Multi-stage stochastic and robust optimization for closed-loop supply chain design

by

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DEDICATION

I dedicate my dissertation work to my family for their patience and support while I was away from them to complete my research.

I also dedicate this dissertation to many friends for their help and support on my research path.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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ABSTRACT

This dissertation focuses on formulating and solving multi-stage decision problems in uncertain environments using stochastic programming and robust optimization approaches. These approaches are applied to the design of closed-loop supply chain (CLSC) networks, which integrate both traditional flow and the reverse flow of products. The uncertainties associated with this application include forward demands, the quantity and quality of used products to be collected, and the carbon tax rate. The design decisions include long-term facility configurations as well as short-term contracts for transportation capacities by various modes that differ according to their variable costs, fixed costs, and emission rates.

This dissertation consists of three papers. The first paper develops a multi-stage stochastic program for a CLSC network design problem with demands and quality of return uncertainties. The second paper focuses on robust optimization; particularly, the question of whether an adjustable robust counterpart (ARC) produces less conservative solutions than the robust counterpart (RC). Using the results of the second paper, a three-stage hybrid robust/stochastic program is proposed in the third paper, in which an ARC is formulated for a mixed integer linear programming model of the CLSC network design problem.

In the first paper, a multi-stage stochastic program is proposed for the CLSC network design problem where facility locations are decided in the first stage and in subsequent stages, the capacities of transportation of different modes are contracted under uncertainty about the amounts of new and return products to transport among facilities. We explore the impact of the uncertain quality of returned products as well as uncertain demands with dependencies between periods. We investigate the stability of the solution obtained from scenario trees of varying granularity using a moment matching method for demands and distribution approximation for the quality of returns. Multi-stage solutions are evaluated in out-of-sample tests using simulated historical data and also compared with two-stage model. We observe an instance

of overfitting, in which a scenario tree including more outcomes at each stage produces a dramatically different solution that has slightly higher average cost, compared to the solution from a less granular tree, when evaluated against the underlying simulated historical data. We also show that when the scenarios include demand dependencies, the solution performs better in out-of-sample simulation.

In the second paper, the ARC of an uncertain linear program extends the RC by allowing some decision variables to adjust to the realizations of some uncertain parameters. The ARC may produce a less conservative solution than the RC does but cases are known in which it does not. While the literature documents some examples of cost savings provided by adjustability (particularly affine adjustability), it is not straightforward to determine in advance whether they will materialize. We establish conditions under which adjustability may lower the optimal cost with a numerical condition that can be checked in small representative instances. The provided conditions include the presence of at least two binding constraints at optimality of the RC formulation, and an adjustable variable that appears in both constraints with implicit bounds from above and below provided by different extreme values in the uncertainty set.

The third paper concerns a CLSC network that is subject to uncertainty in demands for both new and returned products. The model structure also accommodates uncertainty in the carbon tax rate. The proposed model combines probabilistic scenarios for the demands and return quantities with an uncertainty set for the carbon tax rate. We constructed a three-stage hybrid robust/stochastic program in which the first stage decisions are long-term facility configurations, the second stage concerns the plan for distributing new and collecting returned products after realization of demands and returns but before realization of the carbon tax rate, and the numbers of transportation units of various modes, as the third stage decisions, are adjustable to the realization of the carbon tax level. For computational tractability, we restrict the transportation capacities to be affine functions of the carbon tax rates. By utilizing our findings in the second paper, we found conditions under which the ARC produces a less conservative solution. To solve the affinely adjustable version, Benders cuts are generated using recent duality developments for robust linear programs. Computational results show that the ability to adjust transportation mode capacities can substitute for building additional facilities as a way to respond to carbon tax uncertainty. The number of opened facilities in ARC solutions are decreased under uncertainty in demands and returns. The results confirm the reduction of total expected cost in the worst case of the carbon tax rate by increasing utilization of transportation modes with higher capacity per unit and lower emission rate.

CHAPTER 1. INTRODUCTION

1.1 Background

It is important for successful enterprises in today's competitive economy to not only be fast and reliable, but also flexible. In linear programming (LP) models optimization that accounts for randomness or uncertainty in application environments yields more flexible solutions. For example, in supply chain applications many factors such as customer demands, travel time, or government decisions cannot be precisely forecasted. Information about the future is most often revealed over time. As an example, only estimates of customer demand are available when decisions are made while the actual demand will be revealed at a later date. Yearly or monthly government decisions might impact the optimal supply chain network design decisions, which are hard to revise once decided. This dissertation considers uncertainties in a mixed-integer LP (MILP) model of CLSC network design, to represent actual situations more realistically than a deterministic model can. The different decision points, such as before or after the realization of uncertain parameters, are called stages. Different stages involve particular decisions. How to deal with a multi-stage decision problem in an uncertain environment is a challenging issue because decisions involved in a stage depend on uncertain parameters realized before that stage.

Stochastic programming (SP) and robust optimization (RO) have evolved as the two primary approaches to deal with uncertain LP that due to randomness in parameters has been studied in many applications and mathematical models. The RO methodology does not require the exact distribution of model uncertainties. However, uncertainties are modeled as random variables with known distributions in SP. If the precise distribution of uncertain quantities is known, optimal solutions yielded by the robust formulation could be overly and unnecessarily conservative (Goh and Sim, 2010). Uncertain parameters with unknown distribution are defined in terms of uncertainty sets in RO, and decisions are optimized in the worst case.

Stochastic optimization can be modeled as two-stage or multi-stage problems. Uncertain parameters in stochastic optimization can be represented by a set of scenarios, each of which specifies both a full set of random variable realizations and a corresponding probability of occurrence. In two-stage stochastic programs, a subset of the decisions that have to be taken without full information of scenarios are called first stage decisions. Once full information is received about the realization of the random parameters (i.e., once the scenarios are observed) the second-stage decisions are taken. Multistage stochastic programs extend the two-stage models by allowing decisions to depend on the realized uncertainties in each stage. The challenge appears with high numbers of scenarios that lead to a dramatic increase in computational difficulty relative to the deterministic case. It is challenging to both obtain a set of probabilistic scenarios that adequately represent the uncertain parameters while not requiring prohibitive computational effort and to evaluate the resulting solution.

Robust optimization, on the other hand, assumes that the uncertain data reside in an uncertainty set and optimizes for the worst-case member of the set. The goal in this optimization approach is to find a best solution that is feasible with respect to the every value in the uncertainty set. RO computational tractability for many classes of uncertainty sets is the reason for its popularity. The challenge appears when solving the robust counterpart (RC) leads to a too conservative solution. One method that has been developed to tackle this problem considers the adjustability of decision variables to the uncertain parameters, by formulating what is called an adjustable robust counterpart (ARC). Similarly to later-stage variables in stochastic programming, adjustable variables tune themselves with uncertain parameters to develop less conservative solutions in ARC. Conditions under which the adjustability may lower the optimal cost of the RC formulation are investigated in this research.

Stochastic programming and robust optimization tools have been applied in many contexts with uncertain parameters. This dissertation focuses on an application that exploits both SP and RO tools in uncertain MILP. This application focuses on designing a closed-loop supply chain (CLSC) networks, which is an MILP under uncertain environment. Few studies examine the ARC and multi-stage stochastic model of this MILP.

The design of CLSC networks integrates both traditional forward flow and the reverse flow of products. Reverse flows manage the recovery of used products for different reasons. One of the most important reasons, due to increased societal awareness, is environmental concerns. Many countries and regions have established legislation to require products to be more environmentally friendly and energy efficient (Zhang et al., 2011). For example, important policies have been issued by the European Union, such as those related to the end of life for automotive products [Directive on End-of-Life Vehicles, 2000/EC] and electrical and electronic equipment parts [Waste from Electrical and Electronic Equipment, 2003/EU] (Zeballos et al., 2012). These regulations would affect the CLSC network design decisions, which include facility configuration that involves a large investment and lead-time, as well as transportation capacities and product movements that would be decided more often.

The MILP formulation of the CLSC design problem studied in this dissertation includes two aspects. In the first aspect, a multi-stage stochastic program is proposed for the design problem with uncertain demands and quality of returned products, in which there are dependencies of demands among periods. A two-stage stochastic program approach has been implemented frequently in CLSC network design. However, the CLSC design must accommodate the constant shifting of customer requirements. Uncertain demands could change every period with dependencies to their previous periods. These dependencies could be the retailers decision on adjusting their next orders based on the history of their customer demands. Also the solution of two-stage stochastic program would not be the optimal one for the constant changes of uncertain parameters in different periods. Adjusting the facility locations would be significantly costly once implemented. The same goes with adjusting transportation units and production flows to the realized demands and quality of returns. The decisions on transportation units may not be responsive to the changes of demands and quality of returns, which may cause shortage or inventory cost. Therefore, the impact of multi-stage stochastic program with uncertain demands and quality of returns on the obtained solution is investigated in this research.

The second aspect of the CLSC application concerns the uncertainty in demands for both new and returned products and also regulation to mitigate the adverse environmental effects of freight transportation, particularly CO_2 emissions. The design and establishment of the supply chain network is a strategic decision whose effect will last for several years, during which the parameters of the business environment such as carbon tax rates and customer demands may change (Pishvaee et al., 2011). Therefore, it is critical to consider these parameters as uncertain in the design stage. Recent research concludes that the earth's average temperature has been increasing significantly over the past century. The cause of this global warming is the build-up of greenhouse gas (GHG) in the earth's atmosphere. Policy-makers have developed regulations concerning carbon emissions that result from industries such as transportation and power generation. One type of regulation is a carbon tax that forces industries or other polluters to pay taxes on their emitted CO_2. Pricing pollution appears to be more successful than other regulatory approaches. Currently, some countries institute carbon taxes with different prices. Because most of the states in U.S have not implemented such a policy, the carbon tax rate is another uncertain parameter considered in the second part of the CLSC application.

1.2 Problem Statement

The problem investigated in this dissertation concerns how to deal with different decisions of multi-stage models in an uncertain environment. Two-stage and multi-stage formulations are well studied in the literature of stochastic programs. However, few studies are related to the use of ARC and multi-stage stochastic program in MILP models of CLSC decision.

The CLSC network design application in an uncertain environment includes long-term decisions of fixed facilities, contracts for transportation capacity by multiple modes and decisions on product flows. Transportation modes differ in operational cost, capacities and emission rates. Interesting research questions regarding the CLSC network design with uncertainty are outlined as follows. What are the efficient combinations of transportation modes among facilities to balance operational against environmental costs? How would historical data of uncertain parameters such as carbon tax affect the choice of transportation modes in order to minimize the overall cost? What design of facility locations and the number and types of transportation modes would be robust regarding these uncertainties in demands and returns? Would the adjustability of transportation modes to uncertain parameters improve the solution?

The multi-stage stochastic program of CLSC network design becomes computationally cumbersome when the number of scenarios rises. How can scenarios be selected efficiently from the distribution of uncertain parameters such as demands and quality of returned products in order to find a high quality solution? How should the solutions obtained be evaluated and compared?

The ARC formulation applied in our CLSC design is a multi-stage approach to robust optimization that allows some decision variables to adjust the uncertain parameters. The ARC formulation might provide a less conservative solution compared to the RC formulation. However, it is not always straightforward to determine how and when the ARC, once reformulated as appropriate tractable model, reduces the conservativeness of the RC.

The three-stage hybrid robust/stochastic program of CLSC network design is a combination of adjustable robust optimization and stochastic programming. We investigate how an ARC model of CLSC design can be incorporated alongside the stochastic programming model to form a multi-stage hybrid robust/stochastic program of CLSC where some variables are adjustable to uncertain parameters? In other words, how can probabilistic scenarios for some parameters be effectively and efficiently combined with uncertainty sets for others?

1.3 Dissertation Structure

Three papers are provided in this dissertation, one in each chapter.

In Chapter 2, a multi-stage stochastic program is proposed to design a CLSC network with the uncertain quality of returned products as well as uncertain demands for new products in which there are dependencies of demands among periods. The network design involves long-term decisions to invest in fixed facilities such as manufacturing/remanufacturing plants, warehouses, and collection facilities. Procurement of transportation capacity among multiple modes is also required before each period's demands are known. We assume that a fixed proportion of products sold in each period are to be collected as returns, and uncertain return qualities are the random outcome of the grading process for those returned products. In this problem facility location is determined in the first stage. The unit transportation capacities are determined at the next stage in each period before realization of uncertain parameters for that period, and the amount of products to transport as well as inventories are recourse decisions for each period after realization of the uncertain parameters. Scenarios have been chosen effectively from the distribution of uncertain parameters to obtain a high quality solution. The results of stochastic problems with scenario trees of varying granularity are evaluated and compared. We test the solutions for both in-sample and out-of-sample stability to identify which scenario trees yield the best solution. We show that Including demand dependencies improves the solution performs in out-of-sample simulation. Also, adjustability of transportation modes in multi-stage model yields a better solution comparing to two-stage model where transportation modes are the first stage decision variables. Under most of the scenario trees, the solutions to the stochastic program reserves more capacity of the transportation mode with a larger unit capacity, which results in less inventory, and satisfies more of the demands on average compared to the solution of the expected value model. However, a more granular scenario tree resulting from overfitting the simulated historical demand data yields an alternative near-optimal solution with far lower investment in facilities and transportation capacity than the others.

Ben-Tal et al. (2004) provided a theorem that indicates conditions under which the objective values of ARC and RC are equivalent. Another challenge in real applications appears when conditions of this theorem are not met by the RC formulation and yet its optimal objective value matches that of the ARC. It is not always straightforward to determine how and when ARC reduces the conservativeness of the RC. In Chapter 3, a proposition concerning the RC model is elaborated to present conditions under which the ARC model leads to a better solution compared to the RC model of an uncertain linear program.

Chapter 4 describes a multi-stage hybrid robust/stochastic MILP model for CLSC network design with uncertain demands, returns and carbon tax rate. The carbon tax rate is modeled with an uncertainty set because of the lack of historical data in the US to fit a distribution. However, the distribution of demands and returns of a new product may be estimated based on historical data for similar products. Therefore, the CLSC model produces decisions that are robust with respect to carbon tax rate, while demands and returns are modeled with probabilistic scenarios. The first stage variables determine long-term facility configurations that are robust to the carbon tax rate. The second stage decisions concern the product flows among the facilities, decided after realization of demands and returns but before realization of carbon tax. At the final stage, the model determines transportation capacities of different modes after realization of the carbon tax rate. For computational tractability of the ARC, we restrict the transportation capacities to be affine functions of the carbon tax rates. Benders cuts are generated using recent duality developments for robust linear programs. Computational results show that the ability to adjust transportation mode capacities can substitute for building additional facilities as a way to respond to carbon tax uncertainty.

Conclusions and possible future research directions are provided in Chapter 5.

CHAPTER 2. A MULTI-STAGE STOCHASTIC PROGRAM FOR A CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN WITH UNCERTAIN DEMANDS AND QUALITY OF RETURNS

2.1 Introduction

Recently, much attention has been directed toward reprocessing returned products to pursue profit or environmental sustainability. Firms collect returned products to gain profit and/or to avoid legislated fees. Much research has combined the reverse channel of returned products with the forward channel to design a comprehensive network with the objective of minimizing transportation costs as well as inventory and manufacturing/remanufacturing costs (Fleischmann et al., 2003). In a closed-loop supply chain (CLSC), forward flow satisfies new demands, and reverse flow includes procurement and remanufacturing or recycling of returned products. The uncertain amounts of demands and returned products pose a significant challenge for the design of such networks.

Uncertain parameters would affect the decision variables depending on the realizations. Decisions can be made in different stages such as before or after realization of uncertain parameters at different point of time. One popular approach to deal with uncertainties in different stages is stochastic programming. At two-stage stochastic program approach has been implemented frequently in CLSC network design. However, the CLSC design must accommodate the constant shifting of customer requirements. Demands and quantity as well as quality of return products could change every period. The realization of uncertain parameters might be dependent to their previous periods. Retailers usually adjust their next orders based on the history of their customer demands. A period can consist of one or more years. The solution of two-stage stochastic program would not be the optimal one for the constant changes of parameters in different periods. Adjusting the facility locations would be significantly costly once implemented. The same goes with adjusting transportation units and production flows to the realized demands and quality of returns. The decisions on transportation units may not be responsive to the changes of demands and quality of returns, which may cause shortage or inventory cost.

The decision variables that should be adjusted before or after realization of each period depend on the nature of the problem. Facility investments have been considered as first stage variables in the current literature. However, in many real situations after realization of demands and returns the product flows, storage and shortage variables should be decided based on the available transportation units. Decisions concerning capacity to transport goods by various modes, either by purchasing or leasing fleets or by contracting with external providers are required before each period's demands are known.

Two-stage and multi-stage stochastic programs are challenging with large scenario trees spanning multiple periods. The computation time can be controlled by reducing the number of scenarios or by generating a small number of outcomes for each period. However, it is not clear beforehand which scenario tree would best represent the problem and give a near global optimal solution? To help select a scenario generation or reduction method, it is crucial to have a strategy on evaluating the solutions obtained from different scenario trees.

The quality of returns might be uncertain. Different levels of quality require different amounts of remanufacturing, with some not being remanufacturable at all. For example, Denizel et al. (2010) relate that in the IBM corporation in Raleigh, NC, for shipments of used laptops to be eligible for resale, the quality level of returned products after remanufacturing or refurbishing must attain a predetermined level of acceptability. In this process, not all used laptops require the same effort to remanufacture. For example, one used laptop might need only to be cleaned, tested, and loaded with the standard software configuration after formatting the hard drive while another might require repairs that take three times as long.

In this paper we do not model return quantity uncertainty directly. Some firms can accurately estimate the quantity of their returned products either because they lease their new products to customers or they offer a trade-in credit when customers return the old product and purchase a new one. For example, IBM and Pitney Bowes offer an option for leasing their products and remanufacture these products after their return. In addition, firms with a trade-in credit option can have an accurate forecast of how many used products they receive by knowing the sales forecast of new ones. However, the variation in the quality of returns remains a challenge for both firms that take back their leased products and those that receive used ones by trade-in credits (Denizel et al., 2010).

We propose a problem formulation to design a CLSC network with the uncertain quality of returned products as well as uncertain demands for new products in which there are dependencies of demands among periods. The network design involves long-term decisions to invest in fixed facilities such as manufacturing/remanufacturing plants, warehouses, and collection facilities. Procurement of transportation capacity among multiple modes is also required before each period's demands are known. We assume that a fixed proportion of products sold in each period are to be collected as returns, and uncertain return qualities are the random outcome of the grading process for those returned products. A trade-off exists between the shortage of new products relative to demands or the loss of uncollected used products, and excess processing or transportation capacity that goes unused. We formulate the problem as a multi-stage stochastic program where facility location is determined in the first stage. The unit transportation capacities are determined at the next stage in each period before realization of uncertain parameters for that period, and the amount of products to transport as well as inventories are recourse decisions for each period after realization of the uncertain parameters. Obtaining a set of probabilistic scenarios that adequately represent the uncertain parameters while not requiring prohibitive computational effort and also evaluating and comparing the obtained stochastic results pose significant challenges. We combine different scenario generation methods for different random variables and test the solutions for both in-sample and out-of-sample stability to identify which scenario trees yield the best solution.

The main contribution of this paper is proposing a multi-stage stochastic program for CLSC network design with uncertain return qualities in addition to demands, in which there are dependencies of demands among periods. In the solution methodology, the procurement of transportation capacity of multiple modes is decided before realization uncertain parameters.

Multi-stage scenario trees are generated using two approaches for approximating distributions of uncertain parameters. Specifically, we create a synthetic dataset of simulated historical demands to use both as a basis for scenario tree generation by moment matching and to evaluate solutions obtained with different scenario trees of different granularities. We observe an instance of overfitting, in which a scenario tree including more outcomes at each stage produces a dramatically different solution that has slightly higher average cost, compared to the solution from a less granular tree, when evaluated against the underlying simulated historical data.

A brief literature review of CLSC network design follows in Section 2. In Section 3, we present the deterministic model and notation definitions. The stochastic program is formulated in Section 4. Uncertain parameters as well as scenario generation methods are described in Section 5 along with computational experiments for deterministic and stochastic versions to validate the model, and finally provide conclusions and topics for future research in Section 6.

2.2 Literature Review

Since Fleischmann et al. (2001) extended the forward product flow with reverse flow in supply chain, CLSC networks have attracted much attention in the literature because of environmental concerns. Researchers have considered uncertain parameters in their quantitative and qualitative analysis.

A few papers used two-stage scenario-based stochastic problem for designing the CLSC network. Listes (2007) presented a generic two-stage stochastic program for the design of a CLSC network, where the alternative scenarios were based on uncertain demand and returns. The location decisions are made in the first stage, and product flows are the second stage after realization of uncertain parameters. He applied an integer L-shaped method to solve it. Francas and Minner (2009) studied a CLSC network design where they examined capacity decisions and expected performance of two network configurations such as hybrid or separated manufacturing and remanufacturing plants. They assumed demands and returns uncertainties where capacity acquisitions of plants in two different network configurations are the first stage variables and unit manufactured and remanufactured products are second stage decision variables. Additionally, Amin and Zhang (2012) investigated the impact of demand and return uncertainties on the CLSC network configuration with a two-stage stochastic program. They minimized a multi-objective function including total cost and environmental factors, and used weighted sums and ϵ-constraint methods to examine the trade-off surfaces of the test instances. The first stage variables are location variables, and product flows are the second stage variables. Zeballos et al. (2012) proposed a two-stage model to simultaneously design and deal with planning decisions for a CLSC network where the quantity and quality of the product flows of the reverse network are uncertain. They used mixed integer linear programming to maximize the expected profit by deciding on the location variables as the first stage and production, distribution and storage variables as the second-stage variables. Gao and Ryan (2014) designed a CLSC network considering operation over multiple periods while considering uncertainties in demands, returns, and potential carbon emission regulations. They formulated the network design in two-stage stochastic program where facility investment decisions are the first-stage and transportation flows are the second stage variables. Baptista et al. (2015) proposed a heuristic algorithm for solving a multi-period, multi-product CLSC with several sources of uncertainties such as demands, quality and quantity of returned products, transportation cost, financial budget, and investment costs. The first stage decision variables are plants configurations at the start of the year, and second stage variables are production, inventory and product flows for every period.

Only Zeballos et al. (2014) addressed a CLSC network design with a multi-stage stochastic programming approach. They considered uncertain supply levels of raw materials and customer demands as uncertain parameters, where the first stage variables are the binary network design decisions and the other stage decision variables include production, distribution and storage variables. They used a scenario reduction method to reduce the number of scenarios and compared their multi-stage model with deterministic one. However, in their multi-stage approach there are no dependencies of demands among periods and they did attempt to evaluate different scenario trees of different granularities. They also did not compare their solution to the two-stage solution to show the superiority of their approach.

All the above papers in stochastic program for CLSC assumed only facility configuration investments as their first stage decisions. However, the type and capacity of transportation also might need to be decided before realization of uncertain parameters. A few articles on the CLSC network designs considered multiple choices of transportation modes. Paksoy et al. (2011) proposed a quite general CLSC network configuration that handles various costs where they also included different modes of transportation. In the Gao and Ryan (2014) CLSC network designed, they considered different transportation modes which produce a large proportion of greenhouse gas emissions. In another study, Sim et al. (2004) developed a CLSC network where, in addition to transportation modes, they considered multiple products in a multi-period model to minimize facility investments, transportation, operating and production/storage costs. They also used a linear programming-based heuristic genetic algorithm instead of mixed-integer programming and compared it to the exact solvers.

Some studies such as Tao et al. (2012) and Zeballos et al. (2012) have considered uncertain quality as well as the quantity of returned products in CLSC network design. However, one important aspect not often considered in the literature is the utilization of multiple transportation modes among facilities. The uncertainty of both quantity and quality of returns will affect the relative efficiency of transportation modes among facilities. Sorting returned products based on their qualities has been studied by Aras et al. (2004). In addition, Guide et al. (2003) developed a simple framework for determining the optimal prices and the corresponding profitability of sorting returned products in a single period, deterministic setting. Galbreth and Blackburn (2006) also considered sorting deterministic returns in a single period with decision variables such as how many used items to acquire and how selective to be during the sorting process. For multi-period production planning, Zhou et al. (2010) studied a single-product periodic-review inventory system with multiple types of returns. They considered stochastic demands and minimized the expected total discounted cost over a finite planning horizon. Denizel et al. (2010) considered production planning when returns have different and uncertain quality levels along with capacity constraints. Keyvanshokooh et al. (2013) addressed a multi-echelon, multi-period CLSC which determines the acquisition price for different quality level of products. In addition, Cai et al. (2013) studied acquisition and production planning for a hybrid manufacturing/remanufacturing system when the quality of cores include two levels. They used stochastic dynamic programming to derive the optimal acquisition pricing and production policy.

Aspects that are not considered in the literature include a method for formulating and evaluating the CLSC network design in multi-stage stochastic program where transportation capacities are decided before realization of uncertain parameters and there is a dependencies between uncertain demands among periods. Multi-stage scenario trees are generated and a synthetic dataset of simulated historical demands is used both as a basis for scenario tree generation and to evaluate solutions obtained with different scenario trees of different granularities. In addition, uncertain return qualities are assumed the random outcome of the grading process for those returned products.

2.3 The Deterministic CLSC Design Model

In this section, our deterministic mathematical model of CLSC network design considering the quality of returned products, multiple transportation modes, and inventories is presented. The assumptions underlying the model include that plants manufacture and remanufacture a single product in multiple periods; that warehouses and collection centers have the ability to manage inventory between periods; that high-quality returned products can be sold at the same price as new products after remanufacturing; and that the locations of potential facilities such as manufacturing/remanufacturing plants, warehouses, and collection facilities are known. To account for the time value of money transportation and inventory cost parameters are represent their present values. Finally, multiple transportation modes have different capacities to carry products between facilities where the mode with larger capacity has higher fixed cost but not necessarily higher variable cost. Therefore, minimizing the use of empty space in transportation can help to reduce fuel cost and CO_2 emissions. For simplicity, however, we assume transportation capacity in each period to be available in continuous quantities. The decision variables include the locations of facilities, capacities for each transportation mode, the volume of products transported among facilities by each mode, and inventories. The objective is to minimize facility configuration investment as well as transportation and inventory costs.

Following are the definitions of model parameters and variables:

Sets:

- $\mathcal{P}$: the set of potential facilities consisting of factories $\mathcal{F}$, new product warehouses $\mathcal{J}$, and collection centers for returned products $\mathcal{L}$; i.e.,$\mathcal{P} = \mathcal{F} \cup \mathcal{J} \cup \mathcal{L}$
- $\mathcal{K}$: the set of retailers
- $\mathcal{M}$: the set of transportation modes
- $\mathcal{R}$: the set of periods
- $\mathcal{A}$: the set of arcs $\equiv \{ij : (i \in \mathcal{F}, j \in \mathcal{J})\} \cup \{ij : (i \in \mathcal{J}, j \in \mathcal{K})\} \cup \{ij : (i \in \mathcal{K}, j \in \mathcal{L})\} \cup \{ij : (i \in \mathcal{L}, j \in \mathcal{F})\}$

Parameters:

- c_i : the total investment cost ($) for building facility $i \in \mathcal{P}$
- β_{ij} : the length (km) of the arc $ij \in \mathcal{A}$
- g^{mr}: the unit transportation cost ($/km-unit of product) for mode $m \in \mathcal{M}$ in period $r \in \mathcal{R}$
- h^{mr} : the approximate fixed operating cost ($/unit of capacity) of transportation mode $m \in \mathcal{M}$ in period $r \in \mathcal{R}$
- Φ_i^r : the inventory cost ($/unit of product) at warehouse $i \in \mathcal{J}$ or collection center $i \in \mathcal{L}$ in period $r \in \mathcal{R}$
- $\tau^r \in [0, 1]$: the rate of product return in period $r \in \mathcal{R}$ as a proportion of demand
- W_m : the weight limit (tons/unit of capacity) of mode $m \in \mathcal{M}$
- ω : the weight (tons/unit of product)
- η_i : the processing capacity (units of product/period) at node $i \in \mathcal{P}$

Random variables:

- D_k^r: the demand (units of product) for new products by retailer $k \in \mathcal{K}$ in period $r \in \mathcal{R}$
- A^r: the rate of quality; i.e., proportion of acceptable products, after grading in period $r \in \mathcal{R}$

Decision variables:

- x_{ij}^{mr}: the amount of units of product transported on arc $ij \in \mathcal{A}$ using transportation mode $m \in \mathcal{M}$ in period $r \in \mathcal{R}$
- t_{ij}^{mr}: the number of units of transportation mode $m \in \mathcal{M}$ for which to contract on arc $ij \in \mathcal{A}$ for period $r \in \mathcal{R}$
- v_i^r: the amount of inventory (units of product) that is held in warehouse $i \in \mathcal{J}$ or collection center $i \in \mathcal{L}$ in period $r \in \mathcal{R}$
- y_i: binary variable equal to 1 if facility $i \in \mathcal{P}$ is opened, and 0 otherwise

Given realized values d_k^r and α^r for D_k^r and A^r, the deterministic mathematical program to minimize the cost is as follows:

$$\min \sum_{i \in \mathcal{P}} c_i y_i + \sum_{r \in \mathcal{R}} \left\{ \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} \left(g^{mr} \beta_{ij} x_{ij}^{mr} + h^{mr} t_{ij}^{mr} \right) + \sum_{j \in \mathcal{J}} \Phi_j^r v_j^r + \sum_{l \in \mathcal{L}} \Phi_l^r v_l^r \right\} \tag{2.1}$$

s.t.:

$$\sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}} x_{fj}^{mr} + v_j^{r-1} - v_j^r - \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}} x_{jk}^{mr} = 0, \qquad \forall r\in\mathcal{R}, j\in\mathcal{J} \tag{2.2}$$

$$\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} x_{jk}^{mr} = d_k^r, \qquad \forall r\in\mathcal{R}, k\in\mathcal{K} \tag{2.3}$$

$$\sum_{i\in\mathcal{L}}\sum_{m\in\mathcal{M}} x_{ki}^{mr} - \tau^r \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} x_{jk}^{mr} = 0, \qquad \forall r\in\mathcal{R}, k\in\mathcal{K} \tag{2.4}$$

$$\alpha^r \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}} x_{kl}^{mr} + v_l^{r-1} - v_l^r - \sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}} x_{lf}^{mr} = 0 \qquad \forall r\in\mathcal{R}, l\in\mathcal{L} \tag{2.5}$$

$$w x_{ij}^{mr} - W_m t_{ij}^{mr} \le 0 \qquad \forall r\in\mathcal{R}, ij\in\mathcal{A}, m\in\mathcal{M} \tag{2.6}$$

$$\sum_{j:ij\in\mathcal{A}}\sum_{m\in\mathcal{M}} x_{ij}^{mr} - \eta_i y_i \le 0 \qquad \forall r\in\mathcal{R}, i\in\mathcal{P} \tag{2.7}$$

$$y\in\{0,1\}^{|\mathcal{P}|}, x\in\mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|\times|\mathcal{R}|}, t\in\mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|\times|\mathcal{R}|}, \quad v\in\mathbb{R}_+^{|\mathcal{J}|\times|\mathcal{R}|}, \quad v\in\mathbb{R}_+^{|\mathcal{L}|\times|\mathcal{R}|} \tag{2.8}$$

The objective (2.1) consists of present value of three costs; namely, facility configuration investments, transportation, and inventory. The transportation cost includes both variable and fixed costs that vary by the mode of transport. Constraint (2.2) expresses the balance between in-bound and out-bound goods of each warehouse j accounting for inventories between periods. Constraint (2.3) and (2.4) ensure that new demands are provided and returned demands are transported to collection centers for every retailer k, respectively. The grading process occurs in collection centers from which a fraction α^r of returns is transferred to the manufacturer in period r; constraint (2.5) ensures conservation of flow in this process and tracks the inventory in collection centers. Constraint (2.6) is the capacity constraint for the weight of products transported by each mode. Constraint (2.7) enforces the capacity constraints of processing nodes, and constraint (2.8) shows the nonnegativity and binary requirements for the variables.

