Investment decision-making in clean energy under uncertainties:
a real options approach

by

Yihua Li

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Industrial Engineering, Statistics

Program of Study Committee:
Guiping Hu, Co-major Professor
Stephen B. Vardeman, Co-major Professor
Sarah M. Ryan
William Q. Meeker
Mingyi Hong

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation/thesis. The Graduate College will ensure this dissertation/thesis is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University
Ames, Iowa
2018

Copyright © Yihua Li, 2018. All rights reserved.
DEDICATION

I would like to dedicate this thesis to my mom and dad for their unconditional love and support.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF TABLES</th>
<th>vi</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>ix</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>x</td>
</tr>
<tr>
<td>CHAPTER 1. GENERAL INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1 Clean energy in transportation sector</td>
<td>1</td>
</tr>
<tr>
<td>1.1.2 Clean energy in electricity sector</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Literature Review</td>
<td>3</td>
</tr>
<tr>
<td>1.2.1 Technical, economic, and policy analyses for clean energy investment</td>
<td>3</td>
</tr>
<tr>
<td>1.2.2 Real options valuation</td>
<td>4</td>
</tr>
<tr>
<td>1.3 Dissertation Structure</td>
<td>7</td>
</tr>
<tr>
<td>CHAPTER 2.  IS NOW A GOOD TIME FOR IOWA TO INVEST IN CELLULOSIC BIO-FUELS?: A REAL OPTIONS APPROACH CONSIDERING CONSTRUCTION LEAD TIMES</td>
<td>9</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>9</td>
</tr>
<tr>
<td>2.2 Problem Formulation</td>
<td>15</td>
</tr>
<tr>
<td>2.3 Optimal Investment Timing</td>
<td>20</td>
</tr>
<tr>
<td>2.4 Case Study</td>
<td>23</td>
</tr>
<tr>
<td>2.4.1 Modeling price uncertainty</td>
<td>23</td>
</tr>
<tr>
<td>2.4.2 Data sources</td>
<td>25</td>
</tr>
</tbody>
</table>
2.4.3 The baseline case .................................................. 27
2.4.4 Impact of price parameters ....................................... 32
2.4.5 Impact of process parameters .................................... 35
2.4.6 Impact of other factors .............................................. 36

2.5 Conclusions ............................................................. 37

CHAPTER 3. DUAL INVESTMENTS SUBJECT TO RISK AVERSION AND LEAD TIMES:
A CASE OF MITIGATING SUPPLY RISK OF CELLULOSIC BIOFUEL PRODUCTION 39

3.1 Introduction ............................................................. 39
3.2 Model ................................................................. 43
  3.2.1 Formulation ...................................................... 45
  3.2.2 Solution procedure ............................................. 47
3.3 Case Study ............................................................ 48
  3.3.1 Underlying uncertainties ...................................... 48
  3.3.2 Operational data and sources .................................. 52
3.4 Numerical Result ..................................................... 53
  3.4.1 Lead time effect for risk-neutral valuation .................. 54
  3.4.2 Utility function .................................................. 55
3.5 Discussion ............................................................. 61
3.6 Extension ............................................................. 62
3.7 Conclusion ............................................................. 65

CHAPTER 4. VALUATION OF CLEAN TECHNOLOGY ADOPTION IN ELECTRICITY
SECTOR CONSIDER POLICY UNCERTAINTY: A REAL OPTIONS APPROACH . . 67

4.1 Introduction ............................................................. 67
4.2 Model Formulation .................................................... 71
  4.2.1 Problem description and assumptions ....................... 71
  4.2.2 Price uncertainty .............................................. 74
  4.2.3 Policy uncertainty ............................................. 75
4.2.4 Solution procedure ........................................... 77
4.3 Case Study .......................................................... 78
  4.3.1 Data sources ..................................................... 78
  4.3.2 Numerical results .............................................. 80
4.4 Conclusions .......................................................... 85

CHAPTER 5. GENERAL CONCLUSIONS ............................... 87

BIBLIOGRAPHY .......................................................... 89

APPENDIX A. CONSTRUCTING A TRINOMIAL LATTICE .......... 100
APPENDIX B. DETAIL OF THE MODEL IN CHAPTER 3 ............ 102
APPENDIX C. TWO-FACTOR LATTICE CONSTRUCTION WITH TIME DEPENDENT
  PARAMETERS AND CORRELATION COEFFICIENT ................. 103
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.1</td>
<td>Values of GMR process parameters</td>
<td>24</td>
</tr>
<tr>
<td>Table 2.2</td>
<td>Values of model parameters</td>
<td>27</td>
</tr>
<tr>
<td>Table 2.3</td>
<td>Project value and investment time vs. initial price</td>
<td>29</td>
</tr>
<tr>
<td>Table 2.4</td>
<td>Project valuation vs. lead time (baseline)</td>
<td>31</td>
</tr>
<tr>
<td>Table 2.5</td>
<td>Regression results for threshold price ($/gallon) and expected project value ($ million) vs. price parameters</td>
<td>34</td>
</tr>
<tr>
<td>Table 2.6</td>
<td>Regression results for threshold price ($/gallon) and the project value ($ million)</td>
<td>36</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Project valuation</td>
<td>61</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Parameter values</td>
<td>63</td>
</tr>
<tr>
<td>Table 4.1</td>
<td>Two cases of policy uncertainty</td>
<td>77</td>
</tr>
<tr>
<td>Table 4.2</td>
<td>Fitted Price Parameters</td>
<td>78</td>
</tr>
<tr>
<td>Table 4.3</td>
<td>Values and sources of model parameters</td>
<td>79</td>
</tr>
<tr>
<td>Table B.1</td>
<td>Profit function $f_t(P^F_t, P^L_t, P^W_t; x_{1t}, x_{2t})$</td>
<td>102</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Corn stover distribution in Iowa</td>
<td>11</td>
</tr>
<tr>
<td>2.2</td>
<td>Biofuel production from fast pyrolysis and further conversion</td>
<td>15</td>
</tr>
<tr>
<td>2.3</td>
<td>Illustration of the biofuel production process and the incurred costs</td>
<td>18</td>
</tr>
<tr>
<td>2.4</td>
<td>The cash flows of the investment</td>
<td>19</td>
</tr>
<tr>
<td>2.5</td>
<td>Using regression to obtain the threshold price $H_t$ at time $t$</td>
<td>22</td>
</tr>
<tr>
<td>2.6</td>
<td>Mean price estimation</td>
<td>24</td>
</tr>
<tr>
<td>2.7</td>
<td>Threshold prices over time (in the decision period)</td>
<td>28</td>
</tr>
<tr>
<td>2.8</td>
<td>Frequency chart of the project value (baseline with $L = 12$ months)</td>
<td>29</td>
</tr>
<tr>
<td>2.9</td>
<td>Frequency chart of the investment time $\tau$ (baseline with $L = 12$ months)</td>
<td>30</td>
</tr>
<tr>
<td>2.10</td>
<td>Project value vs. the initial fuel price</td>
<td>32</td>
</tr>
<tr>
<td>2.11</td>
<td>Threshold price ($/gallon$) and expected project value ($$ million$) vs. parallel shift in $m_t$</td>
<td>34</td>
</tr>
<tr>
<td>2.12</td>
<td>Regression of threshold price and the expected project value on conversion rate and biomass price</td>
<td>38</td>
</tr>
<tr>
<td>3.1</td>
<td>Biofuel production through fast pyrolysis and further conversion</td>
<td>44</td>
</tr>
<tr>
<td>3.2</td>
<td>Investment state space and transition relationship</td>
<td>46</td>
</tr>
<tr>
<td>3.3</td>
<td>Mean fuel price estimation</td>
<td>49</td>
</tr>
<tr>
<td>3.4</td>
<td>Land price</td>
<td>50</td>
</tr>
<tr>
<td>3.5</td>
<td>Corn stover yield can be decomposed to time effect, the linear function in (a), and weather effect in (b)</td>
<td>51</td>
</tr>
<tr>
<td>3.6</td>
<td>Optimal strategy at $t = 0$ with different lead times and yield levels ($\Delta = 1/0$ for high/low)</td>
<td>55</td>
</tr>
<tr>
<td>Figure 3.7</td>
<td>Utility function $U_\gamma(z)$ representing risk-aversion</td>
<td>56</td>
</tr>
<tr>
<td>Figure 3.8</td>
<td>Threshold of different optimal immediate actions when changing $\gamma$ in $U_\gamma(z)$</td>
<td>57</td>
</tr>
<tr>
<td>Figure 3.9</td>
<td>Risk-aversion effect on hedging decision ($\gamma = 0.995$)</td>
<td>58</td>
</tr>
<tr>
<td>Figure 3.10</td>
<td>Project value frequency charts at $P_0^F = $3.0/gallon, $P_0^L = $5,000/acre: (a) risk neutral (b) risk averse with $\gamma = 0.995$</td>
<td>61</td>
</tr>
<tr>
<td>Figure 3.11</td>
<td>Threshold of different optimal immediate actions when land drift $\mu_L = 0.05$</td>
<td>62</td>
</tr>
<tr>
<td>Figure 3.12</td>
<td>Threshold at $t = 0$ for the simple example. 1–wait, 2–facility only, 3–addition only, 4–both facility and addition</td>
<td>64</td>
</tr>
<tr>
<td>Figure 3.13</td>
<td>Relative project utility values under different strategies vs. “Both”</td>
<td>65</td>
</tr>
<tr>
<td>Figure 4.1</td>
<td>The profile of the future carbon prices</td>
<td>75</td>
</tr>
<tr>
<td>Figure 4.2</td>
<td>The profile of the future carbon prices with policy change occurred at $t$ from carbon tax to emission permit</td>
<td>76</td>
</tr>
<tr>
<td>Figure 4.3</td>
<td>Expected wait time $E[\tau^*]$ against the first year average carbon price $\bar{y}_0$</td>
<td>81</td>
</tr>
<tr>
<td>Figure 4.4</td>
<td>Illustration of entry conditions when repealing an existing policy</td>
<td>82</td>
</tr>
<tr>
<td>Figure 4.5</td>
<td>The threshold prices against proposed repeal time $T_p$</td>
<td>83</td>
</tr>
<tr>
<td>Figure 4.6</td>
<td>Illustration of entry conditions considering a policy change</td>
<td>84</td>
</tr>
<tr>
<td>Figure 4.7</td>
<td>The threshold prices against proposed policy change time $T_p$</td>
<td>85</td>
</tr>
<tr>
<td>Figure C.1</td>
<td>Branching probability</td>
<td>104</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

I would like to take this opportunity to express my thanks to those who made this work possible. First of all, I would like to thank my major professor, Dr. Guiping Hu, for her support and patience throughout the course of my Ph.D. study. My gratitude also goes to my co-major advisor, Dr. Stephen Vardeman, for his encouragement, suggestions and support. I’m extremely grateful to Dr. Chung-Li Tseng from University of New South Wales for his valuable time, continuous guidance, and insightful comments on all three pieces of work included in this dissertation, as well as his sharing of life experiences and thoughts.

I would like to express gratitude to my committee members, Dr. Sarah Ryan, Dr. William Meeker, and Dr. Mingyi Hong for their efforts, suggestions and contribution to this dissertation. I would like to thank other collaborators, Dr. Wei-Chung Miao (National Taiwan University of Science and Technology) and Dr. Yihsu Chen (University of California Santa Cruz) for their help in my research study.

Many thanks to all faculty and staff in Department of IMSE and Statistics, and all course instructors, who made my life at ISU much easier and fulfilled. My sincere thanks also go to my friends. Thanks to their encouragement and inspiration to my study, as well as their constant accompany and emotional support; all these made my years of studying so real and memorable.

In the end, I would like to send my deepest gratitude and love to my parents for their endless and unconditional love, especially during the most stressful times. I cannot express how thankful I am that you choose to endure my bad temper when you have all the reasons to be mad at me; how thankful I am that you trust me, support me, and be proud of me at all times.
International commitments on emission reduction and the deterioration of fossil energy resources have caused more research attention to clean energy production. Getting the optimal investment portfolio in infrastructure for energy supply and consumption is a minimum requirement to enable the transition towards a sustainable energy system. Due to their environmental benefits, advanced biofuel and clean power generation are expected to play an important role in the future in transportation sector and electricity sector, respectively. In this dissertation, a real options approach is adopted for valuating clean technology investment portfolios under uncertainty, exploring managerial insights, and examining policy implications. The dissertation consists three parts discussing problems on clean energy investment.

Biofuel production investment, motivated by consumption volume mandates in revised Renewable Fuel Standard, is a long-term irreversible investment facing revenue uncertainty given volatile fuel market. Iowa, rich in agricultural residues like corn stover, is a major player in the fulfillment of the cellulosic biofuels mandate. In this first part, we aim to answer the question: Is now a good time for Iowa to start investing in cellulosic biofuels? Using a fast pyrolysis facility as an example, we present a real options approach for valuating the investment of a new technology for producing cellulosic biofuels subject to construction lead time and uncertain fuel price. We conduct a case study, in which the profitability of the project, optimal investment timing, and the impact of project lead time are investigated.

The second part extended the previous work by incorporating supply risk and dual sourcing. While corn stover is an abundant source of feedstock for biofuels production in Iowa, there is a potential supply risk due to the following reasons: (1) lack of market; (2) low percentage of farm participation; and (3) yield uncertainty due to the changing weather conditions. The decision maker would consider investing in a land to grow his own feedstock, in addition to the investment
of biofuel facility. Land option with the growing of dedicated energy crops has a value-adding effect when operating with the fast pyrolysis facility. And with dual sourcing, the impact from supply uncertainty could be mitigated. A real options approach is used to analyze the optimal investment timing and benefits of the dual sourcing. Risk-aversion has an unexpected effect on investment decision-making, which may cause the investment decision of the value-adding option can be very sensitive to the primary underlying uncertainty, and the immediate action towards land investment can no longer be described with a single fuel price threshold.

Policy is deemed as one of the top decisive external factor that impacts the interest of a power producer. All energy projects are prone to policy risk, yet such eventualities are difficult to predict and therefore expensive to insure. In the third part of the study, we extend the uncertainty to the scope of government policy, in addition to considering the critical uncertainty of commodity prices. In this work, we want to examine the timing that an owner of a traditional coal-fired generator adopts in a clean technology when facing two realistic policy uncertainty cases: risk of repealing an existing policy, and risk of a policy change. The investment of a natural gas generator is considered in order to meet the load obligation while maximizing its expected long-run profit with regulated emission-related costs considered. The price uncertainties in electricity, natural gas, and carbon emission, together with policy uncertainty jointly affect profitability and decision-making of the clean technology adoption. A real options approach is applied to investigate the optimal investment decision. The producers are risk avoiding when facing uncertain future policy environment; and this reflects in delaying investment plan and creating a future investment plan that is stubborn to current carbon price. To a risk-neutral price-taking power producer, emission trading is a more effective instrument compared to carbon tax, and shifting from carbon tax to emission permits could more effectively inducing immediate investment in clean technology.
CHAPTER 1. GENERAL INTRODUCTION

1.1 Background

The debate on clean energies continues to attract a significant amount of attention within the academic, managerial and policy making communities. The wide application of clean energies is considered as one of the most effective solutions to enhance national energy security and curb greenhouse gas emissions. In this dissertation, we focus on clean energy technologies for transportation and electricity sectors.

1.1.1 Clean energy in transportation sector

Biofuels are substitutes of fossil fuels. Biofuels are categorized into first generation biofuels (or conventional biofuels) made from sources like starch, sugar, animal fats, and vegetable oil; second generation biofuels (or advanced biofuels) made from non-edible biomass, such as, lignocellulosic biomass, woody crops, agricultural residues or waste; and third generation biofuels mainly extract from oil of algae.

Various government policies have been made to stimulate national biofuel production and consumption. Renewable Fuel Standards (RFS) was proposed by US Environmental Protection Agency (EPA) in 2005. RFS requires that at least 7.5 billion gallons per year (BGY) of renewable fuels be blended with conventional gasoline by the year 2012. The revised RFS (RFS2) in 2007 further promotes advanced biofuels by requiring that at least 21 BGY of advanced biofuels are produced, and out of which at least 16 BGY must come from cellulosic biofuel by 2022 (RFS2, 2014). To encourage farmers’ participation, Biomass Crop Assistance Program (BCAP) provides financial assistance to owners and operators of agricultural and non-industrial private forest land who wish to establish, produce, and deliver biomass feedstock. The Volumetric Ethanol Excise Tax Credit
(VEETC) is the largest subsidy to corn ethanol with a tax credit of 45¢/gallon of ethanol blended with gasoline (VEETC, 2011).

Besides biofuels, electric vehicles and plug-in hybrids using low-carbon electricity and hydrogen may also play a role in the future as vehicle fuels.

1.1.2 Clean energy in electricity sector

Electricity in current power systems is mainly generated from coal-fire generators, which is believed to be among the main culprits for global warming. Current power systems have to be overhauled to create one that is cleaner and more sustainable. Similarly in electricity sector, numerous environmental policies and regulations have emerged and been imposed on the power systems, either restrict the pollutant emissions or require a minimum supply come from renewable resources. Associated programs have been implemented to incentivize clean generator investments.

The carbon tax and cap-and-trade policies are two types of environmental policies that differ in approach, both having the potential to reduce greenhouse gas emissions and promote renewable energy. In both policies, there is a cost associated with emissions created during energy generation. In the carbon tax policy, the emission price is predetermined, but the total actual emissions are determined by the market. In cap-and-trade policy, the total emissions are capped, but the emission price is determined by the market. Renewable Portfolio Standard (RPS), also called renewable electricity standard (RES), is a regulation that requires the energy production from the renewable energy source, such as wind, solar, biomass, and geothermal to be increased. RPS also requires that the electricity supply companies produce a specified fraction of their electricity from renewable energy sources.

The total nameplate capacity projected through 2040 shows that renewables grow fastest, coal use plateaus, natural gas surpasses coal by 2030, and oil maintains its leading share, which shows a great potential of clean energy adoption in the near future.
1.2 Literature Review

1.2.1 Technical, economic, and policy analyses for clean energy investment

Investments in renewable energy technologies were still negligible until the early 2000s, with non-governmental expenditures representing a minor share. Since then, they have recored a substantial growth (BNEF, 2016), given their many environmental, economic and social advantages.

Before any renewable technologies can be utilized in commercial applications, evaluating the environmental, technical and economic viability of these technologies are inevitable.

Life cycle analysis (LCA) is a process to analyze and assess the environmental impacts of a product, process or activity over its whole life cycle (extracting and process raw materials, manufacturing, transportation and distribution, use, re-use, maintenance, recycling and final disposal). LCA identifies and quantifies energy and materials used and wastes released to the environment and assesses the impact of those inputs and outputs searching for environmental improvements (Consoli et al., 1993). Using an LCA methodology, environmental performance indicators, including energy intensity, energy payback time, can be determined for various energy technologies (Huo et al., 2009; Fthenakis and Kim, 2009; Tremeac and Meunier, 2009; Yang et al., 2011).

Techno-economic analysis (TEA) focuses on the technical and economic feasibility analysis. In such analysis, the technical aspects of the project are developed based on experimental results with performance of mass and energy balance. And the economic aspects determine the fixed and variable costs of project investment, as well as fuels production (Swanson et al., 2010). The TEA study has been performed on different renewable energy production pathways (Celik, 2003; Zoulis and Lymberopoulos, 2007; Wright et al., 2010; Swanson et al., 2010; Thilakaratne et al., 2014). However, the TEA method applied either the net present value (NPV) approach or the internal rate of rate (IRR) approach, which is rather primitive and does not account for necessary logistics of the entire supply chain, risk and uncertainty, or the flexibility of decision-making, which we argue are too important to overlook. These shortcomings could be overcome with a real options valuation approach, which will be detailed reviewed in Section 1.2.2.
Besides the aforementioned analyses, research studies have been conducted related to policy impact and policy design. Policy instruments, at this stage, is still playing an essential role in attracting investments in clean energy. These studies aim in energy policy and management, and such articles evaluate energy systems, national or regional, with the purpose of guiding the development and formulation of energy policy (Beach and McCarl, 2010; Chen, 2010; Walls et al., 2011; Kim, 2015; Alizamir et al., 2016). Yet, current policy may be based on untrustworthy analysis when failed to account for certain perspectives in the unpredictable world. Policy is subject to changes, which forms an uncertainty that could affect the profitability and decision-making of investors. The impact of policy risk on investment decision has also been discussed (Blyth et al., 2007; Zhou et al., 2010; Reuter et al., 2012; Iychettire et al., 2017).

1.2.2 Real options valuation

An investment opportunity is analogous to a call option on a common stock. It gives us the right, but not the obligation, to make an investment expenditure (the exercise price of the option), and receive a project (a share of stock) the value of which fluctuates stochastically (Dixit and Pindyck, 1994). While the standard NPV rule takes care of the profitability measure of the project, it overlooks the opportunity cost of the instant investment. When facing an irreversible investment with the ability to invest in the future as an alternative to investing today, opportunity cost should be introduced. Applications of real options can be found in numerous areas, including, manufacturing, inventory, natural resources, research & development, strategic decisions, technology, and stock valuation (Miller and Park, 2005; Li and Rajagopalan, 2008; Feng, 2010).

