left(-\sigma_C^4 + 2\alpha_C\sigma_C^2 - 4\rho\sigma_C^2 + 4\Delta\right)^2 = -4\rho\sigma_C^4\left(\rho - \alpha_C\right) < 0$$

Therefore, we have $\frac{\partial\beta_1}{\partial\sigma_C} < 0$. For the sign of $\frac{\partial C^*}{\partial\beta_1}$, $\frac{\partial C^*}{\partial\beta_1} = -\frac{(PK/\rho + W)(\rho - \alpha_C)}{(\beta_1 - 1)^2 K} < 0$.

The other remaining properties can be proved similarly.

Proof of Proposition 3. First we use Equation (10) to re-form Equation (13) into a function of the entry threshold $\bar{C}_0$ as

$$F(\bar{C}_0) = \frac{\left(\bar{C}_0\right)^{\beta_1} K}{(\rho - \alpha_C)\,\beta_1\,(C^*)^{\beta_1 - 1}} - \bar{C}_0 K/(\rho - \alpha_C) + PK/\rho - I$$

When $\bar{C}_0 = 0$ and C^*, the value of $F(\bar{C}_0)$ can be calculated as

$$F(\bar{C}_0 = 0) = PK/\rho - I > 0$$
$$F(\bar{C}_0 = C^*) = -W - I < 0$$

By taking the partial derivative of $F(\bar{C}_0)$ with respect to $\bar{C}_0$ we also have that $F(\bar{C}_0)$ is monotonically decreasing; that is,

$$\frac{\partial F(\bar{C}_0)}{\partial\bar{C}_0} = K\frac{\left(\bar{C}_0/C^*\right)^{\beta_1 - 1} - 1}{\rho - \alpha_C} < 0 \text{ given } \bar{C}_0 < C^*$$

Hence, there exists a unique solution of $\bar{C}_0$.

Proof of Proposition 4. For Proposition 4 we briefly present the proof for $\frac{\partial\bar{C}_0}{\partial W} < 0$ here.

Recall $F(\bar{C}_0)$ in the proof of Proposition 3; it can be further (fully) expanded by using Equation (9) and then we get

$$F(\bar{C}_0) = \frac{\left(\bar{C}_0\right)^{\beta_1} K^{\beta_1}(\beta_1 - 1)^{\beta_1 - 1}}{(\rho - \alpha_C)^{\beta_1}\,\beta_1^{\beta_1}(PK/\rho + W)^{\beta_1 - 1}} - \bar{C}_0 K/(\rho - \alpha_C) + PK/\rho - I$$

As shown in the proof of Proposition 3, $\frac{\partial F(\bar{C}_0)}{\partial\bar{C}_0} \neq 0$ given $\bar{C}_0 < C^*$, which indicates that the implicit function theorem can be applied to Equation (13). We then differentiate

Equation (13) with respect to W in the following form:

$$\frac{\partial A_1}{\partial W}\left(\bar{C}_0\right)^{\beta_1} + \left(A_1\beta_1\left(\bar{C}_0\right)^{\beta_1-1} - \frac{K}{\rho-\alpha_C}\right)\frac{\partial \bar{C}_0}{\partial W} = 0$$

$$\frac{\partial \bar{C}_0}{\partial W} = -\frac{\frac{\partial A_1}{\partial W}\left(\bar{C}_0\right)^{\beta_1}}{A_1\beta_1\left(\bar{C}_0\right)^{\beta_1-1} - \frac{K}{\rho-\alpha_C}}$$

As in Equation (10), $A_1 = \frac{K}{(\rho-\alpha_C)\beta_1(C^*)^{\beta_1-1}}$, and after substitution, we have

$$\frac{\partial \bar{C}_0}{\partial W} = -\frac{\frac{\partial A_1}{\partial W}\left(\bar{C}_0\right)^{\beta_1}}{\frac{K}{\rho-\alpha_C}\left[\left(\frac{\bar{C}_0}{C^*}\right)^{\beta_1-1} - 1\right]}$$

Since $\bar{C}_0 < C^*$, the denominator above as $\frac{K}{\rho-\alpha_C}[(\frac{\bar{C}_0}{C^*})^{\beta_1-1} - 1] < 0$.

Also, it could be mathematically proved that the numerator $\frac{\partial A_1}{\partial W}(\bar{C}_0)^{\beta_1} < 0$ because $\frac{\partial A_1}{\partial W} < 0$. Therefore, $\frac{\partial \bar{C}_0}{\partial W} < 0$.

The other remaining properties can be proved similarly.

Biographical Sketches

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