2.4 Stochastic Program

2.4.1 Multi-Stage Model

In this section, we present a multi-stage stochastic program to minimize expected costs with uncertainty in the demands and the quality of returned products. The location and the number of facilities ($y_i, i \in \mathcal{P}$) are binary decisions to be taken before the realization of any uncertainty for all periods. In each period $r \in \mathcal{R}$, transportation capacities ($t_{ij}^{mr}, ij \in \mathcal{A}, m \in \mathcal{M}$) must be determined before the realization of demands and quality rates. The other decision variables are determined after the realization of uncertainties in each period.

We represent nodes in a scenario tree as $\left(d^{r(\lambda)}, \alpha^{r(\mu)}\right), \lambda = 1, ..., u_r, \mu = 1, ..., z_r$, where $d^{r(\lambda)}$ is a realization of $D^r = \left(D_1^r, ..., D_{|\mathcal{K}|}^r\right)$ and we have u_r values for demands and z_r values for the quality of returns in period r. Therefore, the number of branches from each node at period $r-1$ is $u_r z_r$ and the corresponding set S, of scenario paths has cardinality $|S| = \prod_{r \in \mathcal{R}} u_r z_r$. Given a conditional probability $\rho_\lambda^r \mu$ for node $\left(d^{r(\lambda)}, \alpha^{r(\mu)}\right)$ in period r, a scenario path consists of nodes $\left\{0, \left(d^{1(\lambda)}, \alpha^{1(\mu)}\right), ..., \left(d^{|\mathcal{R}|(\lambda)}, \alpha^{|\mathcal{R}|(\mu)}\right)\right\}$ with its probability computed as $p^s = \rho_{\lambda\mu}^1 ... \rho_{\lambda\mu}^{|\mathcal{R}|}$
.

Figure 2.1 illustrates an example of a scenario tree for the set of periods $\mathcal{R} = \{1, 2, 3\}$ with $u_r = 2$ values for demands and $z_r = 2$ values for quality. In this figure, decision variables y and t^1 are the first-stage variables that must be decided before any realization of uncertainty for all periods. In addition, the decision variables x^r, v^r, and t^{r+1} are determined after realization of uncertain parameters in every period r. This scenario tree consists of $\prod_{r \in \mathcal{R}} u_r z_r = 4^3 = 64$ scenario paths.

To express the extensive form of the deterministic equivalent of this multi-stage stochastic program, we add a superscript ($s \in S$) in deterministic formulation (2.1)-(2.8) to every decision variable and parameter that depends on the scenario path. The probabilities of scenario paths are also included in the objective to determine the expected costs. In addition, to provide complete recourse, we introduce new decision variables for unmet demands and uncollected used products in the case of insufficient transportation or facility capacity. A collection of nodes $b \in B(r)$ where all scenarios $s \in b$ share the same nodes in periods $1, ..., r$ is called a

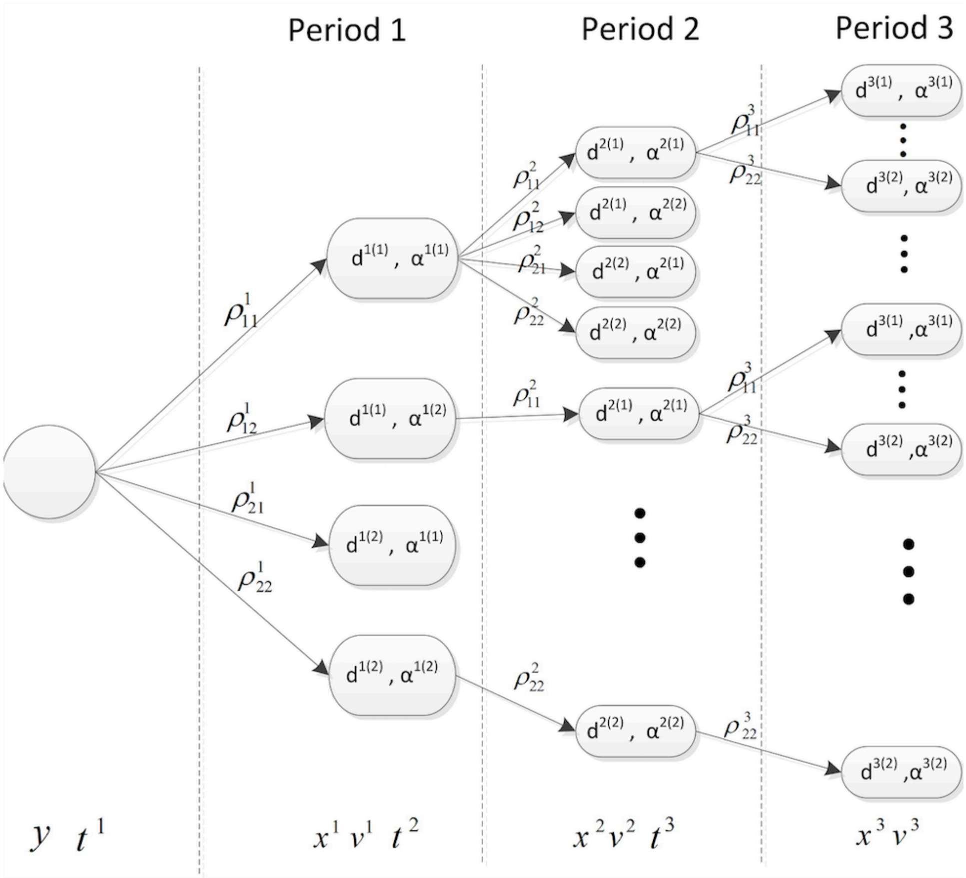

Figure 2.1 Representation of scenario paths for three periods where each node $\left(d^{r(\lambda)}, \alpha^{r(\mu)}\right)$ specifies a combination of demand values at retailers and return quality in period r, and the decision variables displayed under each period can be decided after realization of the random variables for that period.

bundle in period r in which $B(r)$ represents the set of bundles.

The extensive form of the stochastic program, where $\chi \equiv \{y, t, x, v, e, e'\}$, is as follows:

$$\min Z_{MS}(\chi, S) = \sum_{i \in \mathcal{P}} c_i y_i + \sum_{r \in \mathcal{R}} \sum_{s \in S} p^s \Bigg\{ \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} \left(g^{mr} \beta_{ij} x_{ij}^{mrs} + h^{mr} t_{ij}^{mrs} \right)$$
$$+ \sum_{j \in \mathcal{J}} \Phi_j^r v_j^{rs} + \sum_{l \in \mathcal{L}} \Phi_l^r v_l^{rs} + \sum_{k \in \mathcal{K}} \left(\Psi_k^r e_k^{rs} + \Psi_k'^r e_k'^{rs} \right) \Bigg\} \tag{2.9}$$

s.t.:

$$\sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}} x_{fj}^{mrs} + v_j^{r-1,s} - v_j^{rs} - \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}} x_{jk}^{mrs} = 0, \qquad \forall r \in \mathcal{R}, j \in \mathcal{J}, s \in S \quad (2.10)$$

$$\sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} x_{jk}^{mrs} + e_k^{rs} = d_k^{rs}, \qquad \forall r \in \mathcal{R}, k \in \mathcal{K}, s \in S \quad (2.11)$$

$$\sum_{i \in \mathcal{L}} \sum_{m \in \mathcal{M}} x_{ki}^{mrs} + e_k'^{rs} - \tau^r \sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} x_{jk}^{mrs} = 0, \qquad \forall r \in \mathcal{R}, k \in \mathcal{K}, s \in S \quad (2.12)$$

$$\alpha^{rs} \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}} x_{kl}^{mrs} + v_l^{r-1,s} - v_l^{rs} - \sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}} x_{lf}^{mrs} = 0 \qquad \forall r \in \mathcal{R}, l \in \mathcal{L}, s \in S \quad (2.13)$$

$$\sum_{j: ij \in \mathcal{A}} \sum_{m \in \mathcal{M}} x_{ij}^{mrs} - \eta_i y_i \leq 0 \qquad \forall r \in \mathcal{R}, i \in \mathcal{P}, s \in S \quad (2.14)$$

$$w x_{ij}^{mrs} - W_m t_{ij}^{mrs} \leq 0 \qquad \forall r \in \mathcal{R}, ij \in \mathcal{A}, m \in \mathcal{M}, s \in S \quad (2.15)$$

Implementability constraints: (2.16)

$$t_{ij}^{mrs} = t_{ij}^{mrs'} \qquad \forall r \in \mathcal{R}, ij \in \mathcal{A}, m \in \mathcal{M}, s, s' \in b, \ \forall b \in B(r-1)$$

$$x_{ij}^{mrs} = x_{ij}^{mrs'} \qquad \forall r \in \mathcal{R}, ij \in \mathcal{A}, m \in \mathcal{M}, s, s' \in b, \ \forall b \in B(r)$$

$$v_j^{mrs} = v_j^{mrs'} \qquad \forall r \in \mathcal{R}, j \in \mathcal{J} \cup \mathcal{L}, m \in \mathcal{M}, s, s' \in b, \ \forall b \in B(r)$$

$$e_k^{mrs} = e_k^{mrs'}, e_k'^{mrs} = e_k'^{mrs'} \qquad \forall r \in \mathcal{R}, k \in \mathcal{K}, m \in \mathcal{M}, s, s' \in b, \ \forall b \in B(r)$$

$$y \in \{0,1\}^{|\mathcal{P}|}, x, t \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}| \times |\mathcal{R}| \times |S|}, \quad v \in \mathbb{R}_+^{|\mathcal{J}| \times |\mathcal{R}| \times |S|},$$
$$v \in \mathbb{R}_+^{|\mathcal{L}| \times |\mathcal{R}| \times |S|}, \quad e, e' \in \mathbb{R}_+^{|\mathcal{K}| \times |\mathcal{R}| \times |S|} \quad (2.17)$$

Decision variables e_k^{rs} and $e_k'^{rs}$ are included in constraints (2.11) and (2.12) to represent the amounts of unmet demands and uncollected returns. Correspondingly, the quantities Ψ_k^r and $\Psi_k'^r$ in the objective (2.9) are the shortage costs and penalties for the uncollected returned products at retailer k in period $r \in \mathcal{R}$, respectively. The implementability (nonanticipativity) constraints of the staged decision variables are shown in (2.16), where these constraint are enforced over each pair of decision variables for period r or $r-1$ if their scenario paths $s \in S$ and $s' \in S$ belong to the same bundle for that period. Finally, (2.17) represents the expanded dimensions of decision variables in the extensive form of the stochastic program.

2.4.2 Two-Stage Model

In our two-stage stochastic program we assume that facilities ($y_i, i \in \mathcal{P}$) and transportation capacities ($t_{ij}^{mr}, ij \in \mathcal{A}, m \in \mathcal{M}, r \in \mathcal{R}$) decision variables for all periods must be determined before the realization of demands and quality rates as the first stage. Therefore, the second stage decision variables include product flows (x^r), inventories (v^r), unmet demands (e^r)and uncollected used products (e'^r) for all periods $r \in \mathcal{R}$. The extensive form of the two-stage stochastic program, where $\chi \equiv \{y, t, x, v, e, e'\}$ with implicit implementability constrains on y and t, is as follows:

$$\min Z_{TS}(\chi, S) = \sum_{i \in \mathcal{P}} c_i y_i + \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} h^{mr} t_{ij}^{mr} +$$

$$\sum_{r \in \mathcal{R}} \sum_{s \in S} \rho_s^r \left\{ \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} g^{mr} \beta_{ij} x_{ij}^{mrs} + \sum_{j \in \mathcal{J}} \Phi_j^r v_j^{rs} + \sum_{l \in \mathcal{L}} \Phi_l^r v_l^{rs} + \sum_{k \in \mathcal{K}} \left(\Psi_k^r e_k^{rs} + \Psi_k'^r e_k'^{rs} \right) \right\} \tag{2.18}$$

s.t.: (2.10) - (2.14)

$$w x_{ij}^{mrs} - W_m t_{ij}^{mr} \leq 0 \qquad \forall r \in \mathcal{R}, ij \in \mathcal{A}, m \in \mathcal{M}, s \in S \tag{2.19}$$

$$y \in \{0,1\}^{|\mathcal{P}|}, t \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}| \times |\mathcal{R}|}, x \in \mathbb{R}_+^{|\mathcal{A}| \times |\mathcal{M}| \times |\mathcal{R}| \times |S|}, \quad v \in \mathbb{R}_+^{|\mathcal{J}| \times |\mathcal{R}| \times |S|},$$
$$v \in \mathbb{R}_+^{|\mathcal{L}| \times |\mathcal{R}| \times |S|}, \quad e, e' \in \mathbb{R}_+^{|\mathcal{K}| \times |\mathcal{R}| \times |S|} \tag{2.20}$$

2.5 Computational Experiment

To compare the solutions of the stochastic program with different granularities of scenario trees, we constructed an instance that consists of three potential locations for plants, four potential warehouses and four potential collection centers to satisfy eight retailers. We formulated the instance for three periods with three transportation modes using equations (2.9)-(2.17) for the stochastic program and the deterministic model as a special case with a single scenario.

More information about the empirical distributions for demands and the parameter settings are provided in the Appendix. Here we describe scenario generation, optimization and stability results.

2.5.1 Scenario Generation

This section briefly describes our procedures to generate scenarios. We review the distribution approximation method for the continuous distribution of return quality, and a moment matching method for multi-dimensional demands over multiple periods with arbitrary statistical specifications. We optimally discretized the distribution of return quality with different levels of granularity and applied moment-matching to generate demand scenarios from simulated historical data.

2.5.1.1 The Quality of Returns

We assume the quality of returned product parameters (α^r) are independent and distributed according to a Beta density in each period:

$$f(\alpha^r) = \frac{\Gamma(\gamma^r + \delta^r)}{\Gamma(\gamma^r)\Gamma(\delta^r)}(\alpha^r)^{\gamma^r - 1}(1 - \alpha^r)^{\delta^r - 1}, \qquad \gamma^r, \delta^r > 0, \tag{2.21}$$

where γ^r and δ^r are Beta function parameters. Because the support for this distribution is the interval $[0, 1]$, it is a good choice for the proportion of acceptable returns. Furthermore, by changing the distribution parameters γ^r and δ^r, a variety of shapes which could be fitted to the real data is obtained. Some cdfs of this distribution for different values of γ^r and δ^r are illustrated in Figure 2.2. In particular, if $\gamma^r = \delta^r = 1$ then it is a uniform distribution.

To generate k discrete outcomes of this continuous distribution, we approximate a discrete distribution using the Wasserstein-distance Δ_1 as in Pflug (2001).

$$\Delta_1(G, \tilde{G}) = \sum_{q=1}^{k} \int_{\frac{z_{q-1}+z_q}{2}}^{\frac{z_q+z_{q+1}}{2}} |\alpha - z_q| dG(\alpha) \tag{2.22}$$

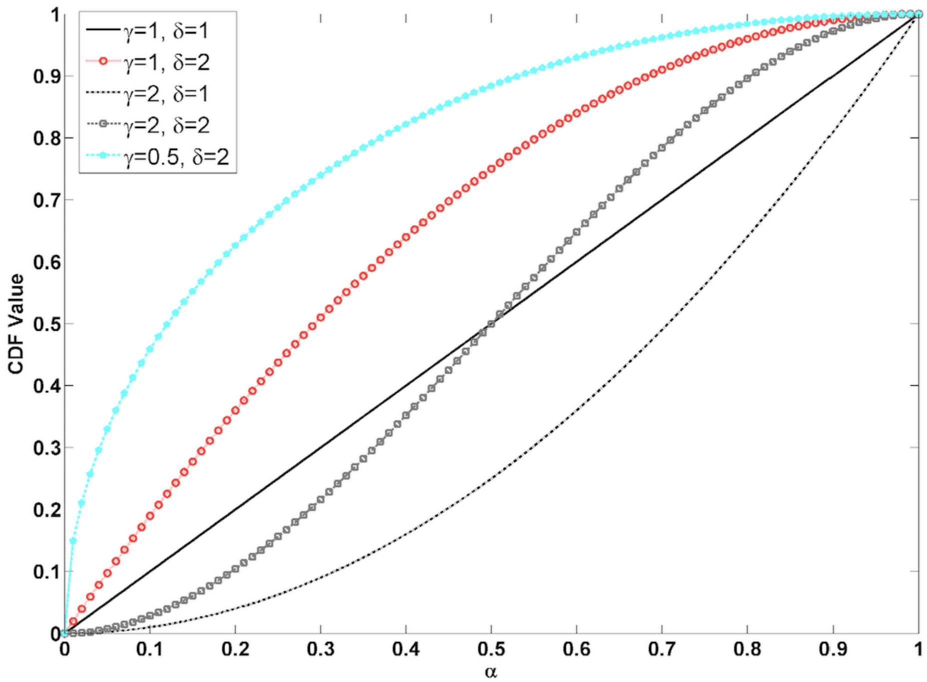

Figure 2.2 CDF of Beta Distribution with different values of γ and δ

where $G(\alpha)$ is the cdf of distribution with density $g(\alpha)$. Here, $z_1, ..., z_k$ form the support for the discrete approximate distribution $\tilde{G}(z)$ with probabilities $P_{z_1} + ... + P_{z_k} = 1, z_0 = 0, z_{k+1} = 1$. The procedure to find $z_1, ..., z_k$ (for example for $k = 2$) is to minimize:

$$\Delta_1(G, \tilde{G}) = \int_0^{\frac{z_1+z_2}{2}} |\alpha - z_1| g(\alpha) d\alpha + \int_{\frac{z_1+z_2}{2}}^1 |\alpha - z_2| g(\alpha) d\alpha \tag{2.23}$$

To find the probability of each z using the property (iii) of Δ_1-distance proven in Theorem 1 of Pflug (2001), we find the masses of the points by:

$$\tilde{G}(x) = \sum_{\{q: z_q \leq x\}} G\left(\frac{z_q + z_{q+1}}{2}\right), \tag{2.24}$$

$$P_{z_q} = G\left(\frac{z_q + z_{q+1}}{2}\right) - G\left(\frac{z_{q-1} + z_q}{2}\right), \tag{2.25}$$

To specify the scenario generation method for quality of returns, we assumed parameters of the Beta distribution for α^r to be $\gamma = 1, \delta = 2$ for every $r \in \mathcal{R}$ so the density function

$g(\alpha^r) = 2(1-\alpha^r)$. Two discrete outcomes from this continuous distribution, to be applied independently for all periods, are generated by minimizing the Wasserstein-distance Δ_1 as in (2.22). The specific procedure to find z_1 and z_2 when $k=2$, substituting $g(\alpha^r)$ in (2.23), is to minimize:

$$\begin{aligned}
\Delta_1\left(G, \tilde{G}^{(2)}\right) &= -4\left(-\frac{z_1^3}{3} + \frac{z_1^2(z_1+1)}{2} - z_1^2\right) \\
&+ 2\left(-\frac{(z_1+z_2)^3}{12} + \frac{(z_1+z_2)^2(z_1+z_2+2)}{8} - \frac{(z_1+z_2)^2}{2}\right) \\
&+ 2(-\frac{1}{3} + \frac{(z_2+1)}{2} - z_2) - 4(-\frac{z_2^3}{3} + \frac{z_2^2(z_2+1)}{2} - z_2^2)
\end{aligned} \tag{2.26}$$

Figure 2.3 illustrates $\Delta_1(G, \tilde{G}^{(2)})$ as a function of z_1 and z_2. Upon applying a non-linear optimization routine in MATLAB, the minimum value of $\Delta_1(G, \tilde{G}^{(2)})$ is found when $z_1 = 0.1554$ and $z_2 = 0.5383$.

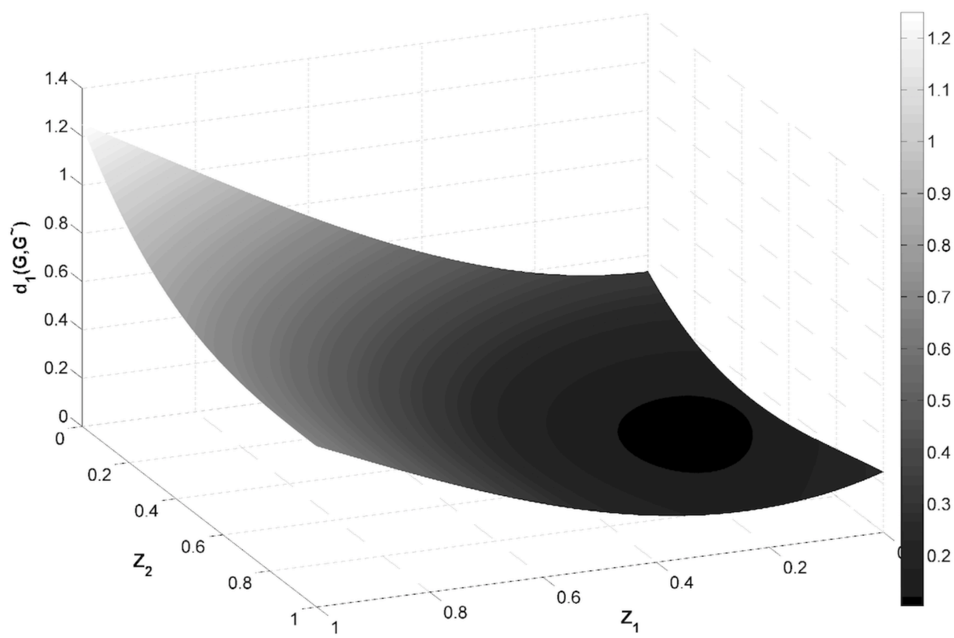

Figure 2.3 Δ_1- distance between discrete and continuous distribution of α for different z_1 and z_2

Finally, the probabilities of each outcome are found below using (2.25) and shown in Table 2.1 as approximate distribution $\tilde{G}^{(2)}$.

$$\tilde{p}_{z_1}^{(2)} = G(\frac{z_1+z_2}{2}) = 0.5734,\ \tilde{p}_{z_2}^{(2)} = G(1) - G(\frac{z_1+z_2}{2}) = 0.4266 \quad (2.27)$$

To explore the stability of the solution with respect to distribution granularity, we also generated more outcomes for the quality of returns using four approximating points instead of two by minimizing:

$$\Delta_1(G, \tilde{G}^{(4)}) = \sum_{q=1}^{k} \int_{\frac{z_{q-1}+z_q}{2}}^{\frac{z_q+z_{q+1}}{2}} |u - z_q| dG(u) = \int_0^{\frac{z_1+z_2}{2}} |u - z_1| g(u) du$$
$$+ \int_{\frac{z_1+z_2}{2}}^{\frac{z_2+z_3}{2}} |u - z_2| g(u) du + \int_{\frac{z_2+z_3}{2}}^{\frac{z_3+z_4}{2}} |u - z_3| g(u) du + \int_{\frac{z_3+z_4}{2}}^{1} |u - z_4| g(u) du. \quad (2.28)$$

The resulting outcomes and probabilities for each period are shown as $\tilde{G}^{(4)}$ in Table 2.1.

Table 2.1 The approximate distributions for quality of returns in each period

Distribution	Values	Probabilities
$\tilde{G}^{(2)}$	0.1554	0.5734
	0.5383	0.4266
$\tilde{G}^{(4)}$	0.0804	0.3085
	0.2565	0.2774
	0.4565	0.2372
	0.7024	0.1769

2.5.1.2 Demands

To simulate a plausible scenario generation process while providing data for out-of-sample stability tests, we first created a dataset of simulated historical demand as $\mathcal{D} = \{\tilde{d}_k^{r\mathfrak{s}}\}$. Here, $\{\tilde{d}_k^{r\mathfrak{s}}\}$ denotes simulated observation $\mathfrak{s}$ of randomly generated demand for retailer $k \in \mathcal{K}$ in period $r = 1, 2, 3., \mathfrak{s} = 1, ..., 250$. The simulated demands for each retailer independently were drawn from Normal distributions $\{\tilde{d}_k^{1\mathfrak{s}}\} \sim N(98, 20)$ in the first period and $\{\tilde{d}_k^{2\mathfrak{s}}\} \sim N(110, 20)$ in the second period. The first two periods' demands of each retailer were independent but the demand of retailer k in the third period was dependent on that retailer's first two periods' demands following:

$$\tilde{d}_k^{3\mathfrak{s}} = \zeta \tilde{d}_k^{1\mathfrak{s}} + \sqrt{1-\zeta^2}\tilde{d}_k^{2\mathfrak{s}} + \epsilon_k^{\mathfrak{s}}, \qquad \mathfrak{s} = 1, ..., 250, \quad k \in \mathcal{K} \tag{2.29}$$

where the ζ parameter was set equal to 0.4 and the random terms $\epsilon_k^{\mathfrak{s}}$ were generated independently from $N(-10, 15)$. In this simulation, we assume that the retailer demands of a product depend on a history of more than one period. An example could be retailers that adjust their orders based on their customers. The first two periods are trials and rest of the orders are based on their past experience of the product.

To generate scenarios for the demands of each retailer k in every period, we used the moment-matching approach of Hyland et al. (2003); specifically, the moment-matching heuristic procedure constructed by Kaut and Mathieu (2012). Hyland and Wallace (2001) presented the general idea of an optimization problem to generate, at each stage, q discrete outcomes for every customer as the decision variables for the demands of the $|\mathcal{K}|$ customers.

Based on simulated historical demands, we computed the first four statistical moments; i.e., mean, variance, skewness, and kurtosis of the marginal distributions for each period. Using the moment-matching scenario generation approach of Hyland et al. (2003), a multi-stage scenario tree with equal weights for all specifications was generated. Rather than generating the whole scenario tree at once, we compute the outcomes of demands at each node and period separately. The mean values between periods are assumed to be state dependent as opposed to the other three specifications. Considering eight retailers and their four properties, a single period includes 32 specifications. The least number of outcomes based on the available degrees of freedom is four outcomes for the demands of each retailer at each period based on Hyland et al. (2003):

$$\min\{q|(I+1)q-1 \geq |B|\} \tag{2.30}$$

In this equation, q is the number of outcomes, B is the set of all specified statistical properties and I is the number of random variables; that is, eight. Therefore, the moment-matching scenario generation consists of $n = 32$ decision variables calculated by four outcomes multiply to eight retailers for each period. Including the conditional probabilities, there were a total of

36 decision variables. Figure 2.4 shows the nodes of multistage scenario tree where the demand outcomes of all retailers are considered as a node and each node has four children in the next period. The connection between periods are based on the mean values of each retailer and obtained using

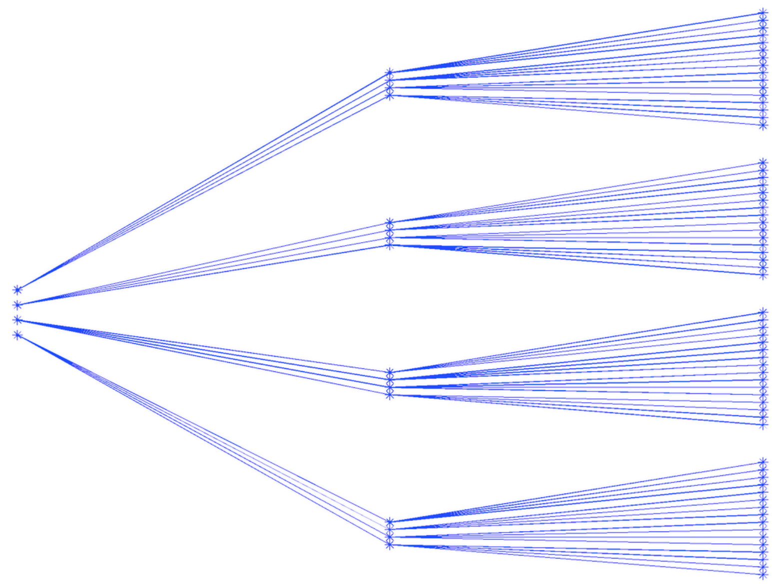

Figure 2.4 Scenario tree representation of three periods and four demand outcomes for each retailer

$$\kappa_k^r(\lambda) = \theta_r \bar{d}_k^r + (1-\theta_r) d_k^{(r-1)\lambda} \tag{2.31}$$

where $\kappa_k^r(\lambda)$ is the expected demand of retailer k in period r over the children of outcome δ in period $r-1$, $d_k^{(r-1)\lambda}$ is the parent node, and $\bar{d}_k^r = \frac{1}{250}\sum_{\mathfrak{s}=1}^{250} \tilde{d}_k^{r\mathfrak{s}}$ that is, the mean value computed from the simulated historical data. Here, θ_r is a constant parameter to combine outcomes of the previous period $d_k^{(r-1)\lambda}$ with the mean value of the current period $\bar{d}_k^r$. We estimate the θ values using

$$\theta_r = \arg\min_{\theta} \sum_{\mathfrak{s}=1}^{250} \left(\left[\theta_r \bar{d}_k^r + (1-\theta_r)\tilde{d}_k^{(r-1)\mathfrak{s}} \right] - \tilde{d}_k^{r\mathfrak{s}} \right)^2, \qquad r > 1 \tag{2.32}$$

that is, the value of θ_r is found by minimizing the sum of squared differences between the forecasted mean demands and the simulated historical demands $\tilde{d}_k^{(r-1)\mathfrak{s}}$. If there were no correlation between period r and $r-1$, θ_r would be equal to one. The values were found by trial and error to be $\theta_2 = 1$ and $\theta_3 \cong 0.5$ for the second and third period, respectively. Therefore, the expected mean value $\kappa_k^r(\lambda)$ is used as the specified mean for retailer k in the children of node $r(\lambda)$ for $r = 2, 3$. The first period means $\bar{d}_k^r$ as well as the other statistical specifications for each retailer are shown in Table A.8.