Real options approach can deal with multiple options related to the project, where the multiple options are connected differently. Smith and Thompson (2008) considered the impact of sequential, yet independent, investment options and active management on the value of a portfolio of real options. The management was applied on an oil exploration project, where a discrete number of related geological prospects are available for drilling. Kwon (2010) modeled a firm’s decision-making when facing deteriorating and highly volatile demand. Possible options include options of
exit the project, a one-time investment option that boosts the project’s profit rate, and exit after the investment is also allowed. Smith and Ulu (2012) presented another work with sequential options, in which the technology adoption can be repeatedly purchase, representing “upgrade” by purchasing new versions of the technology. Maier (2017) studied a portfolio of options - to defer investment, to stage investment, to temporarily halt expansion, to temporarily mothball the operation, and to abandon the project - under conditions of four underlying uncertainties. Exogenous uncertainties, annual operating revenues and their growth rate, and endogenous uncertainties, decision-dependent cost to completion and state-dependent salvage value, were incorporated.

Performance of risk-neutral or risk-averse players can be captured in real options approach through different methods. One common method is to consider risk premium as compensation for the investors. Kouvelis and Tian (2014) studied firm’s interdependent decisions in investing in flexible capacity, capacity allocation to individual products, and the eventual production quantities and pricing in meeting uncertain demand. They applied considered the firm as a risk neutral value maximizer but owned by risk-averse investors, and applied risk premium to make-up for investors. Treville et al. (2014) also implemented risk-averse with a risk-adjusted drift rate, which includes an idiosyncratic-risk premium when developing the optimal production and sourcing choices under evolutionary supply-chain risk. Another often-used way to address risk aversion is applying utility function. Henderson and Hobson (2013) modeled the behavior of a risk-averse agent seeking to maximize expected utility over timing option of selling an indivisible asset with an increasing, concave utility function. They showed that, contrary to intuition, optimal behavior for such a risk-averse agent can include risk-increasing gambles. Kazaz and Webster (2011) studied a firm leasing farm space for fruit supply, and can convert supply to final product, purchase additional supplies from other growers, or sell some (or all) supply in the open market without converting. They have similar findings, that the firm might lease a larger farm space under risk aversion. Kazaz and Webster (2015) found that for price-setting newsvendor problem with uncertain demand of a perishable product, concavity of the objective function is preserved under the introduction of risk
aversion using utility function if the source of uncertainty is demand, yet not necessarily preserved if the source of uncertainty is supply.

Besides risk-preference, value of project lead time has also been studied. Treville et al. (2014) demonstrated the potential value of lead time reduction by modeling lead time as an endogenous decision variable. It is showed that value of lead time reduction depends crucially on the term structure of supply chain risk. Also incremental lead time reduction is often of modest value, while the greatest value comes in reducing lead time enough to permit production to order. Yet, lead time reduction may not always apply in reality.

Recently, as renewable energy became popular, real options approaches have also been utilized in the investments of renewable resources. Such applications involve valuing the real options associated with renewable energy projects and provide optimal investment strategies and/or obtain insights on policy planning. Schmit et al. (2011) investigated the effects of the U.S. ethanol policy on a firm-level investment decision on an infinite time horizon. Maxwell and Davison (2014) extended the analysis of Schmit et al. (2011) by considering entry into the project within a finite time horizon and looking into the optimal operation once the project is initiated. Lin and Thome (2014); Lin and Yi (2014a,b) employed a structural econometric model of a dynamic game to analyze the decision to invest in ethanol plants of corn or various feedstocks in the U.S., Canada and Europe, respectively. Furthermore, Yi et al. (2014) analyzed the effects of government policy on entry, exit, investment, and production decisions of ethanol producers in the U.S.

Bockman et al. (2008) used a real options approach to value continuous investment of small hydropower plants subject to uncertain prices. Lee and Shih (2010) evaluated the renewable energy policy for developing renewable energy in Taiwan using the concept of policy return on investment (PROI). With the analysis, it is found that a higher feed-in tariff policy does not necessarily impact PROI and policy benefit positively, and internalizing external costs does not necessarily impact PROI negatively. Chen and Tseng (2011) compared two policies, tax and cap-and-trade, to see which policy would induce an earlier adoption of clean technology. Under each policy, the investment timing of clean technology was determined using a real-options approach. Boomsma et
al. (2012) proposed a real options approach to investigate investment timing and capacity choice of a renewable energy project under different support schemes including feed-in tariffs and renewable energy certificate trading. Zhou et al. (2010) presented a real options model incorporating policy uncertainty described by carbon price scenarios (including stochasticity), allowing for possible technological change. The model was used to determine the best strategy for investing in CCS in an uncertain environment in China. Reuter et al. (2012) used a real options model in discrete time with lumpy multiple investments to analyze the decisions of an electricity producer to invest into new power generating capacity. The framework was used to analyze energy policy, as well as the reaction of producers to uncertainty in the political and regulatory framework.

1.3 Dissertation Structure

The remaining of the dissertation is organized as follows. Chapter 2 presents a real options valuation of an advanced biofuel technology - fast pyrolysis, upgrading and refining of cellulosic biomass - under fuel price uncertainty. A case study on Iowa is conducted to discuss the optimal investment timing of this new technology for producing cellulosic biofuels from corn stover. A real options approach is proposed to valuate the project subject to construction lead time and uncertain fuel price. The profitability of the project, optimal investment timing and the impact of project lead time are investigated.

Chapter 3 continues the previous discussion and provide the decision maker the flexibility of investing in a land to grow his own feedstock, in addition to the investment of biofuel facility. This new option mitigates the supply risk from single feedstock (corn stover), adds value to the advanced biofuel production, yet has a volatile investment cost. A real options approach is used to analyze the optimal investment timing and benefits of the dual sourcing. The affect of other critical factors (e.g., risk aversion and lead times) on investment decisions are also investigated.

In Chapter 4, we extend the uncertain to the scope of government policy, in addition to considering the critical uncertainties in production and market. In this work, we consider a capacity investment faced by a producer who owns an existing coal plant. The producer considers to adopt
clean generation with natural gas in the future in order to meet the load obligation, as well as to maximize expected long-run profit. Uncertain factors considered include price uncertainties (including, electricity price, natural gas price, and carbon price), and policy uncertainty. A real options approach is conducted on a per MW analysis to investigate producer’s behavior when facing potential policy changes, and provide insights for policymakers in policy selection and design.

Chapter 5 concludes the dissertation with a summary of research findings and contributions.
CHAPTER 2. IS NOW A GOOD TIME FOR IOWA TO INVEST IN CELLULOSIC BIOFUELS?: A REAL OPTIONS APPROACH CONSIDERING CONSTRUCTION LEAD TIMES

The revised Renewable Fuel Standard of the U.S. mandates a production of 16 billion gallons per year by 2022 from cellulosic biofuels. Iowa, rich in agricultural residues like corn stover, is a major player in contributing to the fulfillment of the cellulosic biofuels mandate. Is it a good time for Iowa to start investing in cellulosic biofuels? Using a fast pyrolysis facility as an example, we present a real options approach for valuing the investment of a new technology for producing cellulosic biofuels subject to construction lead times and uncertain fuel price. We conduct a case study on Iowa, in which the decision maker finds the optimal investment time for the fast pyrolysis facility subject to production and distribution constraints. Our result indicates that the project is profitable if the facility is invested now; but could be more profitable if invested later. Namely, now is not the optimal time for Iowa to start constructing the fast pyrolysis facility. We also find that the impact of the lead time on the project value is too significant to overlook. The profitability of the project is sensitive to the outlook of fuel price. If the future retail fuel price drops just 7% lower than forecasted by the Energy Information Administration, the investment may not be profitable. The impact of the technology improvement (production yield) and biomass feedstock price is also analyzed using regression.

2.1 Introduction

The many eco-benefits of replacing petroleum fuels with biofuels have attracted global attention. For example, biofuels are considered environmentally sustainable for reducing greenhouse gas emissions and air pollution, compared to fossil fuels. The production of biofuels could also assist
in regional economic development, especially in rural areas. Furthermore, biofuels could improve energy security through diverse energy sources. Because of these reasons, biofuels have become increasingly popular worldwide, especially in the transportation sector.

Currently, ethanol and biodiesel are the two main types of biofuels used in transportation. Ethanol is normally blended with gasoline, while biodiesel is used as a substitute for diesel. The fact that these biofuels (e.g., sugar/starch-based ethanol and vegetable oil/soy-based biodiesel) are made by the commodities that can also be used for food limits their capability of substituting petroleum fuels. The production of these traditional biofuels is in competition with the food industry, and the increasing production level has provoked debates about the impact on the food market. In recent years, more focus has turned to developing potential techniques to produce advanced biofuels from non-edible feedstock.

The Energy Independence and Security Act of 2007 (EISA 2007) revised the 2005 Renewable Fuel Standard (RFS) by emphasizing new categories of renewable fuels with greenhouse gas thresholds (EPA, 2010). The revised RFS (RFS2) mandates a production of 36 billion gallons per year (BGY) by 2022, from cellulosic biofuels - fuels produced from cellulosic materials, including dedicated energy crops, forestry residues, agricultural residues and urban sources of waste - accomplishing 16 BGY (EPA, 2010). According to the U.S. Energy Information Administration (EIA), renewable sources lead in a rise of primary energy consumption. The renewable share of total energy use is expected to increase from 8% in 2008 to 14% in 2035, in response to renewable fuels policies, including EISA 2007 RFS, tax credits for renewable electricity, and RPS programs. In the transportation sector, biofuels account for more than 80 percent of the growth in liquid fuel consumption (EIA, 2010). Unlike conventional biofuels like corn ethanol, cellulosic biofuels are facing difficulties in meeting the requirements of the revised fuel consumption mandates. The U.S. Environmental Protection Agency (EPA) reduced the mandate dramatically from 2010 to 2013 due to the lack of sufficient production capacity of cellulosic biofuels (CRS, 2013).

Iowa is an agricultural state where biofuels, especially corn ethanol and soy biodiesel, are mostly produced using traditional agricultural crops. In addition to federal support and regulations on
biofuel usage, Iowa state government also provided financial incentives for the production and usage of biofuels, including a biodiesel blend retailer tax credit, a biodiesel producer tax refund, alternative fuel production tax credits, and biofuel infrastructure grants. On the other hand, Iowa is also rich in agricultural residues like corn stover (as shown in Figure 2.1), which can contribute to the fulfillment of the RFS2 cellulosic biofuels mandate if the residues can be utilized with productive processes.

![Figure 2.1 Corn stover distribution in Iowa](image)

To increase the production level of cellulosic biofuels to meet the RFS2 mandates in future years, various studies on different cellulosic biofuel pathways, producing cellulosic ethanol, and biomass-to-liquid fuels, have been conducted. In general, cellulosic biofuels can be made through hydrolysis and fermentation, gasification and further conversion, or liquefaction with further upgrading of cellulose materials. So far these advanced biofuel processing approaches face excessive costs due to low energy density of solid biomass, and highly dispersion of biomass with rather low annual yields.
Fast pyrolysis (or flash pyrolysis), is a thermochemical process that heats compact solid biomass in the absence of oxygen at a temperature between 450 and 500 degrees Celsius, followed by a very rapid cooling and condensing of produced vapor to generate high yields of pyrolysis oil with roughly half the heating value of fossil fuels. This process can serve as a pre-treatment step, creating a uniform, liquid intermediate product with significant increase in energy density. Pyrolysis bio-oil could be produced at a practical scale, matching the amount of biomass collection within reasonable costs. The intermediate product from fast pyrolysis could then be stored and shipped cost-effectively to a central site for further conversion to desirable final products, such as industrial heat, electricity, chemicals, and transportation fuels. Details of fast pyrolysis technology are available in literature (Bridgewater, A. V., 1999, 2002, 2005; Piskorz et al., 1988; Babu, B. V., 2008; Wright et al., 2010; Brown et al., 2013).

Currently, fast pyrolysis technology is starting to gain technology efficiency and going commercial scale. Some companies in the U.S. and Canada are attempting to commercialize the technology and hope to convert biomass (such as wood residues) to transportation fuels with a high yield.

Before any biofuels technologies can be utilized in commercial applications, evaluating the economic feasibility is inevitable. One typical and often adopted method is the techno-economic analysis (TEA). In such analysis, the technical aspects of the project are developed based on experimental results with performance of mass and energy balance. And the economic aspects determine the fixed and variable costs of project investment, as well as fuels production (Swanson et al., 2010). The TEA study on fast pyrolysis and hydroprocessing technology suggests this pathway to be economically feasible, attaining a minimum fuel selling price (MFSP) of $2.57/gal gasoline-equivalent (gge) (Brown et al., 2013). However, the TEA method is rather primitive and does not account for risk and uncertainty or the flexibility of decision-making, which we argue are too important to overlook.

In this work, we use a real options approach to value the irreversible investment of a pyrolysis facility in Iowa subject to a construction lead time and uncertain fuel price. Real options valuation is known to be able to capture the value of flexibility that arises in the decision-making and
operational processes. It has been used in valuing a wide range of investments in natural resources, real estate, R&D and manufacturing, etc. Different types of real options, such as options to defer, abandon, and alter operating scale, are discussed extensively in literature (Cortazar et al., 1998; Huchzermeier and Loch, 2001; Bengtsson and Olhager, 2002; Dimakopoulou et al., 2014).

Recently, as renewable energy became popular, real options approaches have also been utilized in the investments of renewable resources. Such applications involve valuing the real options associated with investment strategies and/or obtain insights on policy planning. Bockman et al. (2008) used a real options approach to value continuous investment of small hydropower plants subject to uncertain prices. Lee and Shih (2010) evaluated the renewable energy policy for developing renewable energy in Taiwan using the concept of policy return on investment (PROI). With the analysis, it is found that a higher feed-in tariff policy does not necessarily impact PROI and policy benefit positively, and internalizing external costs does not necessarily impact PROI negatively. Chen and Tseng (2011) compared two policies, tax and cap-and-trade, to see which policy would induce an earlier adoption of clean technology. Under each policy, the investment timing of clean technology was determined using a real-options approach. Boomsma et al. (2012) proposed a real options approach to investigate investment timing and capacity choice of a renewable energy project under different support schemes including feed-in tariffs and renewable energy certificate trading. Cook and Lin (2014) used a dynamic structural econometric model to examine shutdown and upgrade decisions of wind turbine owners in Denmark.

Bioethanol plant investment has also been valued using real options. Schmit et al. (2011) investigated the effects of the U.S. ethanol policy on a firm-level investment decision on an infinite time horizon. Maxwell and Davison (2014) extended the analysis of Schmit et al. (2011) by considering entry into the project within a finite time horizon and looking into the optimal operation once the project is initiated. Lin and Thome (2014); Lin and Yi (2014a,b) employed a structural econometric model of a dynamic game to analyze the decision to invest in ethanol plants of corn or various feedstocks in the U.S., Canada and Europe, respectively. Furthermore, Yi et al. (2014) analyzed the effects of government policy on entry, exit, investment, and production decisions of
ethanol producers in the U.S. These articles focus on bioethanol production. The technology for producing bioethanol is rather mature with data on plant investment, operations and productions available. In this work we value a cellulosic biofuel facility that will produce liquid transportation fuels. In our case, the production technology is still in an infant stage of commercialization, with very limited plant data available.

In this work, we present a risk-neutral valuation method for the investment of a new technology for producing cellulosic biofuels. A real options approach is used to value the investment subject to a construction lead time and uncertain fuel price. We conduct a case study on Iowa, in which the decision maker (DM), finds the optimal investment time for the fast pyrolysis facility subject to production and distribution constraints. Furthermore, we use regression analysis to study the impact of the technology improvement (production yield) and biomass feedstock price.

The contributions of this work are twofold. In terms of the application, we are the first one to use real options to value a pyrolysis plant, which is an emerging technology for producing advanced biofuels. Our analysis sheds light on the profit and risk of the investment of cellulosic bioenergy production, which has an impact on the sustainable future of the nation’s renewable energy development. In terms of the methodology, we incorporate the operational constraints that impact the valuation and are constantly overlooked by other researchers. These operational constraints include the construction lead time and the production and distribution constraints. As we will demonstrate in this work, overlooking these constraints, especially the lead time, will lead to a significant overvaluation of the asset, which should be avoided in evaluating an investment decision.

The remainder of the chapter is structured as follows. Section 2.2 models the fast pyrolysis facility investment as a multi-stage stochastic programming problem. We present the solution procedure in Section 2.3. We discuss the results from the case study in Section 2.4. Section 2.5 concludes the work.
2.2 Problem Formulation

In this work, the fast pyrolysis facility owner purchases cellulosic biomass from farmers and converts it to hydroprocess bio-oil at the facility with fast pyrolysis and simple hydrotreating. The hydroprocessed bio-oil is then shipped to an existing biorefinery. After further conversion, including hydrocracking and refining, the hydroprocessed bio-oil is converted to final products, such as liquid transportation fuels (gasoline and diesel). For simplicity, we consider one transportation fuel (gasoline) as the output of the pyrolysis process in this work. The facility owner ships and sells the transportation fuel to distributed customers. Besides the transportation fuel, the fast pyrolysis process also produces by-products (primarily fuel gas). Because the revenues from the by-products are relatively small compared with the revenue from the transportation fuel, we do not consider the by-products in this work. The supply chain, from feedstock input to fuel output at customers, is depicted in Figure 2.2.

![Figure 2.2 Biofuel production from fast pyrolysis and further conversion](image)

Assume that the DM is considering building a new fast pyrolysis facility at a pre-determined location in Iowa. Our focus is to value the project considering the optimal investment timing. First we introduce the notation to be used in the model.

**Notation**

\[ t \] \quad \text{index for time (month)}
\( i \) index for gasoline demand locations \( (i = 1, 2, \ldots, N) \)

\( L \) construction lead time for the facility (month)

\( \tau \) unknown future investment time (month)

\( T \) the length of the planning horizon (month)

\( \hat{T} \) the length of the decision period (month)

\( I_t \) capital cost of a fast pyrolysis facility at \( t \) ($)

\( \theta_1 \) conversion rate from dry basis biomass to hydroprocessed bio-oil

\( \theta_2 \) conversion rate from hydroprocessed bio-oil to transportation fuels

\( Q_t \) decision variable for biomass operating level (metric ton)

\( Q^\text{max} \) maximum biomass operating capacity (metric ton)

\( q_{it} \) quantity of fuel output shipped from biorefinery to demand location \( i \) at time \( t \) (metric ton)

\( d_{it} \) fuel demand at location \( i \) at time \( t \) (metric ton)

\( P_t \) fuel market price at time \( t \) ($/gallon)

\( H_t \) threshold price for fuel at time \( t \) ($/gallon)

\( C^B \) biomass feedstock price ($/dry metric ton)

\( C^O \) unit operating cost ($/gallon)

\( C^H \) shipping cost for hydroprocessed bio-oil ($/metric ton-mile)

\( C^F \) shipping cost for fuel ($/metric ton-mile)

\( C^T \) fuel tax ($/gallon)

\( l \) shipping distance from fast pyrolysis facility to biorefinery (mile)
The major underlying uncertainty of this project valuation is the fuel price $P_t$. The other uncertainties, such as production technology (biofuel yield) and biomass feedstock price, although also important to the valuation, are not as volatile as the fuel price. Because the biomass supply chain is still being developed, there are limited data for them. Therefore, we focus on the major uncertainty of the fuel price and treat the other uncertainties as parameters rather than stochastic variables. Their effects on the investment decision and timing will be studied using regression analysis in the case study. Assume that $P_t$ evolves according to the following stochastic process.

$$dP_t = \mu(P_t, t)dt + \sigma dB_t,$$

where $B_t$ is a Wiener process, $\mu$ is the drift function, and $\sigma$ is the volatility. The cash flows associated with the biofuel production at time $t$ include revenue from fuel sales $R_t(Q_t; P_t)$ and operating costs incurred $C_t(Q_t, q_t; P_t)$, where $q_t \equiv (q_{1t}, q_{2t}, \ldots, q_{Nt})$ is a vector of the quantities of fuel shipped to all demand locations at time $t$. Let $\pi_t(\cdot)$ denote the total profit at time $t$. We have

$$\pi_t(Q_t, q_t; P_t) = R_t(Q_t; P_t) - C_t(Q_t, q_t; P_t).$$

The revenue $R_t(\cdot)$ can be represented by

$$R_t(Q_t; P_t) = P_t \theta_1 \theta_2 Q_t,$$

where $\theta_1 \theta_2 Q_t$ is the output quantity of the biofuel production process, as illustrated in Figure 2.3.

Figure 2.3 also shows the cost components incurred in different stages of the production process, including biomass purchase cost, facility operating cost, transportation fees of hydropyrolysis bio-oil and gasoline, and fuel tax. In the valuation, we focus on the investment of the fast pyrolysis facility, not including the biorefinery plant because the refinery technology is mature and its production process is very predictable. Note that the facility operating cost considers conversion costs at both the fast pyrolysis facility and biorefinery. The sum of all these cost components is denoted by $C_t(\cdot)$ as follows.

$$C_t(Q_t, q_t; P_t) = C_B Q_t + C^H \theta_1 Q_t l + C^O \theta_1 \theta_2 Q_t + \sum_{i=1}^N C^F q_{it} \ell_i + C^T \theta_1 \theta_2 Q_t.$$
In (2.4), the facility operating level \((Q_t)\) and the product distribution quantities \((q_{it})\) are decision variables, subject to the following operating constraints:

\[
Q_t \leq Q_{\text{max}} \quad (2.5)
\]
\[
q_{it} \leq d_{it}, \forall i \quad (2.6)
\]
\[
\sum_{i=1}^{N} q_{it} = \theta_1 \theta_2 Q_t \quad (2.7)
\]
\[
Q_t, q_{it} \geq 0, \forall i, t \quad (2.8)
\]

Equation (2.5) shows the plant capacity limitation. Equation (2.6) implies that the amount of fuel products delivered to a location does not exceed the demand level, and (2.7) is the conversion balance constraint.