After finding the relation for mean values between periods, we follow the description of scenario generation to generate four outcomes, as shown in Figure 2.4. The four outcomes with probabilities for eight retailers in Table 2.3 are generated based on specifications of Table 2.2 for the first period. The generated outcomes and specifications of all demands and periods are shown in Table A.6 and A.8 in Appendix.

Table 2.2 Demand specifications for period one

Retailer	Mean	Variance	Skewness	Kurtosis
1	95.54	442.13	-0.067	3.23
2	97.33	433.15	-0.124	3.37
3	99.45	370.26	0.170	2.78
4	96.84	354.61	0.009	2.76
5	96.12	421.22	0.093	3.40
6	99.18	372.27	0.031	2.76
7	97.22	455.82	-0.188	2.94
8	97.48	401.63	-0.085	2.66

Table 2.3 The result of moment matching method with four outcomes for period one

Retailer Probability	1	2	3	4	5	6	7	8
0.3617	121.7	93.1	92.8	92.8	88.0	94.7	122.9	94.8
0.3098	76.1	76.0	81.6	77.3	78.4	79.4	74.5	75.5
0.0064	5.7	8.5	24.7	23.8	8.5	24.2	9.8	21.0
0.3221	86.6	124.4	125.6	121.6	124	124.6	92.0	123.1

Combining the four outcomes (Table 2.3) for demands independently with the two outcomes for return quality (Table 2.1) yields the eight scenario tree nodes for one period shown in Table 2.4. The demands of all retailers should be combined; in this table, however, the demand of only one retailer is shown.

Table 2.4 The nodes of the scenario tree for the first period

Probability	Demand	Quality
0.2074	121.7	0.1554
0.1543	121.7	0.5383
0.1776	76.1	0.1554
0.1321	76.1	0.5383
0.0036	5.7	0.1554
0.0027	5.7	0.5383
0.1847	86.6	0.1554
0.1374	86.6	0.5383

In this three-period instance, the combination of four demand outcomes and two possible quality rates of returns results in a total of $(4 \times 2)^3 = 512$ scenario paths. The representation of scenario paths for the average demands over all retailers and the quality of returns are shown in Figure 2.5, separately, because representing the combined return quality and demand would be confusing. Figure 2.5(a) is another representation of Figure 2.4 where the vertical axis shows the average scenario demands. As we can see, since the first and second periods are independent ($\theta_2 = 1$) the demand scenario nodes at period two coincide for all four paths as opposed to the third period.

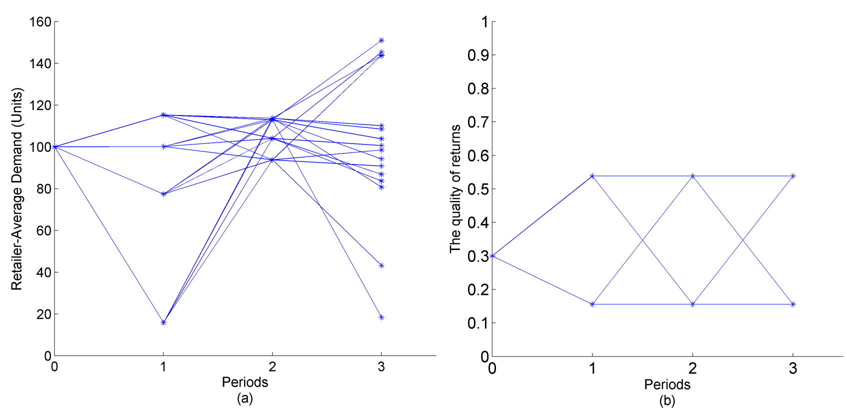

Figure 2.5 Scenario path representation of three periods with four demand outcomes (a) and two outcomes for the quality of returns (b)

2.5.2 Computational Results

We obtained and evaluated solutions to deterministic and stochastic versions of the CLSC design problem with various scenario trees for uncertain demands and quality of returns. The required time for solving this problem is exponentially increasing by the increase of period numbers. We reduce the number of scenarios by generating a small number of outcomes for each period. In this experimental design, we define the scenario trees generated by these outcomes that are solved using multi-stage formulation. We evaluate their solutions using historical simulated data to identify which scenario tree would best represent the problem and give a near global optimal solution. The importance of demand dependencies between periods is presented. We consider the dependencies of uncertain demands to their previous periods and compare it to the cases where there are no dependencies to see how the solution would perform in out-of-sample simulation. Also, the solution to the two-stage model is compared with the multi-stage solution to assess the value of the more complicated multi-stage model. Finally, facility investments and transportation unit solutions are compared to identify the changes of solutions among the recourse problems with different scenarios and deterministic model. The experiments were implemented with the MIP solver of CPLEX 12.5 in the C++ environment on a shared remote servers with 126 GB RAM and 32 Core CPU (Intel$_{®}$ Xeon$_{®}$ 2.00 GHz).

The scenario trees evaluated in this computational experiment differed according to the granularity of approximations of the quality of return and scenario demand outcomes. The deterministic scenario model is represented by $\bar{S}$ where a single scenario consisting of the expected values is used so that $|\bar{S}| = 1$. We denote the simulated observations of demand combined with the four outcomes for quality of return as S_0, which has dimension $(u(S_0) \times z(S_0))^{|\mathcal{R}|} = (250 \times 4)^3 = |S_0|$. The scenario set S_i has dimension $(u(S_i) \times z(S_i))^3$, including $u(S_i)$ demands and $z(S_i)$ qualities of return as an approximation of the original scenarios S_0.

The optimal value of the deterministic problem can be expressed as $\text{EV}_{\bar{S}} = \min_\chi Z(\chi, \bar{S})$, where $\chi \equiv \{y, t, x, v, e, e'\}$ is the vector of all decision variables and $\chi \in X(\bar{S}) \equiv \{X : (2.10) - (2.17) | \bar{S}\}$. Its optimal design is denoted by $(y_{\bar{S}}, t_{\bar{S}}) \equiv \arg\min_{(y,t)} Z(\chi, \bar{S})$. The value $\text{EEV}_{\bar{S}} = E_{S_0}(z(\xi, S_0 | y_{\bar{S}}, t_{\bar{S}}))$ where $\xi \equiv \{x, v, e, e'\}$ and $\xi \in \Xi(S_0, \bar{S}) \equiv \{\xi : (2.10) - (2.17) | y =$

$y_{\bar{S}}; t = t_{\bar{S}}; S_0\}$ represents the evaluation of the performance of the design found from solving the deterministic expected value problem against the simulated historical data (S_0). Here, $z(\xi, S_0|y_{\bar{S}}, t_{\bar{S}})$ is the expected cost evaluated according to equation (2.9). The design variables y and t are fixed to the values $(y_{\bar{S}}, t_{\bar{S}})$ found from the expected value solution, and the recourse variables x, v, e, and e' are optimized. For the stochastic program or recourse problems (RP), the optimal value is represented as $\text{RP}_{S_i} = \min_{\chi} E_{S_i} Z(\chi_i, S_i)$ where $\chi_i \in X(S_i), i = 1, ..., 4$. We denote their expected values with respect to S_0 as $\text{ERP}_{S_i} = E_{S_0}(z(\xi, S_0|y_{S_i}, t_{S_i}))$ where $\xi \in \Xi(S_0, S_i)$ and $(y_{S_i}, t_{S_i}) \equiv \arg\min_{y,t} Z(\chi, S_i)$.

An intricate aspect of evaluating the solutions found with different scenario trees against the simulated historical demand data was to map the data paths to each scenario tree so that, in reverse, the optimal values of decision variables t could be applied to the simulated observations. We used a nearest neighbor approach. First, the simulated demand paths were partitioned into u_1 sets by, for each observation path, identifying the scenario tree node for period one with the smallest Euclidean distance from the period one observed demand vector. Then, within each set for period one, the observed paths were partitioned into u_2 subsets according to minimum Euclidean distance from the corresponding child nodes in period two. A similar step for period three completed the mapping of observed paths to scenario tree paths. For some scenario trees, this process resulted in some scenario tree paths not having any observation paths mapped to them. Figure 2.6 illustrates the partitions of the observations among the three demand outcomes in each period for scenario tree S_1. The dark-colored paths are the ones to which some observed path was mapped. Conversely, the solution values corresponding to nodes in the light-colored tree paths were not applied to any of the observed paths in the evaluation process.

The simulated observed demands were crossed with the four quality of return levels shown in Table 2.1 to complete the granular evaluation set S_0. To evaluate a solution derived from a scenario tree with only two outcomes of return quality in each period, the solution values corresponding for the lower of the two return qualities were applied to paths in S_0 with the two lowest (of four) quality values and solution values for the higher of the two return qualities were applied to the paths in S_0 with the two highest (of four) quality values.

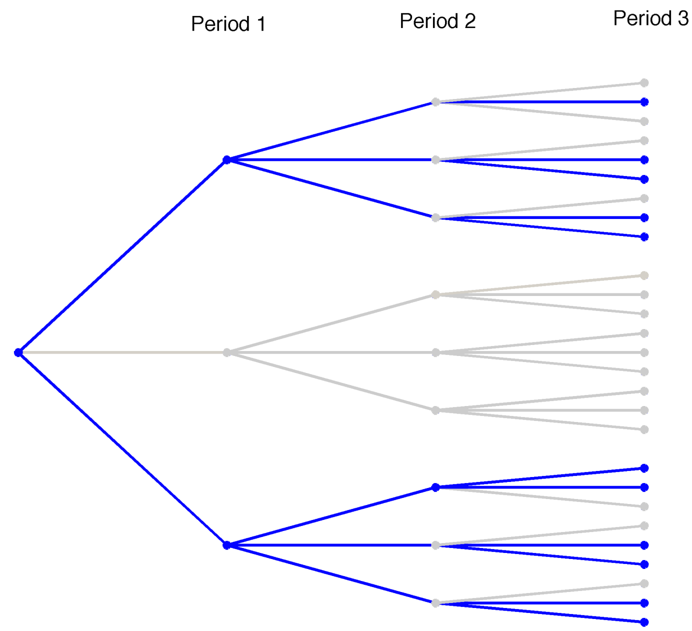

Figure 2.6 The classification of demand paths in simulated historical data for three demand outcomes

In section 2.5.1.2, the least number of outcomes based on the available degrees of freedom is found to be four. Therefore, we first compared the scenario set S_2, in which four demand outcomes along with two return qualities are considered, with the solution of the expected value model. The amount of savings from solving the stochastic model, or the relative improvement over evaluated cost of the EV solution, $\text{EEV}_{\bar{S}} - \text{ERP}_{S_2}$, is 4.2% of the ERP_{S_2} cost. The comparison between the cost components of the $\bar{S}$ and S_2 solutions in Table 2.6 shows that the $\bar{S}$ solution underinvests in transportation capacity, which leads to higher average inventory and shortage costs over the simulated historical data.

To test the stability of solutions with respect to the scenario generation, different combinations of demands and the quality of returns were analyzed as shown in Table 2.5 with their computational times. We decrease the number of demand outcomes in S_2 from four to three in a tree denoted as S_1 having 216 paths. In addition, to show the difference of changing scenario trees by altering the quality of returns, we increased the number of return quality outcomes from two to four in S_3. Further analysis is performed on increasing the number of outcomes of demands from four in to six branches from each node of scenario tree (S_4), while applying the same two quality of return outcomes, which resulted in 1,728 paths. The result of evaluating the solution of these scenario sets S_1, S_3, and S_4 against the simulated historical data are also shown in Table 2.6. In addition to the total cost percent savings relative to the expected value solution, the percentages of cost attributed to different components of total cost are shown in Table 2.6.

Table 2.5 The sets of scenario combinations of demands and quality of return evaluated in this experiment.

Scenario Sets	$\bar{S}$	S_0	S_1	S_2	S_3	S_4
Demand Outcomes	1	250	3	4	3	6
Quality of Return Outcomes	1	4	2	2	4	2
Total Scenarios	1	10^9	216	512	1728	1728
Computational Time (minutes)	1/60	-	31	107	2504	1678

Table 2.6 The evaluation of deterministic and stochastic solutions against simulated historical data, with category costs as % of total cost.

	$\text{EEV}_{\bar{S}}$	ERP_{S_1}	ERP_{S_2}	ERP_{S_3}	ERP_{S_4}
Total Cost ($1000)	2206.18	2118.00	2114.20	2117.58	2151.66
Savings From EEV%	-	4.00	4.17	4.02	2.47
Facility Cost	64.82	64.45	64.56	64.46	33.23
Fixed Transportation	8.73	10.22	10.21	10.13	5.70
Variable Transportation	13.76	15.79	15.88	15.85	9.60
Inventory Cost	3.43	0.54	0.50	0.44	3.00
Shortage Cost	9.26	9.00	8.84	9.12	48.47

Compared to ERP_{S_2}, ERP_{S_1} shows an increase of 0.18% in the objective but still it is 4.00% lower than $\text{EEV}_{\bar{S}}$. This comparison is intuitive because optimizing against more scenarios might improve the solution. However, more experimentation reveals that it is not always the case. The combination of four outcomes for both demands and quality of returns, which results in 4,096 paths, was left unsolved because of the expensive computational time. Instead, the scenario tree (S_3) with $|S_3| = 1,728$ paths comprised of three demands and four quality of returns was evaluated to test the increase in quality of return. The objective ERP_{S_3} was only 0.02% lower than ERP_{S_1} with two return qualities and was also 4.02% smaller than $\text{EEV}_{\bar{S}}$, the cost of deterministic solution. The improvement was not significant relative to its computational time as shown in Table 2.5. In this case, increasing the number of outcomes for the quality of return might not improve the solution.

Further analysis is performed on increasing the number of branches from four to six for demands at each stage of scenario tree (S_4) because this resulted in the largest number of scenarios that we were able to solve in the extensive form of the stochastic program. Applying the same two return quality outcomes results in 1,728 paths where its computational time is about 1,678 minutes shown in Table 2.5. Using more demand or return outcomes would require more than a week to be solved. A scenario tree composed of five demand outcomes and two quality of return levels yielded nearly the same solution as S_2. The evaluated cost ERP_{S_4} in Table 2.6 is actually increased by 1.74% compared to ERP_{S_2} but was still 2.47% lower than deterministic solution $\text{EEV}_{\bar{S}}$. The difference between the solution of S_4 and other scenario trees of S_1, S_2, and S_3 is the significant reduction of opened facilities in the network configuration that results in higher shortage cost in the evaluation ERP_{S_4}. Apparently, increasing the number of demand outcomes by two from its minimum of four found using equation (2.30) resulted in an over-fit that actually deteriorated the solution.

In this section, we solved the multi-stage model for scenario trees S_1' and S_2' assuming there is no dependencies among the periods. The connections between periods for demands are shown in equation (2.31) by value θ_r for period $r \in \mathcal{R}$. When $\theta_r = 1$ there is no dependencies between period r and $r-1$. The estimated values for period two and three using equation (2.32) were found to be $\theta_2 = 1$ and $\theta_3 \cong 0.5$. Period two is independent from period one. To generate outcomes for period three in order to be independent from period two, we calculated the average outcomes of dependent case of period three for every retailer as we have shown in Table A.7 as an example for S_2'. Table 2.7 shows the results of evaluation of their solution. The results are better than deterministic evaluation by 3.4% and 3.6% for scenario trees S_1' and S_2', respectively. However, they are both 0.6% lower than ERP_{S_1} and ERP_{S_2} in Table 2.6 which indicates that considering dependencies of demands among the periods would yield a better solution.

Table 2.8 also compares the optimal cost with the cost of two-stage solution in multi-stage formulation with scenario tree S_1 (ETRP_{S_1}). The facility configurations of both solution are the same. The optimal objective value of ETRP_{S_1} is \$3,783 higher than the multi-stage RP_{S_1}

Table 2.7 The evaluation of deterministic and stochastic solutions against simulated historical data, with category costs as % of total cost when there is no dependencies between periods.

	$\text{EEV}_{\bar{S}}$	$\text{ERP}_{S_1'}$	$\text{ERP}_{S_2'}$
Total Cost ($1000)	2206.18	2131.92	2126.91
Savings From EEV%	-	3.37	3.59
Facility Cost	64.82	33.54	33.62
Fixed Transportation	8.73	5.95	5.85
Variable Transportation	13.76	9.82	9.36
Inventory Cost	3.43	2.25	2.61
Shortage Cost	9.26	48.44	48.56

optimal solution, which this difference indicates the value of formulating and solving the multi-stage version. Multi-stage solution has more adjustability for the use of transportation capacity by adjusting in different periods. Table 2.9 shows the difference between the expected numbers of units of each transportation mode between multi-stage RP_{S_1} and two-stage RP_{S_1} solution. The percentage of contracts for mode two in the multi-stage solution is higher than in the two-stage solution which shows that the former uses transportation units with higher capacity.

Table 2.8 The comparison of two-stage and multi-stage solutions with scenario tree S_1 and category costs as % of total cost where ETRP is the evaluation of two-stage RP solution in multi-stage RP formulation.

	Multi-stage RP_{S_1}	ETRP_{S_1}
Total Cost ($1000)	2020.31	2024.10
Facility Cost	67.56	67.44
Fixed Transportation	10.68	10.67
Variable Transportation	16.57	16.58
Inventory Cost	0.00	0.00
Shortage Cost	5.18	5.31

The facility investments of the $\bar{S}, S_1, ..., S_4$ solutions are compared in Figure 2.7. Figure 2.7(a) depicts the retailer locations, and Figure 2.7(b) shows the potential locations of three plants, four warehouses, and four collection centers. In the deterministic solution $\bar{S}$ (Figure 2.7(c)) we have two opened locations for every facility. A similar configuration exists in the S_1 solution (Figure 2.7(d)) where the only difference is the number and position of opened collection centers. The solutions for S_2 and S_3 both have the configuration shown in Figure

Table 2.9 The comparison between the expected number of units of each transportation mode contracted in multi-stage solution RP_{S1} and two-stage solutions RP_{S_1}

	Multi-stage RP_{S_1}			Two-stage RP_{S_1}		
Modes	1	2	3	1	2	3
Period 1	50.8	99.3	14.8	51.2	99.2	14.7
Period 2	65.2	117.0	15.8	72.5	113.5	15.9
Period 3	73.0	137.0	21.1	70.6	136.8	21.3
% usage of modes	31.8	59.5	8.7	32.6	58.7	8.7

2.7(e), in which their only difference from the S_1 solution is the position of a collection center. Figure 2.7(f) illustrates the facility configuration investments from S_4 which has only one opened facility of each type (plant, warehouse, and collection center). The facility investment of S_4 with six outcomes for demand has the most significant difference among the other scenario trees.

Table 2.10 shows the difference between the expected numbers of units of transportation modes contracted in the $\bar{S}$ solution and the four stochastic solutions within different periods. The use of mode two with more capacity and higher cost compared to the first mode increases from 49.62% in the $\bar{S}$ solution to over 56% in all stochastic solutions. However, usage of mode three does not change as much as the other modes. Overall, the stochastic solutions indicate that the percentage usage of the mode with higher capacity and fixed cost increases in an uncertain environment. Moreover, due to more uncertainty in demands and returns, there is overall more transportation capacity reserved in the stochastic solutions as shown in Table 2.10. However, the solution from S_4 reserves less transportation capacity, consistent with its fewer opened facilities as shown in Figure 2.7(f).

Overall, the qualitative conclusion is that using more scenarios might not result in a better solution but finding the best number of outcomes representing the underlying distribution is more effective. However, the solutions obtained from different scenario trees show two near-optimal investments that, when evaluated by historical simulated data, are both superior to the deterministic solution.

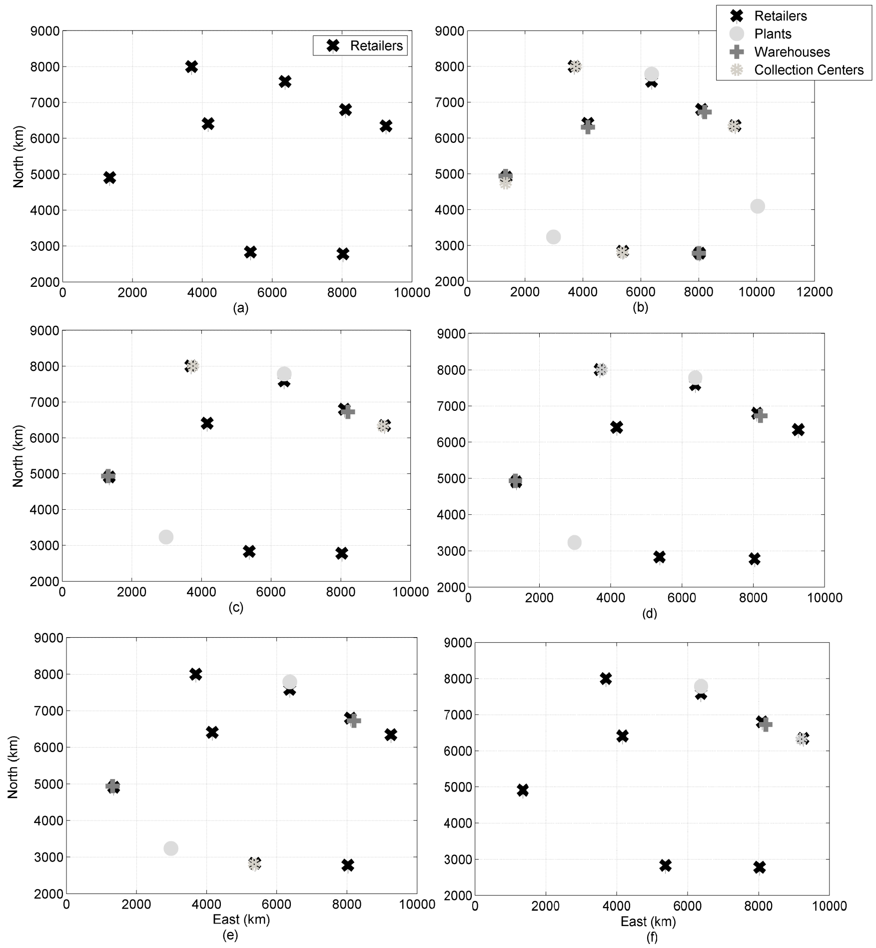

Figure 2.7 The retailers' locations and the potential facility locations are shown in (a) and (b), respectively. The facility configurations in the above figure is as follows: (c): $\bar{S}$, (d): S_1, (e): both S_2 and S_3 , and (f): S_4

2.6 Conclusion

In this paper, we proposed a multi-stage stochastic program when quality of returned products as well as demands are uncertain in a CLSC network design problem that involves long-term decisions to invest in fixed facilities such as manufacturing/remanufacturing plants, warehouses, and collection facilities. In addition, decisions concerning capacities of different modes to transport products were included before realization of uncertain parameters in each stage. The proposed multi-stage stochastic program manages the trade-off between the shortage of demands or the loss of used products, and excess processing or transportation capacity that goes unused.

Table 2.10 The comparison between the expected number of units of each transportation mode contracted in $\bar{S}$ and stochastic solutions: S_1, S_2, S_3 and S_4

	$\bar{S}$			S_1			S_2			S_3			S_4		
Modes	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Period 1	58.2	76.7	13.2	50.8	99.3	14.8	53.0	109.0	15.3	51.2	100.0	15.4	39.7	61.9	8.0
Period 2	72.0	89.1	15.9	65.2	117.0	15.8	70.3	110.0	16.0	71.9	113.0	15.9	39.7	72.4	8.2
Period 3	96.1	107.0	21.8	73.0	137.0	21.1	88.5	142.0	20.6	73.4	136.0	20.9	52.8	70.3	12.1
% usage of modes	41.1	49.6	9.2	31.8	59.5	8.7	33.9	57.8	8.3	32.8	58.5	8.7	36.2	56.1	7.7

A moment matching method was applied to generate the scenario tree for demands, and distribution approximation was used to generate discrete outcomes from a continuous distribution of the uncertain returns quality.

A numerical instance illustrated how uncertainty in demands and quality of returns changes the solution concerning the type of transportation modes and facility investments. All solutions found from scenario trees with different granularities were evaluated in an out-of sample simulation. The underinvestment in transportation capacities of the solution to the deterministic expected value model results in more expected inventory and shortage cost compared to the stochastic program solutions. When uncertainty is taken into account, more transportation capacity is contracted to satisfy more demands while the use of high capacity modes with more fixed cost increased. Different levels of granularity of scenarios demonstrated the existence of a significantly dissimilar alternative near-optimal solution. Increasing the number of outcomes of return quality results in a small improvement in cost. By decreasing the number of demand outcomes from its minimum according to the degrees of freedom available, the solution slightly degraded. The solution was also deteriorated significantly by increasing the number of demand outcomes from its minimal value. Thus, some scenario increments might not necessarily improve the solution due to overfitting. The results of multi-stage solution when there is no dependencies of demands among periods shows a reduction on solution quality comparing to the scenario tree with dependent demands among periods. Finally, the solution of a two-stage stochastic problem has less adjustability for the use of transportation capacity across different periods comparing to the multi-stage solution.

Multi-stage stochastic programming poses challenges in both formulation and computation. Future work is warranted to model the relationships among uncertain variables over time and generate accurate scenario trees that are not too large. In addition, the application of further improved methods for generating multi-stage trees could be investigated and compared. Finally, the solution of larger-scale instances may require decomposition approaches such as progressive hedging or the nested L-shaped method.

CHAPTER 3. CONDITIONS UNDER WHICH ADJUSTABILITY LOWERS THE COST OF A ROBUST LINEAR PROGRAM

3.1 Introduction

Robust optimization is a modeling strategy in which an uncertainty set describes the possible values of some parameters of a mathematical program. The goal in this optimization approach is to find a best solution that is feasible for all parameter values within the uncertainty set. In the original formulation by Soyster (1972) the solution was often observed to be very conservative. The approach was further developed by Ben-Tal and Nemirovski (1998, 1999, 2000) as well as Ghaoui and Lebret (1997) and Ghaoui et al. (1998) independently. These papers proposed tractable solution approaches to special cases of the robust counterpart (RC) in the form of conic quadratic problems with less conservative results.

In the RC formulation, the values of all decision variables are determined before the realization of uncertain parameters (i.e., treated as "here and now" decisions). However, there are applications in which some variables, including auxiliary variables such as slack or surplus variables, could be decided after realization of (some of) the uncertain parameters ("wait and see" decisions). Ben-Tal et al. (2004) proposed an adjustable robust counterpart (ARC) for models with adjustable variables that tune themselves with uncertain parameters. They introduced the ARC concept with two types of recourse; fixed, where the coefficients of adjustable variables are deterministic, and uncertain, where they are not. Because ARC formulations may not be computationally tractable, they also proposed an affinely adjustable robust counterpart (AARC) to approximate the ARC by restricting the adjustable variables to be affine functions of the uncertain parameters. Similar techniques of considering linear adjustability to uncertain parameters have also been employed for tractability in linear stochastic optimization under the label of linear decision rules such as in Kuhn et al. (2009); Bertsimas et al. (2010); Chen and Zhang (2009); and Bertsimas et al. (2013).

The ARC formulation is appealing because it avoids unnecessary conservatism by allowing adjustability. The challenge is that it is not always straightforward to determine when the ARC or AARC might be less conservative than the RC formulation in real applications. Several published cost minimization applications where $Z_{AARC} < Z_{RC}$ include project management (Cohen et al., 2007), inventory control (Ben-Tal et al., 2009), telecommunication (Ouorou, 2013), and production planning (Solyali, 2014). But several papers establish conditions under which $Z_{ARC} = Z_{RC}$ or $Z_{AARC} = Z_{RC}$. Ben-Tal et al. (2004), Bertsimas and Goyal (2010), Bertsimas et al. (2011), Bertsimas and Goyal (2013), Bertsimas et al. (2015), and Marandi and den Hertog (2015) defined conditions under which $Z_{ARC} = Z_{RC}$. However, in some important applications that are not covered by these papers' assumptions, we find $Z_{AARC} = Z_{RC}$, while $Z_{AARC} < Z_{RC}$ in others.

The goal of this paper is to help determine whether Z_{ARC} may be less than Z_{RC} in an application that, without loss of generality, we assume has a cost minimization objective. Because AARC is more tractable than ARC and $Z_{ARC} \leq Z_{AARC}$, we study conditions under which $Z_{AARC} < Z_{RC}$ as a sufficient condition for $Z_{ARC} < Z_{RC}$. Our conditions include the presence of at least two binding constraints at optimality of the RC formulation, and an adjustable variable in both constraints with implicit bounds from above and below with different extreme values in the uncertainty set. Using the dual of the RC, which is explored in Beck and Ben-Tal (2009), we show how RC formulations can be tested in small instances in order to identify whether affine adjustability matters. In this paper, we restrict attention to models with fixed recourse and box uncertainty sets.

In the next section, the required preliminary definitions and explanations are presented. Section 3.3 provides the proposition in detail with illustrative examples. Examples taken from applications in the literature are illustrated in Section 3.4. Conclusions and future research directions are provided in Section 3.5.

3.2 Preliminaries

Consider a linear program (LP):

$$\min_{w \geq 0} c^T w : \quad A' w \leq b, \tag{3.1}$$

where $w \in \mathbb{R}^n_+, c \in \mathbb{R}^n, A' \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m$. The RC of (3.1) was proposed by Ben-Tal et al. (2004) as follows:

$$\min_{w \geq 0} \max_{\zeta \in \mathcal{Z}} \left\{ c^T w : \quad A' w - b \leq 0, \quad \forall \zeta = [c, A', b] \in \mathcal{Z} \right\},$$

where $\mathcal{Z} \subset \mathbb{R}^n \times \mathbb{R}^{m \times n} \times \mathbb{R}^m$ is a given uncertainty set. We can decompose the decision variables w into non-adjustable variables x and adjustable variables y. In addition, if the costs of some non-adjustable variables are affected by uncertainty then we reformulate as in (3.2) to move all uncertainty to the constraints:

$$\min_{u,x,y \geq 0} \left\{ u : c_x^T x + c_y^T y - u \leq 0, Ax + Dy \leq b, \ \forall \zeta = [c, A, D, b] \in \mathcal{Z} \right\}, \tag{3.2}$$

where $x \in \mathbb{R}^{n-p}_+, y \in \mathbb{R}^p_+, A \in \mathbb{R}^{m \times (n-p)}, D \in \mathbb{R}^{m \times p}, b \in \mathbb{R}^m, \mathcal{Z} \subset \mathbb{R}^n \times \mathbb{R}^{m \times (n-p)} \times \mathbb{R}^{m \times p} \times \mathbb{R}^m$. Upon this reformulation (if necessary), we can state the robust counterpart as:

$$Z_{RC} = \min_{x,y \geq 0} \left\{ c_x^T x + c_y^T y : Ax + Dy \leq b, \ \forall \zeta = [A, D, b] \in \mathcal{Z} \right\}. \tag{3.3}$$

Henceforth, we assume all uncertain parameters appear in the constraints. The ARC corresponding to (3.3), where the adjustable variable y is decided after realization of the uncertain parameters, is:

$$Z_{ARC} = \min_{x, y(\zeta) \geq 0, \forall \zeta \in \mathcal{Z}} \left\{ c_x^T x + \max_{\zeta \in \mathcal{Z}} c_y^T y(\zeta) : Ax + Dy(\zeta) \leq b, \ \forall \zeta = [A, D, b] \in \mathcal{Z} \right\}. \tag{3.4}$$

Ben-Tal et al. (2004) assumed, without loss of generality, that the uncertainty set $\mathcal{Z}$ is affinely parameterized by a perturbation vector ξ varying in a given non-empty convex compact perturbation set $\chi \subset \mathbb{R}^L$:

$$\mathcal{Z} = \left\{ [A, D, b] = \left[A^0, D^0, b^0\right] + \sum_{l=1}^{L} \xi^l \left[A^l, D^l, b^l\right] : \xi \in \chi \right\}. \tag{3.5}$$

In the case of fixed recourse, the coefficients of the adjustable variables are deterministic (i.e., $D^l = 0$ for $l = 1, ..., L$). If we define $a_i^l \in \mathbb{R}^{n-p}$ as the i^{th} row of A^l, $d_i \in \mathbb{R}^p$ as the i^{th} row of D^0 and $b_i^l \in \mathbb{R}$ as the i^{th} element of vector b^l, the RC formulation with fixed recourse is as follows:

$$Z_{RC} = \min_{x,y \geq 0}$$
$$\left\{ c_x^T x + c_y^T y : \left(a_i^0 + \sum_{l=1}^{L} \xi^l a_i^l \right) x + d_i y \leq b_i^0 + \sum_{l=1}^{L} \xi^l b_i^l,\ \forall \xi \in \chi,\ i = 1, ..., m \right\}, \tag{3.6}$$

and the fixed recourse version of ARC is:

$$Z_{ARC} = \min_{x,y(\xi) \geq 0, \forall \xi \in \chi} \left\{ c_x^T x + \max_{\xi \in \chi} c_y^T y(\xi) : \right.$$
$$\left. \left(a_i^0 + \sum_{l=1}^{L} \xi^l a_i^l \right) x + d_i y(\xi) \leq b_i^0 + \sum_{l=1}^{L} \xi^l b_i^l,\ \forall \xi \in \chi,\ i = 1, ..., m \right\}. \tag{3.7}$$

The AARC is an approximation of the ARC in which the adjustable variables are restricted to be affine functions of the uncertain parameters. In this approximation, if $\mathcal{Z}$ is affinely parameterized as defined in equation (3.5), the adjustable variables y are restricted to affinely depend on ξ:

$$y = \pi^0 + \sum_{l=1}^{L} \xi^l \pi^l \geq 0, \tag{3.8}$$

where $\pi^l \in \mathbb{R}^p$ for $l = 0, ..., L$. The fixed recourse AARC formulation corresponding to (3.7) is:

$$Z_{AARC} = \min_{x \geq 0, \pi} \left\{ c^T x + \max_{\xi \in \chi} c_y^T \left(\pi^0 + \sum_{l=1}^{L} \xi^l \pi^l \right) : \right.$$
$$\left(a_i^0 + \sum_{l=1}^{L} \xi^l a_i^l \right) x + d_i \left(\pi^0 + \sum_{l=1}^{L} \xi^l \pi^l \right) \leq b_i^0 + \sum_{l=1}^{L} \xi^l b_i^l, \ \forall \xi \in \chi, \quad i = 1, ..., m;$$
$$\left. \pi^0 + \sum_{l=1}^{L} \xi^l \pi^l \geq 0, \ \forall \xi \in \chi \right\}. \tag{3.9}$$

In practice, π^l would be forced to equal zero if y is not adjustable to the l^{th} perturbation for some $l \in \{1, ..., L\}$. The AARC (3.9) is computationally tractable. Even when the coefficients of the adjustable variables are uncertain, it can be approximated by an explicit semi-definite program if the uncertainty set is an intersection of concentric ellipsoids (Ben-Tal et al., 2004). However, the AARC formulation with uncertainty-affected recourse imposes more computational challenge that is not considered in this paper. In addition, only box uncertainty sets (3.10) are considered here to avoid the complexity of interactions among uncertainties. That is, we define

$$\chi = \left\{ \xi : |\xi^l| \leq \rho^l, l = 1, ..., L \right\}, \tag{3.10}$$

where, without loss of generality, we assume that $\rho^l = 1$ for all $l = 1, ..., L$.

3.3 Conditions For $Z_{ARC} < Z_{RC}$

Because $Z_{ARC} \leq Z_{AARC} \leq Z_{RC}$, conditions under which $Z_{AARC} < Z_{RC}$ are sufficient for $Z_{ARC} < Z_{RC}$ as well. The behavior of the solution to the AARC formulation depends on how the uncertain parameters interact in the RC constraints. As detailed below, adjustability may lower the cost if there are at least two constraints that are binding at an optimal RC solution for different values of the same uncertain parameter. In addition, a decision variable that could be made adjustable appears in both constraints, one of which bounds it from above at one extreme of the uncertainty set and the other bounds it from below at the opposite extreme of the uncertainty set. By allowing the variable to adjust to that uncertain parameter, there is a possible improvement from using AARC formulation and, therefore, the more general ARC formulation.

Several papers provided conditions to show when adjustability does not matter or provided bounds on Z_{ARC} based on Z_{RC}. Ben-Tal et al. (2004) and Marandi and den Hertog (2015) proved that for models with constraint-wise uncertainty, $Z_{RC} = Z_{ARC}$. But they did not explicate how the interaction of the same uncertain parameter in separate constraints might allow adjustability to lower the optimal cost.

In other papers, some limitations prevent identification of models with inequality between Z_{RC} and Z_{ARC}. Bertsimas and Goyal (2010) and Bertsimas et al. (2011) approximated a two-stage stochastic model and an adjustable robust counterpart with the robust counterpart. They considered both objective coefficient and constraint right-hand side uncertainty. Bertsimas and Goyal (2010) proved that, for hypercube uncertainty set when uncertainty is in the objective and right-hand side, the robust solution is equal to fully adjustable solution. Using a generalized notion of symmetry for general convex uncertainty sets, Bertsimas et al. (2011) extend the Bertsimas and Goyal (2010) static robust solution performance in two-stage stochastic optimization problems. Bertsimas and Goyal (2011) also compared the optimal affine policy to the optimal fully adaptable solution with no comparison between RC and ARC. The limitation of these studies includes the right-hand side nonnegativity of non-strict "greater than" constraints, which prevents them from modeling upper bounds on decision variables.

Bertsimas and Goyal (2013) and Bertsimas et al. (2015) extended the uncertainty to be in constraint and objective coefficients. They approximated the Z_{ARC} with the Z_{RC} to handle packing constraints such as in revenue management or resource allocation problems. Their limitation, however, does not allow the adjustable variable to have a lower-bound because of the assumptions of non-strict "less than" constraints. Moreover, Bertsimas and Goyal (2013) assumed that objective and constraints are convex and the constraint functions should be convex regarding positive decision variables, and also concave and increasing with respect to uncertain parameters of the positive compact convex set. Bertsimas et al. (2015) assumed a linear objective and constraint functions with tighter bounds and fewer positivity restricted parameters compared to Bertsimas and Goyal (2013). However, they still assumed constraint coefficients, second-stage objective coefficients and decision variables to be positive which rules out lower bounds on second-stage decision variables. Table 3.1 compares the restrictions existing in the literature to our model for the comparison between RC and ARC optimal objective values in linear programming. The implicit lower and upper bounds imposed by constraints on adjustable decision variables are important aspects of the conditions for $Z_{AARC} < Z_{RC}$ to be stated below.

Table 3.1 The limitations considered in the papers and this research for the comparison between RC and ARC objectives in LP

Paper	Uncertain parameters	Limitations of the comparison between RC and ARC
Ben-Tal et al. (2004)	All parameters	Constraint-wise uncertainty
Marandi and den Hertog (2015)	All parameters	Constraint-wise uncertainty
Bertsimas and Goyal (2011, 2010)	b and c_y	$x, y \geq 0$, and $c, b \geq 0$
Bertsimas et al. (2011)	b and c_y	$b \geq 0$, and $\mathcal{Z} \geq 0$
Bertsimas and Goyal (2013)	D and c_y	$x, y \geq 0$, and $c, A, D, b \geq 0$
Bertsimas et al. (2015)	D and c_y	$y \geq 0$,and $c, D \geq 0$
This paper	A and b	Box uncertainty set and $x, y \geq 0$

We identify numerical conditions under which the use of the AARC formulation produces less conservative solutions than the RC. To be able to apply these conditions, we must solve the RC in a representative instance for its optimal primal and dual values. Duality in robust optimization has been studied recently by Beck and Ben-Tal (2009), Soyster and Murphy (2013), Soyster and Murphy (2014), and Bertsimas and Ruiter (2015). The dual of (3.6) can be written as (Beck and Ben-Tal, 2009):

$$D_{RC} = \max_{\lambda} \left\{ \sum_{i=1}^{m} \lambda_i \left[b_i^0 + \sum_{l=1}^{L} \hat{\xi}_i^l b_i^l \right] : \right.$$
$$\left. \lambda a^s \leq c_x^s, \quad \lambda d^{s'} \leq c_y^{s'}, \quad \lambda \leq 0, \quad s = 1, ..., n-p, s' = 1, ...p \right\}, \tag{3.11}$$

where a^s and $d^{s'}$, respectively, denote column s of $A^0 + \sum_{l=1}^{L} \hat{\xi}^l A^l$ and column s' of D, and $\hat{\xi}_i^l$ is the value of ξ for which constraint i is binding (see Definition 2) in the optimal solution to the

RC. The dual of the RC is the same as the optimistic counterpart of the dual of the original linear program (3.1), as mentioned in Beck and Ben-Tal (2009).

The feasible region of the RC (3.6) can be expressed as a convex set $\bigcap_{i=0}^{m} \mathcal{F}_{RC}^{i}$ where (Beck and Ben-Tal, 2009):

$$\mathcal{F}_{RC}^{i} = \left\{ x, y \geq 0 : \left(a_i^0 + \sum_{l=1}^{L} \xi^l a_i^l \right) x + d_i y \leq b_i^0 + \sum_{l=1}^{L} \xi^l b_i^l, \ \forall \xi \in \chi \right\},$$
$$i = 1, ..., m. \tag{3.12}$$

Likewise, the feasible region of AARC (3.9) is given by $\bigcap_{i=0}^{m} \mathcal{F}_{AARC}^{i}$, where

$$\mathcal{F}_{AARC}^{i} = \left\{ x \geq 0, \pi : \left(a_i^0 + \sum_{l=1}^{L} \xi^l a_i^l \right) x + d_i \left(\pi^0 + \sum_{l=1}^{L} \xi^l \pi^l \right) \leq b_i^0 + \sum_{l=1}^{L} \xi^l b_i^l, \right.$$
$$\left. \forall \xi \in \chi; \ \left(\pi^0 + \sum_{l=1}^{L} \xi^l \pi^l \right) \geq 0, \ \forall \xi \in \chi \right\}, i = 1, ..., m. \tag{3.13}$$

From Ben-Tal et al. (2004) we know that $\bigcap_{i=0}^{m} \mathcal{F}_{RC}^{i} \subseteq \bigcap_{i=0}^{m} \mathcal{F}_{AARC}^{i}$ because the AARC differs from the RC only by the inclusion of variables $\pi^l, l = 1, ..., L$. Moreover, (3.12) can be obtained from (3.13) by forcing π^l for $l = 1, ..., L$ to be zero. However, a larger robust feasible set does not necessarily improve the objective. If the parameters of distinct constraints are affected by different perturbations, the (affine) adjustable counterpart may be equivalent to the robust counterpart. The following definition formalizes this concept.

Definition 1. (Ben-Tal et al., 2004) Uncertainty in the RC is **constraint-wise** if $[A, b] \in \mathcal{Z}$ consists of non-overlapping sub-vectors $(a_i^l, b_i^l)_{l=1}^{L}$ for $i = 1, ..., m$ such that $(a_i^0 + \sum_{l=1}^{L} \xi^l a_i^l)x + d_i y \leq b_i^0 + \sum_{l=1}^{L} \xi^l b_i^l$ depends on $(a_i^l, b_i^l)_{l=1}^{L}$ only. Moreover, if $\exists l \in \{1, ..., L\} : [a_i^l, b_i^l] \neq 0$,then $[a_j^l, b_j^l] = 0 \ \forall j \neq i$.

The following result identified some conditions under which the RC and ARC are equivalent.

Theorem 1 (See Theorem 2.1 of Ben-Tal et al. (2004))**.** *The objective values of RC (3.3) and ARC (3.4) are equal if:*

- *The uncertainty is constraint-wise, and*

- *Whenever x is feasible for ARC (3.4), there exists a compact set V_x such that for every A, D, b where $\zeta \in \mathcal{Z}$, the relation $Ax + Dy \leq b$ implies that $x \in V_x$.*

However, the more interesting question of when adjustability would result in $Z_{ARC} < Z_{RC}$ was not explored. The challenge of determining whether the ARC is more advantageous than the RC formulation in real applications is compelling because it is not always evidently determined beforehand. Up to now, it can be determined only by directly solving the full-scale AARC formulation. In some cases the AARC does not produce any better solution than the RC formulation. The proposition below establishes conditions under which the objective values of AARC (3.9) and RC (3.6) are not equal. The following are definitions necessary for stating the conditions.

Definition 2. If $(\hat{x}^{RC}, \hat{y}^{RC})$ is any optimal solution of the RC (3.6), we say that constraint $i \in \{1, ..., m\}$ is **binding** at $(\hat{x}^{RC}, \hat{y}^{RC})$ if $a_i^0 \hat{x}^{RC} + d_i \hat{y}^{RC} = b_i^0 + \sum_{l=1}^{L} \hat{\xi}_i^l (b_i^l - a_i^l \hat{x}^{RC})$ where $\hat{\xi}_i \equiv \text{argmin}_\xi \left(b_i^0 + \sum_{l=1}^{L} \xi^l (b_i^l - a_i^l \hat{x}^{RC}) \right)$ is the worst-case value of ξ with respect to constraint i.

When the uncertainty is not constraint-wise, at least one component $l = 1, ..., L$ is involved in more than one constraint. However, the worst-case value of ξ^l can differ across constraints.

Definition 3. Constraint i is said to be **relaxed** by changing some parameter values a_i^l, d_i, b_i^l to $a_i'^l, d_i', b_i'^l$, if the result is a feasible region $\mathcal{F}_{RC}'^i \subset \mathcal{F}_{RC}^i$.

Proposition 1. *Considering the RC formulation of equation (3.6) and $(\hat{x}^{RC}, \hat{y}^{RC})$ to be any optimal solution, suppose:*

1. *There exist two binding constraints indexed by $j, k \in \{1, ..., m\}, j \neq k$, where relaxing either of these constraints strictly improves Z_{RC}, and $\hat{\xi}_j \neq \hat{\xi}_k$, where $\hat{\xi}_j$ and $\hat{\xi}_k$ are defined in Definition 2.*

2. *The uncertainty is not constraint-wise with respect to the constraints j and k identified in condition 1. Specifically, $\exists q \in \{1, ..., L\}$ such that the q^{th} parameters are non-zero in both constraints: $[a_j^q, b_j^q] \neq 0$ and $[a_k^q, b_k^q] \neq 0$.*

3. There is a component y_r with objective coefficient $c_{y_r} \in \mathbb{R}$ that is basic in $(\hat{x}^{RC}, \hat{y}^{RC})$ such that

a. $d_{jr} d_{kr} < 0$ for the constraints j and k identified in condition 1, and

b. y_r is adjustable to the perturbation ξ^q in AARC where q is defined in condition 2. In other words, in equation (3.8) $\pi_r^q \in \mathbb{R}$ is not forced to be zero.

Assume that $[a_i^q, b_i^q] = 0$ for $i \neq \{j,k\}$. Let λ^* be an optimal dual solution corresponding to $(\hat{x}^{RC}, \hat{y}^{RC})$ as defined by Beck and Ben-Tal (2009), and j and k index two constraints of RC as defined in condition 1. Then

$$\begin{aligned}
&\left|\lambda_j^*(b_j^q - a_j^q \hat{x}^{RC})\right| + \left|\lambda_k^*(b_k^q - a_k^q \hat{x}^{RC})\right| > \left|\lambda_j^*(b_j^q - a_j^q \hat{x}^{RC} - d_{jr}\delta)\right| \\
&+ \left|\lambda_k^*(b_k^q - a_k^q \hat{x}^{RC} - d_{kr}\delta)\right| + |c_{y_r}\delta| + \sum_{\substack{i=1 \\ i \neq \{j,k\}}}^{m} |\delta \lambda_i^* d_{ir}| \,. \qquad (3.14)
\end{aligned}$$

for some $\delta \neq 0$ implies $Z_{RC} > Z_{AARC}$.

Proof. Consider the intersection of the feasible regions defined by constraints j and $k, \mathcal{F}_{RC}^i \cap \mathcal{F}_{RC}^k$, and focus on the perturbation ξ^q. Condition 1 implies:

$$a_j^0 \hat{x}^{RC} + d_j \hat{y}^{RC} = b_j^0 + \sum_{\substack{l=1 \\ l \neq q}}^{L} \hat{\xi}_j^l \left(b_j^l - a_j^l \hat{x}^{RC}\right) + \hat{\xi}_j^q \left(b_j^q - a_j^q \hat{x}^{RC}\right) \text{ and}$$

$$a_k^0 \hat{x}^{RC} + d_k \hat{y}^{RC} = b_k^0 + \sum_{\substack{l=1 \\ l \neq q}}^{L} \hat{\xi}_k^l \left(b_k^l - a_k^l \hat{x}^{RC}\right) + \hat{\xi}_k^q \left(b_k^q - a_k^q \hat{x}^{RC}\right),$$

where, based on condition 1, $\hat{\xi}_j^q \neq \hat{\xi}_k^q$.

From Bazaraa et al. (2010), if surplus variables s are added to linear program (3.1) converting the inequalities to equalities, we have

$$z^* = \min_{w' \geq 0} c^T w' : \quad A'w' = b, \qquad (3.15)$$

where $w'^T = [w, s]$ can be partitioned into w'_B as basic variables and w'_N as non-basic variables in a given basic solution. In addition, if B^* and N are the corresponding optimal basic and non-basic matrices, respectively, the objective and the optimal values of the basic variables can be written as $z^* - c_N^T w'_N = c_{B^*}^T w'_{B^*}$, where $w'_{B^*} = B^{*-1}b - B^{*-1}Nw'_N$. That is, we can rewrite objective z^* as

$$z^* = c_{B^*}^T B^{*-1} b + w_N'^T (c_N - c_{B^*}^T B^{*-1} N) = \lambda^* b + w_N'^T (c_N - \lambda^* N), \tag{3.16}$$

where $\lambda^* = c_{B^*}^T B^{*-1}$ is an optimal dual vector corresponding to the particular optimal solution w'^*. If we define $\Delta_{B^*} = (x_{B^*}, y_{B^*})$ as basic variables and $\Delta_N = (x_N, y_N)$ as the non-basic variables in $(\hat{x}^{RC}, \hat{y}^{RC})$ then we can write:

$$Z_{RC}(\Delta_N) = \sum_{i=1}^{m} \lambda_i^* \left[b_i^0 + \sum_{l=1}^{L} \hat{\xi}_i^l b_i^l \right] + \Delta_N (c_N - \lambda^* N), \tag{3.17}$$

where $Z_{RC}(\Delta_N)$ is the objective value of RC as a function of non-basic variables Δ_N and $Z_{RC}(0) = Z_{RC}$. By subtracting the constant $\sum_{i=1}^{m} \lambda_i^* \left[\sum_{l=1}^{L} \hat{\xi}_i^l (a_i^l \hat{x}^{RC}) \right]$ from $Z_{RC}(\Delta_N)$, we have:

$$z(\Delta_N) \equiv Z_{RC}(\Delta_N) - \sum_{i=1}^{m} \lambda_i^* \left[\sum_{l=1}^{L} \hat{\xi}_i^l (a_i^l \hat{x}^{RC}) \right] =$$

$$\sum_{i=1}^{m} \lambda_i^* \left[b_i^0 + \sum_{l=1}^{L} \hat{\xi}_i^l \left(b_i^l - a_i^l \hat{x}^{RC} \right) \right] + \Delta_N \left(c_N - \lambda^* N \right). \tag{3.18}$$

From condition 3, we know $y_r \in \Delta_{B^*}$. Therefore, based on (3.6) and (3.9) we can identify π_r^q as a new variable with constraint column N_r and objective coefficient $\mathcal{C}_r$ as follows:

$$N_r = \begin{bmatrix} \dots & d_{jr}\hat{\xi}_j^q & \dots & d_{kr}\hat{\xi}_k^q & \dots \end{bmatrix}^T, \mathcal{C}_r = \xi^q c_{y_r}. \tag{3.19}$$

Recall that $\hat{\xi}_j^q$ and $\hat{\xi}_k^q$ are the worst-case values of ξ in equation (3.6) for constraints j and k, respectively, and c_{y_r} is the objective function coefficient of y_r. Based on equation (3.18), for $\Delta_N = (0, ..., \pi_r^q)^T$ where π_r^q has been appended to the set of non-basic variables, we have:

$$z\left((0, ..., \pi_r^q)^T\right) = \sum_{i=1}^{m} \lambda_i^* \left[b_i^0 + \sum_{l=1}^{L} \hat{\xi}_i^l (b_i^l - a_i^l \hat{x}^{RC}) \right] + \pi_r^q (\mathcal{C}_r - \lambda^* N_r). \tag{3.20}$$

Following equation (3.20) we can isolate j and k and also substitute (3.19):

$$z\left((0,...,\pi_r^q)^T\right) = \sum_{\substack{i=1\\ i\neq\{j,k\}}}^{m} \lambda_i^* \left[b_i^0 + \sum_{l=1}^{L} \hat{\xi}_i^l (b_i^l - a_i^l \hat{x}^{RC})\right] +$$
$$\lambda_j^* \left[b_j^0 + \sum_{\substack{l=1\\ l\neq q}}^{L} \hat{\xi}_j^l (b_j^l - a_j^l \hat{x}^{RC}) + \hat{\xi}_j^q (b_j^q - a_j^q \hat{x}^{RC})\right] +$$
$$\lambda_k^* \left[b_k^0 + \sum_{\substack{l=1\\ l\neq q}}^{L} \hat{\xi}_k^l (b_k^l - a_k^l \hat{x}^{RC}) + \hat{\xi}_k^q (b_k^q - a_k^q \hat{x}^{RC})\right] +$$
$$\pi_r^q \left(\xi^q c_{y_r} - \lambda^* \begin{bmatrix} ... & d_{jr}\hat{\xi}_j^q & ... & d_{kr}\hat{\xi}_k^q & ... \end{bmatrix}^T\right) \tag{3.21}$$

Upon rearranging terms, denoting a value of π_r^q by δ, and also based on the assumption of $[a_i^q, b_i^q] = 0$ for $i \neq \{j,k\}$, we have:

$$z\left((0,...,\delta)^T\right) = \sum_{\substack{i=1\\ i\neq\{j,k\}}}^{m} \lambda_i^* \left[b_i^0 + \sum_{l=1}^{L} \hat{\xi}_i^l (b_i^l - a_i^l \hat{x}^{RC})\right] +$$
$$\lambda_j^* \left[b_j^0 + \sum_{\substack{l=1\\ l\neq q}}^{L} \hat{\xi}_j^l (b_j^l - a_j^l \hat{x}^{RC}) + \hat{\xi}_j^q (b_j^q - a_j^q \hat{x}^{RC} - d_{jr}\delta)\right] +$$
$$\lambda_k^* \left[b_k^0 + \sum_{\substack{l=1\\ l\neq q}}^{L} \hat{\xi}_k^l (b_k^l - a_k^l \hat{x}^{RC}) + \hat{\xi}_k^q (b_k^q - a_k^q \hat{x}^{RC} - d_{kr}\delta)\right]$$
$$+ \delta \xi^q c_{y_r} - \sum_{\substack{i=1\\ i\neq\{j,k\}}}^{m} \delta \lambda_i^* d_{ir} \xi_i^q. \tag{3.22}$$

Let $z(0) = Z_{RC} - \sum_{i=1}^{m} \lambda_i^* \left[\sum_{l=1}^{L} \hat{\xi}_i^l (a_i^l \hat{x}^{RC})\right] \equiv z\left((0,...,\delta)^T\right)$ for $\delta = 0$. The inequality $z(0) > z\left((0,...,\delta)^T\right)$ is equivalent to:

$$\hat{\xi}_j^q \lambda_j^*(b_j^q - a_j^q \hat{x}^{RC}) + \hat{\xi}_k^q \lambda_k^*(b_k^q - a_k^q \hat{x}^{RC}) > \hat{\xi}_j^q \lambda_j^*(b_j^q - a_j^q \hat{x}^{RC} - d_{jr}\delta)$$
$$+ \hat{\xi}_k^q \lambda_k^*(b_k^q - a_k^q \hat{x}^{RC} - d_{kr}\delta) + \xi c_{y_r}\delta - \sum_{\substack{i=1 \\ i \neq \{j,k\}}}^{m} \xi_i^q \delta \lambda_i^* d_{ir}. \tag{3.23}$$

Based on Definition 2 and assumption (3.10), $\hat{\xi}_i^q(b_i^q - a_i^q \hat{x}^{RC}) = -\left|b_i^q - a_i^q \hat{x}^{RC}\right|$. Then, since $\lambda_i \leq 0$ in the dual of RC (3.11), we have

$$\hat{\xi}_j^q \lambda_j^*(b_j^q - a_j^q \hat{x}^{RC}) + \hat{\xi}_k^q \lambda_k^*(b_k^q - a_k^q \hat{x}^{RC}) = \left|\lambda_j^*(b_j^q - a_j^q \hat{x}^{RC})\right| + \left|\lambda_k^*(b_k^q - a_k^q \hat{x}^{RC})\right| \tag{3.24}$$

In addition,

$$\hat{\xi}_j^q \lambda_j^*(b_j^q - a_j^q \hat{x}^{RC} - d_{jr}\delta) + \hat{\xi}_k^q \lambda_k^*(b_k^q - a_k^q \hat{x}^{RC} - d_{kr}\delta) + \xi c_{y_r}\delta - \sum_{\substack{i=1 \\ i \neq \{j,k\}}}^{m} \xi_i^q \delta \lambda_i^* d_{ir}$$
$$\leq \left|\lambda_j^*(b_j^q - a_j^q \hat{x}^{RC} - d_{jr}\delta)\right| + \left|\lambda_k^*(b_k^q - a_k^q \hat{x}^{RC} - d_{kr}\delta)\right| + |c_{y_r}\delta| + \sum_{\substack{i=1 \\ i \neq \{j,k\}}}^{m} |\delta \lambda_i^* d_{ir}| \tag{3.25}$$

Therefore, from the right-hand sides of (3.24) and (3.25), if (3.14) holds considering box uncertainty set (3.10), there exists $\delta \neq 0$ such that $z(0) > z\left((0, ..., \delta)^T\right)$(expressed as inequality (3.23)). Recall that $z(0) + \sum_{i=1}^{m} \lambda_i^* \left[\sum_{l=1}^{L} \hat{\xi}_i^l (a_i^l \hat{x}^{RC})\right] = Z_{RC}$. Because the AARC could have multiple adjustable variables, $Z_{AARC} \leq z\left((0, ..., \delta)^T\right) +$ $\sum_{i=1}^{m} \lambda_i^* \left[\sum_{l=1}^{L} \hat{\xi}_i^l (a_i^l \hat{x}^{RC})\right]$. Therefore, inequality (3.14) implies $Z_{RC} > Z_{AARC}$. □ □

Remark 1. For simplicity in the proof, we focus on only two constraints j and k that have the same uncertain parameter in (3.14), and consider y_r as adjustable to a single perturbation ξ^q where $[a_i^q, b_i^q] = 0$ for $i \neq \{j, k\}$. The result can be extended using the same intuition if there exist similar constraints to j or k that satisfy conditions 1 - 3 with no assumption that $[a_\kappa^q, b_\kappa^q] = 0$. Expressions of the form $|\lambda_\kappa^*(b_\kappa^q - a_\kappa^q \hat{x}^{RC})|$ and $|\lambda_\kappa^*(b_\kappa^q - a_\kappa^q \hat{x}^{RC} - d_{\kappa r}\delta)|$ for such constraints $\kappa \in \{1, ..., m\}$ would be added to the left- and right-hand sides of (3.14), respectively, and index κ should be excluded from the sum $\sum_{\substack{i=1 \\ i \neq \{j,k\}}}^{m} |\delta \lambda_i^* d_{ir}|$. The extension of (3.14) when considering all constraints is as follows:

$$\sum_{i=1}^{m} \left|\lambda_i^*(b_i^q - a_i^q \hat{x}^{RC})\right| > \sum_{i=1}^{m} \left|\lambda_i^*(b_i^q - a_i^q \hat{x}^{RC} - d_{ir}\delta)\right| + |c_{y_r}\delta| \tag{3.26}$$

Examples 4 and 5 illustrate the use of this expanded inequality.