At each time \(t\), the facility finds its optimal operating decisions \((Q_t^*, q_{t}^*)\) based on the fuel price \(P_t\) at time \(t\) by solving:

\[
(Q_t^*, q_{t}^*) = \arg \max_{Q_t, q_{t}} \{ \pi_t(Q_t, q_{t}; P_t) : (2.5)-(2.8) \} \quad (2.9)
\]

Let the project value for investing in the fast pyrolysis facility be denoted by \(F(P_0)\), where \(P_0\) is the current fuel price. We want to maximize its expected net present value (NPV) as follows:

\[
F(P_0) = \max_{\tau \in [0, T]} E \left[ -I_{\tau} e^{-r\tau} + \int_{\tau+L}^{T} e^{-r\tau} \pi_s(Q_s^*, q_{s}^*; P_s) ds \right]^+, \quad (2.10)
\]
where $E$ is the expectation operator, $r$ is the discount rate, $\tau$ is the future investment time, and we use the notation $x^+ \equiv \max(x, 0)$. In (2.10) $I_\tau$ is the equivalent (one-time) capital cost of the pyrolysis facility incurred at time $\tau$, which is equal to the worth of all the (monthly) debt repayments at time $\tau$. In (2.10) we maximize the expected NPV of the total profits (made through the operations of the pyrolysis facility) over a fixed planning horizon $[0, T]$ (e.g., $T=20$ years). Equation (2.10) also describes that the facility requires $L$ time periods to become operational. This lead time $L$ may include the time required for construction, environmental evaluation, and permitting.

Figure 2.4 shows the cash flows associated with the investment. This investment problem is somewhat similar to valuing an American call option on a dividend-paying stock at a prespecified price. In this context, $\hat{T}$ represents the expiration date of this call option. By exercising the call option, the DM buys the stock, which corresponds to constructing the facility. The dividends correspond to the profits $\pi_t$ generated by the facility. However, there are two major differences from valuing a call option in reality. First, there is a delay of $L$ time periods between the time that the option is exercised and the time that the ownership of the stock is successfully transferred to the option holder. Second, the stock loses its value after $T$. The reasons for the design of having a finite $T$ and $\hat{T}$ are further elaborated in the next paragraph. Our problem is to determine the optimal timing for exercising the above call option.
As shown in Figure 2.4, the investment time \( \tau \) is only considered over a decision period \([0, \hat{T}]\), where \( \hat{T} \) is chosen to be much smaller than the fixed planning horizon \( T \). We choose \( T = 20 \) years and \( \hat{T} = 5 \) years in our case study. This design has practical reasons. First, if \( L \) is the life of the facility, the life cycle of the facility starts at \( \tau \) and ends at \( \tau + L + \mathcal{L} \). This duration of life varies with \( \tau \), which complicates the analysis. To simplify, we focus on all the cash flows occurring within \([0, T]\) (20 years), regardless of \( \tau \). That means the cash flows occurring beyond \( T \) are truncated. Second, since we are limiting the value of \( T \), the option for investing in the facility should not be long-lived. Furthermore, by limiting the decision period to be less than 5 years, we also ensure that the present value of the truncated cash flows that occur beyond \( T \) is not significant.

Furthermore, investing in a facility is a big decision. Instead of assuming that the investment decision is evaluated continuously over time, it is more reasonable to assume that the investment decision is evaluated periodically (e.g., at the beginning of each month) until it is made. That is, we assume the investment time \( \tau \) is an integer over \([0, \hat{T}]\). It is also possible that the project may not be profitable and, therefore, the facility may not be built and the project value is zero. This possibility is also accounted for in the formulation.

### 2.3 Optimal Investment Timing

The investment timing \( \tau \) in (2.10) is a random variable if the facility is not built immediately at time 0. Therefore, if \( \tau > 0 \), the (future) investment decision simply depends on the fuel price \( P_\tau \) at \( \tau \), which is uncertain. Intuitively, at each time \( t \) there is a threshold price, denoted by \( H_t \), such that building the facility is optimal if \( P_t \geq H_t \); otherwise, it is better off to wait.

In general, a threshold price is not the break-even price at which the project value is zero. It is well known in literature that waiting has value (Dixit and Pindyck, 1994). In this case, the effects of delaying the investment of the pyrolysis plant are twofold. First, it delays the disbursement of the capital, which earns interest. Second, it also delays the time that the facility starts to collect revenues. The future revenues depend on the evolution of fuel price \( P_t \). To make it more complicated, the future costs may also increase (e.g., labor or materials costs) if the project is
delayed. However, for a new technology, such as the biofuel pyrolysis plant considered in this work, delaying the investment may allow the DM to adopt a newer technology (e.g., with a higher yield) later. Therefore, even if building the facility immediately can yield a positive project value, it may not be immediately clear whether delaying the project would be better off. In this work, we do consider that the capital cost \( I_t \) may change over time. While we do not consider the evolution of the new technology, we will later use regression analysis to estimate the effect of the pyrolysis yield to the project value. Next we discuss our approach for solving (2.10).

**The solution procedure**

To solve (2.10), the key is to determine the threshold prices at all possible decision-making points. Our approach is to approximate the continuous process of \( P_t \) in (2.1) by a discrete price lattice (such as a binomial or trinomial lattice). We use a trinomial lattice as it is able to handle the processes with a drift function that is state dependent (e.g., a mean-reverting process whose drift \( \mu \) is a linear function of \( P_t \)).

To approximate a continuous process using a discrete lattice, we use a small step size \( \Delta t \), for building the lattice, noting that an investment decision can only be made at certain time periods (e.g., corresponding to the beginning of each month before \( T \)). Then using backward stochastic dynamic programming (SDP) steps, we can obtain the project value at time 0.

After the entire lattice has been evaluated (from \( T \) to 0 backward), for each time \( t \) where an investment decision is available, we collect the information associated with each lattice node \( (j = 1, \ldots, J) \): \( \{(P_t^j, G_t^j(P_t^j), F_t^j(P_t^j))\}_{j=1}^J \), where \( P_t^j \) is the price of node \( j \); and \( G_t^j(P_t^j) \) is the corresponding value-to-go at node \( j \) representing the cumulative project value from \( t \) to \( T \) when the fuel price at node \( j \) is \( P_t^j \) assuming that the facility has been in place. That is,

\[
G_t^j(P_t^j) = E \left[ \sum_{s=t+L}^{T} e^{-r(s-t)} \pi_s(Q^*_s, q^*_s; P_s) \right]. \tag{2.11}
\]
We use $F^j_t(P^j_t)$ to represent the cumulative project value at node $j$ from $t$ to $T$ when the fuel price at $t$ is $P^j_t$ as follows.

\[
F^j_t(P^j_t) = \begin{cases} 
\max \left\{ G^j_t(P^j_t) - I_t, e^{-r}E[F_{t+1}(P_{t+1})] \right\} & t < \hat{T} \\
\max \{ G^j_t(P^j_t) - I_t, 0 \} & t = \hat{T}
\end{cases}
\]  

(2.12)

It can be seen in (2.12) that the DM faces two alternatives at each time $t$: to invest immediately or to wait. Consider the difference of the values associated with these two alternatives, and denote it by $D^j_t(P^j_t)$ as follows.

\[
D^j_t(P^j_t) = \begin{cases} 
G^j_t(P^j_t) - I_t - e^{-r}E[F_{t+1}(P_{t+1})] & t < \hat{T} \\
G^j_t(P^j_t) - I_t & t = \hat{T}
\end{cases}
\]  

(2.13)

We then regress $\{D^j_t(P^j_t)\}_{j=1}^J$ on $\{P^j_t\}_{j=1}^J$ to obtain a functional relationship $\tilde{D}_t(P_t)$. The threshold price $H_t$ is where $\tilde{D}_t(\cdot)$ intersects with the price axis. That is, $\tilde{D}_t(H_t) = 0$. This process is illustrated in Figure 2.5, from which it becomes clear that when $P_t \geq H_t$ at time $t$, it is optimal to build the facility immediately; otherwise it is better off to wait.

![Figure 2.5 Using regression to obtain the threshold price $H_t$ at time $t$](image)

In the last step we use Monte Carlo simulations to determine the distribution of the optimal investment time $\tau$. Given the threshold prices obtained $H_t$, $t = 1, \cdots, \hat{T}$, one can generate sample price paths $\{P_t(k) | P_0(k) = P_0, k = 1, \cdots, K\}$ that approximate (2.1), where the superscript $k$ is the
index of simulation runs. In the \( k \)-th simulation run, the corresponding investment time, denoted by \( \tau^{(k)} \), is the first time when the simulated fuel price is greater than the threshold price. That is,

\[
\tau^{(k)} = \min\{t|P_t^{(k)} \geq H, t = 0, 1, 2, \ldots, \hat{T}\}.
\]

Then we can estimate the (discrete) distribution of the investment time \( \tau \). The expected optimal investment time (or delay) is estimated by

\[
E[\tau] = \frac{\sum_{k=1}^{K} \tau^{(k)}}{K}.
\]

2.4 Case Study

In this section we present a case study to demonstrate the proposed valuation approach. Although the case is fictitious, all parameter values used here are obtained from literature and are believed to be close to those in reality.

2.4.1 Modeling price uncertainty

The general form of the fuel price is described in (2.1). In this case study, we use the following geometric mean-reverting (GMR) process to capture the evolution of the fuel (gasoline) price.

\[
d\ln P_t = \lambda_t (m_t - \ln P_t) dt + \sigma_t dB_t
\]

where \( \lambda_t \) is the reverting coefficient, \( m_t \) is the mean level of gasoline price at time \( t \), \( \sigma_t \) is the volatility, and \( B_t \) is a Weiner process.

Cellulosic biofuels production is still in an early stage of commercialization. There are currently no derivative markets trading cellulosic biofuels. To estimate the process of the fuel price, we turn to the gasoline retail prices published by the U.S. EIA.

Using the historical data of monthly retail gasoline prices of the Midwest region, obtained from the EIA (EIA, 2016a), we estimate the values of the parameters, \( \lambda_t \), \( m_t \), and \( \sigma_t \) in (2.16). The historical price data are depicted as the solid line, ranging from 1994 to 2014, in Figure 2.6. On the other hand, EIA has also provided its forecast on the annual retail price up to 2040 (EIA, 2016b),
shown in Figure 2.6 as the dashed line. The price parameter $m_t$ is estimated with a polynomial regression over historical and predicted gasoline retail prices from 1994 to 2040 to capture the fuel price trend, and the fitted values of $m_t$ as shown in Figure 2.6 with the dotted line. The values of $m_t$ for the next 20 years are given in Table 2.1. Note that in determining the trend of the mean price $m_t$, there is no specific monthly or seasonal patterns observed. As a result, we have one mean value $m_t$ for each year in Table 2.1.

The values of $\lambda_t$ and $\sigma_t$ are estimated based on a fitted $m_t$ using the method of maximum likelihood (Tseng and Barz, 2002). Since there are no sufficient data to support a monthly or annual $\lambda_t$ and $\sigma_t$, we have a constant $\lambda$ and $\sigma$ for the entire planning horizon. The fitted values of all price parameters are listed in Table 2.1.

![Graph showing mean price estimation with historical, EIA forecast, and fitted lines.](image)

**Figure 2.6 Mean price estimation**

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_t = 0.0700$</th>
<th>$\sigma_t = 0.0781$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>2015</td>
<td>2016</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.09</td>
<td>1.11</td>
</tr>
<tr>
<td>Year</td>
<td>2017</td>
<td>2018</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.13</td>
<td>1.15</td>
</tr>
<tr>
<td>Year</td>
<td>2019</td>
<td>2020</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.16</td>
<td>1.18</td>
</tr>
<tr>
<td>Year</td>
<td>2021</td>
<td>2022</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.19</td>
<td>1.20</td>
</tr>
<tr>
<td>Year</td>
<td>2023</td>
<td>2024</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.21</td>
<td>1.22</td>
</tr>
<tr>
<td>Year</td>
<td>2025</td>
<td>2026</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.23</td>
<td>1.24</td>
</tr>
<tr>
<td>Year</td>
<td>2027</td>
<td>2028</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.24</td>
<td>1.25</td>
</tr>
<tr>
<td>Year</td>
<td>2029</td>
<td>2030</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Year</td>
<td>2031</td>
<td>2032</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Year</td>
<td>2033</td>
<td>2034</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.26</td>
<td>1.26</td>
</tr>
</tbody>
</table>

**Table 2.1 Values of GMR process parameters**
Under the risk-neutral valuation paradigm, we set the discount rate \( r \) to the risk-free rate \( r_f \) in (2.10)-(2.13). The price process (2.16) is also risk-neutralized by subtracting the normalized risk-premium from the mean level of the gasoline price. The risk-neutral price process then has a drift function
\[
\lambda_t(m_t - (r_a - r_f)/\lambda_t - \ln P_t),
\]
where \( r_a \) is the risk-adjusted discount rate (Schwartz and Smith, 2000).

Given the risk-neutral GMR process for the fuel price, we use the trinomial price lattice proposed in Tseng and Lin (2007) to approximate this process. Similar to the lattice developed by Hull and White (1993), the lattice nodes in our implementation are predetermined and fixed. Each lattice node maps to three adjacent nodes in the next time period following certain branching patterns. The branching probabilities are then determined such that the mean and the variance of the price change are matched. This method is straightforward in handling processes with general drift functions, including our case where the mean level \( m_t \) is time dependent. A brief review of the trinomial lattice is given in the Appendix A.

Since our historical data are monthly prices, the parameters are estimated on a monthly basis. In our implementation, we use a time step size of the lattice being \( \Delta t = 1/30 \), corresponding to each day, to better approximate the continuous price process.

### 2.4.2 Data sources

The candidate location for the fast pyrolysis facility is assumed to be at the center of Story County, Iowa. To fulfill the volume requirement of cellulosic biofuels in RFS2, large-scale productions of biofuels would be the future trend. Iowa, as a state rich in biomass supply, will play an important role. Thus, a local biorefinery is assumed to be constructed for economic efficiency, compared with outsourcing the refining process to out-of-state refineries. The location of the refinery is considered to be the center of Kossuth County, which maximizes the long-term profit of the whole biofuel production in Iowa (Li et al., 2014). Overall, these two pyrolysis and refinery facilities will provide gasoline to the businesses and residents in Iowa.
The gasoline demand in Iowa is considered of a metropolitan statistical areas (MSAs) level, while the demand location is modeled at all MSA centers. The demand (quantity) is assumed to be proportional to the population. The total gasoline demand of Iowa is obtained from EIA, using 2011 state-level gasoline consumption statistics (EIA, 2013), and the population of Iowa MSAs is from U.S. Census Bureau 2013 (US Census Bureau, 2013).

Since Iowa is an agricultural state rich in corn production, the residues, corn stover is considered to be the feedstock of the plant. Assume the facility purchases corn stover from local farmers at $58.5/dry metric ton, which includes delivery cost (OEERE, 2011).

The technology data are based on the revised TEA of a fast pyrolysis plant with bio-oil upgrading using 2,000 metric tons per day (MT/day) of corn stover feedstock (Brown et al., 2013). The main products after fast pyrolysis, bio-oil upgrading and refining are naphtha-range and diesel-range fuels, which can be used in the transportation sector. The fuel amount is measured by gasoline gallon equivalent. The unit operating cost \( C^O \) of liquid fuel for such a facility is $1.091/gallon excluding feedstock cost. The bio-oil conversion rate \( \theta_1 \) from dry basis corn stover feedstock is 0.63, and the fuel conversion rate \( \theta_2 \) from bio-oil is 0.414. Capital cost \( I_t \) is estimated to be $429 million that increases by an inflation rate of 1.5% per year (BLS, 2017). A tax credit from facility depreciation is also included.

The transportation method chosen for hydrotreated bio-oil and gasoline is by truck. The variable transportation cost \( C^H \) and \( C^F \) is assumed to be $0.26/ton-mile, the national average truck shipping cost (BTS, 2012). The shipping distances are estimated by great circle distances, which are the shortest distance between any two locations on a sphere, modified by a circuity factor. The circuity factor for truck transportation mode is 1.22 (CBO, 1982).

When estimating the profit from fuel production, the excise tax imposed on the fuel sale must be included. The fuel tax of gasoline is assumed to be $0.404/gallon, which is the fuel tax rate in Iowa in 2013 (API, 2017).

We set the risk-free interest rate to be 5% because the average long-term U.S. treasury yield over the last 20 years (from 1995 to 2014) is approximately 5% (US Treasury, 2014).
discount rate reflecting the DM’s risk preference is set to be \( r_a = 10\% \). We also assume the project is financed by a commercial loan with a loan rate of 7.5\%, which is consistent with that used in (Brown et al., 2013).

A summary of major model parameter values is listed in Table 2.2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>0.63</td>
<td>Brown et al. (2013)</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>0.414</td>
<td>Brown et al. (2013)</td>
</tr>
<tr>
<td>( I_t )</td>
<td>$429 million</td>
<td>Brown et al. (2013)</td>
</tr>
<tr>
<td>( C^B )</td>
<td>$58.5/dry metric ton</td>
<td>OEERE (2011)</td>
</tr>
<tr>
<td>( C^O )</td>
<td>$1.091/gallon</td>
<td>Brown et al. (2013)</td>
</tr>
<tr>
<td>( C^H )</td>
<td>$0.26/ton-mile</td>
<td>BTS (2012)</td>
</tr>
<tr>
<td>( C^F )</td>
<td>$0.26/ton-mile</td>
<td>BTS (2012)</td>
</tr>
<tr>
<td>( C^T )</td>
<td>$0.404/gallon</td>
<td>API (2017)</td>
</tr>
</tbody>
</table>

2.4.3 The baseline case

The baseline case uses all the parameter values given in Sections 2.4.1 and 2.4.2. Furthermore, we assume the planning horizon is \( T = 20 \) years, and the investment decision is only considered in the beginning of each month in the first 5 years (i.e., \( \hat{T} = 5 \) years). First we evaluate the investment project considering a lead time \( L = 12 \) months. The threshold price is \( H_0 = $4.48/gallon \) at \( t = 0 \). Assume that the initial fuel price is \( P_0 = $3.0/gallon \) (the mean price level \( m_t \) of 2015), the expected project value is $140.7 million (net present value). Since the initial condition \( P_0 < H_0 \), the optimal strategy is to invest later, and the expected waiting time is about \( E[\tau] = 42.0 \) months. We also evaluate the project value without the delay option (i.e. invest immediately), the expected NPV is $120.4 million. Thus, it is optimal to wait rather than to invest immediately. There are two main reasons. First, deferring the project can lower the investment costs, but also delay the revenue flows. When the gain from waiting exceeds the cost of the foregone revenues from delaying the investment, it is better to wait. Second, the mean level of the retail gasoline price \( P_t \) in (2.16) is
increasing over time based on the EIA forecast in Figure 2.6, which implies a potential growth in the investment value. This growth creates a value to waiting.

In Figure 2.7, the threshold prices $H_t$ for the first five years are shown. It can be seen that the threshold price decreases over time, indicating that the optimal exercise condition (for building the pyrolysis facility) becomes easier to meet over time. Also considering the increasing trend of the fuel price forecasted in Figure 2.6, apparently there is an optimal time ahead to invest in the pyrolysis facility when the increasing gasoline price meets the declining threshold price. Since the investment expenditures are at least partially irreversible, by deferring the investment the DM retains the right to gain from building the facility in more favorable condition (with a higher gasoline price). The DM also retains the option to forego the investment if the gasoline price does not turn out to increase as forecasted. This interpretation assumes there is no competition in supplying the same type of cellulosic biofuels.

![Threshold prices over time (in the decision period)](image)

The risk associated with the investment is shown in Figure 2.8. The frequency chart in Figure 2.8 is obtained using the Monte Carlo simulation described in Section 2.3. The optimal investment time can also be obtained from the same simulation. Figure 2.9 shows the frequency chart of the investment time $\tau$. It can be seen that the distribution of the project value is bell-shaped. The
Value-at-Risk (VaR) highlighted in Figure 2.8 shows that the DM has a 10% chance of losing more than $67.3 million, while having 79.0% chance to profit from this project (with a positive project value). The distribution of $\tau$ in Figure 2.9 is more irregular. The probability of building the facility has a sudden increase at $\hat{T}$ because $\hat{T}$ is the last time to decide to build or to abandon. At this time, no waiting is possible and it would build as long as the project value is positive.

Figure 2.8  Frequency chart of the project value (baseline with $L = 12$ months)

Table 2.3  Project value and investment time vs. initial price

<table>
<thead>
<tr>
<th>$P_0$ ($$/gallon)</th>
<th>F($P_0$) ($ million)</th>
<th>$E[\tau]$ (month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>140.7</td>
<td>42.0</td>
</tr>
<tr>
<td>3.50</td>
<td>142.4</td>
<td>36.1</td>
</tr>
<tr>
<td>3.90</td>
<td>144.3</td>
<td>31.7</td>
</tr>
<tr>
<td>4.10</td>
<td>145.6</td>
<td>26.0</td>
</tr>
<tr>
<td>4.30</td>
<td>147.3</td>
<td>22.0</td>
</tr>
<tr>
<td>4.40</td>
<td>148.3</td>
<td>17.2</td>
</tr>
<tr>
<td>4.48</td>
<td>149.4</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.3 illustrates the idea of the threshold price at $t = 0$. As the initial price $P_0$ increases, the expected project value increases while the expected waiting time decreases. When $P_0$ is about $4.48/gallon, the expected waiting time reduces to zero for the first time. This price is called the
threshold price at $t = 0$. As argued earlier, the threshold price is not the break-even price (its project value is positive); it is actually much greater than the break-even price.