Next we show the importance of condition 1, which has been ruled out in the literature. Condition 1 holds if there are two binding constraints with different values of the uncertain parameter at the optimal RC solution. The variable that is adjustable to the uncertain parameter in both constraints is effectively bounded above and below by these constraints based on condition 3. One of these bounds is unfavorable for the objective but can be relaxed by adjustability in a direction that lowers the objective value.

Remark 2. Suppose conditions 2 and 3 of Proposition 1 hold but $\hat{\xi}_j^q = \hat{\xi}_k^q = \hat{\xi}^q$. The coefficient of π_r^q in (3.20) is reformulated by inserting (3.19) as:

$$(\mathcal{C}_r - \lambda^* N_r) = \left(\xi^q c_{y_r} - c_{B^*} B^{*-1} \begin{bmatrix} \dots & d_{jr}\hat{\xi}_j^q & \dots & d_{kr}\hat{\xi}_k^q & \dots \end{bmatrix}^T\right). \tag{3.27}$$

The left-hand-side of (3.27) equals:

$$\left(\xi^q c_{y_r} - c_{B^*} B^{*-1} \begin{bmatrix} \dots & d_{jr} & \dots & d_{kr} & \dots \end{bmatrix}^T \hat{\xi}^q\right). \tag{3.28}$$

If $N_r = N_r' \hat{\xi}^q$ in (3.28) where $N_r' = \begin{bmatrix} \dots & d_{jr} & \dots & d_{kr} & \dots \end{bmatrix}^T$, since N_r' equals the r^{th} column of B^*, multiplying B^{*-1} and N_r' yields the r^{th} column of identity matrix I_n. Following (3.28) and since the r^{th} element of c_{B^*} is c_{y_r}, we have:

$$\left(\xi^q c_{y_r} - c_{B^*}^T \begin{bmatrix} 0 & \dots & \hat{\xi}^q & \dots & 0 \end{bmatrix}^T\right) = c_{y_r}(\xi^q - \hat{\xi}^q), \tag{3.29}$$

where (3.29) expresses the coefficient of π_r^q in (3.20) as a function of ξ. The parameter ξ^q can take on a value that forces the coefficient of π_r^q in (3.20) to equal 0. Therefore, for any value of π_r^q, $z\left((0, ..., \pi_r^q)^T\right) = z(0)$ and $Z_{AARC} = Z_{RC}$.

Note also that if condition 2 does not hold, then the uncertainty is constraint-wise, and Z_{ARC} equals Z_{RC} (Ben-Tal et al., 2004; Marandi and den Hertog, 2015).

To illustrate the proposition, Examples 1-3 are provided based on the following LP formulation:

$$\min_{x,y\geq 0} c_x x + c_y y : \quad a_1 x + d_1 y \leq b_1, a_2 x + d_2 y \leq b_2, \tag{3.30}$$

where $a_i = a_i^0 + \xi a_i^1$ and $b_i = b_i^0 + \xi b_i^1$ are the uncertain parameters in constraints $i = 1, 2$.

Example 1. This example illustrates equivalence of RC and AARC objective values based on Remark 2. If the parameter values of (3.30) are $a_1^0 = -3, a_1^1 = -1, a_2^0 = 0, a_2^1 = -1, b_1^0 = -6, b_1^1 = -1, b_2^0 = 1, b_2^1 = -1, c_x = c_y = 1, d_1 = 1$ and $d_2 = -1$ where $\xi \in [-1, 1]$, the RC formulation is as follows:

$$\begin{aligned}
&Z_{RC} = \min_{x,y\geq 0} x + y : \\
&(i = 1)\ -(3+\xi)x + y \leq -6 - \xi, \quad \forall \xi \in [-1, 1] \\
&(i = 2)\ -\xi x - y \leq 1 - \xi, \quad \forall \xi \in [-1, 1]
\end{aligned} \tag{3.31}$$

Figure 3.1(a) illustrates the RC feasible region formed by the constraints in their respective most restrictive cases. Since the uncertainty sets are polyhedral, the RC can be converted to an explicit LP by defining additional constraints and variables $v_1 = -\min_{-1\leq\xi\leq 1} \xi(x-1)$ in constraint 1 and $v_2 = -\min_{-1\leq\xi\leq 1} \xi(x-1)$ in constraint 2 as follows (Ben-Tal et al., 2004):

$$\begin{aligned}
Z_{RC} = \min_{x,y,v_1,v_2\geq 0} x + y : &-3x + y \leq -6 - v_1, \quad -v_1 \leq x - 1 \leq v_1, \\
&-y \leq 1 - v_2, \quad -v_2 \leq x - 1 \leq v_2.
\end{aligned} \tag{3.32}$$

The optimal values of the RC variables by solving (3.32) are $\hat{x}^{RC} = 3, \hat{y}^{RC} = 1, Z_{RC} = 4$. Note that the optimal solution for this particular instance could be identified with only one auxiliary variable $v \equiv v_1 = v_2$. We can identify $j = 1$ and $k = 2$ in (3.31) as satisfying conditions 1-3 of the proposition. The values of λ^* can be easily found using the deterministic formulation (3.32) for all corresponding constraints. For example, constraint $-3x+y \leq -6-v_1$ in (3.32) corresponds to $i=1$ in (3.31). The optimal basic variables of RC (3.31) are x and y. Their cost coefficients and the optimal values of the dual variables are $c_{B^*}^T = \begin{pmatrix} 1 & 1 \end{pmatrix}$ and $\lambda^* = c_{B^*}^T B^{*-1} = \begin{pmatrix} \lambda_1^* & \lambda_2^* \end{pmatrix} = \begin{pmatrix} -2 & -3 \end{pmatrix}$, respectively.

We can also obtain the values of ξ at the optimal solution in constraints j and k, by substituting the optimal values of $\hat{x}^{RC}$ and $\hat{y}^{RC}$ into constraints $j=1$ and $k=2$ of formulation (3.31) and identifying the values of ξ where the constraints hold as equalities. In this instance, we obtain $\hat{\xi}_1 = -1, \hat{\xi}_2 = -1$, which do not satisfy condition 1.

Considering the adjustable variable as an affine function $y = \pi^0 + \xi\delta$, and by inserting the corresponding parameter values and the new variable δ into (3.14) we obtain:

$$10 > 2\left|2-\delta\right| + 3\left|2+\delta\right| + \left|\delta\right|. \tag{3.33}$$

The inequality (3.33) cannot be satisfied, because its right-hand side is a convex piecewise linear expression with minimum value 10. Indeed by solving the AARC with $y = \pi^0 + \xi\delta$, we find $\hat{x}^{AARC} = 3, \hat{\pi}^0 = 1, \hat{\delta} = 0$,and $Z_{AARC} = 4 = Z_{RC}$.

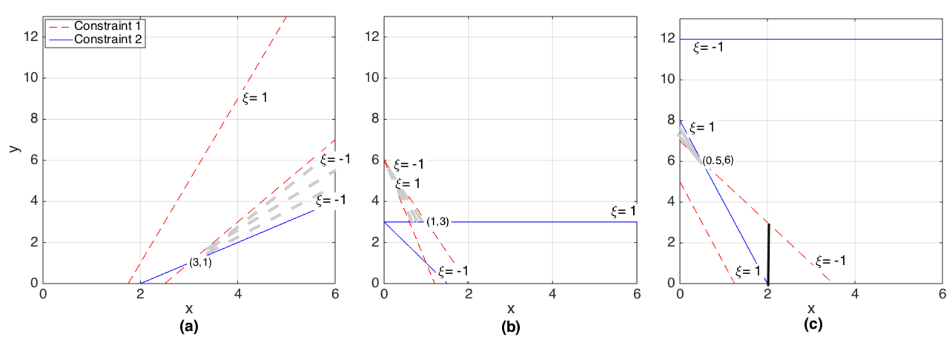

Figure 3.1 The feasible regions of the RC constraints within uncertainty set $\xi \in [-1,1]$ for (a) Example 1, (b) Example 2, (c) Example 3 are shaded with gray lines. The thick black line in (c) is $y = \frac{3}{2} - \frac{3}{2}\xi$ for $\xi \in [-1,1]$.

Example 2. This instance shows that if condition 3(a) is not satisfied (i.e., $d_{jr}d_{kr} > 0$ but still conditions 1, 2 and 3(b) are satisfied) the objective values of RC and AARC are equal.

The RC formulation of (3.30) with $a_1^0 = -4, a_1^1 = -1, a_2^0 = -1, a_2^1 = 1, b_1^0 = -6, b_1^1 = 0, b_2^0 = -3, b_2^1 = 0, c_x = c_y = 1, d_1 = -1$ and $d_2 = -1$ where $\xi \in [-1, 1]$ is:

$$
\begin{aligned}
& Z_{RC} = \min_{x,y \geq 0} x + y : \\
& (i = 1) \; -(4 + \xi)x - y \leq -6, \quad \forall \xi \in [-1, 1] \\
& (i = 2) \; (-1 + \xi)x - y \leq -3, \quad \forall \xi \in [-1, 1]
\end{aligned} \tag{3.34}
$$

The optimal values of RC variables following the same method of Example 1, in which we converted the RC problem to its deterministic formulation (3.32), are $\hat{x}^{RC} = 1, \hat{y}^{RC} = 3, Z_{RC} = 4$ (see Figure 3.1(b)). The two constraints $j = 1$ and $k = 2$ satisfy conditions 1, 2 and 3(b) but not 3(a). Moreover, the coefficients of the adjustable variable y for the two constraints are $d_1 = -1, d_2 = -1$. Also, $\hat{\xi}_1 = -1$ and $\hat{\xi}_2 = 1$. The optimal values of dual variables are $\begin{pmatrix} \lambda_1^* & \lambda_2^* \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & -\frac{2}{3} \end{pmatrix}$. After inserting the corresponding parameter values in equation (3.14), we have:

$$
1 > \frac{1}{3} \left| 1 + \delta \right| + \frac{2}{3} \left| -1 + \delta \right| + \left| \delta \right| . \tag{3.35}
$$

Again, the right-hand side of inequality (3.35) is a convex piecewise linear expression whose minimum value is 1. The optimal values of AARC variables when $y = \pi^0 + \xi\delta$ are $\hat{x}^{AARC} = 1$, $\pi^0 = 3, \delta = 0$, and $Z_{AARC} = 4 = Z_{RC}$.

Example 3. This example illustrates the case in which all conditions of the proposition are satisfied along with (3.14) so that $Z_{RC} > Z_{AARC}$. In this instance, the parameter values of (3.30) are $a_1^0 = -3, a_1^1 = -1, a_2^0 = 1, a_2^1 = 1, b_1^0 = -6, b_1^1 = 1, b_2^0 = 5, b_2^1 = -1, c_x = c_y = 1, d_1 = -1$ and $d_2 = \frac{1}{2}$, where $\xi \in [-1, 1]$. The RC formulation is:

$$
\begin{aligned}
& Z_{RC} = \min_{x,y \geq 0} x + y : \\
& (i = 1) \; -(3 + \xi)x - y \leq -6 + \xi, \quad \forall \xi \in [-1, 1] \\
& (i = 2) \; (1 + \xi)x + \frac{1}{2}y \leq 5 - \xi, \quad \forall \xi \in [-1, 1]
\end{aligned} \tag{3.36}
$$

Here the optimal values of the RC variables are: $\hat{x}^{RC} = \frac{1}{2}, \hat{y}^{RC} = 6, Z_{RC} = \frac{13}{2}$. Figure 3.1(c) illustrates the feasible region as well as the optimal solution of the adjustable variable $y = \pi^0 + \xi\delta = \frac{3}{2} - \frac{3}{2}\xi$ for $\xi \in [-1, 1]$. In line with condition 1, the two constraints $j = 1$ and $k = 2$ are binding at $\hat{\xi}_1 = -1$ and $\hat{\xi}_2 = 1$; that is, $\hat{\xi}_1 \neq \hat{\xi}_2$. Condition 2 holds because at least one parameter depends on ξ in these two constraints. In the ARC formulation of (3.36), y is adjustable to ξ which has non-zero coefficients in both constraints that satisfy condition 3. The objective coefficient vector of the basic variables is $c_{B^*}^T = \begin{pmatrix} 1 & 1 \end{pmatrix}$ and the optimal dual variables of RC are $\lambda^* = c_{B^*}^T B^{*-1} = \begin{pmatrix} \lambda_1^* & \lambda_2^* \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} & -1 \end{pmatrix}$. Inequality (3.14) is:

$$\frac{15}{4} > \frac{3}{2}\left|\frac{3}{2} + \delta\right| + \left|-\frac{3}{2} - \frac{1}{2}\delta\right| + |\delta|. \tag{3.37}$$

If $\delta = -\frac{3}{2}$ then (3.37) is satisfied as $\frac{15}{4} > \frac{9}{4}$. The optimal values of the AARC variables are $\hat{x}^{AARC} = 2, y = \frac{3}{2} - \frac{3}{2}\xi$, and $Z_{AARC} = 5 < Z_{RC}$.

3.4 Applications

To evaluate the potential for affine adjustability to lower the cost in any application, inequality (3.26) (that is, the extension of (3.14)) can be tested in a small instance. The following two examples illustrate this evaluation process in applications where the AARC approach has been applied successfully. Note that these applications are evaluated using inequality (3.26) before reformulation as AARC.

Example 4 (Inventory model)**.** Multi-stage inventory management has been solved by the AARC approach frequently (Ben-Tal et al., 2004, 2009; Adida and Perakis, 2010). Ben-Tal et al. (2004) formulated it as:

$$Z_{RC} = \min_{p_N, p_A} \sum_{j=1}^{J} \sum_{t=1}^{T} c_j(t) p_j(t)$$

$$0 \le p_j(t) \le P_j(t), \quad j = 1, ..., J, \; t = 1, ..., T$$

$$\sum_{t=1}^{T} p_j(t) \le Q(j), \quad j = 1, ..., J$$

$$V_{min} \le v(1) + \sum_{j=1}^{J} \sum_{s=1}^{t} p_j(s) - \sum_{s=1}^{t} \tilde{\theta}_s(\xi) \le V_{max}, \quad \forall \xi \in \chi, \quad t = 1, ..., T. \tag{3.38}$$

Here, J and T are the numbers of factories and periods, respectively, $p = \{p_j(t)\}$ denotes the production quantities with costs $c_j(t)$ and $P = \{P_j(t)\}$ are the production capacities of factory j in period t. The subsets of adjustable and non-adjustable variables for the AARC formulation are $p_A = \{p_j(s) | s \in \{1, ..., t\}\}$, $t = 1, ..., T$ and $p_N = p \setminus p_A$, respectively. In addition, $Q(j)$ represents the maximum cumulative capacity of factory $j, v(1)$ stands for the amount of available product at the beginning of the horizon, and V_{min} (V_{max}) are the minimum (maximum) storage capacity of the warehouse. We assume that $\tilde{\theta}_t(\xi)$, the demand in period t, is uncertain and lies in a box uncertainty set $\tilde{\theta}_t(\xi) = \bar{\theta}_t + \xi^t \hat{\theta}_t$ where $|\xi^t| \le \rho_t$.

Consider a simple instance where $T = 2, J = 2$ and the parameter values are:

$$c(1) = \begin{bmatrix} 9 \\ 8 \end{bmatrix}, c(2) = \begin{bmatrix} 10 \\ 9 \end{bmatrix}, P(1) = P(2) = \begin{bmatrix} 20 \\ 20 \end{bmatrix}, Q = \begin{bmatrix} 50 \\ 20 \end{bmatrix}, V_{min} = 0, V_{max} = 10$$

If the uncertain demands for two periods are $\tilde{\theta}_1(\xi) = 10 + \xi^1 3$ and $\tilde{\theta}_2(\xi) = 10 + \xi^2 2$ where $|\xi^1| \le 1$ and $|\xi^2| \le 1$, then the optimal solution to (3.38) using the same process as in Example 1 are $\hat{p}^{RC}(1) = \begin{bmatrix} 0 & 17 \end{bmatrix}^T, \hat{p}^{RC}(2) = \begin{bmatrix} 5 & 3 \end{bmatrix}^T$ with $Z_{RC} = 213$.

By considering $p_1(1)$ as adjustable to the first perturbation ξ^1, the following represents how to evaluate the RC optimum solution based on the general inequality (3.26).

Only three constraints have non-zero corresponding dual values $\lambda^* =$

$(-1, -10, -1)^T$ as follows:

$(i = 1)$ $v(1) + p_1(1) + p_2(1) \leq (\bar{\theta}_1 + \xi^1 \hat{\theta}_1) + V_{max}$

$(i = 2)$ $-v(1) - p_1(1) - p_2(1) - p_1(2) - p_2(2) \leq -(\bar{\theta}_1 + \xi^1 \hat{\theta}_1) - (\bar{\theta}_2 + \xi^2 \hat{\theta}_2) - V_{min}$

$(i = 3)$ $p_2(1) + p_2(2) \leq Q(2)$

The coefficients a_i^l equal zero for all i and l while $b_1^1 = \hat{\theta}_1 = 3, b_2^1 = -\hat{\theta}_1 = -3$. Also, the coefficient vector of adjustable variable $p_1(1)$ in these constraints is $d = (1, -1, 0)^T$. Finally, the coefficient of $p_1(1)$ in the objective, denoted c_{y_r} in (3.26), is 9. Therefore, considering the affine function $p_1(1) = \pi^0 + \xi^1 \delta$, (3.26) is:

$$33 > |3 - \delta| + 10|-3 + \delta| + 9|\delta| \tag{3.39}$$

Inequality (3.39) holds for $\delta = 3$, for example. Therefore, the conservatism of problem (3.38) would be reduced by AARC formulation. When only $p_1(1)$ is adjustable, $Z_{AARC} = 208$. The AARC formulation when $p_i(1)$ is adjustable to ξ^1 and $p_i(2)$ is adjustable to both ξ^1 and ξ^2 yields the optimal objective value of $Z_{AARC} = 207$ in this instance.

However, a single modification to this instance renders adjustability ineffective. If V_{max} changes to 100, then the new RC solution with $Z_{RC} = 205$ is $p(1) = \begin{bmatrix} 5 & 20 \end{bmatrix}^T, p(2) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. The non-zero dual values are $\lambda_2^* = -9$ and $\lambda_4^* = -1$ where constraint $i = 4$ is $p_2(1) \leq P_2(1)$. After this change, conditions 1 and 2 in the proposition do not hold only one of the binding constraints involves an uncertain parameter. Therefore, $Z_{RC} = Z_{AARC}$.

Example 5 (Project management)**.** A time-cost tradeoff problem (TCTP) in project management with uncertainty in time duration was another application proposed by Cohen et al. (2007) to solve with AARC. The following is the RC formulation of TCTP:

$$Z_{RC} = \min_{x,y\geq 0} \max_{\xi\in\chi} \sum_{ij} \mu_{ij}\tilde{T}_{ij}(\xi) + \sum_{ij} \Phi_{ij}y_{ij} + Cx_n$$

$$-(x_j - x_i + y_{ij}) \leq -\tilde{T}_{ij}(\xi), \quad \forall\xi \in \chi, \ \ \forall j, \forall i \in \mathcal{P}_j$$

$$y_{ij} \leq \tilde{T}_{ij}(\xi) - M_{ij}, \quad \forall\xi \in \chi, \ \ \forall j, \forall i \in \mathcal{P}_j$$

$$x_1 = 0, \ x_n \leq \mathcal{D}, \tag{3.40}$$

where x_i denotes the start time of node $i \in \{1, ..., n\}$ in which n is the final node. When $x_1 = 0$ then x_n is the project duration with overhead cost C, and $\mathcal{D}$ denotes its predetermined due date. The immediate predecessors set of node j is shown as $\mathcal{P}_j$. The decision variable y_{ij} represents the crashing of activity $ij \in \{1, ..., n\}$ with a constant marginal cost Φ_{ij}. The uncertain normal duration of each activity $\tilde{T}_{ij}(\xi)$ is assumed to belong a symmetric interval with objective coefficient μ_{ij} as the compensation of the contractor. In this example, we assume $\tilde{T}_{ij}(\xi) = \bar{T}_{ij} + \xi^{ij}\hat{T}_{ij}$ where $|\xi^{ij}| \leq \rho_{ij}$. In addition, M_{ij} represents the lower bound of activity duration ij. In the AARC each variable is adjustable to a portion of the uncertain parameters. A small instance of the problem with only three nodes $n = 3$ and two arcs $ij = \{(1,2),(2,3)\}$ is specified with the following parameter values, extracted from the same instance as in Cohen et al. (2007) limited to three nodes and two sequential activities:

$$\mu = \begin{bmatrix} 5 \\ 5 \end{bmatrix}, \Phi = \begin{bmatrix} 15 \\ 2 \end{bmatrix}, M = \begin{bmatrix} 1.3 \\ 1.9 \end{bmatrix}, \bar{T} = \begin{bmatrix} 3 \\ 4.4 \end{bmatrix}, \hat{T} = \begin{bmatrix} 0.3 \\ 0.44 \end{bmatrix}, C = 15.$$

Assuming the uncertainty set surrounding the duration of each activity is $|\xi^{ij}| \leq 1$ for $ij \in \{(1,2),(2,3)\}$, then the optimal values of the RC variables following the same process of Example 1 are $\hat{x}^{RC} = (0.0, 3.3, 6.08)^T, \hat{y}^{RC} = (0, 2.06)^T$, with $Z_{RC} = 136.02$.

We select y_{23} as the affine adjustable variable to ξ_{23}, that is, $y_{23} = \pi^0 + \xi^{23}\delta$. The constraints with corresponding non-zero optimal dual values $\lambda^* =$
$(-1, -15, -15, -13)^T$ of RC (3.40) are as follows:

$$(i=1)\ \ -u + Cx_n + \sum_{ij} \Phi_{ij} y_{ij} \leq -\sum_{ij} \mu_{ij} (\bar{T}_{ij} + \xi^{ij} \hat{T}_{ij})$$

$$(i=2)\ \ -(x_2 - x_1 + y_{12}) \leq -(\bar{T}_{12} + \xi^{12} \hat{T}_{12})$$

$$(i=3)\ \ -(x_3 - x_2 + y_{23}) \leq -(\bar{T}_{23} + \xi^{23} \hat{T}_{23})$$

$$(i=4)\ y_{23} \leq (\bar{T}_{23} + \xi^{23} \hat{T}_{23}) - M_{23}$$

where u is an auxiliary variable after converting the objective of (3.40) to constraint $i = 1$. All coefficients a_i^l equal zero, while $b^2 = (-2.2, 0, -0.44, 0.44)^T$. In addition, the coefficient vector d of adjustable variable y_{23} in the constraints is $(2, 0, -1, 1)^T$. Finally, the coefficient of y_{23} in the objective c_{y_r} equals zero. Substituting into inequality (3.26), we obtain:

$$14.52 > |-2.2 - 2\delta| + 15|-0.44 + \delta| + 13|0.44 - \delta| \tag{3.41}$$

In this instance, the right-hand-side of inequality (3.41) equals 3.08 for $\delta = 0.44$. Since the adjustability of a single variable would reduce the RC objective function, making more variables adjustable might reduce it more. Indeed, the AARC optimal objective value when both activity durations are adjustable (i.e., $y_{ij} = \pi_{ij}^0 + \xi^{ij}\pi_{ij}^1$) is $Z_{AARC} = 124.58$ based on $\hat{x}^{AARC} = (0.0, 3.4, 5.3)^T, \pi^0 = (0, 2.5)^T, \pi^1 = (0.3, 0.44)^T$.

These examples illustrate how RC formulations can be tested in small-scale instances using optimal primal and dual solutions in order to identify whether AARC and therefore ARC might be advantageous.

3.5 Conclusion

In some situations, uncertain linear programs can be solved by the ARC or the more tractable AARC instead of the RC formulation to provide a less conservative solution. The proposition provided in this paper identifies conditions under which the objective values of ARC and RC of uncertain linear program are not equivalent by using the AARC formulation. In the provided conditions, the RC formulation includes at least two constraints that are binding at the optimal RC solution for different values of the same uncertain parameter. In addition, a variable to be made adjustable appears in both constraints and is bounded from above by one constraint at one extreme of the uncertainty interval and bounded from below by the other at the opposite extreme of the uncertainty interval. One of these bounds is unfavorable for the objective. By relaxing this bound, adjustability increases the feasible region of the RC in a direction that lowers the objective value.

Besides providing insights into formulations where adjustability is beneficial, we show how RC formulations can be tested in small-scale instances using dual variables of RC in order to identify whether the ARC is advantageous. Some small instances demonstrate different situations of RC formulations. The examples illustrate that, although the models are not covered by the previous papers' conditions for $Z_{ARC} = Z_{RC}$, nevertheless Z_{AARC} is equal to Z_{RC}. A third example and two applications from the literature demonstrate the use of this proposition to establish that $Z_{ARC} < Z_{RC}$.

In this paper we only considered the fixed recourse case. For uncertainty-affected recourse a similar approach would require more computational complexity that is a subject for future research. Another extension could be including more complex uncertainty sets beyond box uncertainty. For instance, ellipsoidal uncertainty, used in many applications, allows interactions among uncertain parameters to be modeled.

CHAPTER 4. CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN WITH MULTIPLE TRANSPORTATION MODES UNDER STOCHASTIC DEMAND AND UNCERTAIN CARBON TAX

4.1 Introduction

With concern over global climate change, regulations over carbon emissions resulting from industries such as transportation and power generation have been developed by policy-makers in different nations. For example, in 2005 the European Union instituted a carbon emission trading scheme (EU ETS) for the energy-intensive industries with the aim of reducing greenhouse gas (GHG) emissions by at least 20% below 1990 levels (BeÜhringer et al., 2009). In addition, China, which is one of the world's largest emitters of GHG, has announced in recent years that the Ministry of Finance may levy taxation policies over CO_2 emissions (Xinhuanet, 2013).

As of January 2011, the US Environmental Protection Agency has power to regulate the carbon emissions of companies operating in the US. In the past, the federal government has tended to emphasize "command and control" regulatory approaches to control pollutants. For the US to reduce its GHG emissions, most environmental policy analysts agree it must use market-based environmental mechanisms. The two main market-based options are a carbon tax and a cap-and-trade system of tradable permits for emissions (Metcalf, 2009), with the tax proposals currently receiving more attention.

According to a survey, 26 percent of CO_2 emissions were generated by transportation activities in 2014 (U.S. Environmental Protection Agency, 2016). International trade liberalization contributes to significantly more transportation of products in global supply chains (Mallidis et al., 2012). These trades employ different modes of transportation such as road, rail, air, and water, each of which has a certain rate of GHG emissions. Among them, road transportation

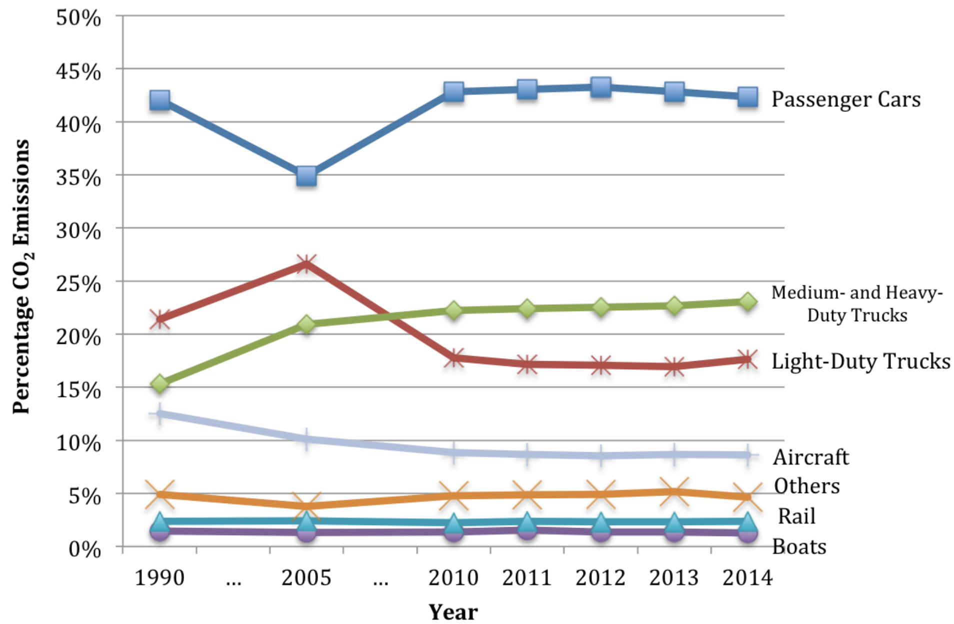

Figure 4.1 CO_2 emissions from fossil fuel combustion in transportation by mode (1990-2014).

modes account for nearly 83% of CO_2 emissions. Light trucks were responsible for 18% of CO_2 emissions while medium- and heavy-duty trucks contributed 23% in 2014 (Figure 4.1)[1].