**Impact of the lead time $L$**

In reality, a pyrolysis facility cannot be built overnight. Therefore, the impact of the lead time to the project valuation should be considered. Unfortunately, most papers using real options valuation ignore the lead time. In Table 2.4, we show the basic information of the project valuation by changing the length of the lead time from 0 to 36 months. In the second column of Table 2.4, it shows that the project value (and the percentage change relative to the no-lead-time case) decreases significantly when the lead time increases. When the lead time increases from 0 to one year, the project value depreciates to about 90% of that without lead time; at two years, it drops to 86.5%. When the lead time is three years, it loses over 15% of the project value without lead time. This shows that any valuation that does not consider lead time risks inflating the project value.

Table 2.4 also shows that the threshold price increases with the lead time. This is reasonable because the lead time increases the risk of the investment and, therefore, the DM would require seeing a higher fuel price (and, therefore, a higher potential profit) to invest in the facility. But
what is the impact of the lead time on waiting? Should the DM wait for a longer or shorter period when the lead time increases? Our result shows that a long lead time would actually push the DM to enter the market sooner. Because of the long lead time, the DM needs to build the facility sooner so that s/he can start collecting revenues sooner while the favorable market lasts.

This is also reflected in Figure 2.7, in which all curves of $H_t$ are decreasing, implying that as time goes, if the facility has not been built, the entry requirement becomes lower and lower. This can be interpreted as follows: At $t = 0$, though the project seems highly profitable, the DM has sufficient time to optimize the entry timing to maximize the expected profit - the focus is on the entry (timing) flexibility. However, as time goes on, if the facility has not yet been built, the timing flexibility decreases (because the time to $\hat{T}$ becomes shorter), and the focus shifts to profitability rather than the flexibility. That is, ultimately the DM wants to enter the market while the profitable opportunity lasts. This situation is exacerbated when the lead time is very long, such as 36 months. In the beginning, it requires a higher $H_0$ to justify the risk due to the long lead time. If $P_t$ is never greater than $H_t$ to justify entry, the long delay will push the time to start collecting revenues further back, which eventually pushes the DM to enter the market while the project value is still positive. Therefore, if a project’s lead time becomes longer, the DM may be required to act sooner while a favorable market lasts.

<table>
<thead>
<tr>
<th>Lead time $L$ (month)</th>
<th>$F(P_0)$ ($\text{million}$)</th>
<th>$H_0$ ($\text{$/gallon}$)</th>
<th>$E[\tau]$ (month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>156.0 (100%)</td>
<td>4.05</td>
<td>42.6</td>
</tr>
<tr>
<td>6</td>
<td>146.3 (93.8%)</td>
<td>4.24</td>
<td>42.1</td>
</tr>
<tr>
<td>12</td>
<td>140.7 (90.2%)</td>
<td>4.48</td>
<td>42.0</td>
</tr>
<tr>
<td>18</td>
<td>137.2 (88.0%)</td>
<td>4.74</td>
<td>40.6</td>
</tr>
<tr>
<td>24</td>
<td>134.9 (86.5%)</td>
<td>5.01</td>
<td>37.9</td>
</tr>
<tr>
<td>30</td>
<td>133.3 (85.5%)</td>
<td>5.23</td>
<td>32.8</td>
</tr>
<tr>
<td>36</td>
<td>132.4 (84.9%)</td>
<td>5.39</td>
<td>26.9</td>
</tr>
</tbody>
</table>

In Figure 2.10, we show the expected project value vs. the initial fuel price $P_0$. It can be seen that the project value is not only smaller when the lead time is longer; it is also less sensitive to the
initial fuel price when the lead time is longer. In Figure 2.10, each curve has a turning point at its threshold price, highlighted by an x. When the initial fuel price is greater than the threshold price, it can be seen that the project value increases much faster than otherwise. This is because the project is “in-the-money” at $t = 0$ to the right side of the threshold price and is “out-of-the-money” to the left.

![Figure 2.10](image)

**Figure 2.10** Project value vs. the initial fuel price

### 2.4.4 Impact of price parameters

In this section we discuss the impact of price parameters on the project value. The three parameters in the GMR process in (2.16) are the mean price level $m_t$, reversion coefficient $\lambda$, and volatility $\sigma$. The investment is evaluated with a lead time of $L = 12$ months.

By repeatedly running our programs with discrete values of $\lambda$, $\sigma$, and all $m_t$ (parallel shifted) at $\pm10\%$ of the corresponding values in the baseline, we use regression analysis to investigate the impact of the price parameters on the project value. Intuitively, the higher the fuel price is, the more profitable the investment becomes, which is reflected by an increase in the project value. In the regression models, we use $\Delta m$ to represent the parallel shift percentage in $m_t$; $\Delta \lambda$ and $\Delta \sigma$ are the percentage changes in $\lambda$ and $\sigma$, respectively. The regression functions for the threshold prices
at $t = 0$ and the project values are obtained as follows:

$$H_0(\Delta_\lambda, \Delta_\sigma, \Delta_m) = \alpha_0 + \alpha_1 \Delta_m + (\alpha_2 \Delta_m + \alpha_3 \Delta_m^2) \Delta_\lambda + (\alpha_4 \Delta_m + \alpha_5 \Delta_m^2) \Delta_\sigma + \gamma (\Delta_m - m_h)^+ + \epsilon$$  \hfill (2.17)

$$F(\Delta_\sigma, \Delta_m; P_0 = 3.0) = \beta_0 + \beta_1 \Delta_m + \beta_2 \Delta_m^2 + (\beta_3 \Delta_m + \beta_4 \Delta_m^2) \Delta_\sigma + \eta (\Delta_m - m_f)^+ + \epsilon$$  \hfill (2.18)

where $\epsilon$ and $\varepsilon$ are mutually independent normal distributions, each with a zero mean. The values of the coefficients in the regression functions can be found in Table 2.5. In general, a positive $\Delta_\lambda$ means that any price deviation from the mean lasts for a shorter time period, and a negative $\Delta_\sigma$ represents smaller price variability. It can be seen from (2.17) that the project value shows that both positive $\Delta_\lambda$ and negative $\Delta_\sigma$ would increase the threshold price and decrease the project value. These suggest that the investment in a pyrolysis plant is more favorable in a more volatile fuel market. However, compared to the influence of $m_t$, $\lambda$ and $\sigma$ have far less impact. The impact of $\lambda$ and $\sigma$ are shown to be independent of each other in the regression function (2.17), yet highly interacted with the mean price level $m_t$. A positive $\Delta_m$ gives $\lambda$ and $\sigma$ larger influence in the threshold price and the project value. The impact of $\lambda$ on project value is almost negligible, thus no terms with $\Delta_\lambda$ are denoted in (2.18).

When the mean price level $m_t$ is shifted down in parallel to some extent, it would trigger a change in the investment decision (from invest to do-not-invest), which is captured by the non-negative operator terms ($x^+$) involving $m_h$ and $m_f$ in (2.17) and (2.18), respectively. We have estimated that $m_h = -0.071$ and $m_f = -0.068$, and the values of both $m_h$ and $m_f$ are very close, indicating a very consistent approximation in the regression analysis. Since the profile of $m_t$ is largely influenced by the EIA forecast (in Figure 2.6), this means that if the future retail fuel price is about 7% lower than the trend predicted by the EIA (parallel shift), the project is no longer profitable. The effect of changing $m_t$ (parallel shift) on the threshold price and the project value
is given in Figure 2.11. Each of the two curves in Figure 2.11 appears to have a turning point when the parallel shift in $m_t$ is at about $-7\%$, corresponding to the values of $m_h$ and $m_f$ in the regression models (2.17) and (2.18), respectively. Again, this indicates that if the price outlook is about $7\%$ lower than expected, the project should not be invested at all.

Table 2.5  Regression results for threshold price ($$/gallon) and expected project value ($$ million) vs. price parameters

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\gamma$</th>
<th>$m_h$</th>
<th>$\text{sd}^\dagger$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-11.5</td>
<td>-205</td>
<td>-437</td>
<td>6103</td>
<td>122</td>
<td>-1520</td>
<td>197</td>
<td>-7.1e-2</td>
<td>3.1</td>
<td>0.82</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
<td>$\beta_3$</td>
<td>$\beta_4$</td>
<td>$\eta$</td>
<td>$m_f$</td>
<td>$\text{sd}^\dagger$</td>
<td>$R^2$</td>
<td></td>
</tr>
<tr>
<td>45.2</td>
<td>891</td>
<td>4290</td>
<td>811</td>
<td>9134</td>
<td>1477</td>
<td>-6.8e-2</td>
<td>14.3</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

$\dagger$: Residual standard deviation

Figure 2.11  Threshold price ($$/gallon) and expected project value ($$ million) vs. parallel shift in $m_t$
2.4.5 Impact of process parameters

Biomass feedstock price and production yield are two major factors in the production process that are influential to the investment decision. For a large-scale facility (2000 metric ton/day in our case), a slight increase in the feedstock price could mean a significant increase in the production cost. Likewise, the production yield, referring to the efficiency of the new technology, could also have an impact on the revenue and the conversion cost. According to Wright et al. (2010) and Brown et al. (2013), fast pyrolysis and further conversion to produce liquid transportation fuel is relatively immature, which leads to a high level of uncertainty in technology efficiency. Therefore, we would like to assess the impact of these two factors on the investment decision.

A polynomial form is selected for the regressions of both decisions: “do not invest” \((n = 0)\) within planning horizon and “invest” \((n = 1)\). Let \(Y\) be the process conversion rate \(Y = \theta_1 \theta_2\), and \(C^B\) be the biomass feedstock price. The regression functions for both the threshold price at \(t = 0\) and the project value are as follows.

\[
H_0(Y, C^B) = a_{n1} + a_{n2} Y + a_{n3} C^B + a_{n4} Y^2 + a_{n5} (C^B)^2 + a_{n6} Y C^B + \epsilon_n \quad (2.19)
\]

\[
F(Y, C^B; P_0 = 3.0) = b_{n1} + b_{n2} Y + b_{n3} C^B + b_{n4} Y^2 + b_{n5} (C^B)^2 + b_{n6} Y C^B + \epsilon_n \quad (2.20)
\]

where the residual terms, \(\epsilon_n\) and \(\epsilon_n\), are each independent normal distributions with means of zero and standard deviations of \(\sigma_{1n}\) and \(\sigma_{2n}\), respectively, for \(n = 1, 2\). The regression parameters and the statistical measures of the fitting are given in Table 2.6.

The regression models are selected based on the statistical measure of \(R^2\), one-way analysis of variance (ANOVA) significance tests, and satisfactory Mean Absolute Percentage Error (MAPE) or Mean Absolute Error (MAE). The regression of the threshold price and the expected project value are illustrated in Figure 2.12. The dashed line in each figure illustrates the “indifference” boundary. On one side of the boundary, Region A represents the region in which investing in the project is favorable, and on the other side, Region B represents the region where the DM should not invest.
Table 2.6  Regression results for threshold price ($/gallon) and the project value ($ million)

<table>
<thead>
<tr>
<th>$/gallon ($ million)</th>
<th>$/gallon ($ million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$/gallon ($ million)</td>
<td>$/gallon ($ million)</td>
</tr>
<tr>
<td>$/gallon ($ million)</td>
<td>$/gallon ($ million)</td>
</tr>
</tbody>
</table>

Intuitively, the project would be very profitable when the conversion rate is high and the biomass feedstock price is low (Region A in Figure 2.12). The significance of the interaction term \((YC_B)\) in the regression models shows that the two factors, biomass feedstock price and the conversion rate, do not affect the investment decision independently. The effects of the conversion rate on the threshold price and the project value are much greater than that of the biomass feedstock price. The indifference boundary highlighted in Figure 2.12 shows the trade-off relation between these two factors. Basically, every 1% increase in the biomass feedstock price can be offset by a 0.25% increase of the conversion rate. This result can be used to price long-term contracts of biomass feedstock to ensure profitability given a conversion rate.

2.4.6 Impact of other factors

Thus far we have discussed how the factors of fuel price, production yield, and feedstock price impact the investment value and timing. There are two other factors that are also relevant to the investment decision-making: project finance and biomass supply chain. In project finance, a capital structure has an implication on project (construction) risks and long-term profitability. On the other hand, supply chain uncertainties, including feedstock availability and logistic costs, if not managed well, can be detrimental to the facility's day-to-day operations. These factors, to some degree, all contribute to the project value and influence the investment timing. To analyze their
impacts on the project value requires specific, technical knowledges, and is outside the scope of this work. However, we can roughly discuss how they affect the investment timing.

In general, a factor that can contribute to the increase of the project value tends to induce a quicker investment, i.e., reduce waiting, and vice versa. Therefore, a higher cost of capital will increase the project cost and lower the project value. It will also delay the investment because it will require a higher threshold price to ensure profitability. Likewise, higher supply chain costs (e.g., low availability of feedstock and high transportation costs) will contribute to the increase of operating costs and the decrease the project value, which increases waiting. We hope to incorporate these factors to the valuation in future research.

2.5 Conclusions

In this work, a real options approach has been proposed to value a fast pyrolysis facility investment under fuel price uncertainty. We intended to answer the question whether now is a good time for Iowa to start investing in cellulosic biofuels. Given that the fuel price outlook is very positive as forecasted by the Energy Information Administration (EIA), the investment in general is very profitable with manageable risks (about 80% chance to be profitable). However, our valuation model also indicates that the trigger price for an immediate investment is $4.48/gallon, which suggests the decision maker should invest later to maximize the net profit. Our result also indicates that the construction lead time is too important to ignore. When the lead time increases from 0 to one year, the project value is 10% lower; at two years, 13.5% lower. This result suggests that a speedy permitting approval process of the facility construction by government can increase the project value and may induce investments. Using regression we also show how technology improvement in production yield may increase the project value and offset the effects from biomass feedstock price change.
Figure 2.12  Regression of threshold price and the expected project value on conversion rate and biomass price
CHAPTER 3. DUAL INVESTMENTS SUBJECT TO RISK AVERSION AND LEAD TIMES: A CASE OF MITIGATING SUPPLY RISK OF CELLULOSIC BIOFUEL PRODUCTION

We consider the investment of a cellulosic biofuel facility using fast pyrolysis with corn stover as its main feedstock, whose market price is subject to yield uncertainty. To mitigate the supply uncertainty, the decision maker (DM) also considers a secondary investment in a land to grow switchgrass. Both investments have nonzero lead times and are subject to uncertainties of biofuel price, land price, and feedstock yield. We then solve the optimal timing for both investments. Because of the lead time, the exercise of the secondary option does not have to take place after the primary facility investment. When the DM is risk-neutral, each of the two investments should be carried out following the traditional threshold method such that it is optimal to invest when the biofuel price exceeds the threshold. However, we show that with risk-aversion the traditional threshold method only holds for the facility investment, while the secondary decision can be very sensitive to the asset’s underlying uncertainty. Under some land price the optimal decision is to alternately invest and not invest in land over as many as five intervals of biofuel price. When there are multiple investments to make, risk-aversion may have an unintuitive influence on the order of the investments undertaken such that the traditional investment rule may not be optimal.

3.1 Introduction

Firms entering new markets face numerous operational challenges. The timing of investment in new capacity has long been recognized as a critical strategic decision. It may be beneficial to delay an investment if time will bring more information from which a firm can learn about the future prospects of the investment, provided the opportunity to invest does not disappear (Dixit, 1992). A real options valuation approach is commonly applied to option valuation techniques to capital
investment decision under uncertainty, given that an investment opportunity can be viewed as an analogue to financial options on a common stock (Dixit and Pindyck, 1994). A real option itself is the right, but not the obligation, to undertake certain business initiatives, such as deferring, abandoning, expanding, staging or contracting a capital investment project.

In recent years, real options has been applied to agricultural related projects. Jones et al. (2001) considered a production-scheduling problem for seed corn when random yields and demands exist. Given two sequential production periods before demand occurs, insights into the considerable marginal benefit from having a second planting period was provided using a real options approach. Kazaz (2004) discussed production planning in an olive oil industry with random yield and demand. While producers can lease farm space from farmers to grow olives, yield risk is mitigated through the option of having a second chance to buy extra olives from other growers after the crop yield is observed.

Performance of risk-neutral or risk-averse players can be captured in real options approach through different methods. One common method is to consider risk premium as compensation for the investors (Kouvelis and Tian, 2014; Treville et al., 2014). Another often-used way to address risk-aversion is to apply utility function. With optimizing over expected utility of the investor, Henderson and Hobson (2013) and Kazaz and Webster (2011) both showed counterintuitive results where risk-pursuing actions are taken by risk-averse decision makers. Kazaz and Webster (2015) stated that for price-setting newsvendor problem with uncertain demand of a perishable product, concavity of the objective function is preserved under the introduction of risk-aversion using utility function if the source of uncertainty is demand, yet not necessarily preserved if the source of uncertainty is supply.

Besides risk-preference, value of project lead time has also been studied. Treville et al. (2014) demonstrated the potential value of lead time reduction by modeling lead time as an endogenous decision variable. Yet, lead time reduction may not always apply in reality. Our previous work in Chapter 2 showed that in a facility investment problem, overlooking production lead time would inflate the project value and have impact on investment timing decision.
This work valuates a cellulosic biofuel project via the real options approach. The promotion of cellulosic biofuels, a green alternative for traditional fossil fuels, is emphasized in the revised renewable fuel standard (RFS2). RFS2 mandates a production of 36 billion gallons per year (BGY) renewable fuel to be blended into transportation fuel by 2022, out of which 16 BGY must be accomplished with cellulosic biofuels, fuels produced from lignocellulose (e.g., corn stover, switchgrass, miscanthus, wood chips) (EPA, 2010).

Fast pyrolysis is a rapid decomposition of organic matter in the absence of oxygen. Conducted at $500^\circ F$, fast pyrolysis of lignocellulosic biomass can produce pyrolysis oil and co-product bio-char, where the former can be refined into usable transportation fuels and the latter can enhance soil quality (Brown et al., 2013; Mullen and Boateng, 2008; Mullen et al., 2010).

Chapter 2 discussed the investment of fast pyrolysis technology in Iowa to produce cellulosic biofuel subject to construction lead time of the facility and uncertain fuel price. Yet, other sources of uncertainty would also affect the project value. Although corn stover is by far the agricultural residue of the largest quantity in Iowa, there is a potential supply risk due to (1) lack of market due to concerns about sustained soil productivity and lack of commercial conversion technologies; (2) low percentage of farm participation; and (3) yield uncertainty due to the changing weather conditions (Koundinya, 2009; Thompson, 1963; Thompson and Tyner, 2014). Since feedstock costs account for a large proportion of total cost in biofuel production, the supply risk could be an important factor to take into consideration.

Dual sourcing is an effective way to mitigate supply risk. Tomlin (2009) stated that dual sourcing and inventory are two prevalent and widely studied strategies firms use to manage yield risk. The author studied a firm’s optimal sourcing and inventory decisions given their forecast of supplier’s yield with a Bayesian learning. Li and Debo (2009) studied the decision of a manufacturer (the buyer) in selecting between sole- and second-sourcing strategies for a noncommodity component. While a second-sourcing strategy allows the buyer to take advantage of alternative sourcing opportunities (option value), the future supplier competition in second-sourcing induces the first supplier to ask for a higher price at the beginning of the horizon (cost of future supplier compe-
Wang et al. (2010) discussed the effect of sourcing from multiple suppliers and/or exert effort to improve supplier reliability.

Aside from agricultural residues like corn stover, bio-oil could also be produced from a few types of potential energy crops, where switchgrass is seen as one with great potential. Switchgrass is a perennial that do not require annual reseeding, has low agricultural inputs (fertilizer and pesticides), and can tolerate annual cutting; it has the ability of obtaining moderate to high yields on marginal farmlands; it’s hardy in poor soil and climate conditions, for example, drought and flooding; and it is capable of producing cellulosic biofuels, which have less impact on agricultural commodity markets compared to traditional biofuels (CIAS, 2001; Mullen and Boateng, 2008). In this work, the DM would consider investing in a land to grow his own feedstock, in addition to the investment of the fast pyrolysis facility. Due to the enormous amount of land required for full feedstock supply, the land option would fulfill only limited degree of vertical integration in feedstock supply.

In this work, we extend our work in Chapter 2 and study the interaction of two irreversible investments, the investment in a cellulosic biofuel facility using corn stover as its main feedstock as a primary option, and the investment in a land to grow switchgrass as a secondary option. The secondary option of growing your own feedstock enables partial vertical integration in biomass supply and thus mitigates supply risk. Due to the existence of lead time, the secondary investment does not have to be exercised after the primary facility investment. The operation capacity of the facility is flexible according to the prospect of the market. We use a case study of investing in biofuel production in Iowa, subject to uncertain fuel price, feedstock supply, and land price, to illustrate our real options valuation. The optimal immediate investment strategy on both the primary investment and secondary option relies on the recent market condition. This is not unusual, due to the crude price collapse, nearly $400 billion spending on gas and oil projects have been canceled (Adams, 2016). Because of the interdependency of operations of both options, the secondary investment decision also needs to be assessed based on the underlying fuel price, as well as its cost.

In the valuation process, different factors may affect the project value and investment decision-making. Lead time impact is found to be too significant to overlook, and in our case study,
both facility and land investment has nonzero lead times. The facility has a lead time, which include the time required for construction, environmental evaluation, and permitting. The land lead time consists the establishment of switchgrass, which is generally slow and difficult. Sometimes reseeding is necessary in order to achieve the peak yield, and it often takes from 2 to 3 years. Risk-aversion should be considered when dealing with capital investments, especially when under risky environment.

With the analysis, we want to address the following research questions: when is the optimal timing for the DM to invest in the fast pyrolysis facility? How about land? Since the two investment options are non-sequential, what could be the order of investment? How would the aforementioned factors, investment lead times and risk-aversion affect the decision-making? The contribution of this work are twofold: (i) The biofuel project is valuated considering two relevant options, facility investment being primary and land/feedstock investment being secondary. The secondary option is included to mitigate the impact of supply risk and it has a value-adding effect when functioning with the primary investment, it can be exercised ahead of the facility due to the lead time consideration. (ii) Risk-aversion effect brings in unintuitive influence in the investment decision-making when there are multiple investments to make, therefore, the traditional threshold investment rule may not be optimal.

The remainder of this Chapter is organized as follows. In Section 3.2, we develop the valuation model of the aforementioned case study in biofuel industry. Section 3.3 presents the numerical results from the case study. Section 3.5 summarizes the implication discussion. Section 3.6 further explored the effect of risk aversion with a separate case with a primary asset and a secondary option. Section 3.7 provides concluding remarks and indicates future areas of research.

### 3.2 Model

The supply chain of biofuel considered in this work, as shown in Figure 3.1, starts with the biomass produced from farmers, which is converted to hydroprocessed bio-oil through the proposed fast pyrolysis and simple hydrotreating facility. The hydroprocessed bio-oil requires further refining
(e.g., hydrocracking and refining) before becoming liquid transportation fuels, such as gasoline and diesel, which are then shipped to consumers at street pumps. We assume that the refining process, which is a mature technology, is outsourced to some existing refinery facility.

Figure 3.1  Biofuel production through fast pyrolysis and further conversion

Consider a decision maker (DM) who is evaluating the investment of a new fast pyrolysis technology to produce cellulosic biofuel. In addition to the main feedstock, corn stover, the DM also considers the real option to invest in a land to grow her own feedstock (e.g., switchgrass) as the second crop to ensure stable supply. In this work, using a real options approach we value the facility investment together with the option of growing one's own feedstock as the second crop. The purpose is to determine the optimal timing for both facility and the second crop to maximize the expected net present utility of the project.

The underlying uncertainties considered in this work consist of (i) fuel price $P^F_t$ that affect fuel sale revenue; (ii) land price $P^L_t$ which is a key factor for growing the second crop; and (iii) yield uncertainty $Y_t$ of the feedstock, which depends on weather conditions (e.g., temperature and precipitation) during the crop growing season.

Incorporating land investment for growing the second crop in the proposed valuation problem can be tricky. Although a land is considered an immovable property, its investment is not necessarily irreversible. Land price can be volatile such that the investment can have high profit potential, but with high risk as well. Since our inclusion of a land investment is to cultivate a second crop to mitigate supply uncertainty, we make the following assumptions about the land investment:
• The land investment is irreversible during the life of the pyrolysis facility.

• Land price is uncertain and volatile. The land has a salvage value based on its market value at the end of the life cycle. However, there is no profit expected purely from buying and selling the land. This can be manipulated by setting the drift (or growth) rate of the land price to be slightly lower than the discount rate.

• The second crop is switchgrass and is developed by a land that is solely dedicated for the proposed pyrolysis facility. The switchgrass that is not used by the facility has no resale value.

Based on these assumptions, the valuation model we can better assess the true value of a second crop in terms of how it mitigates the supply uncertainty. Furthermore, we impose lead times for both investment decisions. The facility has a lead time for construction, environmental evaluation, and permitting, while growing the second crop has a lead time for land establishment and re-seeding to obtain maximum acreage yield of switchgrass.

3.2.1 Formulation

Consider a DM whose utility function is $U_\gamma$ parameterized by $\gamma$ measuring the level of the DM’s risk aversion. The DM would evaluate the investment decisions every $\Delta t$ during the decision period $[0, \hat{T}]$ considering the status of both investment decisions, $x_{1t}$ and $x_{2t}$, where $x_{1t}$ represents the investment status of the pyrolysis facility at $t$, and $x_{2t}$ represents the investment status of the land development at $t$. Let $V_t^\gamma(P_t^F, P_t^L, Y_t; x_{1t}, x_{2t})$ denote the project value of the remaining planning horizon $(t, \hat{T}]$ given investment status $(x_{1t}, x_{2t})$. Then we have the following recursive relationship, $\forall t \in [0, T]$:

$$
V_t^\gamma(P_t^F, P_t^L, Y_t; x_{1t}, x_{2t}) = U_\gamma(f_t(P_t^F, P_t^L, Y_t; x_{1t}, x_{2t})) \Delta t + \max_{u_{1t}, u_{2t} \in \{0, 1\}} \left\{U_\gamma(I_t(x_{1t}, x_{2t}, u_{1t}, u_{2t})) + e^{-r \Delta t} \mathbb{E}_t \left[V_{t+\Delta t}^\gamma(P_{t+\Delta t}^F, P_{t+\Delta t}^L, Y_{t+\Delta t}; x_{1,t+\Delta t}, x_{2,t+\Delta t}) \right]\right\}.
$$

(3.1)
where for $i = 1, 2$

$$x_{i,t+\Delta t} = \begin{cases} u_{it}, & \text{if } x_{it} = 0 \\ \min(N_i, x_{it} + 1), & \text{otherwise} \end{cases} \quad (3.2)$$

$$\sum_{t=1}^{\hat{T}} u_{it} \leq 1; \ u_{it} = 0, \forall t > \hat{T} \quad (3.3)$$

In Equation (3.1), $E_t$ denotes the expectation operator given the information at $t$; $f_t$ represents the profit in period $t$; and $I_t$ is a lump-sum capital cost incurred from the investments. Equation (3.2) describes the transition of state variables $x_{it}$ given the investment decision $u_{it}$, where $N_i = L_i/\Delta t$, $i = 1, 2$. The state transition is mainly used to describe the effect of lead times $L_i$, $i = 1, 2$. Equation (3.3) states that each investment decision $u_{it} = 1$ is only considered during the decision period $[0, \hat{T}]$, and is irreversible such that it can be at most exercised once. Overall, the formulation of the investment problems is subject to the following boundary condition for the project’s salvage value. We assume that the facility has no salvage value at the end of its life; however, the land, if it is invested, carries a salvage value $S_T$ based on the land’s market value (i.e., land price $P_T^L$ multiplied by the size of the land).

$$V_T^\gamma(P_T^E, P_T^L, Y_T; x_{1T}, x_{2T}) = U_\gamma(S_T) 1(x_{2T} > 0) \quad (3.4)$$

$$x_{i,0} = 0, i = 1, 2 \quad (3.5)$$

The transition diagram of $x_{it}$ is given in Figure 3.2.
The profit function $f_t$ consists of three components, revenue from fuel sale, biofuel production costs, and land related costs. The revenue from biofuel sale can only be collected after the facility is in operation (i.e., after $x_1 t$ reaches $N_1$). The biofuel production costs include feedstock cost, shipping cost of intermediate product (hydroprocessed bio-oil) to the refinery, transportation cost shipping final product (gasoline) to the demand locations, operating cost, and fuel tax. The land related cost needs to be included with land operations (after $x_2 t$ reaches $N_2$), including, production, labor, and trucking and loading costs. The actual profit is calculated at the optimal production quantity, which is obtained based on the profitability of biofuel under current market condition. The formulation of the profit function is given in the appendix.

3.2.2 Solution procedure

Standard backward stochastic dynamic programming (SDP) steps are used to solve the project valuation problem following the recursive relationship in (3.1) with the underlying uncertainties $P^F_t$ and $P^L_t$ modeled using discrete price lattices. The project value for investing in the fast pyrolysis facility with the second crop option is $V_0^\gamma(P^F_0, P^L_0, Y_0; x_{1,0} = 0, x_{2,0} = 0)$, indicating that both the facility and the land are open for investment at $t = 0$.

After using SDP steps to determine the optimal investment strategy at each time, Monte Carlo simulations are used to determine the distributions of the optimal investment times $\tau_1^*$ (for the facility) and $\tau_2^*$ (for the second crop). In the simulation step, we first generate sample paths of the underlying uncertainties, including the fuel price, the land price, and weather (to obtain the yield and feedstock price), following their stochastic processes. Then on each sample path, use the optimal investment strategy previously obtained from the SDP steps to decide whether to invest or not at each time. With these, we can obtain the distributions of the optimal investment times $\tau_1^*$ and $\tau_2^*$, as well as the distribution of the project value.

**Proposition 1.** If the lead time for land development is shorter than that of the facility, i.e., $L_2 < L_1$, it is never optimal to invest in the land before the facility. On the other hand, if $L_2 \geq L_1$, it is possible that the land is invested even before the facility.
Proof. When \( L_2 < L_1 \), there is no benefit to invest in the land before the facility. Doing so, the land is developed before the facility is operational. Based on the assumptions about the land investment, the second crop produced cannot be used and is worthless while the land still needs to be maintained. Furthermore, by the assumptions, the expected net profit of the land investment is negative, which is another reason not to invest it early. When \( L_2 \geq L_1 \), the land needs longer time to develop than the time to construct the facility. Therefore, early cost-saving resulted from the second crop and the opportunity cost of the land investment come into play. The land should be invested earlier than the facility if its early cost-saving outweighs the opportunity cost of the land investment. □

3.3 Case Study

In this section, we present a case study to demonstrate the proposed valuation approach. Modeling of the underlying uncertain factors, references of parameter values, and the numerical result of the biofuel facility investment with the second crop option are presented.

3.3.1 Underlying uncertainties

3.3.1.1 Fuel price uncertainty

In this case study, we assume that the evolution of fuel price can be captured by a geometric mean-reverting (GMR) process:

\[
d\ln(P_t^F) = -\mu^F (\ln(P_t^F) - m_t^F) \, dt + \sigma_t^F dW_t^F, \tag{3.6}
\]

where \( \mu^F \) is the reverting coefficient, \( m_t^F \) is the mean level of gasoline price at pump at time \( t \), \( \sigma_t^F \) is the volatility, and \( W_t^F \) is a Weiner process.

To estimate the mean level of the fuel price \( m_t^F \), we obtained historical (1995-2017) weekly retail gasoline price of the Midwest region from the EIA (EIA, 2016a), together with the EIA annual retail price forecast up to 2040, as shown in Figure 3.3 (EIA, 2016b). The exponential of the mean level \( \exp(m_t^F) \) from 1995 to 2040 is estimated using a polynomial regression with an order
of three, with the regression result depicted as the dotted line in Figure 3.3. The price parameters \( \mu^F \) and \( \sigma^F \) are estimated based on historical weekly retail gasoline price and the fitted \( m^F_t \) using the maximum likelihood method. The estimated parameter values are \( \hat{\mu}^F_T = 0.4147 \) and \( \hat{\sigma}^F_t = 0.1200 \). We discretize the fitted GMR process for the fuel price using a trinomial price lattice proposed by Hull and White (Hull and White, 1993).

![Figure 3.3 Mean fuel price estimation](image)

### 3.3.1.2 Land price uncertainty

The evolution of land price \( P^L_t \) is modeled with a geometric Brownian motion (GBM),

\[
dP^L_t = \mu^L P^L_t dt + \sigma^L P^L_t dW^L_t, \tag{3.7}
\]

where \( \mu^L \) is the drift rate, \( \sigma^L \) is the percentage volatility, and \( W^L_t \) is a Weiner process.

Use the historical farmland value in a Midwest state from 1950 to 2015 (Zhang, 2016), we fitted the land price parameters with the maximum likelihood method and obtain \( \hat{\mu}^L = 0.0576 \) and \( \hat{\sigma}^L = 0.1148 \). Although the fitted drift rate suggests the land will increase at an annual rate of 5.76%, the actual land value in Midwest region started to drop from 2015. Also based on an assumption mentioned previously, we do not want the land value growth overshadows the cost saving of the second crop, in this case study, we assume the land price increases by 2% per
year, which is lower than the discount rate 5% assumed. This continuous price process is then approximated with a binomial lattice proposed by Rubinstein (1994).

![Graph showing land price over time](image)

Figure 3.4 Land price

### 3.3.1.3 Supply uncertainty

Thompson (1963) interpreted that besides rapid technological progress in agriculture, weather (especially rainfall and temperature in the period of the growing season) also serves as an important impact variable in crop yield. The yield of corn stover is highly correlated to the corn yield, and is estimated based on the corn yield considering a residue-to-grain ratio (Heid, 1984). Thompson (1963) suggests to regress corn stover yield on time, precipitation, and temperature, while the trend over time indicates the effect of technology,

\[ \text{Yield}_t = \beta_0 + \beta_1 t + \beta_2 \cdot \text{Precipitation} + \beta_3 \cdot \text{Temperature} + \varepsilon_t \]

The historical data of the yield is shown in Figure 3.5(a). In this work, we do not include the technology effect in the valuation because the fast paralysis technology itself is in an infant stage of development and its future advancement is largely unknown. We model the weather effect of the yield uncertainty, which refers to the last three terms of \( \text{Yield}_t \), by two separate underlying regimes – low yield \((\Delta = 0)\) and high yield \((\Delta = 1)\), with a two-component Gaussian mixture as follows.

\[ Y_t \approx \beta_2 \cdot \text{Precipitation} + \beta_3 \cdot \text{Temperature} + \varepsilon_t = (1 - \Delta) \cdot Y_1 + \Delta \cdot Y_2, \quad (3.8) \]
where $Y_1 \sim N(\mu_1, \sigma_1^2)$ and $Y_2 \sim N(\mu_2, \sigma_2^2)$, $\Delta \in \{0, 1\}$ with $\Pr(\Delta = 1) = \pi$. The data of $Y_t$ is shown in Figure 3.5(b).

The data are extracted from the historical monthly climate data obtained from NOAA (NOAA, 2016), we use the average precipitation and temperature during corn growing season (by month) from 1960 to 2015. An expectation maximization (EM) algorithm is used to estimate the parameters in the process (Hastie et al., 2009), where we get $\hat{\mu}_1 = 3.1632$, $\hat{\sigma}_1 = 9.9743$, $\hat{\mu}_2 = -36.9772$, $\hat{\sigma}_2 = 12$.

![Figure 3.5](image)

Figure 3.5 corn stover yield can be decomposed to time effect, the linear function in (a), and weather effect in (b).

The supply uncertainty in this work is modeled in terms of market price of corn stover driven by the uncertain yield. When the yield is high, with plenty of supply, the market price is expected to be low. Since there is no existing markets for corn stover, we assume its price follows a linear function of the yield $Y_t$ with a negative slope. Brown et al. (2013) has estimated that the cost of feedstock (corn stover) used is $83$/metric ton. We then use this as the corn stover price when the yield is high at 4.7 MT/acre. On average, high yield is about 15% higher than low yield. We further assume that the price and yield are inversely proportional at the two high/low yields. That is, the stover price when the yield is low (at 4.7 (1-15%) MT/acre) is $83/(1 - 15\%)$ per MT. Therefore,
the following relation is obtained for price of corn stover $P^C$ ($/\text{metric ton})$: $P^C = 179.89 - 20.75Y_t$, where $Y_t$ is in the unit of MT/acre. Estimate the probability that the price is negative negligible $10^{-15}$.

Based on a study by University of Kentucky (2013), the peak yield of switchgrass is assumed to be at 6 MT/acre in a high-yield year, and the acreage yield decreases 15% in a low-yield year. Here we assume the yield of switchgrass is also subject to the same underlying weather condition for $Y_t$. Similarly, the total acreage operational cost in a low-yield year is $1/(1 - 15\%)$ times that of a high-yield year. This assumption is rather conservative since switchgrass is a type of energy crop with high potential due to its relative stable yield against tough climate conditions. Since both corn stover and switchgrass are all subject to the yield uncertainty driven by weather, as opposed to the variable feedstock costs for corn stover driven by (continuous) weather effect, the advantage of using switchgrass lies in its stable costs at the two high/low yield states.

3.3.2 Operational data and sources

In this study, the candidate location for the fast pyrolysis facility is assumed to be at the center of Wayne County, Iowa, where farmlands are of lower rank, an ideal location for the land investment. The capacity of our proposed fast pyrolysis facility is considered capable of processing 100 metric ton of feedstock per day (MT/day). Since the biofuel production is a long-term goal, and Iowa, being one of the greatest farming states in the United States would contribute significantly in the cellulosic biofuel market, the refinery location is assumed to be at the center of Kossuth County, which maximizes the long-term profitability of the whole biofuel production in Iowa according to Li et al. (2014). The gasoline demand location is considered at a metropolitan statistical area, Mason City, which is the closest one from the refinery location.

A land of 2,000 acres is to be invested to grow switchgrass as the second source of the facility’s feedstock, which supports approximately 30% of the feedstock in a high-yield year. Switchgrass related data are obtained from a research conducted by University of Kentucky (2013), which estimates the land establishment cost to be $295/acre during the lead time, including labor, trucking
and loading, production, reseeding, and fertilizer costs. After the land lead time, the operational cost for the land is $312/acre.

The technology data for a fast pyrolysis facility are based on Brown et al. (2013). The final product after fast pyrolysis, hydroprocessing, and refining are naphtha-range and diesel-range liquid transportation fuels, and the fuel amount is measured by gasoline gallon equivalent (gge). The unit operating cost of fuel is $1.091/gallon excluding feedstock cost. Since analyses on fast pyrolysis conclude that the conversion rate of corn stover and switchgrass to pyrolysis oil are very close (Mullen and Boateng, 2008; Mullen et al., 2010), in this work, we assume stover-based and switchgrass-based biofuel share the same unit operating cost. The bio-oil conversion rate from the feedstock is 0.63, and the fuel conversion rate from bio-oil is 0.414. Capital cost of a facility with 100 MT/day is estimated to be $39 million (BLS, 2017), and increases with an inflation rate of 1.5% per year. A tax credit from facility depreciation is also included.

Besides the operating costs, logistic costs and federal and state excise taxes on gasoline are included through the supply chain. The logistic costs include transportation of hydroprocessed bio-oil from fast pyrolysis facility to refinery and gasoline from refinery to the demand location via truck. The variable transportation cost is estimated a the national average truck shipping cost, $0.26/ton-mile (BTS, 2012). The shipping distances are estimated by great circle distances, the shortest distance between any two locations on a sphere, times a circuity factor 1.22 for truck transportation mode (CBO, 1982). The excise tax imposed on the fuel sale is assumed to be $0.491/gallon, which is the fuel tax rate in Iowa in 2017 (API, 2017).

The risk-free interest rate is set to be 5%, since the average long-term U.S. treasury yield over the last 20 years is approximately 5%.

### 3.4 Numerical Result

Given the two investment opportunities, the facility and the land for the second crop, the optimal investment strategy refers to the optimal immediate actions at $t = 0$, which have four possibilities: (i) do nothing (Wait for short), (ii) invest in facility now and land later (P or Primary
for short), (iii) invest in land now and facility later (S or Secondary for short), and (iv) invest in both the facility and land now (Both for short). The optimal strategy depends on three values at $t = 0$: the fuel price $P_0^F$, the land price $P_0^L$, and the yield level $Y_0$ (high or low). The minimum prices of $P_0^F$ and $P_0^L$ that prompt an immediate action are called threshold prices for that action. I’m not sure if we could still use this term, especially when this is no longer valid in risk-averse cases.

3.4.1 Lead time effect for risk-neutral valuation

Proposition 1 states that the optimal investment strategy depends on the lead times $L_1$ and $L_2$. The result for three pairs of lead times are shown in Fig. 3.6 under two different yield levels ($\Delta = 1/0$ for high/low yield). With $L_2$ fixed at 2 years, by changing $L_1$ from 3 to 2, and to 1, these three cases capture the relationship of three different relative sizes for $L_1$ and $L_2$.

By comparing Fig. 3.6(a) - (c) with the sub-figures (d) - (f), one can see the effect of the yield. Overall, when the yield is low, one requires a more favorable market condition (higher fuel price and/or lower land price) to prompt the same investment taken when the yield is high. Since the effect of yield on the optimal investment strategy is subtle, to save space we will only report the result when the yield of the current year is low in the sequel.

In Fig. 3.6(a) and (d), where $L_1 > L_2$, the only two optimal actions are Wait and Primary. The result is consistent with Proposition 1, which states that the action of Secondary is never optimal to be taken prior to Primary. One can see the actions involving the land investment in the other sub-figures. In Fig. 3.6(b) and (e), where $L_1 = L_2$, the facility and the land can be invested if the fuel price is high and land price is low, respectively. Since there is no lapse of the lead times, both the facility and land can be invested simultaneously such that the second crop can be ready to use as soon as the facility is operational. In Fig. 3.6(c) and (f), land can be invested even before the facility because it has a longer lead time.

When the DM takes the Primary action to invest in the facility, this does not mean that she has forgone the land investment. Using simulation, the expected optimal land investment time $E[\tau_2^*]$ is
superimposed to the sub-figures. It can be seen that while the DM takes the Primary action, she has in mind to invest in the land at a later time.

Figure 3.6 Optimal strategy at \( t = 0 \) with different lead times and yield levels (\( \Delta = 1/0 \) for high/low)

3.4.2 Utility function

In this work, we consider the following utility function \( U_\gamma \) parameterized by \( \gamma \in (0, 1] \) for describing the risk aversion of the DM.

\[
U_\gamma(z) = \begin{cases} 
  z^\gamma, & \text{if } z > 1 \\
  z, & \text{if } 1 \geq z > -1 \\
  2z + (-z)^\gamma, & \text{if } z \leq -1 
\end{cases}
\]  (3.9)
\( U_\gamma(z), \gamma \in (0,1] \) in (3.9) is continuous, monotonically increase, and concave (Figure 3.7). Also, the smaller the value of \( \gamma \) the more risk-averse is the DM, and when \( \gamma \to 1 \), (3.9) converges to the identity function, which represents a risk-neutral DM.