Designing a closed-loop supply chain (CLSC) involves long-term decisions to invest in fixed facilities such as manufacturing or remanufacturing plants, warehouses, and collection facilities. Somewhat more flexible are decisions concerning capacity to transport goods by various modes, either by purchasing or leasing fleets or by contracting with external providers. To reduce the negative environmental consequences from supply chains, legislation and social concerns have been motivating firms to plan their supply chain structures and find ways to handle both forward and reverse product flows. The reverse flows include the recycling or manufacturing of returned products that occur due to commercial and consumer returns, extended producer responsibility legislation, or the potential profits derived from remanufacturing and resale. Much research has been proposed to mitigate the inverse environmental effects of freight transportation, particularly CO_2 emissions (Hickman and Banister, 2011). One approach involves decisions concerning the choice among modes with varying emission rates, capacities,

[1]Source: U.S. Environmental Protection Agency, 2016. *Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990-2014*, available at (https://www.epa.gov/sites/production/files/2016-04/documents/us-ghg-inventory-2016-main-text.pdf, Aug. 2016)

and costs (Mallidis et al., 2012). How uncertainty concerning emission tax rates should affect the choice of modes and product flows while minimizing the overall cost is worth investigation.

In this paper we formulate and solve a tractable closed-loop supply chain network design problem that includes: facility configuration that is robust to carbon tax regulation and optimizes the expected cost of satisfying demands and collecting returns; product flows that optimally balance the tradeoffs between transportation cost and emission-related operational costs in the worst case of carbon tax rate; and transportation capacities of various modes that respond to the carbon tax rate. The results of numerical case studies show how the optimal number and locations of opened facilities respond to uncertainty in demands and returns. In addition, we observe the choices of modes based on different carbon tax uncertainty levels and the extent to which adjustability of transportation capacities to carbon tax rates is beneficial.

Overall, our model optimizes the facility configuration to minimize the expected cost over probabilistic scenarios for demands and returns, where the worst case of the uncertain carbon tax rate is considered in each scenario. A large number of scenarios for demands and returns in large-scale instances renders the solution procedure computationally cumbersome. Therefore, we apply Benders decomposition (BD) to solve the hybrid robust/stochastic model. Benders cuts are formulated using the dual solutions of robust counterpart (RC) and affinely adjustable robust counterpart (AARC) sub-problems which we obtain using recent duality results.

A brief literature review of the recent work follows in Section 2. In Section 3, we introduce our CLSC network structure by considering carbon tax policy. The CLSC design formulation with stochastic demands and returns is provided in Section 3.1. The proposed robust formulation and our tractable solution approach for the AARC formulation with uncertain tax rate policy is explained in Section 3.2. The hybrid robust/stochastic model that combines the stochastic demands and returns with uncertainty sets for the carbon tax rate is provided in Section 3.3. We present computational results in Section 4 and finally conclusions as well as future research directions in Section 5.

4.2 Literature Review

Environmental concerns have prompted many nations to devise penalties or incentives to reduce their carbon footprints. Much attention has been devoted to reducing the GHG emissions of transport activities and facilities in supply chains. As an example, cap-and-trade and carbon taxes are regulations that have been studied in the literature and practiced in some countries. For example, Fransoo and van Houtum (2010) investigated the effect of cap-and-trade and company-wide (hard constraint on emissions) regulation on transportation mode decisions. Furthermore, they analyzed the effect of considering emission costs or emission in their model, and they found that emission cost penalties have only a small effect on transport mode selection compared to constraints. However, they did not consider the effect of transportation mode decisions emission cost parameters. Benjaafar et al. (2013) presented and modified traditional supply chain models to include carbon footprint along with other costs. They also examined different regulatory emissions such as cap-and-trade and carbon tax and presented the effect of their parameters on costs and emissions. Fu and Kelly (2012) evaluated the impacts of different transportation tax policies for carbon emission in Ireland. Their results suggested that the fuel based carbon tax is better than either a vehicle registration tax or motor tax in terms of tax revenue, carbon emission reductions, and social welfare, but worse than the latter in terms of household utility and production costs. Zakeri et al. (2015) presented an analytical supply chain planning model to examine the supply chain performance under carbon taxes and carbon emissions trading. They found that the carbon tax is more worthwhile from an uncertainty perspective as emissions trading costs depend on numerous uncertain market conditions. These studies have not considered the choice among transportation modes in supply chains.

Some models did include different modes of transportation. Forkenbrock (2001) examined and estimated the air pollution and GHG emissions of different types of railroad companies, and compared them to freight trucking. Pan et al. (2010) explored the environmental impact of pooling of supply chain resources at a strategic level and extracted the emission functions of two transport modes, rail and road, using a French case study. Paksoy et al. (2011) also proposed a general CLSC network configuration that handles various costs including emission costs for transportation activities in a completely deterministic environment for all parameters. More research includes the investigation of Bloemhof-Ruwaard et al. (2011) on the environmental impact of inland navigation (transportation by canals or rivers) compared to inland transport modes, which identified that the road transport mode is the biggest contributor of hazardous gas emission. However, the effect of uncertain carbon tax rate on the choice of transportation modes has not been investigated.

To model an uncertain carbon tax rate, we formulate the RC of the optimization problem with uncertain parameters whose distribution functions are unknown or difficult to determine. This approach was first proposed by Soyster (1972) and further developed by Ben-Tal and Nemirovski (1998, 1999, 2000) as well as Ghaoui and Lebret (1997); Ghaoui et al. (1998) independently. The more recent papers proposed tractable solution approaches to special cases of robust counterparts in the form of conic quadratic problems with less conservative solutions than the Soyster (1972) approach. Ben-Tal et al. (2004) defined the adjustable robust counterpart (ARC) and more tractable AARC models with adjustable variables that tune themselves to the values of uncertain parameters described by certain forms of uncertainty sets. They defined conditions under which the solutions of RC and ARC are equal. Haddad-Sisakht and Ryan (2016) established conditions under which affine adjustability may lower the optimal cost of the RC solution.

Along with GHG emissions, we also consider product returns because of environmental concerns. Several strategies have been introduced to solve deterministic and stochastic versions of CLSC network design. Zeballos et al. (2012) proposed a two-stage scenario-based model for a CLSC design problem in which the quantity and the quality of returned product flows are uncertain. Vahdani et al. (2012) and Pishvaee et al. (2012) also designed bi-objective CLSCs, the former combining robust optimization and queuing theory to solve their model with fuzzy multiple objectives, and the latter by applying robust possibilistic programming to cope with their model uncertainties. Amin and Zhang (2012) investigated the impact of demand and return uncertainties on the CLSC network configuration with a scenario-based stochastic program. Georgiadis and Athanasiou (2013) dealt with long-term two-capacity planning strategies of a CLSC network with uncertainty in actual demand, sales patterns, quality and timing of end-of-use product returns. They also considered two sequential product-types in network design and solved with a simulation-based system dynamics optimization approach. Vahdani and Mohammadi (2015) developed a bi-objective model for CLSC network design to minimize total cost and waiting time under multiple uncertain parameters. They proposed a hybrid solution approach based on interval programming, stochastic programming, robust optimization, and fuzzy multi-objective programming. Keyvanshokooh et al. (2016) developed a hybrid robust-stochastic programming approach for a profit maximizing CLSC network design under stochastic scenarios for transportation costs and polyhedral uncertainty sets for demands and returns.

Gao and Ryan (2014) considered a robust formulation of a multi-period capacitated CLSC network design problem while considering two regulations for carbon emissions. They integrated stochastic programming and robust optimization to deal with uncertainty in demands and returns as well as carbon regulation parameters caused by different transport modes. They observed that, as the uncertainty level in the carbon tax increases, more facilities are opened and more capacity of modes with lower emission rates is used. Their model did not allow for the allocation of capacity among transportation models to adjust to the carbon tax rate. The contributions of this paper include incorporation of this adjustability to obtain a less conservative model. We show that by adjustability, the same number of facilities can accommodate the increase of uncertainty. A methodological contribution is to integrate a scenario-based optimization for product uncertainties with AARC for tax uncertainty in a three-stage model. To our knowledge, the generation of Benders cuts from the duals of the RC and AARC formulations has not been done previously.

4.3 CLSC Model

In our model, the first stage variables determine long-term facility configurations that are robust to both types of uncertainty. High uncertainty of future carbon tax costs must be considered to identify necessary changes in the structure of supply chain. We assume carbon tax policy rather than cap-and-trade system, since this is politically more likely in the US. Moreover, it may be the only feasible way to regulate emissions from transportation because of the large number of entities involved. To model the tax rate uncertainty we use uncertainty sets because of the lack of data with which to estimate distributions. However, the distributions of demands and return quantities for a new product may be estimated based on historical data for the similar given product.

The second stage decisions concern a plan for the product flows among facilities after realization of demands and returns but before realization of carbon tax. Finally, transportation capacities of different modes are decided after realization of carbon tax. We use an ARC model in which the transportation capacities of various modes are adjustable to carbon tax rates. For tractability we adopt the AARC, in which the adjustable variables are restricted to be affine functions of the tax rate.

The closed-loop supply chain network is denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of arcs. The node set $\mathcal{N} = \mathcal{P} \cup \mathcal{K}$, where $\mathcal{P}$ is a set of potential facilities consisting of factories $\mathcal{I}$, new product warehouses $\mathcal{J}$, collection centers for returned products $\mathcal{L}$; i.e., $\mathcal{P} = \mathcal{I} \cup \mathcal{J} \cup \mathcal{L}$; and $\mathcal{K}$ is the set of retailers. Let $\mathcal{M}$ be the set of transportation modes available for the supply chain. The arc set $\mathcal{A} = \{ij : (i \in \mathcal{I}, j \in \mathcal{J}), (i \in \mathcal{J}, j \in \mathcal{K}), (i \in \mathcal{K}, j \in \mathcal{L}), (i \in \mathcal{L}, j \in \mathcal{I})\}$ (see Figure 4.2 for the network topology). The closed-loop supply chain configuration decisions consist of determining which of the processing facilities to open. Let binary variable y_i be the decision to open the processing facility $i \in \mathcal{P}$ and x_{ij}^m be the number of units of product transported from node i to node j using transportation mode m, where $ij \in \mathcal{A}$ and $m \in \mathcal{M}$. Decision variables t_{ij}^m denote the number of units transportation mode $m \in \mathcal{M}$ for which to contract on arc $ij \in \mathcal{A}$.

In addition, the unmet demands and discarded returns decision variables are denoted as z_k and e_k units of products respectively, for customer k. In this model, we do not consider keeping inventory in facilities across periods. We assume that manufacturers are responsible for processing returns after receiving them from collection centers, and we only consider a single product. The deterministic mathematical model for CLSC network design can be stated as follows:

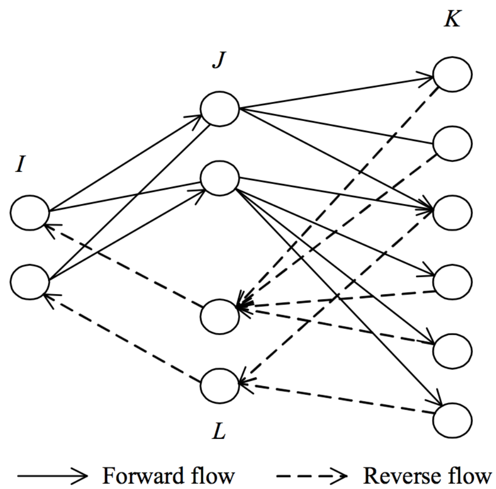

Figure 4.2 Closed-loop supply chain network structure

$$\min \sum_{i\in\mathcal{P}} c_i y_i + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m t_{ij}^m + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m \beta_{ij} x_{ij}^m + \sum_{k\in\mathcal{K}} (\theta z_k + \zeta e_k)$$
$$+ w\alpha \sum_{ij\in\mathcal{A}} \beta_{ij} \sum_{m\in\mathcal{M}} \tau^m x_{ij}^m, \tag{4.1}$$

$$\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} x_{jk}^m + z_k = d_k^n, \qquad \forall k\in\mathcal{K} \tag{4.2}$$

$$\sum_{i\in\mathcal{L}}\sum_{m\in\mathcal{M}} x_{ki}^m + e_k = d_k^o, \qquad \forall k\in\mathcal{K} \tag{4.3}$$

$$\sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} x_{ji}^m - \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} x_{ij}^m = 0 \qquad \forall j\in\mathcal{J} \tag{4.4}$$

$$\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} x_{ji}^m - \sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} x_{ij}^m = 0 \qquad \forall j\in\mathcal{L} \tag{4.5}$$

$$w x_{ij}^m - W_m t_{ij}^m \le 0 \qquad \forall ij\in\mathcal{A}, m\in\mathcal{M} \tag{4.6}$$

$$\sum_{j:ij\in\mathcal{A}}\sum_{m\in\mathcal{M}} x_{ij}^m - \eta_i y_i \le 0 \qquad \forall i\in\mathcal{P} \tag{4.7}$$

$$y \in \{0,1\}^{|\mathcal{P}|}, x \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|}, t \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|}, \quad z, e \in \mathbb{R}_+^{|\mathcal{K}|} \tag{4.8}$$

In this model, c_i denotes the investment cost (\$) for building facility $i \in \mathcal{P}, h^m$ is the approximate fixed operating cost (\$/units of transportation) per unit of capacity of transportation mode m, g^m is the unit transportation cost (\$/units of product-km) of mode m, and β_{ij} is the distance (km) from node i to node j. The unmet demand cost is θ (\$/units of products) and the corresponding cost for discarded returns is ζ. In addition, α is the carbon tax rate (\$/ton) subject to uncertain policy decision. In the last term of the objective function, w is the weight of product (tons/units of product), and τ^m is the carbon emission factor (ton/tons-km) for transportation mode m.

Constraints (4.2) and (4.3) compute met or unmet demands as well as returned products, where d_k^n is the demand (units of product) for new products and d_k^o is the quantity of returns (units of product). Constraints (4.4) and (4.5) ensure that the warehouse and collection centers will not carry stocks across periods or incur backlogs. Constraint (4.6) requires that the product's weight does not exceed the total capacity of transportation mode m from node i to node j, where W_m denotes the weight limit (tons/units of transportation capacity) of mode m. Constraint (4.7) enforces capacity constraints of the processing nodes, where η_i denotes the capacity at node $i \in \mathcal{P}$. Finally, variable restrictions are given in (4.8).

The proposed model is a three-stage hybrid robust/stochastic program with multiple scenarios for the demands and returns. In the first stage, the decisions pertain to the long-term strategy of finding facility configurations because changing facilities in the short-term or adjusting them to values of uncertain parameters is usually costly. The second stage decisions concern the plan for distributing new and collecting returned products after realization of demands and returns by the customers but before realization of the carbon tax. The scenarios represent "macro" - level descriptions of product acceptance and consumer behavior rather than high-frequency variability. At the final stage, the model decides on capacities of each transportation mode after the realization of carbon tax level, reflecting the fact that legislation usually takes a long time to be decided and implemented.

In the following sections, we first introduce the stochastic program (SP) for CLSC design, then the robust optimization of the recourse problem, and finally the three-stage hybrid robust/stochastic program.

4.3.1 Stochastic Program For CLSC Design

In our model, the first stage variables are binary decisions y for facility configuration. In this subsection, we incorporate probabilistic scenarios for demands and return quantities. If $s \in \mathcal{S}$ is a given realization with probability P_s, the stochastic programming extension of (4.1)-(4.8) is as follows:

$$\min_y \sum_{i \in \mathcal{P}} c_i y_i + \sum_{s \in \mathcal{S}} P_s Q_N(y, s) \tag{4.9}$$

$$y \in \{0, 1\}^{|\mathcal{P}|}$$

where the second stage of the stochastic program optimizes cost in a given scenario, assuming the carbon tax rate is at its nominal value, $\bar{\alpha}$:

$$Q_N(y, s) = \min_{x_s, t_s, z_s, e_s} \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} h^m t^m_{ijs} + \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} g^m \beta_{ij} x^m_{ijs}$$

$$+ \sum_{k \in \mathcal{K}} (\theta z_{ks} + \zeta e_{ks}) + w\bar{\alpha} \sum_{ij \in \mathcal{A}} \beta_{ij} \sum_{m \in \mathcal{M}} \tau^m x^m_{ijs}, \tag{4.10}$$

$$\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}} x^m_{jks} + z_{ks} = d^n_{ks}, \qquad \forall k \in \mathcal{K} \tag{4.11}$$

$$\sum_{i\in\mathcal{L}}\sum_{m\in\mathcal{M}} x^m_{kis} + e_{ks} = d^o_{ks}, \qquad \forall k \in \mathcal{K} \tag{4.12}$$

$$\sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} x^m_{jis} - \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} x^m_{ijs} = 0 \qquad \forall j \in \mathcal{J} \tag{4.13}$$

$$\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}} x^m_{jis} - \sum_{i\in\mathcal{K}}\sum_{m\in\mathcal{M}} x^m_{ijs} = 0 \qquad \forall j \in \mathcal{L} \tag{4.14}$$

$$\sum_{j:ij\in\mathcal{A}}\sum_{m\in\mathcal{M}} x^m_{ijs} - \eta_i y_i \leq 0 \qquad \forall i \in \mathcal{P} \tag{4.15}$$

$$w x^m_{ijs} - W_m t^m_{ijs} \leq 0 \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{4.16}$$

$$x_s \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|},\ t_s \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|},\ z_s, e_s \in \mathbb{R}_+^{|\mathcal{K}|}. \tag{4.17}$$

Note that relatively complete recourse is provided by the slack variables in (4.11) and (4.12). To incorporate the third-stage and consider the carbon tax uncertainty, we introduce the RC and AARC formulation of the recourse problem in the following section.

4.3.2 Robust Counterpart and Affinely Adjustable Robust Counterpart of Recourse Problems

The robust counterpart of the recourse problem is to find an optimal solution that satisfies all constraints for any carbon tax $\tilde{\alpha} \in \mathcal{U}$. We define the RC of (4.10) - (4.17) as:

$$Q_{RC}(y, s) = \min_{u_s, x_s, t_s, z_s, e_s} u_s, \quad \text{such that } \forall \tilde{\alpha} \in \mathcal{U}, \tag{4.18}$$

$$\sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m t^m_{ijs} + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m \beta_{ij} x^m_{ijs} + \sum_{k\in\mathcal{K}} (\theta z_{ks} + \zeta e_{ks})$$
$$+ w\tilde{\alpha} \sum_{ij\in\mathcal{A}} \beta_{ij} \sum_{m\in\mathcal{M}} \tau^m x^m_{ijs} \leq u_s, \tag{4.19}$$

$$(4.11) \ - \ (4.17), \tag{4.20}$$

where $u_s \in \mathbb{R}, x_s, t_s$ and the slack variables z_s and e_s are all here-and-now decisions regarding carbon tax uncertainty. However, the RC formulation may provide an overly conservative solution by requiring all decision variables to be feasible for all values of $\tilde{\alpha}$ in the uncertainty set. To obtain a less conservative solution, we assume that t_s is an adjustable variable; i.e., its value can be determined after the tax rate uncertainty is resolved (Ben-Tal et al., 2004). The ARC is as follows:

$$Q_{ARC}(y, s) = \min_{u_s, x_s, z_s, e_s} u_s, \quad \text{such that } \forall \tilde{\alpha} \in \mathcal{U}, \exists t_s(\tilde{\alpha}) \text{ such that} \tag{4.21}$$

$$(4.11) \ - \ (4.17) \text{ and } (4.19), \tag{4.22}$$

where variable t_s is a function of the uncertain parameter $\tilde{\alpha}$. Usually, ARC models cannot be solved efficiently even in fixed recourse cases. A tractable approximation is provided by the AARC, where adjustable variables are restricted to be affine functions of the uncertainties (Ben-Tal et al., 2004). Setting $t^m_{ijs} = \pi^m_{ij(0)s} + \tilde{\alpha}\pi^m_{ij(1)s}$, where $\pi_{(0)s}$ and $\pi_{(1)s}$ are non-adjustable variables, allows the t_s variables to depend on $\tilde{\alpha}$. Under this restriction, the transportation capacity decisions in the ARC (4.21) - (4.22) are replaced by an AARC given by:

$$Q_{AARC}(y, s) = \min_{u_s, x_s, \pi_s, z_s, e_s} u_s, \quad \text{such that } \forall \tilde{\alpha} \in \mathcal{U}, \tag{4.23}$$

$$\sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} h^m \left(\pi^m_{ij(0)s} + \tilde{\alpha}\pi^m_{ij(1)s} \right) + \sum_{m \in \mathcal{M}} \sum_{ij \in \mathcal{A}} g^m \beta_{ij} x^m_{ijs}$$

$$+ \sum_{k \in \mathcal{K}} (\theta z_{ks} + \zeta e_{ks}) + w\tilde{\alpha} \sum_{ij \in \mathcal{A}} \beta_{ij} \sum_{m \in \mathcal{M}} \tau^m x^m_{ijs} \leq u_s, \tag{4.24}$$

$$(4.11) - (4.15), \text{ and} \tag{4.25}$$

$$w x^m_{ijs} - W_m \left(\pi^m_{ij(0)s} + \tilde{\alpha}\pi^m_{ij(1)s} \right) \leq 0 \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{4.26}$$

$$x_s \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{M}|}_+, \ \pi_s \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{M}|}, \ z_s, e_s \in \mathbb{R}^{|\mathcal{K}|}_+, \ u_s \in \mathbb{R}. \tag{4.27}$$

Here, all the decisions are second-stage decision variables once t_s has been replaced by its affine function of $\tilde{\alpha}$. Note that also adjusting product flows to the carbon tax rate introduce uncertain recourse and convert the AARC into a semi-definite program. When combined with the binary first-stage decisions, such a formulation is currently intractable. Therefore, in this paper, we require product flows to be robust to the carbon tax rate.

The purpose of the AARC formulation is to produce less conservative solutions than the RC. However, because uncertainty affects only (4.19) and is thus constraint-wise, RC (4.18)-(4.20) satisfies Theorem 2.1 of Ben-Tal et al. (2004) which defines conditions under which the objectives of RC and ARC are equal. However, by introducing new constraints, certain conditions listed in Proposition 1 of Haddad-Sisakht and Ryan (2016) are satisfied, and therefore, the AARC could be less conservative than the RC formulation.

It is not always straightforward to determine when the ARC might be less conservative compared to the RC formulation. Haddad-Sisakht and Ryan (2016) described specific departures from constraint-wise uncertainty that allow a difference between the optimal objective values of the RC and ARC formulations. One example for the RC (4.18)-(4.20) is to incorporate lower bound on the transportation and emission cost of each mode as follows:

$$Q^L_{RC}(y,s) = \min_{u_s,x_s,t_s,z_s,e_s} u_s, \quad \text{such that } \forall \tilde{\alpha} \in \mathcal{U}, \tag{4.28}$$

$$\sum_{ij \in \mathcal{A}} \left(h^m t^m_{ijs} + g^m \beta_{ij} x^m_{ijs} + w \tilde{\alpha} \beta_{ij} \tau^m x^m_{ijs} \right) \geq L^m, \ \forall m \in \mathcal{M} \tag{4.29}$$

$$(4.11) \ - \ (4.17), \text{ and } (4.19) \tag{4.30}$$

where L^m is a lower bound on the cost of mode m determined by management. Assuming $\tilde{\alpha}$ belongs to a box uncertainty set, the RC of (4.28)-(4.30) with adjustable variable t satisfies the conditions of Haddad-Sisakht and Ryan (2016), which are loosely described as: the model contains at least two binding constraints at optimality of the RC formulation and an adjustable variable in both constraints with implicit bounds from above and below for different extreme values in the uncertainty set. Therefore, the following affinely adjustable modification $Q^L_{AARC}(y,s)$ could result in a less conservative solution than to $Q^L_{RC}(y,s)$, depending on the parameter values.

$$Q^L_{AARC}(y,s) = \min_{u_s,x_s,\pi_s,z_s,e_s} u_s, \quad \text{such that } \forall\tilde{\alpha} \in \mathcal{U}, \tag{4.31}$$

$$\sum_{ij\in\mathcal{A}} \left(h^m \left(\pi^m_{ij(0)s} + \tilde{\alpha}\pi^m_{ij(1)s}\right) + g^m\beta_{ij}x^m_{ijs} + w\tilde{\alpha}\beta_{ij}\tau^m x^m_{ijs}\right) \geq L^m, \ \forall m \in \mathcal{M} \tag{4.32}$$

$$(4.24) \ - \ (4.27), \tag{4.33}$$

Using the dual of the RC, Haddad-Sisakht and Ryan (2016) described how small instances can be used to identify whether affine adjustability reduces the optimal cost. A constraint, such as (4.29), that guarantees at least minimal use of some transportation mode might reflect units of capacity already procured (Yuzhong and Guangming, 2012) or the desire to guarantee access to a mode that provides rapid delivery despite its higher emissions and cost (Turban et al., 2015). Contractual provisions might cause reluctance to change usage dramatically from previous periods. Or, usage above a threshold might gain a quantity discount.

A lower bound on the cost of using a transportation mode is used, instead of a direct lower bound on t, because considering a minimal number of transportation units procured does not necessarily guarantee the use of that available mode for transportation. Considering a lower bound based on cost, as opposed to the number of transportation units, also could reflect how much a manager would like to spend on internal capacity rather than outsourcing. In addition, constraining cost as a continuous quantity is compatible with our neglect of integer restrictions on the units of transportation capacity to avoid computational complications.

4.3.3 Integration of Robust Optimization And Stochastic Programming

In the proposed hybrid robust/stochastic optimization model, the first stage variables are binary decisions y for facility configuration and the second stage decisions are product flows x after realization of demands and returns before realization of carbon tax. The third stage decisions are unit transportation capacities t that should be decided after realization of the carbon tax rate. We assume the uncertain $\tilde{\alpha}$ falls in a box uncertainty set. Specifically, $\tilde{\alpha} = \bar{\alpha} + \xi\hat{\alpha}$, where the perturbation scalar ξ varies set:

$$\chi_p \equiv \{\xi|\ |\xi| \leq \rho\}\,. \tag{4.34}$$

Without loss of generality, the adjustable variable can be adjusted to perturbation scalar ξ instead of $\tilde{\alpha}$ as $t^m_{ijs} = \pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}$ (Ben-Tal et al., 2004). Our hybrid robust/stochastic CLSC design model is as follows:

$$Z_{RC} = \min_y \sum_{i\in\mathcal{P}} c_i y_i + \sum_{s\in\mathcal{S}} P_s Q^L_{RC}(y,s) \tag{4.35}$$

$$y \in \{0,1\}^{|\mathcal{P}|},$$

and the affine adjustable version is:

$$Z_{AARC} = \min_y \sum_{i\in\mathcal{P}} c_i y_i + \sum_{s\in\mathcal{S}} P_s \min_{u_s,x_s,\pi_s,z_s,e_s} u_s, \quad \text{such that } \forall \xi \in \chi_p, \tag{4.36}$$

$$\sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} h^m \left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) + \sum_{m\in\mathcal{M}}\sum_{ij\in\mathcal{A}} g^m \beta_{ij} x^m_{ijs}$$

$$+\sum_{k\in\mathcal{K}}(\theta z_{ks} + \zeta e_{ks}) + w(\bar{\alpha} + \xi\hat{\alpha}) \sum_{ij\in\mathcal{A}} \beta_{ij} \sum_{m\in\mathcal{M}} \tau^m x^m_{ijs} \leq u_s, \tag{4.37}$$

$$\sum_{ij\in\mathcal{A}} \left(h^m \left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) + g^m \beta_{ij} x^m_{ijs}\right)$$

$$+ \sum_{ij\in\mathcal{A}} \left(w(\bar{\alpha} + \xi\hat{\alpha})\beta_{ij}\tau^m x^m_{ijs}\right) \geq L^m,\ \forall m \in \mathcal{M} \tag{4.38}$$

$$(4.11) - (4.15),\ \text{and} \tag{4.39}$$

$$w x^m_{ijs} - W_m \left(\pi^m_{ij(0)s} + \xi\pi^m_{ij(1)s}\right) \leq 0 \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{4.40}$$

$$y \in \{0,1\}^{|\mathcal{P}|},\ x_s \in \mathbb{R}_+^{|\mathcal{A}|\times|\mathcal{M}|},\ \pi_s \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{M}|},\ z_s, e_s \in \mathbb{R}_+^{|\mathcal{K}|},\ u_s \in \mathbb{R}. \tag{4.41}$$

Problems (4.35) and (4.36)-(4.41) can be solved directly as mixed integer programs; however, with large numbers of scenarios and potential facilities, this approach would become computationally cumbersome. We use a multi-cut version of Benders decomposition (BD) (Benders, 1962) to decompose the problem into master and sub-problems Birge and Louveaux (2011).

Because the recourse problem is always feasible since it has relatively complete recourse, the only optimality cuts are generated. The master problem is:

$$\min_{y,\delta_s} \sum_{i\in\mathcal{P}} c_i y_i + \sum_{s\in\mathcal{S}} \delta_s, \tag{4.42}$$

Optimality cuts

$y \in \{0,1\}^{|\mathcal{P}|},\ \ \delta_s \in \mathbb{R}$

where δ_s is a lower bound on the objective value for sub-problem s.

The decision variables in the master problem are binary variables y for facility configuration. The sub-problems for each scenario $s \in \mathcal{S}$ with optimal objective value Σ_s (where $\Sigma_s = Q^L_{RC}(y,s)$ or $\Sigma_s = Q^L_{AARC}(y,s)$ for the RC or AARC formulation, respectively) minimize upper bounds on transportation, shortage and emission costs for given $\hat{y}$. The BD algorithm solves the master problem and sub-problems iteratively. If $\Sigma_s > \delta_s$ in master problem (4.42), an optimality cut is added. The algorithm continues until $\Sigma_s \leq \delta_s$ for all scenarios $s \in \mathcal{S}$ (Birge and Louveaux, 2011).

An optimality cut for a scenario is obtained using the dual objective value of the corresponding sub-problem. Each sub-problem is an AARC or RC formulation with carbon tax uncertainty set whose dual can be obtained using the approach of Beck and Ben-Tal (2009). By denoting the dual variables of constraints (4.24), (4.32), (4.11) - (4.15), and (4.26), respectively, as λ_1 to λ_8, the dual of sub-problem (4.31) - (4.33) is as follows:

$$\Sigma^D_s = \max_{\lambda} \sum_{i\in\mathcal{P}} \eta_i y_i \lambda_{7i} + \sum_{k\in\mathcal{K}} (d^n_{ks}\lambda_{3k} + d^o_{ks}\lambda_{4k}) + \sum_{m\in\mathcal{M}} L^m \lambda_{2m}, \tag{4.43}$$

$$-\lambda_1 = P_s, \tag{4.44}$$

$$h^m(\lambda_1 + \lambda_{2m}) - W_m\lambda^m_{8ij} = 0, \qquad \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{4.45}$$

$$h^m\tilde{\alpha}(\lambda_1 + \lambda_{2m}) - W_m\tilde{\alpha}\lambda^m_{8ij} = 0, \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall ij \in \mathcal{A}, m \in \mathcal{M} \tag{4.46}$$

$$\theta\lambda_1 + \lambda_{3k} \leq 0, \qquad \forall k \in \mathcal{K} \tag{4.47}$$

$$\zeta\lambda_1 + \lambda_{4k} \leq 0, \qquad \forall k \in \mathcal{K} \tag{4.48}$$

$$(g^m\beta_{ij} + \tilde{\alpha}w\beta_{ij}\tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda^m_{8ij}$$

$$+ \lambda_{5j} + \lambda_{7i} \leq 0 \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall ij \in (\mathcal{I}, \mathcal{J}), m \in \mathcal{M} \tag{4.49}$$

$$(g^m\beta_{jk} + \tilde{\alpha}w\beta_{jk}\tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda^m_{8jk}$$

$$+ \lambda_{3k} - \lambda_{5j} \leq 0 \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall jk \in (\mathcal{J}, \mathcal{K}), m \in \mathcal{M} \tag{4.50}$$

$$(g^m\beta_{kl} + \tilde{\alpha}w\beta_{kl}\tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda^m_{8kl}$$

$$+ \lambda_{4k} + \lambda_{6l} + \lambda_{7l} \leq 0 \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall kl \in (\mathcal{K}, \mathcal{L}), m \in \mathcal{M} \tag{4.51}$$

$$(g^m\beta_{li} + \tilde{\alpha}w\beta_{li}\tau^m)(\lambda_1 + \lambda_{2m}) + w\lambda^m_{8li}$$

$$- \lambda_{6l} + \lambda_{7i} \leq 0 \qquad \text{for some } \tilde{\alpha} \in \mathcal{U}, \forall li \in (\mathcal{L}, \mathcal{I}), m \in \mathcal{M} \tag{4.52}$$

$$\lambda_1 \in \mathbb{R}_-, \lambda_2 \in \mathbb{R}_+^{|\mathcal{M}|}, \lambda_3, \lambda_4 \in \mathbb{R}^{|\mathcal{K}|}, \lambda_5 \in \mathbb{R}^{|\mathcal{J}|}, \lambda_6 \in \mathbb{R}^{|\mathcal{L}|}, \lambda_7 \in \mathbb{R}_-^{|\mathcal{P}|}, \lambda_8 \in \mathbb{R}_-^{|\mathcal{A}|\times|\mathcal{M}|} \tag{4.53}$$

If $\Sigma_s > \delta_s$, the following optimality cut is added to the master problem for the next iteration:

$$\sum_{i\in\mathcal{P}} \eta_i y_i \lambda^*_{7i} + \sum_{k\in\mathcal{K}} (d^n_{ks}\lambda^*_{3k} + d^o_{ks}\lambda^*_{4k}) + \sum_{m\in\mathcal{M}} L^m\lambda^*_{2m} \leq \delta_s \tag{4.54}$$

where the left-hand-side is Σ^D_s from (4.43).

4.4 Computational Experiments

The questions addressed in this section include: What are the effects of adjustability and non-adjustability over decisions in CLSC design with multiple transportation modes? How does carbon tax rate uncertainty affect the solutions? Do the uncertainties in demands and return quantities significantly affect the AARC solution? To validate the model and solution approach, we present a computational experiment based on randomly generated instances with realistic parameter values. We compare the results between the nominal, non-adjustable RC and AARC models with various sizes of the uncertainty set. We also show the effect of scenarios on the strategic aspects of the solution in terms of choosing transportation modes, facility locations, and satisfying demands and collecting returns.

In our computational experiment, the locations of potential facilities are randomly selected from a 3500 km $\times$ 2000 km rectangle, and the Euclidian distance is used. The uniform distributions of data generators for the fixed costs c_i of potential factories, warehouses and collection centers and for their capacities η_i are shown in Table 4.1.

Table 4.1 The generator distributions for fixed cost and capacities of potential facilities

	Fixed Cost c_i(\$1000)	Capacities η_i (units of product)
Factories	Uniform$[1000, 4000]$	Uniform$[3000, 6000]$
Warehouses	Uniform$[500, 1500]$	Uniform$[3000, 7000]$
Collection Centers	Uniform$[500, 1500]$	Uniform$[600, 900]$

Based on research studies such as Levinson et al. (2004) and Mallidis et al. (2010), many approaches have been used to estimate truck operating costs which depend on fuel, repair and maintenance, tire, depreciation, and labor cost. Levinson et al. (2004) conducted a survey to identify the average cost per kilometer for the average truckload, which they found \$0.69/km. In addition, several sources such as Coyle et al. (2011) and a white paper by Armstrong Associates Inc. (2009) approximate that 70 to 90 percent of truck operating costs are variable and 10 to 30 percent are fixed costs. More specifically, the latter stated that variable costs include those parameters changing within a year, such as direct labor, fuel, insurance, rented equipment, and maintenance. Fixed costs, which include depreciation, building leased/purchased, management/salespeople, and overhead, are usually steady over a year.

In our computational experiment, only road transport modes are considered, including light, mid-size and heavy trucks. From U.S. government documents, the estimated weights W_m of light, mid-size, and heavy trucks are shown in Table 4.2 (U.S. Department of Transportation, 2000). The estimated unit transportation costs of the modes g^m (per km per ton) for the trucks calculated based on Byrne et al. (2006) are also shown in Table 4.2. We assume each unit is a pallet with 1.1 ton weight. The fixed operating cost h^m per unit of capacity for each road mode is calculated based on approximately 20% of total truck operating costs (Coyle et al., 2011). Moreover, we calculate the total cost of each truck by multiplying the average distance between facilities by the maximum weight of each truck divided by 0.80. Therefore, the fixed costs for different instances depend on the randomly generated distances. The h^m values for the deterministic instances of Section 4.1 are provided in Table 4.2.

Table 4.2 The estimated parameters of mode transportations

Mode (Truck Type)	W_m (tons)	g^m($/units of product-km)	h^m($/unit of transportation)
Light	8.9	0.0213	68
Mid-size	15.2	0.0211	115
Heavy-duty	19.6	0.0240	169

The demands d_k^n are generated according to a normal distribution with mean value 400 units and standard deviation 100; i.e. N(400,100). We have three scenarios for demands: low, medium and high demands. We assume the medium and high demands are 100 and 200 units more than low demands, respectively. Independent of demands, returned products d_k^o are obtained by multiplying the rate of return Rt_k generated from N(0.2 , 0.1) by demands: $d_k^o = Rt_k.d_k^n$. Shortage costs θ and ζ for unmet demands and uncollected returned products usually exceed other components such as production and transportation costs (Absi and Kedad-Sidhoum, 2008). Therefore, after calculating the maximum cost for transporting one unit to a customer, shortage cost are randomly generated according to Unif[1000 -1500].

The carbon emission factor, τ^m, of road transport mode m depends on the mode as well as its vehicle condition, maintenance, roads, type of fuel, and many other factors. The factor values that we used in this experiment, shown in Table 3, are based on data from The Network for Transport and Environment (2014). Heavy trucks usually have lower emission rate per ton but more capacity than light trucks.

For the nominal values of uncertain carbon tax $\bar{\alpha}$, the carbon tax rate of British Columbia in 2012 (Sumner et al., 2009) is used. The instances are solved by CPLEX on a computer with

Table 4.3 The carbon emission rate of different modes (tons/km-ton)

Modes (Trucks)	Light (m=1)	Mid-size (m=2)	Heavy-duty (m=3)
τ^m	0.00025	0.00018	0.00012

8 GB RAM and Intel Core i7 2.00 GHz CPU.

4.4.1 RC and AARC Comparison

In this computational experiment, we assume there are five potential facilities for each of plants, warehouses, and collection centers. The goal is to satisfy 20 customers in different locations. The carbon tax uncertainty set is $\tilde{\alpha} = \bar{\alpha} + \xi\hat{\alpha}$ where the nominal value $\bar{\alpha} = 30$ and the deviation value $\hat{\alpha}$ range from 0 to 30 with $|\xi| \leq 1$. The deterministic model of carbon tax uncertainty has $\hat{\alpha} = 0$, and deterministic demands and returns are assumed by considering one scenario with expected value of demand and return scenarios for each customer. Figure 4.3 shows the facility configuration of the solution of AARC (4.36)-(4.41) when demands and returns are deterministic and $\hat{\alpha} = 10$. In addition, the lower bounds on transportation and emission costs for all three modes are assumed to be zero. In this instance, three plants, three warehouses, and two collection centers are opened.

The RC (4.35) and AARC (4.36)-(4.41) solutions for different values of $\hat{\alpha}$ with $\bar{\alpha} = 30$ and $L^1 = L^2 = L^3 = 0$ are compared in Table 4.4. In this table, the total use of three modes by summing over total product flows of all arcs are shown to be the same for both RC and AARC formulations. As shown in the last column, there is no difference between the RC and the AARC solution since uncertainty is constraint-wise. As the uncertainty of carbon tax increases, the use of transportation mode three with less emission cost increases. Mode two is used in most cases when there is no lower bound on transportation cost.

To generate Table 4.5, we assumed the lower bound, L_1, on transportation and emission costs of mode one is \$1M but $L^2 = L^3 = 0$. The RC and the AARC solutions of different tax uncertainty set are compared for $\bar{\alpha} = 30$. The facility configuration is the same for both RC and AARC. The difference between the RC and AARC objective values increases with the uncertainty of the carbon tax rate. In all of these instances, the use of mode two or three with

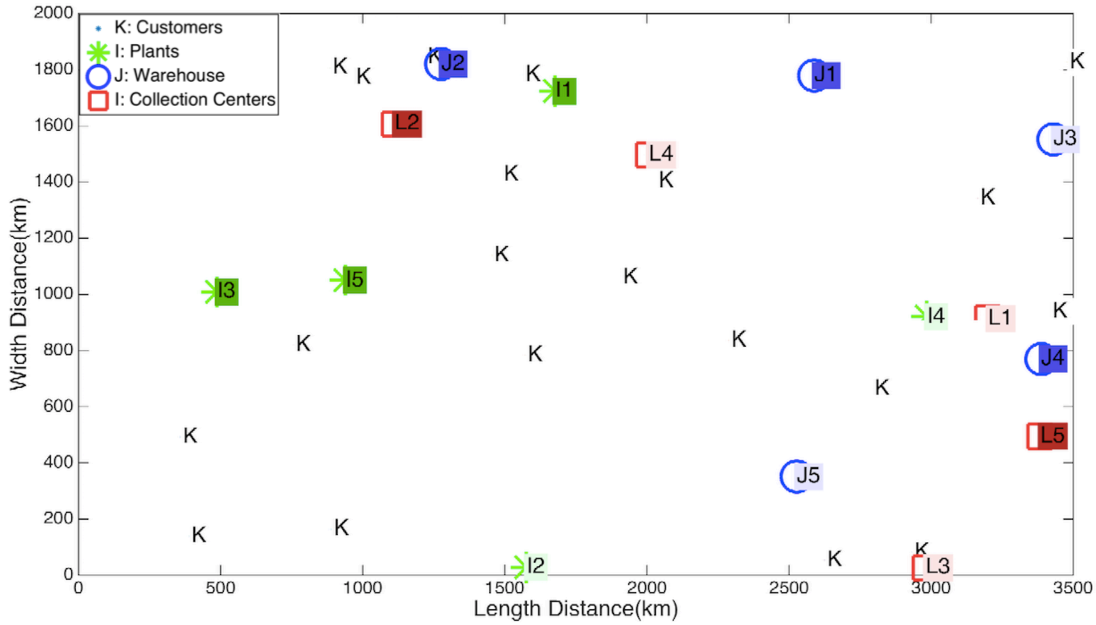

Figure 4.3 Facility configuration of RC or AARC solution when demands and returns are deterministic and $\hat{\alpha} = 10$ and $L^1 = L^2 = L^3 = 0$. Opened facilities are shown in darker color.

less emission cost is higher in the AARC solution than in the RC solution.

Tables 4.6 and 4.7 illustrate the differences between RC and AARC solutions and optimal objective values when the lower bound on transportation cost of mode one and three, respectively, vary from \$100,000 to \$1M. The AARC reduces the conservativeness of the RC solution as the lower bound on the cost of either transportation mode increases. However, the RC and AARC objective differences with the mode one lower bound (Table 4.6) are higher than with the mode three lower bound (Table 4.7) because mode one has the higher emission rate.

We also incorporated uncertainty in demands and returns in the robust CLSC and solved the hybrid robust/stochastic problems (4.35) and (4.36)-(4.41). We used BD to solve the problem with three demand and return scenarios: low, medium and high.

To implement the deterministic model, the average values of assumed scenarios of the uncertain demands and returns are used. If $\tilde{d}$ denotes the uncertain demands and returns, and $\bar{d}$ is the expected value of their distributions, then the optimal value of the deterministic problem can be expressed as EV= Z_{AARC} from (4.36)-(4.41) with deterministic $\bar{d}$. The EV

Table 4.4 The comparison between RC and AARC when $\bar{\alpha} = 30$, and $L^1 = L^2 = L^3 = 0$ for different values of $\hat{\alpha}$. The % use of mode m is $\sum_{ij\in\mathcal{A}} x_{ij}^m / \left(\sum_{\mu\in\mathcal{M}} \sum_{ij\in\mathcal{A}} x_{ij}^\mu\right)$ %.

	% use of mode				
$\hat{\alpha}$	m=1	m=2	m=3	Z_{AARC}	$(Z_{RC} - Z_{AARC})/Z_{RC}$%
0	0	100	0	11643265	0
10	0	100	0	11700130	0
15	0	100	0	11728563	0
20	0	100	0	11756995	0
25	0	77	23	11782611	0
30	0	70	30	11806341	0

solution for the facility configuration is denoted by $\bar{y}(\bar{d})$. For the stochastic program or recourse problem (RP), the optimal value is denoted as RP= Z_{AARC} using the three scenarios. When the performance of the deterministic solution $\bar{y}(\bar{d})$ is evaluated in the stochastic model, we obtain $EEV = \sum_{i\in\mathcal{P}} c_i\bar{y}_i(\bar{d}) + \sum_{s\in\mathcal{S}} P_s Q^L_{AARC}(\bar{y}(\bar{d}), s)$.

The amount of savings that results from solving the stochastic model, called the value of the stochastic solution (VSS), equals EEV−RP (Birge and Louveaux, 2011). The costs of RP and EEV and their comparisons for the AARC model are shown in Tables 4.8 and 4.9. For example, the VSS with the nominal value of the carbon tax rate $\hat{\alpha} = 0$ and $L^1 = 0$ in Table 4.8, is EEV − RP $= 12,432,293 - 12,391,806 = 40,487$ which is 0.33% of RP.

The results in Table 4.8 indicate that the savings from finding the stochastic solution compared to the deterministic solution decreases as the carbon tax rate uncertainty increases. Table 4.9 shows the cost savings of the stochastic solution for different values of lower bounds on modes one and three. The highest cost savings are observed for the highest values of each lower bound.

To evaluate facility configurations and the use of modes under both types of uncertainty, we compared the solutions as the nominal carbon tax $\bar{\alpha}$ increases from 20 to 50 in Table 4.10. For each carbon tax uncertainty level, we randomly generated ten instances of demands, returns, fixed costs, and capacities from their distributions, maintaining a fixed number, 20, of potential facilities of each type to satisfy 70 customers. The results in Table 4.10 show that by increasing the nominal value of the carbon tax rate, the use of modes with lower emission rate would

Table 4.5 The comparison between RC and AARC when $\bar{\alpha} = 30, L^2 = L^3 = 0$, and $L^1 = 1,000,000$ for different values of $\hat{\alpha}$. The % use of mode m is $\sum_{ij\in\mathcal{A}} x_{ij}^m / \left(\sum_{\mu\in\mathcal{M}} \sum_{ij\in\mathcal{A}} x_{ij}^{\mu}\right)\%$.

		% use of mode			$(Z_{RC} - Z_{AARC})/Z_{RC}\%$
$\hat{\alpha}$	Types	m=1	m=2	m=3	
0	RC	97	3	0	0.00
	AARC	97	3	0	
10	RC	100	0	0	0.24
	AARC	94	6	0	
15	RC	100	0	0	0.62
	AARC	92	8	0	
20	RC	100	0	0	0.99
	AARC	91	9	0	
25	RC	96	0	4	1.23
	AARC	90	0	10	
30	RC	99	0	1	1.48
	AARC	88	0	12	

significantly increase. However, unlike the results found in Gao and Ryan (2014), the number of opened facilities do not significantly change.

Table 4.11 shows the results for 20 trials of the same experiment to compare the solutions for stochastic and deterministic demands and returns of the AARC formulation. We assumed randomly generated the probabilities of scenarios 1 and 2 from Unif$[0.3, 0.35]$ and set $P_3 = 1 - (P_1 + P_2)$. The results show that the stochastic solution opens fewer facilities compared to the deterministic one but the use of modes with lower capacity or higher emission rate increases. Figure 4.4 shows the facility configuration of the same instance as in Figure 4.3 but with stochastic demands and returns. In the stochastic solution the numbers of both warehouses and collection centers are decreased from three to two facilities, and one plant has moved to a different location compared to deterministic one in Figure 4.3.

To see how the number of opened facilities is affected by adjustability, Figure 4.5 shows the total number of opened facilities for four different randomly generated instances. We assumed higher demands to represent longer periods by setting the mean and standard deviation of demands to be 100 and 10 thousand units, respectively, and the demands in the medium and high scenarios are 10000 and 20000 units, respectively, more than those in the low scenario. Also

Table 4.6 The comparison between RC and AARC when $\bar{\alpha} = 30, \hat{\alpha} = 10, L^2 = L^3 = 0$ for different values of L^1.

L^1($1000)	Types	% use of mode m=1	m=2	m=3	$(Z_{RC} - Z_{AARC})/Z_{RC}\%$
100	RC	20	80	0	0.01
	AARC	19	81	0	
250	RC	40	60	0	0.02
	AARC	36	64	0	
500	RC	69	31	0	0.05
	AARC	62	38	0	
750	RC	88	12	0	0.09
	AARC	82	18	0	
1000	RC	100	0	0	0.24
	AARC	94	06	0	

the facility capacities for plants and warehouses were randomly generated from Unif$[1M, 2M]$, and for the collection centers from Unif$[100000, 200000]$. The results in Figure 4.5 indicates that by increasing the nominal value of the carbon tax rate, the number of opened facilities are increased. However, there are values of $\bar{\alpha}$ for which AARC would open fewer facilities compared to the RC solution.

4.5 Conclusion

In this paper, we formulated a hybrid robust/stochastic model for CLSC network design that is subject to uncertainty in demands and returned products. We used probabilistic scenarios for the quantities of demands and returned products where the first stage decisions are facility configuration and product flows are determined in the second stage after demand and return quantities are realized. The model structure accommodates carbon tax policy by ensuring that the resulting solutions of facility configuration and product flows are robust to the uncertain carbon tax rate. The transportation capacities as the third stage decisions are assumed to be affine functions of the carbon tax rate for tractable yet less conservative solution to the problem.

In computational experiments, we illustrated the reduced conservatism provided by affine adjustability in the robust counterpart. We analyzed the solutions of the RC and AARC for-

Table 4.7 The comparison between RC and AARC when $\bar{\alpha} = 30, \hat{\alpha} = 10, L^1 = L^2 = 0$ for different values of L^3.

L^3(\$1000)	Types	% use of mode m=1	m=2	m=3	$(Z_{RC} - Z_{AARC})/Z_{RC}\%$
100	RC	0	95	05	0.00
	AARC	0	95	05	
250	RC	0	87	13	0.00
	AARC	0	88	12	
500	RC	0	68	32	0.01
	AARC	0	72	28	
750	RC	0	40	60	0.02
	AARC	0	46	54	
1000	RC	0	3	97	0.04
	AARC	0	17	83	

Table 4.8 Evaluating hybrid robust/stochastic AARC solution with robust AARC solution when $\bar{\alpha} = 30$, and $L^2 = L^3 = 0$, for different values of $\hat{\alpha}$.

	$L^1 = 0$			$L^1 = 1,000,000$		
$\hat{\alpha}$	Stochastic (RP)	EEV	$\frac{VSS}{RP}\%$	Stochastic (RP)	EEV	$\frac{VSS}{RP}\%$
0	12,391,806	12,432,293	0.33	12,463,191	12,555,984	0.74
10	12,448,955	12,483,124	0.27	12,534,298	12,601,637	0.53
15	12,475,871	12,508,539	0.26	12,567,575	12,623,956	0.45
20	12,502,786	12,533,954	0.25	12,600,414	12,645,983	0.36
25	12,527,455	12,557,362	0.24	12,631,885	12,666,767	0.28
30	12,548,517	12,578,992	0.24	12,662,188	12,687,443	0.20

mulations with different levels of uncertainty in the carbon tax rate with lower bounds on the transportation and emission costs of different modes. The results confirm the intuitive understanding that the total expected cost in the worst case of the carbon tax rate is decreased by increasing utilization of transportation modes with higher capacity per unit and lower emission rate. This behavior is consistent across different levels of the lower bounds on transportation and emission costs by mode. Imposing a lower bound on the mode with highest emission rate, maximizes the cost difference between the RC and AARC solutions. The number of opened facilities in AARC solutions are decreased under uncertainty in demands and returns, which indicates the potential for over-investment in facilities if this source of uncertainty is ignored. When there is uncertainty in demands and returns, the numbers of opened facilities do not

Table 4.9 Evaluating hybrid robust/stochastic AARC solution with robust AARC solution when $\bar{\alpha} = 30, \hat{\alpha} = 10$, and $L^2 = 0$ for different values of L^1 and L^3.

L(\$1000)		Stochastic (RP)	EEV	$\frac{VSS}{RP}\%$
	100	12,455,944	12,489,521	0.27
	250	12,467,874	12,501,327	0.27
$(L^3 = 0), L^1:$	500	12,489,066	12,521,598	0.26
	750	12,511,519	12,543,165	0.25
	1000	12,534,298	12,601,637	0.53
	100	12,451,178	12,485,342	0.27
	250	12,454,847	12,488,945	0.27
$(L^1 = 0), L^3:$	500	12,461,284	12,496,485	0.28
	750	12,469,413	12,506,054	0.29
	1000	12,486,465	12,568,429	0.65

Table 4.10 The comparison among "mean ± standard error" of the AARC solutions of ten randomly generated instances of parameters with different values of $\bar{\alpha}$ when $L^1 = \$1.5M, L^2 = L^3 = 0$ and $\hat{\alpha} = 10$.

	Average use of modes(%)			Average opened facilities		
$\bar{\alpha}$	m=1	m=2	m=3	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{K}\rvert$
20	91 ± 1.2	9 ± 1.2	0 ± 0.0	8.1 ± 0.2	7.6 ± 0.2	4.4 ± 0.7
35	87 ± 1.9	13 ± 1.9	0 ± 0.0	7.8 ± 0.2	7.5 ± 0.2	4.1 ± 0.7
50	85 ± 0.9	6 ± 1.3	9 ± 1.4	8.1 ± 0.3	7.6 ± 0.2	3.7 ± 0.6

vary with the nominal value of carbon tax, but the optimal use of modes with lower emission rates increases. In addition, the AARC solution opens fewer facilities and more highly utilizes modes with lower emission rates than the RC solution.

Suggestions for future research including considering product flows to be adjustable the carbon tax rate, which would make the AARC problem significantly harder to solve due to its uncertain recourse. Another possibility is to model parameter variations over multiple periods

Table 4.11 The comparison among "mean ± standard error" of the AARC solutions of ten randomly generated instances of parameters between deterministic and stochastic demands and returns when $L^1 = \$1.5M, L^2 = L^3 = 0$,$\bar{\alpha} = 50$ and $\hat{\alpha} = 30$.

	Average use of modes(%)			Average opened facilities		
	m=1	m=2	m=3	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{K}\rvert$
Stochastic	96 ± 0.6	0 ± 0.0	4 ± 0.6	8.35 ± 0.2	7.95 ± 0.1	3.4 ± 0.6
Deterministic	91 ± 1.0	0 ± 0.0	9 ± 1.0	9.25 ± 0.2	8.75 ± 0.1	3.4 ± 0.5

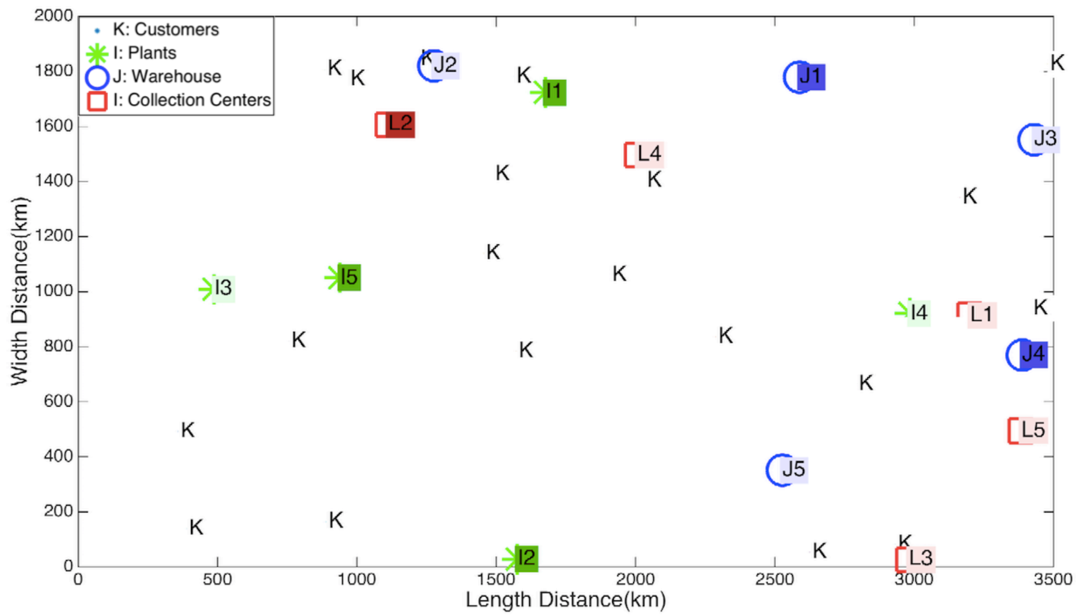

Figure 4.4 Facility configuration of RC or AARC solution when demands and returns are uncertain and $\hat{\alpha} = 10$ and $L^1 = L^2 = L^3 = 0$. Opened facilities are shown in darker color.

of operation for the CLSC network design. In addition, explicitly modeling inventories in the facilities to the problem could be a useful extension to examine the tradeoff between emission and inventory costs.

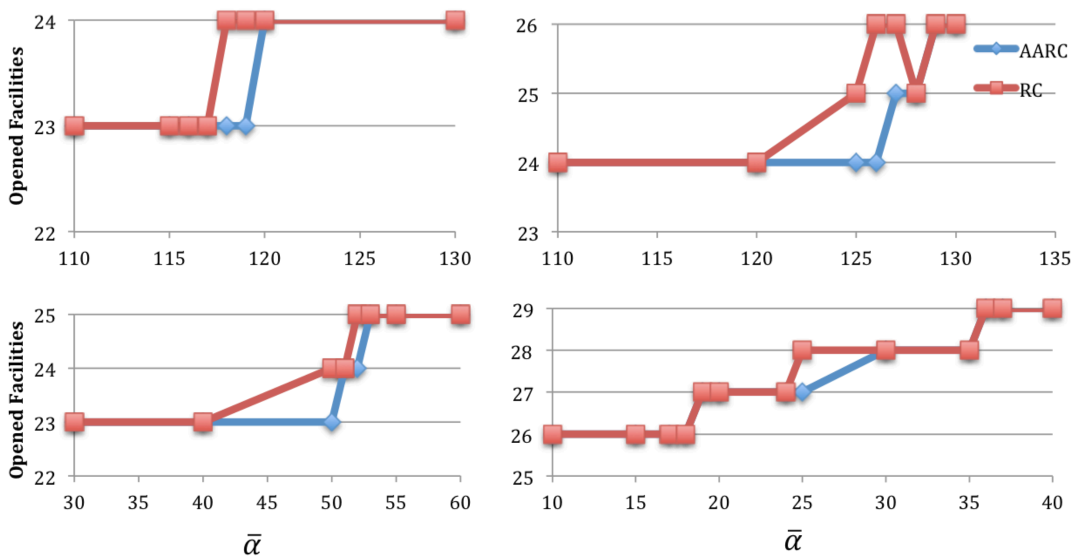

Figure 4.5 Total number of opened facilities of RC and AARC solution when $\bar{\alpha}$ is increasing in horizontal axes and $\hat{\alpha} = 10$, $L^1 = \$100M, L^2 = L^3 = 0$.