![Figure 3.7 Utility function \( U_\gamma(z) \) representing risk-aversion](image)

In the remainder of this work, we will focus on the case where \( L_1 = 1 \) year, and \( L_2 = 2 \) years, in which all four immediate actions are possible to be optimal. In Fig. 3.8(a)-(f), risk aversion of the DM is imposed by increasingly decreasing the value of \( \gamma \) using the proposed utility function \( U_\gamma \). In each sub-figure, the dotted lines show the result of the previous sub-figure (for Fig. 3.8(a) it refers to the case \( \gamma = 1 \), which is Fig. 3.6(c)) to illustrate the changes.

It can be seen that the more risk-averse (smaller \( \gamma \)), the DM is less motivated towards making investments in the sense that all regions corresponds to making investments are moved towards southeast indicating higher threshold price for biofuel and lower threshold price for land. Also the regions involving land investment shrink and eventually disappear when \( \gamma \) is small, indicating the DM is not willing to take the risks of the land investment. More interestingly, as soon as the risk aversion is imposed, the boundary for land investments is no longer smooth. For example, the boundary for regions “S” and “Both” in Fig. 3.6(c) is smooth where \( \gamma = 1 \). However, a kink or change of slope can be seen in Fig. 3.8(a)-(d) of the corresponding boundary. The region around
Figure 3.8  Threshold of different optimal immediate actions when changing γ in $U_\gamma(z)$

the kink in Fig. 3.8(b) is enlarged in Fig. 3.9. In this figure, when the land price is $8,500/\text{acre}$, changing the fuel price corresponds to the horizontal line in Fig. 3.9 that crosses points A to F. Along this line, the DM’s optimal strategy for the facility investment is to invest when the fuel price is to the right of point C. For the land investment, the optimal strategy is less intuitive: one should invest when the fuel price falls within the intervals [B, C] and [D, E]. That is, under some land price the optimal decision is to alternately invest and not invest in land over as many as five intervals of biofuel price.

To interpret the result in Fig. 3.9, we introduce two properties in the following two propositions that are associated with the risk aversion behavior of the DM characterized by a smooth utility function $U_\gamma(\cdot)$. Note that both propositions consider a styled, two-period cash flow stream, with a
cost component $C < 0$ invested at $t = 0$ and a benefit $B > 0$ is obtained at $t = 1$. We also do not consider budget constraints of the DM in this context.

**Proposition 2** (Simultaneous investments) Given two investment opportunities whose aggregate costs and benefits are $B_i > 0$ and $C_i < 0$, $i = 1, 2$, if the DM is indifferent to investing in any or none of them, the following two statements are true: (i) A risk-neutral DM remains indifferent to investing in both opportunities simultaneously ($\gamma = 1$). (ii) A risk-averse DM would not want to invest in both opportunities simultaneously.

**Proof.** When $\gamma = 1$, $U_\gamma(z) = z$. Apparently, $U_\gamma(B_1 + B_2) + U_\gamma(C_1 + C_2) = U_\gamma(B_1) + U_\gamma(B_2) + U_\gamma(C_1) + U_\gamma(C_2)$. Since there is no change in the valuation, the DM remains indifferent to none or any combination of these two investment opportunities without any budget constraints. When the DM is risk averse ($\gamma < 1$), $U_\gamma$ is concave and we have $U_\gamma(B_1 + B_2) < U_\gamma(B_1) + U_\gamma(B_2)$ and $U_\gamma(C_1 + C_2) < U_\gamma(C_1) + U_\gamma(C_2)$. These two inequalities imply that

$$U_\gamma(B_1 + B_2) + U_\gamma(C_1 + C_2) < U_\gamma(B_1) + U_\gamma(C_1) + U_\gamma(B_2) + U_\gamma(C_2).$$

(3.10)

That is, the DM is losing value for investing in both opportunities simultaneously. Given that the DM is originally indifferent to investing in any or none of them, at least one of these two opportunities is no longer investable. \(\square\)
There are at least two implications that can be derived from Proposition 2.

- When $\gamma = 1$, it is possible that regions Wait, P, S, and Both coincide, as shown in Fig. 3.6(c), in which there is one point where the DM is indifferent to wait, primary opportunity, secondary opportunity, and both.

- When $\gamma < 1$, regions Wait and Both will no longer be adjacent but be separated by either regions P or S, as shown in Fig. 3.8(a) - (d).

- When risk aversion is imposed, the DM becomes more “calculating”. Proposition 2 shows that the DM will not invest in two projects simultaneously without seeing any synergy, which is required to reverse the inequality of (3.10). In this context, the synergy comes from the ability that the second crop can reduce the production cost of the primary facility.

While synergy is appreciated, the next proposition shows that unless the synergy can keep pace with market flourishing, the appreciation for any limited synergy will eventually diminish as market flourishes.

**Proposition 3** (Diminishing appreciation of limited synergy) Consider a project with aggregate benefit and cost $B > 0$ and $C < 0$. Suppose that an ancillary project can yield a fixed cost saving $\Delta C > 0$ to the original project. The appreciation of this cost saving $U_\gamma(B + C + \Delta C) - U_\gamma(B + C)$ (i) is indifferent to a risk-neutral DM regardless of the value of $B + C$; and (ii) is decreasing to a risk averse DM as $B + C$ increases.

**Proof.** When $\gamma = 1$, $U_\gamma(B + C + \Delta C) - U_\gamma(B + C) = \Delta C$, which is a constant. But when $\gamma < 1$, $U_\gamma(B + C + \Delta C) - U_\gamma(B + C) = U'_\gamma(\xi)\Delta C$, where $\xi \in (B + C, B + C + \Delta C)$. Since $U'_\gamma(\cdot)$ is concave when $\gamma < 1$, $U'_\gamma(\cdot)$ is a decreasing function. The property is proved.

Consider the horizontal line in Fig. 3.9 crossing points A to F, as the fuel price increases, according to Proposition 1 due to the longer lead time of the land investment, it is possible that the optimal strategy is to invest in “land first and facility later”, which explains the interval $[B, C]$. When the fuel price continues to increase, the facility value increases. A natural outcome would be...
to invest in both simultaneously, which unfortunately is not preferred with risk aversion as shown in Proposition 2. Therefore, the DM’s preference has to change to investing in “facility first and land later,” which creates the price gap \([C, D]\). While the land and the second crop can bring some cost saving, as the fuel price increases, the facility produces more and eventually reaches the max capacity, so does the cost saving. As the revenue continues to increase, the value of the cost saving is diluted, and eventually the land option is not attractive as described in Proposition 3. This explains while the land the second crop are only of interest within a finite interval \([D, E]\). The fact that the optimal investment strategy for the second crop occurs in disjointed price intervals indicates that the traditional threshold method is no longer optimal in this case.

**Mitigating supply uncertainty**

Fig. 3.10 (a) and (b) show the project values of the proposed pyrolysis facility investment under risk neutral and risk aversion using simulation. In each sub-figure, we show the histograms of the project value (PV) with and without the second crop. The corresponding statistics of these cases are summarized in Table 3.1. Using simulation, the evolution of the underlying uncertainties are simulated following (3.6) - (3.8). At each time period, the DM makes the investment decisions based on the optimal criteria obtained for each time period, similar to the one in Fig. 3.8 (b) that is for \(t = 0\). Even with risk aversion in Fig. 3.10 (b) we obtain the real project value (\$) instead of the utility value; noting it’s the utility value that was maximized. When the initial market condition of the fuel price is \(P_0^F = \$3.0/\text{gallon}\) and land price is \(P_0^L = \$5000/\text{acre}\), it can be seen from Table 3.1 that the second crop does increase the expected project value and mitigate the supply risk by lowering the 5% Value-at-Risk (VaR) and conditional VaR (CVaR) in the risk neutral case. When risk aversion \((\gamma = 0.995)\) is imposed, while the second crop still increases the expected project value, the 2nd crop does not lower the VaR. This is because there is a probability as high as 70% that the DM completely abandon the project, i.e., \(\text{Prob}(x_{1T} = x_{2T} = 0) = 70\%\). On the other hand, with the second crop, the averse DM feels very comfortable to invest in the project.
Figure 3.10  Project value frequency charts at \( P_F^L = 3.00 \) gallon, \( P_L^0 = 5,000 \) /acre: (a) risk neutral (b) risk averse with \( \gamma = 0.995 \)

Table 3.1  Project valuation

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>2nd Crop</th>
<th>( E[PV] )</th>
<th>( \text{Std}[PV] )</th>
<th>VaR (5%)</th>
<th>CVaR (5%)</th>
<th>Prob(( x_{1T} = 0 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yes</td>
<td>$7.18M</td>
<td>$15.3M</td>
<td>-$17.34M</td>
<td>-$19.60M</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>$2.78M</td>
<td>$13.0M</td>
<td>-$18.76M</td>
<td>-$20.97M</td>
<td>9%</td>
</tr>
<tr>
<td>0.995</td>
<td>Yes</td>
<td>$7.05M</td>
<td>$14.8M</td>
<td>-$16.58M</td>
<td>-$18.43M</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>$2.04M</td>
<td>$8.1M</td>
<td>-$8.69M</td>
<td>-$14.24M</td>
<td>70%</td>
</tr>
</tbody>
</table>

3.5 Discussion

The anormal result presented in Fig. 3.9 is a result of three main factors of two interrelated investment options: the difference of their lead times, DM’s risk aversion, and the limited cost saving of the “secondary” option on the primary option, as described in Propositions 1-3, respectively. However, two assumptions made in this work also contribute indirectly to the “discontinuity” of the land investment decision: (i) land is a money-losing investment, and (ii) land is an irreversible investment. In reality, these two assumptions are not necessarily true. When these two assumptions
are relaxed, the land investment becomes very popular because its benefit is no longer limited to the cost saving but also land value growth. In Figure 3.11, the threshold of optimal immediate decisions show that, when land drift is as high as the risk-free-rate (5%), the land investment is always favorable regardless of the initial land price, and the “discontinuity” phenomenon is no longer observed in this case. The question is: can we obtain similar results in more realistic settings? Does the “discontinuity only occur in the threshold price of the secondary investment, and can it occur to the primary investment? It turns out that the answers are all “yes”. We discuss another example in the next section.

![Figure 3.11 Threshold of different optimal immediate actions when land drift $\mu^L = 0.05$](image)

(a) Risk-neutral  
(b) Risk-averse

3.6 Extension

To duplicate the “discontinuity of the threshold prices, we focus on the interplay between risk aversion and lead times and consider a broader class of investment problems that meets the following three realistic conditions under risk aversion: (i) There is a primary asset to invest. (ii) A secondary (ancillary) option is available that adds value to the primary investment. (iii) The lead time of the secondary option is longer than the primary one. Note that in (ii) the added value from the secondary option does not have to be limited. If it is limited, this may create an additional cap of the threshold price for the secondary option as point E in Fig. 3.9.
A firm is considering investing in a production facility (primary) that would produce a gadget at a fixed rate of $Q$ units per year. Assume the capital cost and the lead time for constructing the facility are $I_1$ and $L_1$. On top of the production facility, the DM can also invest in an addition (secondary), which increase the production rate from $Q$ to $KQ$ with $K > 1$, which requires a capital cost $I_2$ and a lead time of $L_2$ to construct it. Assume the gadget’s price $P_t$ follows a GBM: $dP_t = \mu P_t dt + \sigma P_t dW_t$ with current price $P_0$. With the production facility and/or the addition, the firm receives revenue from product sale $Q \times P_t$ or $KQ \times P_t$ in time period $t$ as appropriate. This project is valued with a life of $T$; and both the facility and the addition have no salvage value. The same utility function $U_\gamma$ is applied in the valuation. Detailed data is given in Table 3.2.

<table>
<thead>
<tr>
<th>Table 3.2 Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$T$</td>
</tr>
<tr>
<td>$L_1$</td>
</tr>
<tr>
<td>$r$</td>
</tr>
<tr>
<td>$Q$</td>
</tr>
</tbody>
</table>

The optimal investment of the facility with the addition option follows the formulation given in (3.5) - (3.1) with the profit function at each period being $Q \times P_t$ or $KQ \times P_t$ as appropriate, where the fixed production cost is assumed to be zero for simplicity. The result on the (immediate) optimal investment decisions are displayed in Fig 3.12 with various $\gamma$ values (decreasing from 1.00 to 0.94). From the result, we can see that in the risk neutral case ($\gamma = 1$), region “W” (wait) and region “Both” are adjacent and are separated by either region “P” (primary) or region “S” (secondary), as illustrated by Proposition 2. Depending on the common boundary between regions “P” and “S”, various “discontinuity” in the investment threshold price is created.

In Fig. 3.12(b) and (c), along the dotted line one can see that it goes across regions “Wait” - “Secondary” - “Primary” - “Both”, where reveals a gap in the intervals of the threshold price for the secondary investment, which is the same as what have been observed previously. Fig. 3.12(d) and (e) are more interesting. The dotted line in each of these two sub-figures go across regions
Figure 3.12  Threshold at $t = 0$ for the simple example. 1–wait, 2–facility only, 3–addition only, 4–both facility and addition

“Wait” - “Primary” - “Secondary” - “Primary” - “Both”, where the discontinuity in threshold price intervals is observed in both the primary and secondary investments.

To further understand the effect of Fig. 3.12(e), we plot the project utility values by different strategies: “Wait”, “Primary” (primary first and secondary later), “Secondary” (secondary first and primary later), and “Both” (invest both primary and secondary) at time 0 in Fig. 3.13. To better illustrate their differences, all four values are plotted against the value corresponding to “Both”, which is now the horizontal axis. From Fig. 3.13 it can be seen that the two curves corresponding to “Primary” and “Secondary” intertwine, which creates the inverted v-shaped common boundary.
between regions S and P, due to the nonlinearity of the aggregate of the utility function values applied over the cash flows.

![Figure 3.13 Relative project utility values under different strategies vs. “Both”](image)

3.7 Conclusion

In this work, a real options approach has been proposed to value a fast pyrolysis facility investment together with a land investment growing feedstock for biofuel production. The valuation is performed under fuel price, land price, and supply uncertainties. We intended to provide the investment strategy for a DM as to taking the optimal investment action given current market condition.

Our results indicates that growing our own feedstock can lower the operating cost and mitigate the impact of supply risk. Investment timing of land is restricted by the land and facility lead times, in that it could not be optimal if the operation of land is for sure prior to the operation of facility. This consideration of risk-aversion creates unintuitive influence on the investment decision. Due to the interdependency of the primary investment and the secondary investment, the investment decision of the value-adding option can be very sensitive to the primary underlying uncertainty. The immediate action towards land investment can no longer be described with a single fuel price.
threshold given a land price, but alternates between invest and not invest in land over as many as five intervals in fuel price.
CHAPTER 4. VALUATION OF CLEAN TECHNOLOGY ADOPTION IN ELECTRICITY SECTOR CONSIDER POLICY UNCERTAINTY: A REAL OPTIONS APPROACH

Policy choice in combating climate change has received long-lasting attention. We consider the impact of two types of policy instruments, carbon tax and emission permit, to the adoption of clean energy generation in power sector. We examine the timing that a producer with an existing coal-fired generator adopts in a clean generation from natural gas, in order to meet the load obligation while maximizing its expected long-run profit with regulated emission-related costs involved. In this work, we consider uncertainties of government policies, in addition to the critical uncertainty of commodity prices. Two basic types of policy scenarios are considered, risk of repealing an existing policy and risk of a policy change. A real options approach is applied to assess the power producer’s optimal adoption decision. Not surprisingly, policy uncertainties always delay adoption decisions. Unless with highly favorable (potential policy environment), power producers would prefer to wait and see until the outcome prevails.

4.1 Introduction

Despite significant environmental and social benefits, clean energy is economically and technically disadvantaged (WEA, 2004). Although the final global trend driving the growth of adoption and investment in clean energies is the improvement in efficiency, reliability of the technology, and long-term sustainability; from today’s standpoint, policy incentives are still the key driver for investing in clean technologies (BNEF, 2013). Getting the right type of investment in infrastructure for energy supply and consumption is a minimum requirement to enable the transition towards a sustainable energy system. One of the key tasks of climate change policy-makers is therefore to create incentives to encourage the necessary investments to be undertaken. Three primary types
of environmental policies (or climate policies) have been implemented in many countries around the world, they are mandates (e.g., renewable portfolio standard), incentives (e.g., capital subsidy), and markets (e.g., tradable green electricity certificates).

For example, a Europe emission trading system (EU ETS) directive was adopted in 2003 and the system was launched in 2005. The EU ETS started from a 3-year pilot of “learning by doing” phase 1 (2005–2007) in preparation for phase 2 (2008–2012); up until now, it has established to phase 3. The EU ETS improved from covering only CO₂ emissions to considering CO₂, N₂O, and perfluorocarbons (PFCs); from considering power generators and energy-intensive industries to mandating more companies into participation. It has successfully established a carbon price and enabled free trading in emission allowances across the EU, and aims to link with other compatible systems. The Regional Greenhouse Gas Initiative (RGGI) is the first mandatory market based program in the United States to reduce greenhouse gas emissions. This scheme caps emissions from power generation in ten north-eastern US states.

In Australia, the policy situation is more complicated. In late 2010, Prime Minister Gillard promised no carbon tax during her campaign. After election, carbon tax was introduced by the Gillard Government and became effective on Jul. 1, 2012. Initially, the price of a permit for one tonne of carbon was fixed at $23 for the 2012–13 financial year, with unlimited permits being available from the Government. The fixed price rose to $24.15 for 2013–14. In 2011, opposition leader Abbott vowed to repeal carbon tax if he was elected in Nov. 2013. After Abbott was elected, carbon tax got repealed by the Australian senate on Jul. 14, 2014. Back in 2012, the government announced a transition to an emissions trading scheme in 2014–15, i.e., a “hybrid” structure with carbon tax implemented at the moment and a potential transition to emission trading in the future, where the available permits will be limited in line with a pollution cap (primarily applied to electricity generators and industrial sectors). In 2012, Australia agreed to connect with Europe’s carbon trading program in 2015, the first step toward a global trading system. At that time, power producers in Australia face two types of policy uncertainties, policy repeal and policy change. One
may wonder how a power producer would react to such uncertainties if one was considering the adoption of clean technology.

Falling behind EU in green policy implementations, in the U.S., Australia and other countries and regions, how their potential climate policies would effect clean technology adoption also attracts academic attentions. A rich body of literature has considered climate policy effects to clean energy generation. Zhou et al. (2011) presented a bilevel optimization approach to design incentive policies for stimulating investments in renewable energy. Their model considered capacity investment in traditional coal-fired and renewable wind generation, where tax would be imposed to the former while subsidy would be applied to the latter during the investment and producing. Kim (2015) offered new perspectives on the problem of environmental regulation enforcement by developing a novel analytical framework that combines law enforcement economics with reliability theory. The regulator determines inspection frequency and penalty amounts to minimize environmental and social costs, performing either random inspections or periodic inspections. Chen and Tseng (2011) explored the optimal investment timing when a coal-fired plant owner considers introducing clean technologies in face of carbon tax and tradable permits via a real options approach. The impact on profitability and investment timing resulting from climate policies has been studied. Although different policy perspectives have been discussed, the potential uncertainty in policy has not been addressed.

As stated in BNEF (2013), all energy projects are prone to policy risks. Retroactive cuts in financial support impacts financing arrangements of investors, yet the impact is declining as technology costs decline. Any unscheduled reductions in the level of financial support or regulation changes may impact investors’ confidence in the sector. Insuring against retroactive or unscheduled policy changes remains a topic of interest to investors seeking highly predictable returns. However, such eventualities are difficult to predict and therefore expensive to insure.

Blyth et al. (2007) described investment decision towards carbon capture and storage (CCS) technology in power generation subject to uncertain future climate policy, which is treated as external risk factor over which the company has no control of. Policy uncertainty is represented
as an exogenous event that creates uncertainty in the carbon price. Zhou et al. (2010) presented a real options model incorporating policy uncertainty described by carbon price scenarios (including stochasticity), allowing for possible technological change. The model was used to determine the best strategy for investing in CCS in an uncertain environment in China. Reuter et al. (2012) used a real options model in discrete time with lumpy multiple investments to analyze the decisions of an electricity producer to invest into new power generating capacity. The framework was used to analyze energy policy, as well as the reaction of producers to uncertainty in the political and regulatory framework. Policy uncertainties were discussed in these articles, yet in a rough manner, where only one type of policy is considered, and the uncertainty was represented as a stochastic process in one single parameter.

Gatzert and Vogl (2016) provided a stochastic model framework to quantify policy risks associated with renewable energy investments, thereby also took into account energy price risk, resource risk, and inflation risk. It made use of expert estimates and fuzzy set theory to quantify policy risks. Iychettire et al. (2017) assessed the impact of policy choices by proposing renewable energy sources for electricity (RES-E) support policy as a combination of components (design elements) such as, price warranty versus quantity warranty, technology specificity versus technology neutrality that are common to all renewable electricity support schemes.

To bridge the above-mentioned gaps, this work assumes that a producer that owns a traditional coal-fired generator and is seeking the opportunity of adopting natural gas generation in the future. The producer faces price uncertainties of electricity, natural gas and carbon emission, as well as policy uncertainty. The optimal investment time is examined with a per MW analysis using a real options approach, subject to construction lead time and optimal dispatching decision during operation. We consider two realistic policy changing scenarios: risk of repealing an existing policy and risk of a policy change; their effects on producer’s behavior are discussed.

The rest of the chapter is organized as follows: In Section 4.2, an economic model is presented with uncertain factors modeling and a brief solving procedure. In Section 4.3, a case study is presented with numerical results. We conclude the chapter in Section 4.4.
4.2 Model Formulation

In this section, detailed model formulation, including the assumptions, uncertain factors, and mathematical model description are provided.

4.2.1 Problem description and assumptions

Due to the mounting pressure for employing clean power-generating technologies, a producer considers to adopt a clean power plant. This work assesses the capacity expansion decision-making of the producer who intends to build a new natural gas (NG) plant in addition to an existing coal-fired plant, subject to price uncertainties in electricity, NG, and carbon prices, as well as the uncertainty in CO$_2$ policy.