CHAPTER 5. GENERAL CONCLUSION

Optimization of multi-stage decision problems under uncertain environments was the focus of this dissertation. We modeled a CLSC network design as a multi-stage stochastic MILP and a hybrid robust/stochastic program. We also investigated when an ARC formulation reduces the conservativeness of the RC formulation.

The proposed CLSC network design problem in this dissertation includes long-term decisions of fixed facilities, decisions of contracts of transportation capacity by multiple modes and decisions of product flows. This application contains two parts.

In the first part of the application, the CLSC network design problem includes uncertain demands and quality of returned products, in which there are dependencies of demands among periods. Using multistage stochastic programming with scenario generation from simulated historical data, the solutions obtained from different scenario trees are evaluated in out-of-sample tests using the same historical data. The underinvestment in transportation capacities of the solution to the deterministic expected value model results in more expected inventory and shortage cost compared to the stochastic program solutions. When uncertainty is taken into account, more transportation capacity is contracted to satisfy a larger proportion of demands while the use of high capacity modes with more fixed cost increased. Different levels of granularity of scenarios demonstrated the existence of a significantly dissimilar alternative near-optimal solution. Some scenario increments might not necessarily improve the solution due to overfitting. The results of multi-stage solution when there is no dependencies of demands among periods show a reduction on solution quality comparing to the scenario tree with dependent demands among periods. The solution of a two-stage stochastic problem has less adjustability for the use of transportation capacity across different periods comparing to the multi-stage solution.

In the next part of the application, the ARC MILP formulation of CLSC network design with tax rate uncertainty. However, the ARC formulation does not always produce a less conservative solution than the RC formulation. Therefore, Chapter 3 provides conditions in which the objective values of ARC and RC are not equivalent. In these conditions, the RC formulation includes at least two constraints that are binding at the optimal RC solution for different values of the same uncertain parameter. In addition, a variable to be made adjustable appears in both constraints and is bounded from above by one constraint at one extreme of the uncertainty interval and bounded from below by the other at the opposite extreme of the uncertainty interval. One of these bounds is unfavorable for the objective. By relaxing this bound, adjustability increases the feasible region of the RC in a direction that lowers the objective value. Using the dual values of the optimal RC solution, we show how RC formulations can be tested in small instances before converting to AARC in order to identify whether adjustability matters.

In the second part of CLSC network design, we consider carbon tax as an environmental regulation with multiple modes of transportation. In computational experiments, we illustrated the reduced conservatism provided by affine adjustability in the robust counterpart. The results also confirm the intuitive understanding that the total expected cost in the worst case of the carbon tax rate is decreased by increasing the utilization of transportation modes with higher capacity per unit and lower emission rate. We identified significant factors in deciding the best numbers and types of transportation modes when there is carbon tax uncertainty. As the nominal or the interval of carbon tax rate uncertainty increases, the use of transportation mode with less emission cost increases. Imposing a lower bound on the mode with highest emission rate maximizes the cost difference between the RC and AARC solutions. The number of opened facilities in AARC solutions is decreased under uncertainty in demands and returns, which indicates the potential for over-investment in facilities if this source of uncertainty is ignored. When there is uncertainty in demands and returns, the numbers of opened facilities do not vary with the nominal value of carbon tax, but the optimal use of modes with lower emission rates increases. In addition, the AARC solution opens fewer facilities and more highly utilizes modes with lower emission rates than the RC solution.

In summary, the contributions of this dissertation include:

- Developing a multi-stage model for designing a CLSC network with integrated uncertain quality of returns and demands with dependencies between periods while different transportation modes are decided before realization of uncertain parameters to offer more efficient solutions compared to the deterministic solution,
- Identifying significant factors in deciding the best numbers and types of transportation modes when there is carbon tax uncertainty in CLSC network design,
- Providing conditions in which the objective values of ARC and RC are not equivalent
- Developing a three-stage hybrid robust stochastic model with transportation capacities that adjust to carbon tax rate in uncertainty sets within probabilistic scenarios for quantities of demands and returned products.

Future research in the CLSC design application could include modeling the relationships among uncertain variables over time. For example, the demands and returned products can be dependent over the periods. More accurate scenario generations can be used to create scenario trees that are not too large. Scenario reductions can also be used for large-scale problems or the ones with longer periods. Finally, the solution of larger-scale instances may require decomposition approaches such as progressive hedging or the nested L-shaped method.

For uncertain carbon tax rate modeling parameter variations over multiple periods of operation could be another extension for the CLSC network design. Product flows can be considered to be adjustable the carbon tax rate, which would make the AARC problem significantly harder to solve due to its uncertain recourse. In addition, explicitly modeling inventories in the facilities to the problem could be a useful extension to examine the tradeoff between emission and inventory costs. For multi-stage adjustable robust optimization, we only considered the fixed recourse case. For uncertainty-affected recourse, a similar approach would require more computational complexity that is a subject for future research.

APPENDIX A. PARAMETER VALUES FOR THE COMPUTATIONAL EXPERIMENTS IN CHAPTER 2

Table A.1 provides the two-dimensional coordinates of retailers and potential locations for plants, warehouses, and collection centers. The coordinates of retailers were generated randomly between 1000 and 12000 km from (0, 0) and the coordinates of potential facilities are located close to retailers where we assume that they are usually constructed in real applications. The distance β_{ij} is the Euclidean distance between two facilities.

Table A.1 The coordinates of retailers and potential locations of facilities (km)

Retailers, $\|\mathcal{K}\| = 8$		Plants, $\|\mathcal{F}\| = 3$		Warehouses, $\|\mathcal{J}\| = 4$		Collection centers, $\|\mathcal{L}\| = 4$	
X	Y	X	Y	X	Y	X	Y
4164	6409	10037	4096	4164	6309	3752	8001
6370	7586	6370	7786	8199	6729	5353	2802
9257	6343	2980	3239	8000	2780	1319	4738
8029	2780			1319	4938	9207	6323
1349	4908						
8099	6799						
5373	2832						
3692	8001						

Table A.2 shows the fixed cost and capacities of the potential facilities, for which the parameter values are equal for all facilities at the same type.

Table A.2 Fixed cost ($) and capacities of potential facilities (unit of product/period)

Facility	$c_i(\forall i \in \mathcal{P})$ ($)	$\eta_i(\forall i \in \mathcal{P})$ ($)
Plants	400,000	550
Warehouse	250,000	600
Collection center	65,000	500

Table A.3 presents the holding costs of each warehouse and collection center which were generated to be in a reasonable proportion with the other costs. We assumed that holding costs are higher in collection centers because returned products lose value more quickly. The shortage costs of demands and uncollected costs of used products, which are equal for each retailer, are shown in this table as well. The randomly generated shortage costs are assumed to be higher than the highest transportation cost of one unit from a plant to a retailer.

Table A.3 Inventory costs ($/unit of product) in warehouses and collection centers and shortage and uncollected returns costs ($/unit of product)

	$\Phi_j (j \in \mathcal{J})$	$\Phi_l (l \in \mathcal{L})$	$\Psi_k = \Psi'_k (k \in \mathcal{K})$
1	452	571	873
2	497	543	885
3	452	583	871
4	481	594	897
5			919
6			923
7			959
8			969

Table A.4 illustrates the properties of three transportation modes with specified capacity, fixed and variable costs for all periods. From U.S. Department of Transportation documents, the estimated weights of light, mid-size and heavy trucks are considered to be 8.9 , 15.2, and 19.6 tons, respectively (The U.S. Department of Transportation, 2000). The estimated unit transportation costs of light, mid-sized, and heavy trucks are $0.0215, $0.022, and $0.024 per km per ton, respectively, that are calculated based on (Byrne et al., 2006). In addition, we consider each unit of product as a pallet with 1.1 ton weight ($w = 1.1$). The per unit capacity of fixed operating cost for each road-transportation mode are calculated based on approximately 20% of total truck operating costs (Coyle et al., 2011).

Table A.4 The amount of capacity (tons/unit mode), variable ($/km-unit of product) and fixed costs ($/unit mode) of transportation modes

Mode	W_m(tons/unit mode)	g^m ($/km-unit of product)	h^m ($/unit mode)
1- Small trucks	8.9	0.0215	248.4
2- Mid-Size trucks	15.2	0.0220	404.3
3- Heavy truck	19.6	0.0240	510.0

Table A.5 shows the return rates in each period for all retailers. We assumed that this rate is an increasing function of periods.

Table A.5 Return rate of retailers at different periods

Period	t
1	0.2
2	0.3
3	0.5

Table A.6 shows demand outcomes for all retailers that were obtained by the moment-matching heuristic explained in section 2.5.1.2. In Table A.6 there is no dependencies between period one and two but period three depends to period two. Table A.7 is demand outcomes of period three when there is no dependencies between period three and two, which is calculated by taking the average values of four scenarios of period three in Table A.6. More specifically, each outcome value in Table A.7 was calculated by summation over four outcomes of multiplying probability of each scenario to their values of period three in Table A.6. Also each probability of Table A.7 is calculated by making the average of four scenario probabilities of period three in Table A.6. Therefore, the data used for the case of no dependencies of demands among periods is period one and two from Table A.6 and period three from Table A.7. Table A.8 is the demand specifications of all retailers.

Table A.6 The four demand outcomes (unit of product) of each eight retailers for different periods.

	Scenario	Retailer Probability	1	2	3	4	5	6	7	8
Period 1	1-1	0.361747	122.0	93.1	92.8	92.8	88.0	94.7	123.0	94.8
	1-2	0.309766	76.1	76.0	81.6	77.3	78.4	79.4	74.5	75.5
	1-3	0.006355	5.7	8.5	24.7	23.8	8.5	24.2	9.79	21.0
	1-4	0.322132	86.6	124.0	126.0	122.0	124.0	125.0	92.0	123.0
Period 2	2-1	0.010119	101.0	185.0	177.0	59.8	63.1	83.7	44.3	36.1
	2-2	0.386865	133.0	82.9	84.8	90.0	86.6	87.8	134.0	133.0
	2-3	0.007629	28.3	126.0	44.4	182.0	171.0	189.0	44.9	123.0
	2-4	0.595387	93.0	123.0	124.0	126.0	121.0	125.0	97.0	95.2
Period 3 (Root Period 2-1)	3-1	0.592752	103.0	142.0	138.0	83.2	81.1	93.7	76.6	70.4
	3-2	0.325370	148.0	191.0	188.0	128.0	124.0	139.0	117.0	115.0
	3-3	0.026237	35.7	81.0	75.0	30.9	23.8	35.0	59.7	4.2
	3-4	0.055640	73.6	145.0	149.0	58.8	128.0	93.3	28.0	51.3
Period 3 (Root Period 2-2)	3-1	0.552377	116.0	90.9	93.3	90.9	94.2	91.7	112.0	115.0
	3-2	0.060496	66.5	47.8	47.2	105.0	65.1	91.8	152.0	93.0
	3-3	0.006875	212.0	194.0	196.0	16.3	14.0	7.6	32.0	22.4
	3-4	0.380252	159.0	134.0	136.0	140.0	136.0	139.0	158.0	161.0
Period 3 (Root Period 2-3)	3-1	0.030786	0.0	52.8	10.5	146.0	82.9	148.0	156.0	48.9
	3-2	0.365464	110.0	159.0	119.0	185.0	167.0	148.0	106.0	156.0
	3-3	0.489904	62.3	112.0	71.8	146.0	133.0	181.0	64.1	111.0
	3-4	0.113840	62.3	112.0	71.8	108.0	190.0	107.0	106.0	111.0
Period 3 (Root Period 2-4)	3-1	0.593040	93.7	107.0	107.0	109.0	110.0	109.0	98.4	95.6
	3-2	0.005154	3.5	11.5	8.2	26.9	40.3	19.2	36.6	0.1
	3-3	0.377057	142.0	157.0	158.0	158.0	153.0	155.0	141.0	142.0
	3-4	0.024748	43.0	89.7	144.0	145.0	59.7	171.0	45.6	55.4

Table A.7 The demand outcomes (unit of product) of each eight retailers for period three when there is no dependencies among periods.

	Scenario	Retailer Probability	1	2	3	4	5	6	7	8
Period 3	3-1	0.442239	180.7	199.6	197.3	168.8	168.1	175.4	170.5	163.5
	3-2	0.189121	92.3	123.2	107.4	115.7	105.4	105.0	86.2	100.2
	3-3	0.225019	86.3	117.4	98.2	132.2	123.5	148.0	86.5	108.3
	3-4	0.143622	72.6	73.9	71.6	72.6	81.8	74.3	74.9	78.1

Table A.8 Demand specifications of each retailer for three periods.

		Demand specifications			
Retailers	Period	Mean	Variance	Skewness.	Kurtosis
1	1	95.54	442.13	-0.067	3.23
	2	$\kappa_1^2(\lambda)$	427.77	0.015	2.69
	3	$\kappa_1^3(\lambda)$	704.17	-0.234	3.18
2	1	97.33	433.15	-0.124	3.37
	2	$\kappa_2^2(\lambda)$	431.40	0.071	2.92
	3	$\kappa_2^3(\lambda)$	679.04	0.024	2.96
3	1	99.45	370.26	0.170	2.78
	2	$\kappa_3^2(\lambda)$	436.04	-0.222	2.65
	3	$\kappa_3^2(\lambda)$	685.87	-0.066	3.15
4	1	96.84	354.61	0.009	2.76
	2	$\kappa_4^2(\lambda)$	370.73	-0.158	2.72
	3	$\kappa_4^2(\lambda)$	612.35	0.064	2.47
5	1	96.12	421.22	0.093	3.40
	2	$\kappa_5^2(\lambda)$	329.70	-0.160	2.40
	3	$\kappa_5^2(\lambda)$	571.30	-0.238	3.02
6	1	99.18	372.27	0.031	2.76
	2	$\kappa_6^2(\lambda)$	374.08	0.102	3.07
	3	$\kappa_6^2(\lambda)$	587.50	-0.027	3.11
7	1	97.22	455.82	-0.188	2.94
	2	$\kappa_7^2(\lambda)$	402.47	-0.169	2.94
	3	$\kappa_7^2(\lambda)$	574.19	-0.222	2.91
8	1	97.48	401.63	-0.085	2.66
	2	$\kappa_8^2(\lambda)$	391.84	-0.080	2.85
	3	$\kappa_8^2(\lambda)$	653.11	-0.195	3.35

Bibliography

Absi, N. and Kedad-Sidhoum, S. (2008). The multi-item capacitated lot-sizing problem with setup times and shortage costs. *European Journal of Operational Research*, 185(3):1351–1374.

Adida, E. and Perakis, G. (2010). Dynamic pricing and inventory control: robust vs.stochastic uncertainty modelsäîa computational study. *Annals of Operations Research*, 181(1):125–157.

Amin, S. H. and Zhang, G. (2012). A multi-objective facility location model for closed-loop supply chain network under uncertain demand and return. *Applied Mathematical Modelling*, 37(6):4165–4176.

Aras, N., Boyaci, T., and Verter, V. (2004). The effect of categorizing returned products in remanufacturing. *IIE Transactions*, 36(4):319–331.

Armstrong Associates Inc. (2009). Carrier procurement insights, trucking company volume, cost and pricing tradeoffs. *Armatrong and Associates, Inc.*

Baptista, S., Gomes, M. I., Escudero, L., Medeiros, P., and Cabrita, F. (2015). *Operations research and big data*, chapter A fix-and-relax algorithm for solving parallel and sequential versions of a multi-period multi-product closed loop supply chain design and operation planning model. Springer.

Bazaraa, M. S., Jarvis, J. J., and Sherali, H. D. (2010). *Linear Programming and network flows*. John Wiley and Sons, Inc., Hoboken, New Jersey, 4 edition.

Beck, A. and Ben-Tal, A. (2009). Duality in robust optimization: Primal worst equals dual best. *Operations Research Letters*, 37(1):1–6.

Ben-Tal, A., Boaz, G., and Shimrit, S. (2009). Robust multi-echelon multi-period inventory control. *European Journal of Operational Research*, 199(3):922–935.

Ben-Tal, A., Goryashkoy, A., Guslitzerz, E., and Nemirovski, A. (2004). Adjustable robust solutions of uncertain linear programs. *Mathematical Programming*, 99(2):351–376.

Ben-Tal, A. and Nemirovski, A. (1998). Robust convex optimization. *Mathematics of Operations Research*, 23(4):769–805.

Ben-Tal, A. and Nemirovski, A. (1999). Robust solutions of uncertain linear programs. *Operations Research Letters*, 25:1–13.

Ben-Tal, A. and Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, 88(3):411–424.

Benders, J. F. (1962). Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4:238 – 252.

Benjaafar, S., Li, Y., and Daskin, M. (2013). Carbon footprint and the management of supply chains:insights from simple models. *IEEE Transactions on Automation Science and Engineering*, 10(1):99–116.

Bertsimas, D. and Goyal, V. (2010). On the power of robust solutions in two-stage stochastic and adaptive optimization problems. *Mathematics of Operations Research*, 35(2):284–305.

Bertsimas, D. and Goyal, V. (2011). On the power and limitations of affine policies in two-stage adaptive optimization. *Mathematical Programming*, 134(2):491–531.

Bertsimas, D. and Goyal, V. (2013). On the approximability of adjustable robust convex optimization under uncertainty. *Mathematical Methods of Operations Research*, 77(3):323–343.

Bertsimas, D., Goyal, V., and Lu, B. Y. (2015). A tight characterization of the performance of static solutions in two-stage adjustable robust linear optimization. *Mathematical Programming*, 150(2):281–319.

Bertsimas, D., Goyal, V., and Sun, X. A. (2011). A geometric characterization of the power of finite adaptability in multistage stochastic and adaptive optimization. *Mathematics of Operations Research*, 36(1):24–54.

Bertsimas, D., Iancu, D. A., and Parrilo, P. A. (2010). Optimality of affine policies in multistage robust optimization. *Mathematics of Operations Research*, 35(2):363–394.

Bertsimas, D. and Ruiter, F. d. (2015). Duality in two-stage adaptive linear optimization: Faster computation and stronger bounds. http://www.optimization-online.org/DB_FILE/2015/08/5053.pdf.

Bertsimas, D., Sim, M., and Zhang, M. (2013). A practicable framework for distributionally robust linear optimization. http://www.optimization-online.org/DB_FILE/2013/07/3954.pdf.

BeÜhringer, C., Rutherford, T. F., and Tol, R. S. J. (2009). The EU 20/20/2020 targets: An overview of the EMF22 assessment. *Energy Economics*, 31:S268–S273.

Birge, J. R. and Louveaux, F. (2011). *Introduction to Stochastic Programming*. Springer Series in Operations Research and Financial Engineering. Springer, London, 2 edition.

Bloemhof-Ruwaard, J., Van der Laan, E., and Beijer, C. (2011). Sustainable inland transportation. *International Journal of Business Insights and Transformation*, 3(3):26 – 33.

Byrne, J., Waegel, A., Haney, B., Tobin, D., Alleng, G., Karki, J., , and Suarez, J. (2006). Pathways to a us hydrogen economy: Vehicle fleet diffusion scenarios. *Technical report, Newark, DE: Center for Energy and Environmental Policy*.

Cai, X., Lai, M., Li, X., Li, Y., and Wu, X. (2013). Optimal acquisition and production policy in a hybrid manufacturing/remanufacturing system with core acquisition at different quality levels. *European Journal of Operational Research*.

Chen, X. and Zhang, Y. (2009). Uncertain linear programs: Extended affinely adjustable robust counterparts. *Operations Research*, 57(6):1469–1482.

Cohen, I., Golany, B., and Shtub, A. (2007). The stochastic timeàìcost tradeoff problem: A robust optimization approach. *Networks*, 49(2):175–188.

Coyle, J. J., Novak, R. A., Gibson, B., and Bardi, E. J. (2011). *Transportation: A Supply Chain Perspective*. Cengage Learning, New York, 7 edition.

Denizel, M. ., Ferguson, M., and Souza, G. (2010). Multiperiod remanufacturing planning with uncertain quality of inputs. *Engineering Management, IEEE Transactions on*, 57(3):394–404.

Fleischmann, M., Beullens, P., Bloemhof-Ruwaard, J., and van Wassenhove, L. (2001). The impact of product recovery on logistics network design. *Production and operations management*, 10(2):156–173.

Fleischmann, M., Nunen, J. V., and Grave, B. (2003). Integrating closed-loop supply chains and spare-parts management at IBM. *Interfaces*, 33(6):44–56.

Forkenbrock, D. J. (2001). Comparison of external costs of rail and truck freight transportation. *Transportation Research Part A*, 35:321–337.

Francas, D. and Minner, S. (2009). Manufacturing network configuration in supply chains with product recovery. *Omega*, 37(4):757–769.

Fransoo, K. H. T. T. J. and van Houtum, G. (2010). Effect of carbon emission regulations on transport mode selection in supply chains. *Flexible Services and Manufacturing Journal*, 26(1):170–195.

Fu, M. and Kelly, J. A. (2012). Carbon related taxation policies for road transport: Efficacy of ownership and usage taxes, and the role of public transport and motorist cost perception on policy outcomes. *Transport Policy*, 22:57–69.

Galbreth, M. and Blackburn, J. (2006). Optimal acquisition and sorting policies for remanufacturing. *Production / operations management*, 15(3):384–392.

Gao, N. and Ryan, S. M. (2014). Robust design of a closed-loop supply chain network for uncertain carbon regulations and random product flows. *EURO Journal on Transportation and Logistics*, 3(1):5–34.

Georgiadis, P. and Athanasiou, E. (2013). Flexible long-term capacity planning in closed-loop supply chains with remanufacturing. *European Journal of Operational Research*, 225(1):44–58.

Ghaoui, L. E. and Lebret, H. (1997). Robust solutions to least-squares problems with uncertain data. *SIAM Journal on Matrix Analysis and Applications*, 18(4):1035–1064.

Ghaoui, L. E., Oustry, F., and Lebret, H. (1998). Robust solutions to uncertain semidefinite programs. *SIAM Journal on Optimization*, 9(1):33–Äì52.

Goh, J. and Sim, M. (2010). Distributionally robust optimization and its tractable approximations. *Operations Research*, 58(4-Part-1):902–917.

Guide, D., Teunter, R., and Van Wassenhove, L. N. (2003). Matching demand and supply to maximize profits from remanufacturing. *Manufacturing and Service Operations Management*, 5(4):303–316.

Haddad-Sisakht, A. and Ryan, S. M. (2016). Conditions under which adjustability lowers the cost of a robust linear program. Available at: http://lib.dr.iastate.edu/imse_reports/3/.

Hickman, Robin Ashiru, O. and Banister, D. (2011). Transitions to low carbon transport futures: strategic conversations from london and delhi. *Journal of Transport Geography*, 19(6):1553–1562.

Hyland, K., Kaut, M., and Wallace, S. W. (2003). A heuristic for moment-matching scenario generation. *Computational Optimization and Applications*, 24(2-3):169–185.

Hyland, K. and Wallace, S. W. (2001). Generating scenario trees for multistage decision problems. *Management Science*, 47(2):295–307.

Kaut, M. and Mathieu, D. (2012). *Chapter 4 of Modeling with Stochastic Programming Springer*, page 99. Springer, London, 1 edition.

Keyvanshokooh, E., Fattahi, M., Seyed-Hosseini, S., and Tavakkoli-Moghaddam, R. (2013). A dynamic pricing approach for returned products in integrated forward/reverse logistics network design. *Applied Mathematical Modelling*, 37:10182–10202.

Keyvanshokooh, E., Ryan, S. M., and Kabir, E. (2016). Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated benders decomposition. *European Journal of Operational Research*, 249(1):76–92.

Kuhn, D., Wiesemann, W., and Georghiou, A. (2009). Primal and dual linear decision rules in stochastic and robust optimization. *Mathematical Programming*, 130(1):177–209.

Levinson, D., Corbett, M., and Hashami., M. (2004). Operating costs for trucks. *http://nexus.umn.edu/papers/truckoperatingcosts.pdf.*

Listes, O. (2007). A generic stochastic model for supply-and-return network design. *Computers and Operations Research*, 34(2):417–442.

Mallidis, I., Dekker, R., and Vlachos, D. (2010). Greening supply chains: Impact on costs and design. *Retrieved from Erasmus University, Econometric Institute website:* *http://repub.eur.nl/res/pub/20375*.

Mallidis, I., Dekker, R., and Vlachos, D. (2012). The impact of greening on supply chain design and cost: a case for a developing region. *Journal of Transport Geography*, 22:118–128.

Marandi, A. and den Hertog, D. (2015). When are static and adjustable robust optimization with constraint-wise uncertainty equivalent? *Tilburg CentER (Center for Economic Research) Discussion Paper*, 2015-045.

Metcalf, G. E. (2009). Market-based policy options to control U.S. greenhouse gas emissions. *Journal of Economic Perspectives*, 23(2):5–27.

Ouorou, A. (2013). Tractable approximations to a robust capacity assignment model in telecommunications under demand uncertainty. *Computers & Operations Research*, 40(1):318–327.

Paksoy, T., Bekta, T., and zceylan, E. (2011). Operational and environmental performance measures in a multi-product closed-loop supply chain. *Transportation Research Part E: Logistics and Transportation Review*, 47(4):532–546.

Pan, S., Ballot, E., and Fontane, F. (2010). The reduction of greenhouse gas emissions from freight transport by pooling supply chains. *International Journal of Production Economics*, 143(1).

Pflug, G. C. (2001). Scenario tree generation for multiperiod financial optimization by optimal discretization. *Mathematical Programming Series*, 89:251–271.

Pishvaee, M. S., Rabbani, M., and Torabi, S. A. (2011). A robust optimization approach to closed-loop supply chain network design under uncertainty. *Applied Mathematical Modelling*, 35(2):637–649.

Pishvaee, M. S., Razmi, J., and Torabi, S. A. (2012). Robust possibilistic programming for socially responsible supply chain network design: A new approach. *Fuzzy Sets and Systems*, 206(1):1–20.

Sim, O., Jung, S., Kim, H., and Park, J. (2004). A generic network design for a closed-loop supply chain using genetic algorithm. In al., K. D. e., editor, *Genetic and Evolutionary Computation: GECCO 2004*, volume 3103, pages 1214–1225. Springer-Verlag Berlin Heidelberg.

Solyali, O. (2014). Production planning with remanufacturing under uncertain demand and returns. *H.UÃà. IÃáktisadi ve IÃádari Bilimler FakuÃàltesi Dergisi*, 32(2):275–296.

Soyster, A. L. (1972). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research*, 21(5):1154–1157.

Soyster, A. L. and Murphy, F. H. (2013). A unifying framework for duality and modeling in robust linear programs. *Omega*, 41(6):984–997.

Soyster, A. L. and Murphy, F. H. (2014). On the integration of row and column uncertainty in robust linear programming. *Journal of Global Optimization*, pages 1–29.

Sumner, J., Bird, L., and Smith, H. (2009). Carbon taxes: A review of experience and policy design considerations. Technical report, National Renewable Energy Laboratory. Available at: http://www.nrel.gov/docs/fy10osti/47312.pdf.

Tao, Z., Zhou, S. X., and Tang, C. S. (2012). Managing a remanufacturing system with random yield: Properties, observations, and heuristics. *Production and Operations Management*, 21(5):797–813.

The Network for Transport and Environment (2014).

The U.S. Department of Transportation (2000). The U.S. Department of Transportation's comprehensive truck size and weight study. *The U.S. Department of Transportation*, Vol. ll.

Turban, E., King, D., Lee, J. K., and Liang, Ting-Peng andTurban, D. C. (2015). *Order Fulfillment Along the Supply Chain*. Springer Texts in Business and Economics.

U.S. Department of Transportation (2000). The U.S. Department of Transportation's comprehensive truck size and weight study. *U.S. Department of Transportation*, 2.

U.S. Environmental Protection Agency (2016). Sources of greenhouse gas emissions, U.S. Environmental Protection Agency.

Vahdani, B. and Mohammadi, M. (2015). A bi-objective interval-stochastic robust optimization model for designing closed loop supply chain network with multi-priority queuing system. *International Journal of Production Economics*, 170(Part A):67–78.

Vahdani, B., Tavakkoli-Moghaddam, R., Modarres, M., and Baboli, A. (2012). Reliable design of a forward/reverse logistics network under uncertainty: A robust-M/M/c queuing model. *Transportation Research Part E: Logistics and Transportation Review*, 48(6):1152–1168.

Xinhuanet (2013). China to introduce carbon tax: official. http://news.xinhuanet.com/english/china/2013-02/19/c_132178898.htm.

Yuzhong, M. and Guangming, Y. (2012). A study of outsourcing and self-run business decision-making in the distribution transportation of international logistics. *Energy Procedia*, 16:965–970.

Zakeri, A., Dehghanian, F., Fahimni, B., and Sarkis, J. (2015). Carbon pricing versus emissions trading: A supply chain planning perspective. *International Journal of Production Economics*, 164:197–205.

Zeballos, L. J., Gomes, M. I., Barbosa-Povoa, A. P., and Novais, A. Q. (2012). Addressing the uncertain quality and quantity of returns in closed-loop supply chains. *Computers and Chemical Engineering*, 47(20):237–247.

Zeballos, L. J., Méndez, C. A., Barbosa-Povoa, A. P., and Novais, A. Q. (2014). Multi-period design and planning of closed-loop supply chains with uncertain supply and demand. *Computers and Chemical Engineering*, 66:151–164.

Zhang, X., Wu, Q., and Tian, Z. (2011). Bullwhip effect reduction in closed-loop supply chains using h-infinity control. *IEEE*.

Zhou, S. X., Tao, Z., and Chao, X. (2010). Optimal control of inventory systems with multiple types of remanufacturable products. *Manufacturing and Service Operations Management*, 13(1):20–34.