The expansion project is valuated under the following assumptions

- The producer is a price taker (producing a small amount of power relative to the entire market).

- The producer is subject to a load obligation, and the NG plant, if invested, is capable of generating clean energy to replace utmost $\bar{\alpha} \in (0, 1]$ of the existing coal-based power generation.

- The expansion decision in the NG unit is irreversible, and is considered at the beginning of each month within the decision period if not exercised yet. The decision period is considered to be 10-year.

- The coal-fired plant is considered to be perpetual with regular, continuous maintenance and upgrade, while the valuation focuses on the life cycle of the NG plant.

Following the approach used in Chen and Tseng (2011), a per MW analysis is used in this model.

The revenue of running a power plant comes from providing electricity at market price $X_t$ ($/\text{MWh}$), where the cost includes fixed and variable O&M (operations and maintenance) cost, fuel cost, and the emission cost. With the existing coal-fired plant, the costs of generating one
MWh electricity at \( t \) include fuel cost \( C_{\text{cl}} \) (\$/MWh, assumed to be fixed), fixed and variable O&M cost \( C_{\text{OM}}^{\text{OM}} \) (\$/MWh) with coal generation, and emission cost.

\[
f^t_1(X_t, Y_t) = f^t_{\text{cl}}(X_t, Y_t) = X_t - C_{\text{cl}} - C_{\text{OM}}^{\text{OM}} - Q^E_1 Y_t \tag{4.1}
\]

where, \( Q^E_1 \) (ton/MWh) is the quantity of CO\(_2\) emission when generating one MWh from coal, and \( Y_t \) (\$/ton) represents the unit carbon emission cost. The process of carbon price \( Y_t \) (\$/ton) depends on the policy in action at \( t \), which will be explained in more details in Section 4.2.2 and 4.2.3. If an NG plant is built and is in operation, using the NG plant to generate one MWh yields a profit of

\[
f^t_{\text{ng}}(X_t, Y_t, G_t) = X_t - G_t H - C_{\text{OM}}^{\text{OM}} - Q^E_2 Y_t \tag{4.2}
\]

where \( G_t \) (\$/MMBtu) is the NG price, \( H \) (MMBtu/MWh) is the heat rate of the NG plant, \( C_{\text{OM}}^{\text{OM}} \) (\$/MWh) is the fixed and variable O&M cost with NG generation, and \( Q^E_2 \) (ton/MWh) is the quantity of CO\(_2\) emission when generating one MWh from NG. With the operation of the new NG plant, the producer has the flexibility of dispatching power generating to both power plants, and here we assume both power plants are operated in the way that the per MWh profit is maximized.

\[
f^t_1(X_t, Y_t, G_t) = \max_{0 \leq \alpha_t \leq \bar{\alpha}} \left[ (1 - \alpha_t) f^t_{\text{cl}}(X_t, Y_t) + \alpha_t f^t_{\text{ng}}(X_t, Y_t, G_t) \right]
\]

\[
= \begin{cases} f^t_{\text{cl}}(X_t, Y_t), & f^t_{\text{cl}}(X_t, Y_t) < f^t_{\text{ng}}(X_t, Y_t, G_t) \\ (1 - \bar{\alpha}) f^t_{\text{cl}}(X_t, Y_t) + \bar{\alpha} f^t_{\text{ng}}(X_t, Y_t, G_t), & f^t_{\text{cl}}(X_t, Y_t) \geq f^t_{\text{ng}}(X_t, Y_t, G_t) \end{cases}
\]

\[
= \max_{\alpha_t \in \{0, \bar{\alpha}\}} \left[ (1 - \alpha_t) f^t_{\text{cl}}(X_t, Y_t) + \alpha_t f^t_{\text{ng}}(X_t, Y_t, G_t) \right] \tag{4.3}
\]

where, \( \alpha_t \) is the dispatch factor at time \( t \) which is limited to the range \([0, \bar{\alpha}]\). Since both \( f^t_{\text{cl}} \) and \( f^t_{\text{ng}} \) are linear in prices, the optimal profit in (4.3) is achieved on the boundary of \( \alpha_t \), i.e., when \( \alpha_t = 0 \) or \( \bar{\alpha} \).

It is assumed that the decision maker considers to build an NG plant in the next 10-year horizon. Let \( \tau \) (month) denote the investment time, which is a random variable. A lump-sum capital cost would be incurred at \( \tau \). The lead time of the NG plant is considered to be \( \nu \) months, which include the amount of time for permitting, environmental evaluation, and facility construction. We
would like to investigate the optimal investment timing of the NG plant. The project value of the
investment of an NG plant given an existing coal-fired plant is

\[
J(x, y, g) = \max_{\tau \in \phi([0, T])} \mathbb{E}_0 \left[ \int_0^{\tau+\nu} f_s^0(X_s, Y_s)e^{-rs}ds - \bar{\alpha}K_\tau e^{-r\tau} + \int_{\tau+\nu}^{\tau+\nu+\ell} (f_s^1(X_s, Y_s, G_s) - f_s^0(X_s, Y_s)) e^{-rs}ds \right] \tag{4.4}
\]

where, \(T\) (month) is the length of the decision period, and \(\phi([0, T])\) is the set of stopping times of the filtration with values in \([0, T]\). In (4.4), \(\ell\) (month) is the life of the NG plant, \(r\) is the risk-free-rate, \(K_\tau\) ($/MW) is the capital investment for the NG plant at \(\tau\), and \((x, y, g) = (X_0, Y_0, G_0)\) are the current values of prices at \(t = 0\). The optimal dispatch option when both plants are available for power generation is embedded in \(f_t^1(\cdot)\) defined in Equation (4.3).

By rearranging the terms in Equation (4.4), we have

\[
J(x, y, g) = \max_{\tau \in \phi([0, T])} \mathbb{E}_0 \left[ \int_0^{\tau+\nu} f_s^0(X_s, Y_s)e^{-rs}ds - \bar{\alpha}K_\tau e^{-r\tau} + \int_{\tau+\nu}^{\tau+\nu+\ell} \max \{f_s^0(X_s, Y_s, G_s) - f_s^1(X_s, Y_s), 0\} e^{-rs}ds \right] \tag{4.5}
\]

where \(C = C^{cl} + C_1^{OM} - C_2^{OM}\), \(Q = Q_1^E - Q_2^E\), and \(\max \{C - G_t H + QY_t, 0\}\) is the payoff function of a spread option (Carmona and Durrleman, 2003). The first term in Equation (4.5) is the profitability of the assuming perpetual coal-fired plant, where the second term refers to the capital investment and the cumulative payoff of the NG plant investment project, considering optimal investment timing and optimal dispatch in operations. The first term in Equation (4.5) is independent of the
expansion decision, therefore, we could consider the simplified problem in Equation (4.6) to focus on the optimal investment timing of the expansion option.

\[
\tilde{J}(y, g) = \max_{\tau \in \phi([0, T])} \mathbb{E}_0 \left[ -\bar{\alpha}K_t e^{-rt} + \bar{\alpha} \int_{\tau+\nu}^{\tau+\nu+\ell} \max \{C - G_s H + QY_s, 0\} e^{-rs} ds \right] \tag{4.6}
\]

Given the assumption that the coal-fired plant is perpetual, together with the per MW analysis, the uncertainty in electricity price drops out of the simplified valuation function in Equation (4.6). The investment decision in the NG plant is related to the trade-off between the additional cost in operating with NG \(\bar{\alpha}(C - G_t H)\), and the cost saving due to less emission \(\bar{\alpha}QY_t\).

### 4.2.2 Price uncertainty

The price uncertainties considered in this work include prices for electricity \(X_t\) (\$/MWh), NG \(G_t\) (\$/MMBtu), and carbon price \(Y_t\) (\$/MWh). As mentioned in Section 4.2.1, with using the per MW analysis, the uncertain electricity price \(X_t\) drops out of the valuation. The remaining two uncertain price factors are considered to evolve following geometric mean-reverting processes:

\[
d \ln G_t = \mu^g_t (m^g_t - \ln G_t) dt + \sigma^g_t dW^g_t \tag{4.7}
\]
\[
d \ln Y_t = \mu^\gamma_t (m^\gamma_t - \ln Y_t) dt + \sigma^\gamma_t dW^\gamma_t \tag{4.8}
\]

where \(\mu^g_t\) and \(\mu^\gamma_t\) are reverting coefficients; \(m^g_t\) and \(m^\gamma_t\) are the mean levels of NG and carbon emission prices; \(\sigma^g_t\) and \(\sigma^\gamma_t\) are the volatilities; and \(W^g_t\) and \(W^\gamma_t\) are correlated Wiener processes, such that

\[
d W^g_t d W^\gamma_t = \rho dt \tag{4.9}
\]

The fitted mean levels \(m^g_t, m^\gamma_t\) capture the trend and seasonality of the price processes.

With emission trading policy, parameters of geometric mean-reverting price process \(Y_t\), \((\mu^\gamma_t, \mu^\gamma_t, \sigma^\gamma_t)\), are fitted from historical data. In our case study, all parameters are estimated by month, where \(m^\gamma_t\) represents a seasonality pattern over the year. Under the taxation policy, the carbon tax is assumed to remain the same within each year, and \(Y_t\) is reduced to a geometric mean-reverting price process with \(m^\gamma_t\) having an identical value for each year and \(\mu^\gamma_t = \sigma^\gamma_t = 0\).
To facilitate the comparison between emission permits and carbon tax, we introduced a parameter $\bar{y}_0$ as the average carbon price of the first year in the planning horizon, and assume it has an annual growth rate of $r_c$. For the taxation policy, the carbon tax of the $n$-th year is set to be a constant $\bar{y}_0 \exp(nr_c)$; and for emission trading, the mean level of carbon permit price is adjusted such that the average of $\exp(m_t^y)$ within the $n$-th year equals $\bar{y}_0 \exp(nr_c)$. See Figure 4.1 for an illustration.

![Image](image-url)

(a) Emission permit  (b) Carbon tax

Figure 4.1  The profile of the future carbon prices

### 4.2.3 Policy uncertainty

All energy projects are prone to policy risks, while public policies are expected to induce clean technology adoptions. Although command-and-control policy instruments can mandate specific controls on technologies to be installed and to stipulate limits on emissions sources, it is studied that market-based instruments tends to result in an equalization cost across all polluters, which is a basic condition for a least-cost outcome. Here we consider two emission control policies: emission permit and carbon tax. Two cases of policy uncertainties are considered: risk of repealing an existing policy, and risk of a policy change. We assume the potential policy change is announced to occur at a future time $T_p$ (month), with probability $\pi$ that the policy change will indeed occur, representing the probability that it actually comes into action (or decision makers’ belief in such a policy change), and the effect of such policy uncertainty is embedded in the carbon price process $Y_t$. 
Case 1. Risk of repealing an existing policy. In Case 1, we consider the possibility that the current emission policy (carbon tax or emission trading) may be repealed at $T_p$.

Case 2. Risk of a policy change. In Case 2, we replicate what happened in Australia in 2012, when the government has implemented carbon tax. But the government also announced the intention to replace the carbon tax by emission trading in a future time. Without knowing the initial price of the emission trading, we introduce a parameter $\gamma$ to denote the ratio of the average carbon price before and after the policy change at $T_p$. To be more specific, if the current carbon tax is $\bar{y}_0$ (at $t = 0$), at $T_p$, the annual average carbon price would be $\bar{y}_0 \exp(r_c T_p)$. Then the average permit prices in the first year after emission trading is implemented should be $\gamma \bar{y}_0 \exp(r_c T_p)$. In general, $\gamma \leq 1$. See Figure 4.2 for an illustration of the average carbon price shift as the policy type changes from carbon tax to emission permit.

(a) $\gamma = 1$

(b) $\gamma < 1$

Figure 4.2 The profile of the future carbon prices with policy change occurred at $t$ from carbon tax to emission permit
Table 4.1 lists the current ($t = 0$) and anticipating ($T_p$) policy under the two policy uncertainty cases. For Case 2, we model specifically the Australian hybrid-policy situation, and consider a possible transition of policy from carbon tax to emission permit.

<table>
<thead>
<tr>
<th>Case 1: Risk of repealing an existing policy</th>
<th>Current policy</th>
<th>Anticipating at $T_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon tax</td>
<td>Null</td>
<td>Emission permit</td>
</tr>
</tbody>
</table>

| Case 2: Risk of a policy change             | Carbon tax     | Emission permit       |

4.2.4 Solution procedure

The expansion decision is visited on a monthly-basis until the capacity increase has been executed, while the dispatch decisions are made at a higher frequency, on a daily-basis. A two-factor lattice with time dependent parameters and correlation are constructed (Hull and White, 1994; Tseng and Lin, 2007) to approximate the continuous process of electricity and carbon prices (see more details in Appendix C). Backward stochastic dynamic programming (SDP) steps are performed with a small step size (daily in our case) to obtain project value at time 0 and to determine the entry conditions of the expansion option at all possible decision-making points (beginning of each month within the first 10-year in our case).

Monte Carlo simulation method is used to determine the distribution of the optimal investment time ($\tau^*$) for the NG plant. Each simulation run generates a realization of the price uncertainties following the evolution processes in Equation (4.7–4.9) for the entire planning horizon. At each decision time (beginning of each month in the decision period), if the new facility has not yet been invested, the expansion decision is either made or deferred based on the current prices ($Y_t, G_t$) and the optimal exercise conditions (boundaries) identified from the SDP steps. After a total of $N$ simulation runs, we could collect the empirical distribution, as well as the expected values of the optimal investment time and the project value. It is possible that under unfavorable market conditions, the expansion option is never exercised during the planning horizon $[0, T]$. Since one
of our focus is to see how an emission policy affects investment of clean energy technology, for the runs where the expansion option expires without being exercised, we record the adoption time to be \( T \), to enable calculation of an expected wait time \( \mathbb{E}[\tau^*] \).

4.3 Case Study

4.3.1 Data sources

Set up in 2005, the Europe emissions trading system (EU ETS) is the world’s first and biggest international emissions trading system, accounting for over three quarters of international carbon trading. The EU ETS is inspiring the development of emission trading in other countries and regions. The EU also aims to link the EU ETS with other compatible systems. The EU ETS is in phase 3 since 2013, and we obtained the phase 3 daily EUA (EU allowance) price data (Business Insider, 2017) to fit the price parameters for emission permit price process. Henry Hub NG spot prices obtained from the EIA (EIA, 2017a) are used for NG price process parameter fitting.

The price parameters are estimated using the maximum likelihood method following steps in Tseng and Barz (2002). The price parameters estimated by months are summarized in Table 4.2. Based on the historical price data from 2013 to 2017, the fitted correlation coefficient between the NG price and emission permit price is \( \hat{\rho} = -0.3571 \).

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{m}_t^x )</td>
<td>3.30</td>
<td>3.25</td>
<td>3.21</td>
<td>3.24</td>
<td>3.27</td>
<td>3.29</td>
<td>3.25</td>
<td>3.24</td>
<td>3.21</td>
<td>3.25</td>
<td>3.26</td>
<td>3.32</td>
</tr>
<tr>
<td>( \hat{\mu}_t^x )</td>
<td>0.06</td>
<td>0.08</td>
<td>0.09</td>
<td>0.05</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.07</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>( \hat{\sigma}_t^x )</td>
<td>0.05</td>
<td>0.08</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>( \hat{m}_t^{y_1} )</td>
<td>1.88</td>
<td>1.84</td>
<td>1.78</td>
<td>1.83</td>
<td>1.82</td>
<td>1.90</td>
<td>2.01</td>
<td>2.06</td>
<td>2.06</td>
<td>2.04</td>
<td>2.06</td>
<td>1.98</td>
</tr>
<tr>
<td>( \hat{\mu}_t^{y_1} )</td>
<td>0.01</td>
<td>0.09</td>
<td>0.06</td>
<td>0.01</td>
<td>0.04</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( \hat{\sigma}_t^{y_1} )</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
<td>0.06</td>
<td>0.04</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The cost components, heat rate and CO\(_2\) emission are obtained from the 2016 EIA report on capital cost estimates for utility scale electricity generating plants (EIA, 2016c), where the existing coal-fired plant is assumed to follow that with the ultra supercritical coal (USC) technology, and
the expecting NG generator follows that with the advanced natural gas combined cycle (ANGCC) technology. The anticipating NG generator is assumed to has a maximum capacity that could replace \( \bar{\alpha} = 50\% \) of the generation of the existing coal-fired generator. The overnight capital cost of the new NG generator is assumed to increase with an inflation rate of 1.5% per year, and the tax credit from depreciation is also taken into account. We assume the construction lead time for the NG generator is one-year.

Based on the weekly data on coal price in different regions (EIA, 2017b), we assume a constant coal price of $30/ton, and the heat content of coal is in the range of 8,000 Btu/lb to 12,000 Btu/lb, in our case study, we consider the heat content of coal to be 10,000 Btu/lb. For existing coal-fired power plants, heat rates are typically in the range of 9,000 Btu/kWh to 11,000 Btu/kWh (Power, 2014), the heat rate reported for the selected USC technology has an average heat rate of 10,300 Btu/kWh (EIA, 2016c). With these, it is calculated that the cost of coal of the existing coal-fired plant is \(~$14/MWh\).

The risk-free-rate is set to be 5%, equals to the average long-term U.S. treasury yield over the last 20 years (US Treasury, 2014).

A summary of main parameter values is listed in Table 4.3, additional transformation may be required to convert values listed to the desired units used in the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C^\text{cl})</td>
<td>$14/MWh</td>
<td>EIA (2017b); Power (2014)</td>
</tr>
<tr>
<td>(C_1^{\text{OM}})</td>
<td>$42.1kW/yr (fixed)</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(C_1^{\text{OM}})</td>
<td>$4.60/MWh (variable)</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(Q_1^{\text{E}})</td>
<td>206lb/MMBtu</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(C_2^{\text{OM}})</td>
<td>$10.0kW/yr (fixed)</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(C_2^{\text{OM}})</td>
<td>$2/MWh (variable)</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(H)</td>
<td>6,300Btu/kWh</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(Q_2^{\text{E}})</td>
<td>117lb/MMBtu</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(K_0)</td>
<td>$1,104,000/MW</td>
<td>EIA (2016c)</td>
</tr>
<tr>
<td>(r)</td>
<td>5%</td>
<td>US Treasury (2014)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>1-year</td>
<td>Assumed</td>
</tr>
<tr>
<td>(\bar{\alpha})</td>
<td>0.5</td>
<td>Assumed</td>
</tr>
</tbody>
</table>
4.3.2 Numerical results

As environmental policy is deemed as one of the top decisive external factor that impacts the interest of a producer. Comparing the operation of the existing coal-fired generator and the expecting NG generator we could see the economics, although it may seem less costly to produce power from coal than NG, with the consideration of carbon prices (either from tax or permits), coal does not necessarily have the cost advantage than other clean technology such as NG.

In this section, we will see under two policy uncertainty cases, how a future policy change would affect the expansion decision, as well as how would decision maker react when the future policy change is uncertain. We focus on the impact of uncertain factor $Y_t$ on investment timing. All analysis and results below are based on an initial NG price at $g = G_0 = $26.0/MMBtu, which is the fitted mean level of NG price at $t = 0$.

4.3.2.1 Case 1. Risk of repealing an existing policy

In Case 1, a currently implementing emission policy, carbon tax or emission trading, is running with an annual average price of $\bar{y}_0$, and it is announced that the current policy will be repealed at a future time $T_p$ with probability $\pi$. In the special case where $\pi = 0$, i.e., repeal is never possible or even considered, this special case is reduced to a scenario where pure carbon tax (or pure emission permit) is imposed as the carbon control policy, as discussed in Chen and Tseng (2011). With our updated parameter setting, the threshold carbon price for adding the new NG plant at $t = 0$ is $\bar{y}_0 \geq $68.5/ton under carbon tax and $\bar{y}_0 \geq $42.4/ton under emission permit. This is consistent to Chen and Tseng (2011), where emission trading triggers adoption of clean technologies at a considerably lower level of carbon prices relative to a tax policy.

Figure 4.3 shows the expected wait time $E[\tau^*]$ against the first year average carbon price $\bar{y}_0$ of carbon tax or emission permit, and the points where the lines cross X-axis (identified with cross markers) indicate the prices above which immediate investment is optimal to perform, i.e., the threshold prices.
Figure 4.3  Expected wait time $\mathbb{E}[\tau^*]$ against the first year average carbon price $\bar{y}_0$

Figure 4.4 illustrates how would a power producer reacts to the possibility of a policy repeal. It is seen that repealing a current carbon tax or emission permit has similar effect on investment decisions of the producer. If the policy repeal will happen with certainty ($\pi = 1$), it would highly discourage the producer to exercise the expansion option, unless the current carbon price is high enough (above price at cross marker 4 in Figure 4.4), such that the benefit gained from less carbon emission power produced with NG before $T_p$ can overcome the capital cost of the investment and the addition production costs. When repeal time is not longer than construction lead time of the NG plant ($T_p \leq \nu$), the threshold price would be infinity, since regardless of current carbon price, the NG plant would not gain any benefit from adding new capacity. If the repealing is not as credible ($0 < \pi < 1$), we observed that the producer has an “wait-and-see” attitude in investment decision-making, which represents in the figure as the flat region to the right of cross marker 2. Within a long interval of $\bar{y}_0$, the decision maker would like to wait until the proposed repeal time $T_p$ and make decision based on if the repeal actually occurs then. If the policy got repealed, no investment is exercised; and if not repealed, prices above the threshold at cross marker 2 will be favorable for investing at $T_p$, and this leads to an expected wait time of $\mathbb{E}[\tau^*] = \pi T_p + (1 - \pi)T$. The threshold price of immediate investment (at cross marker 3) will be at a much higher level, indicating that only with a high enough incentive will the decision maker take the risk of investing immediately despite of future policy repeal. The invested NG plant would not contribute any clean
generation after the carbon policy repeal, since the producer is making the dispatching decision such that the profit is maximized.

Figure 4.4 Illustration of entry conditions when repealing an existing policy

Figure 4.5 presents the threshold price of immediate investment against proposed repeal time. The threshold price decreases if proposed repeal time is farther in the future, because of longer guaranteed length of time to benefit with a clean technology from current emission policy; and the threshold increases rapidly with $\pi$, which shows that the repealing of existing emission policy defers the expansion decision enormously. It is noticed that the threshold price triggering an immediate investment with emission permit is lower than that with carbon tax. Even if the policy is facing possible repealing, emission trading holds an advantage in clean technology adoption.
4.3.2.2 Case 2. Risk of a policy change

Given the fact that emission trading has the ability of encouraging clean technology adoption at a lower carbon price level, can a government like the Australian government then in 2012, currently implementing carbon tax, lower the threshold price by announcing a future policy change to emission permit? The answer turns out to be “not much”.

Recall that we introduced a parameter $\gamma \leq 1$ representing a shift in average CO$_2$ price level as policy changes from carbon tax to emission permit. In Figure 4.6, threshold prices when facing the potential policy change are presented. When $\gamma$ is close to 1, it indeed slightly lowers the threshold price for immediate adoption of the clean technology, compared with that with pure carbon tax. When $\pi = 0$, this scenario reduced to a case with pure carbon tax without uncertainty. When the policy change is happening for sure ($\pi = 1$), we observed a price interval with $E[\tau^*] = T_p - \nu$. Suppose $T_p > \nu$, this means that the policy change triggers future actions (at $T_p - \nu$) unless the carbon price is significantly high. When less credible ($0 < \pi < 1$), the “wait-and-see” phenomenon is observed, indicating the timing for future action is also uncertain. The producer will wait until $T_p$ to see actual policy realization and decide then. This leads to an expected wait time for the
“wait-and-see” price interval to be $\mathbb{E}[\tau^*] = \pi T_p + (1 - \pi)T$. As $\gamma$ decreases, the threshold price increases, and the curve shifts to the right, approaching to that with pure carbon tax.

As the ratio $\gamma$ continues to decrease, the threshold price for immediate investment would go above the threshold price under pure carbon tax, which indicates that the proposed change of policy design is discouraging immediate clean energy adoption.

As shown in Figure 4.7, the proposed policy change to emission trading with a high enough $\gamma$ decreases the threshold price of adopting the clean technology compared to pure carbon tax. The extent of threshold price reduction is affected by the proposed policy change time and the credibility of the policy change.

In both cases, when the anticipating policy has uncertainty ($0 < \pi < 1$), i.e., it is doubtful whether the anticipating policy will come into action or not, the producer tends to take a “wait-and-see” attitude and delay investment action. That is, the producer would rather wait until $T_p$ to
observe the outcome of the policy change, then make the expansion decision. The future investment action at $T_p$ does not need to face “uncertainty” and the threshold price reduces dramatically without policy risk.

4.4 Conclusions

As a key drive to clean technology adoption, the implementation of proper emission policy is a critical topic in combating climate change. In this work, the optimal expansion decision of an NG generation replacing traditional coal-fired generation is investigated under price and policy uncertainty. Two types of instruments, carbon tax and emission permit, are considered under two realistic policy uncertainty cases: risk of repealing an existing policy; and risk of a policy transition. A real options approach is conducted to study the optimal exercise decision with a per MW analysis.

Since carbon policy is a key driver to clean technology adoption, a possible repealing of the current policy would greatly discourage the producer’s expansion decision. It is observed that to a risk-neutral price-taking power producer, emission permit triggers immediate clean energy adoption at a relatively lower carbon price level than carbon tax. Therefore a change of policy from carbon tax to emission permit could more effectively inducing immediate investment in clean technology.
Our results indicate that a policy uncertainty delays the producer’s investment action, even if the probability of the announced future policy coming into action is small. When facing policy uncertainty, the producer would create a plan for future investment action, and the future plan is insensitive to the current carbon price. The influencing factors to the producer from the policy uncertainty include the proposed policy change time and the credibility of the policy change.
CHAPTER 5. GENERAL CONCLUSIONS

Making use of clean energy resources has become an attractive topic during the past few decades, due to the increasing concerns related to energy security, fuel price volatility, and climate challenge. Getting the optimal investment portfolio in infrastructure for energy supply and consumption is a minimum requirement to enable the transition towards a sustainable energy system. In this report, three topics are presented to shed light on the biofuel technology adoption and clean power generation investment under various sources of uncertainty.

In the first work, the investment of a cellulosic biofuel production technology (fast pyrolysis) is discussed. The project is valuated via a real options approach, given operational constraints of the biofuel facility and uncertain fuel price. The contributions of this work are twofold. In terms of the application, we are the first one to use real options to value a pyrolysis plant, which is an emerging technology for producing advanced biofuels. Our analysis sheds light on the profit and risk of the investment of cellulosic bioenergy production, which has an impact on the sustainable future of the nation’s renewable energy development. In terms of the methodology, we incorporate the operational constraints that impact the valuation and are constantly overlooked by other researchers. These operational constraints include the construction lead time and the production and distribution constraints. As we will demonstrate in this work, overlooking these constraints, especially the lead time, will lead to a significant over-valuation of the asset, which should be avoided in evaluating an investment decision.

In the second piece, a secondary option - land investment for feedstock growing - is provided to the decision-maker, in addition to the primary facility investment for biofuel production. The optimal investment timings for facility and land are investigated subject to nonzero lead times of both investments and production operational constraints. The contribution of this work includes:

(i) Two options related to the biofuel project are considered, with facility investment being primary
and land/feedstock investment being secondary. While the secondary option has a value-adding effect when function with the primary option, it can be exercised ahead of the primary one due to lead time effect. (ii) Multiple sources of uncertainty are considered, including fuel price, land price, and supply uncertainty. (iii) Risk aversion effect brings in unintuitive influence in investment decision-making when there are multiple investments to make, therefore, the traditional threshold investment rule may not be optimal.

All energy projects are prone to policy risk, yet such eventualities are difficult to predict and therefore expensive to insure. Policy uncertainty may refer to uncertainty about monetary or fiscal policy, and the tax or regulatory regime, which will affect spending and investment towards leading businesses. In the third part, we consider a decision-making problem concerning the capacity investment faced by a producer who owns an existing coal plant. As the pressure for employing cleaner power-generating technologies is mounting, the producer considers to adopt clean generation in the future. We want to examine the timing that the producer adopts in a clean generation in order to meet the load obligation while maximizing her expected long-run profit with regulated emission-related costs considered. A real options approach is applied to assess the power producer’s optimal investment decision under price and policy uncertainties. It is observed that a possible repealing of the current policy would greatly discourage the producer’s expansion decision; and a change of policy from carbon tax to emission permit could more effectively inducing immediate investment in clean technology. A policy uncertainty delays the producer’s investment action, even if the probability of the announced future policy coming into action is small. When facing policy uncertainty, the producer would create a plan for future investment action, and the future plan is insensitive to the current carbon price. The policymakers could make use of the findings from the work to predict the behavior of producers due to policy change, predict the impact of policy adjustment, and select proper instruments to stipulate the clean technology adoption.
[https://www.ft.com/content/50bbaec2-ba0e-11e5-bf7e-8a339b6f2164?mhq5j=e1/](https://www.ft.com/content/50bbaec2-ba0e-11e5-bf7e-8a339b6f2164?mhq5j=e1/)


Environmental Protection Agency (EPA), 2010. Regulation of fuels and fuel additive changes to renewable fuel standard program; final rule. Federal Register 2010; Vol. 75.


Lin, C.-Y.C., Yi, F., 2014b. What factors affect the decision to invest in a fuel ethanol plant?: A structural model of the ethanol investment timing game. Working paper, University of California at Davis.

Maier, S. 2017. An approach for valuing portfolios of interdependent real options under both exogenous and endogenous uncertainties. Submitted to 21th Annual International Conference on Real Options.


Thompson, L. M. 1963. Weather and technology in the production of corn and soybeans. Iowa State University, Center for Agricultural and Rural Development Reports.


University of Kentucky. 2013. Switchgrass for bioenergy.


In this appendix, we briefly describe how the risk neutral probabilities are obtained, which is based on Tseng and Lin (2007). To simplify the notation, we consider the following general stochastic process under the risk-neutral space:

$$ dy = \mu(y, t) dt + \sigma dB, \quad (A.1) $$

where $B$ is a Wiener process. To apply (A.1) to the risk-neutral process of (2.16), one can have $y = \ln P_t$ and $\mu(y, t) = \lambda_t (m_t - (r_a - r_f)/\lambda_t - y)$.

To describe the lattice, we use the notation node $(j,t)$ to represent the $j$-th node at time $t$, corresponding to a price $y_{j,t}$ and a drift $\mu_{j,t} = \mu(y_{j,t}, t)$. Denote the price jump of the lattice as a constant $\Delta y$, which equals to $y_{j+1,t} - y_{j,t}$, for any node $(j,t)$. Following Hull and White (1993), $\Delta y$ is set to be $\sigma \sqrt{3\Delta t}$.

The idea of the trinomial lattice is that any arbitrary node $(j,t)$ maps to three adjacent nodes $(j + \kappa_{j,t}, t + \Delta t)$, $(j + \kappa_{j,t}, t + \Delta t)$, and $(j + \kappa_{j,t} - 1, t + \Delta t)$ in the next time period $t + \Delta t$, where $\kappa_{j,t}$ is selected such that $\kappa_{j,t} \Delta y$ approximates the expected price deviation $\mu_{j,t} \Delta t$ with

$$ \kappa_{j,t} = \text{nint} \left( \frac{\mu_{j,t} \Delta t}{\Delta y} \right), \quad (A.2) $$

where nint($x$) is the function that rounds $x$ to the nearest integer. When $x$ is a half integer, the function rounds up (e.g., nint(1.5)$=2$). At each discrete time $t$, we have the following approximation of the diffusion:

$$ E[y(t + \Delta t)] = y(t) + \mu(y, t) \Delta t \quad (A.3) $$

$$ V[y(t + \Delta t)] = E[y(t + \Delta t)^2] - E[y(t + \Delta t)]^2 = \sigma^2 \Delta t \quad (A.4) $$
At node \((j, t)\), the branching probabilities \(p_{j,t}^u\), \(p_{j,t}^m\), \(p_{j,t}^d\) are determined to match the first two moments of the price change at \(y_{j,t}\) as follows.

\[
p_{j,t}^u(\kappa_{j,t} + 1)\Delta y + p_{j,t}^m\kappa_{j,t}\Delta y + p_{j,t}^d(\kappa_{j,t} - 1)\Delta y = \mu_{j,t}\Delta t
\]
(A.5)

\[
p_{j,t}^u(\kappa_{j,t} + 1)^2(\Delta y)^2 + p_{j,t}^m\kappa_{j,t}^2(\Delta y)^2 + p_{j,t}^d(\kappa_{j,t} - 1)^2(\Delta y)^2 = \mu_{j,t}^2(\Delta t)^2 + \sigma^2\Delta t
\]
(A.6)

\[
p_{j,t}^u + p_{j,t}^m + p_{j,t}^d = 1
\]
(A.7)

The solution of (A.5) - (A.7) is

\[
p_{j,t}^u = \frac{1}{2} \left( \frac{1}{3} + \frac{\mu_{j,t}\Delta t}{\Delta y} - \kappa_{j,t} \right) + \left( \frac{\mu_{j,t}\Delta t}{\Delta y} - \kappa_{j,t} \right)^2
\]
(A.8)

\[
p_{j,t}^d = \frac{1}{2} \left( \frac{1}{3} - \frac{\mu_{j,t}\Delta t}{\Delta y} - \kappa_{j,t} \right) + \left( \frac{\mu_{j,t}\Delta t}{\Delta y} - \kappa_{j,t} \right)^2
\]
(A.9)

\[
p_{j,t}^m = 1 - p_{j,t}^u - p_{j,t}^d
\]
(A.10)

It can be shown that \(p_{j,t}^u\), \(p_{j,t}^m\) and \(p_{j,t}^d\) are always between 0 and 1 (Tseng and Lin, 2007).
APPENDIX B. DETAIL OF THE MODEL IN CHAPTER 3

The profit function \( f_t \) varies from state to state. It is summarized in the following table.

<table>
<thead>
<tr>
<th>State of Investment</th>
<th>Profit Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not invested ((x_1t = 0))</td>
<td>( Q \left[ P_t^F - C_t^C(Y_t) \right]^+ )</td>
</tr>
<tr>
<td>Facility construction ( (0 &lt; x_1t &lt; N_1) )</td>
<td>(-\eta Q C_t^{S1} -\eta Q C_t^{S1} -\eta Q C_t^{S1} + Q \left[ P_t^F - C_t^C(Y_t) \right]^+ )</td>
</tr>
<tr>
<td>Facility operation ( (x_1t = N_1) )</td>
<td>(-\eta Q C_t^{S2} -\eta Q C_t^{S2} -\eta Q C_t^{S2} + \eta Q \left[ P_t^F - C_t^S \right]^+ )</td>
</tr>
<tr>
<td>Not invested ((x_2t = 0))</td>
<td>((1 - \eta)Q \left[ P_t^F - C_t^C(Y_t) \right]^+ )</td>
</tr>
<tr>
<td>Land preparation ( (0 &lt; x_2t &lt; N_2) )</td>
<td>(-\eta Q C_t^{S1} -\eta Q C_t^{S1} -\eta Q C_t^{S1} + Q \left[ P_t^F - C_t^C(Y_t) \right]^+ )</td>
</tr>
<tr>
<td>Land operation ( (x_2t = N_2) )</td>
<td>(-\eta Q C_t^{S2} -\eta Q C_t^{S2} -\eta Q C_t^{S2} + \eta Q \left[ P_t^F - C_t^S \right]^+ )</td>
</tr>
</tbody>
</table>

The additional parameters used in the above table are defined below with their values.

\( \eta \) The ratio of the production supported by switchgrass when available (= 0.5)

\( Q \) The production capacity of the fast pyrolysis facility (gge/year)

\( C_t^C \) The production cost of stover-based biofuel at \( t \) ($/gge), including stover cost, shipping cost, operating cost, and fuel tax. This conversion cost is a function of the yield condition \( Y_t \), which would affect stover cost.

\( C_t^S \) The production cost of switchgrass-based biofuel excluding land operational cost ($/gge)

\( C_t^{S1} \) The land preparation cost ($/gge)

\( C_t^{S2} \) The land operational cost ($/gge), including switchgrass growing, trucking/loading, labor costs, etc.

From the profit function, it is clear that production quantity follows a bang-bang solution and is either 0 or the maximum capacity \( Q \) due to the linear term of the objective function.
APPENDIX C. TWO-FACTOR LATTICE CONSTRUCTION WITH TIME DEPENDENT PARAMETERS AND CORRELATION COEFFICIENT

In Appendix A, the construction of a one-factor trinomial lattice and the calculation of transition probability with constant drift and volatility are presented. When facing time-dependent drift and volatility, representing periodic patterns in the data series, we use an extended approach represented in Appendix D in Tseng and Lin (2007) to construct trinomial lattice. The approach is briefly summarized below.

Include an time index to consider time dependency, such that the drift function is \( \mu_t(y) \), and volatility is \( \sigma_t \). To determine the lattice grid current price node maps to in the next time period, we have

\[
y_t + \mu_t(y_t) \Delta t \in \left[ y_t + \left( \kappa - \frac{1}{2} \right) h_{t+1}, y_t + \left( \kappa + \frac{1}{2} \right) h_{t+1} \right]
\]

where price increment \( h_t \) is defined as

\[
h_t = c \sigma_t \sqrt{\Delta t}
\]

Given the fact that when facing time dependent volatility, the price increment varies over time, instead of a relative position shift \( \kappa \), an absolute position with a common base point zero in used. Therefore

\[
\kappa_{t+1} \equiv \left[ \frac{y_t + \mu_t(y_t)}{h_{t+1}} + \frac{1}{2} \right]
\]

i.e., with current price at \( y_t \), it branches into \( (\kappa_{t+1} - 1)h_{t+1}, \kappa_{t+1}h_{t+1}, \) and \( (\kappa_{t+1} + 1)h_{t+1} \) at \( t + 1 \), and

\[
\epsilon_{t+1} \equiv \frac{y_t + \mu_t(y_t)}{h_{t+1}} - \kappa_{t+1}
\]
And the transition probability is calculated as

\[
\begin{align*}
p_{tu} &= \frac{1}{2} \left( \frac{1}{3} + \epsilon_{t+1} + \epsilon_{t+1}^2 \right) \\
p_{td} &= \frac{1}{2} \left( \frac{1}{3} - \epsilon_{t+1} + \epsilon_{t+1}^2 \right) \\
p_{tm} &= 1 - p_{tu} - p_{td}
\end{align*}
\] (C.5-C.7)

When constructing a two-factor trinomial lattice, each price node branches into $3 \times 3 = 9$ nodes in a predetermined price lattice of the next time period, as shown in Figure C.1 (Figure 2 in Chen and Tseng (2011)). We use \((\tilde{p}_{1u}, \tilde{p}_{1m}, \tilde{p}_{1d})\) and \((\tilde{p}_{2u}, \tilde{p}_{2m}, \tilde{p}_{2d})\) to denote the branching probability calculated following Equation (C.5-C.7) of the first and second factor, respectively; time index $t$ is omitted for simplicity. If the two factors are uncorrelated, we could easily calculate the transition matrix as

\[
\begin{bmatrix}
p_{du} & \tilde{p}_{1d} \tilde{p}_{2u} & \tilde{p}_{1d} \tilde{p}_{2u} \\
p_{mu} & \tilde{p}_{1m} \tilde{p}_{2u} & \tilde{p}_{1m} \tilde{p}_{2u} \\
p_{uu} & \tilde{p}_{1u} \tilde{p}_{2u} & \tilde{p}_{1u} \tilde{p}_{2u}
\end{bmatrix} =
\begin{bmatrix}
p_{dm} & \tilde{p}_{1d} \tilde{p}_{2m} & \tilde{p}_{1d} \tilde{p}_{2m} \\
p_{mm} & \tilde{p}_{1m} \tilde{p}_{2m} & \tilde{p}_{1m} \tilde{p}_{2m} \\
p_{mm} & \tilde{p}_{1u} \tilde{p}_{2m} & \tilde{p}_{1u} \tilde{p}_{2m}
\end{bmatrix}
\] (C.8)

Figure C.1 Branching probability
When there is a correlation between the two factors, in Hull and White (1994), the authors proposed to use the following adjustment matrix when correlation coefficient $\rho > 0$

$$
\begin{bmatrix}
  r_{du} & r_{mu} & r_{uu} \\
  r_{dm} & r_{mm} & r_{um} \\
  r_{dd} & r_{md} & r_{ud}
\end{bmatrix}
= 
\begin{bmatrix}
  -\delta & -4\delta & +5\delta \\
  -4\delta & +8\delta & -4\delta \\
  +5\delta & -4\delta & -\delta
\end{bmatrix}
$$ (C.9)

By choosing $c_1 = c_2 = \sqrt{3}$, the authors suggest setting $\delta$ equal to $\rho/36$. Similarly, when $\rho < 0$

$$
\begin{bmatrix}
  r_{du} & r_{mu} & r_{uu} \\
  r_{dm} & r_{mm} & r_{um} \\
  r_{dd} & r_{md} & r_{ud}
\end{bmatrix}
= 
\begin{bmatrix}
  +5\delta & -4\delta & -\delta \\
  -4\delta & +8\delta & -4\delta \\
  -\delta & -4\delta & +5\delta
\end{bmatrix},
$$ (C.10)

where, $\delta = -\rho/36$. The advantage of this approach is its computational efficiency. Though adequate in practice when valuation is the main focus, it is possible that there occurs a feasibility issue, i.e., getting negative or greater than 1 probability. In Chapter 4, we follow the optimization-based approach in Tseng and Lin (2007) to calculate the branching probability matrix. The transition probability $p_{ij}, i, j \in \Omega \equiv \{u, d, m\}$ is obtained by solving the following optimization problem

$$
\min \sum_{i,j \in \Omega} (p_{ij} - \tilde{f}_{ij})^2 \quad \text{(C.11)}
$$

s.t.

$$
\sum_{i \in \Omega} r_{ij} = 0, \forall j \in \Omega \quad \text{(C.12)}
$$

$$
\sum_{j \in \Omega} r_{ij} = 0, \forall i \in \Omega \quad \text{(C.13)}
$$

$$
r_{uu} - r_{ud} - r_{du} + r_{dd} = \rho \frac{c_1 c_2}{c_1 c_2} \quad \text{(C.14)}
$$

$$
0 \leq p_{ij} \leq 1, \forall i, j \in \Omega \quad \text{(C.15)}
$$

Here,

$$
r_{ij} \equiv p_{ij} - \tilde{p}_{ii} \tilde{p}_{2j}
$$

$$
\tilde{f}_{ij} \equiv \int_{\phi(z_{ij})} f(w)dw
$$
where, $f$ is the p.d.f. of the underlying processes at the given node. $\phi(z_{ij})$ follows an intuitive way for assigning surrounding area of the $3 \times 3$ price nodes as illustrated in Figure C.1 (Figure 4 in Chen and Tseng (2011)), $h_1$ and $h_2$ in the figure are price increment of the first and second factor defined as in Equation (C.2).