Optimization model to increase resilience, with application to the electric power network

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Disaster resilience

• Disaster resilience is the ability to (Bruneau et al. 2003)
  – Reduce the chances of a shock
  – Absorb a shock if it occurs
  – Recover quickly after it occurs

• Nonlinear disaster recovery (Zobel 2014)


Quantifying disaster resilience

\[ R_*(\beta, X, T) = 1 - \frac{\beta X T}{T^*} \]
Quantifying disaster resilience

\[ R_*(\bar{X}, T) = 1 - \frac{\bar{X}T}{T^*} \]
Research questions

1. How should a decision maker allocate resources between reducing loss and decreasing time in order to maximize resilience?

2. How should the allocation change based on the assumptions in the allocation functions?

3. Does the optimal decision change when there is uncertainty?

4. How can this theoretical model be applied to a real-world disruption?
Resource allocation model

\[ R_*(\bar{X}, T) = 1 - \frac{\bar{X}T}{T^*} \]

Factor as a function of resource allocation decision

maximize \( R_*(\bar{X}(z_{\bar{X}}), T(z_T)) \)

minimize \( \bar{X}(z_{\bar{X}}) \times T(z_T) \)

subject to \( z_{\bar{X}} + z_T \leq Z \)

\( z_{\bar{X}}, z_T \geq 0 \)

Budget

November 6, 2015
SafeLife-X
Allocation functions

- $\bar{X}(z_{\bar{X}})$ and $T(z_{T})$ describe ability to allocate resources to reduce each factor of resilience

- Requirements
  - Factor should decrease as more resources are allocated: $\frac{d\bar{X}}{dz_{\bar{X}}}$ and $\frac{dT}{dz_{T}}$ are less than 0
  - Constant returns or marginal decreasing improvements as more resources are allocated: $\frac{d^2\bar{X}}{dz_{\bar{X}}^2}$ and $\frac{d^2T}{dz_{T}^2}$ are greater than or equal to 0
Four allocation functions

1. Linear
2. Exponential
3. Quadratic
4. Logarithmic
Linear allocation function

\[
\bar{X}(z_{\bar{X}}) = \hat{X} - a_{\bar{X}}z_{\bar{X}}
\]
\[
T(z_T) = \hat{T} - a_Tz_T
\]

- Decision maker should only allocate resources to reduce one resilience factor based on \[\max \left\{ \frac{a_{\bar{X}}}{\hat{X}}, \frac{a_T}{\hat{T}} \right\} \]
- Focuses resources on the factor whose initial parameter is already small and where effectiveness is large
Exponential allocation function

\[
\bar{X}(z_X) = \bar{X} \exp(-a_X z_X)
\]
\[
T(z_T) = \hat{T} \exp(-a_T z_T)
\]

- Decision maker should only allocate resources to reduce one resilience factor based on \(\max\{a_X, a_T\}\)
- Decision depends only the effectiveness and not the initial values
Quadratic allocation function

\[
\bar{X}(z_X) = \bar{X} - b_X z_X + a_X z_X^2
\]

\[
T(z_T) = \bar{T} - b_T z_T + a_T z_T^2
\]

- Assume \( z_X \leq \frac{b_X}{2a_X} \), \( z_T \leq \frac{b_T}{2a_T} \) so that functions are always decreasing
Logarithmic allocation functions

\[ \bar{X}(z_{\bar{X}}) = \hat{X} - a_{\bar{X}} \log(1 + b_{\bar{X}} z_{\bar{X}}) \]

\[ T(z_T) = \hat{T} - a_T \log(1 + b_T z_T) \]
Uncertainty with independence

• Assume $\hat{X}$, $\hat{T}$, $a_X$, $b_X$, $a_T$, $b_T$ have known distributions

• Assume independence

• Maximize expected resilience $E[R_*(\bar{X}, T)]$

• Linear, quadratic, and logarithmic allocation functions: May be optimal to allocate to both factors
Exponential allocation, uncertainty

- Always a convex optimization problem

\[ E[(a_T - a_{\bar{X}})\exp([a_T - a_{\bar{X}}]z_{\bar{X}} - a_T Z)] = 0 \]
Uncertainty without probabilities

• Each parameter is bounded above and below, i.e. $\underline{X} \leq \hat{X} \leq \bar{X}$ and $\underline{aX} \leq a\hat{X} \leq \bar{aX}$

• Maximin approach
  
  $\text{maximize } \min \ R_*(\bar{X}(z\bar{X}), T(z_T))$

• Same rules as the case with certainty but choose worst-case parameters to determine allocation, i.e. $\hat{X}$ and $a\bar{X}$
Superstorm Sandy

• October 2012
  – East coast of the U.S.
  – Second costliest hurricane in U.S. history

• ConEdison Electric Utility
  – 670,000 New York city customers without electricity
  – Approximately 1/5 of ConEdison’s customers
  – Duration: 13 days
ConEdison’s Post-Sandy Plan

• $1 billion over 4 years to increase resilience of electric power network
• Hardening activities (reduce $\bar{X}$)
  – Trimming trees around power lines
  – Building higher flood plains
  – Backup power for substations
• Restoration activities (reduce $T$)
  – Smart-grid technologies
  – Preemptively shutting down steam plants
  – Deploying advance steam before the storm

Model parameters

• From Zobel (2014) and Johnson (2005)

<table>
<thead>
<tr>
<th></th>
<th>Most likely</th>
<th>Minimum</th>
<th>Maximum</th>
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</thead>
<tbody>
<tr>
<td>$\hat{X}$</td>
<td>0.073</td>
<td>0.030</td>
<td>0.22</td>
</tr>
<tr>
<td>$T$</td>
<td>13</td>
<td>3</td>
<td>26</td>
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</table>

• Assume triangular distribution


Model parameters

Effectiveness parameters for different allocation functions from


Optimal amount (in millions of dollars) to allocate to increase hardness from a $1 billion budget

<table>
<thead>
<tr>
<th>Certainty</th>
<th>Allocation function</th>
<th>Amt</th>
</tr>
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<tbody>
<tr>
<td>Linear</td>
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<tr>
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<tr>
<td>Quadratic</td>
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<tr>
<td>Logarith</td>
<td>648</td>
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<table>
<thead>
<tr>
<th>Uncertainty with independence</th>
<th>Allocation function</th>
<th>Amt</th>
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<tr>
<td>Linear</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>Expon</td>
<td>1000</td>
<td></td>
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<tr>
<td>Quadratic</td>
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<tr>
<td>Logarith</td>
<td>470</td>
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<table>
<thead>
<tr>
<th>Robust allocation</th>
<th>Allocation function</th>
<th>Amt</th>
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<tbody>
<tr>
<td>Linear</td>
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<td></td>
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<tr>
<td>Expon</td>
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<td></td>
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<tr>
<td>Quadratic</td>
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<tr>
<td>Logarith</td>
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<td></td>
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</table>
## Resilience results

### Resilience given optimal allocation from a $1 billion budget

<table>
<thead>
<tr>
<th>Allocation function</th>
<th>Resilience</th>
<th>Allocation function</th>
<th>Resilience</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.963</td>
<td>None</td>
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<tr>
<td>Linear</td>
<td>0.986</td>
<td>Linear</td>
<td>0.965</td>
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<tr>
<td>Expon</td>
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<td>Quadratic</td>
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<td>Quadratic</td>
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<tr>
<td>Logarith</td>
<td>0.989</td>
<td>Logarith</td>
<td>0.969</td>
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</table>

<table>
<thead>
<tr>
<th>Uncertainty with independence</th>
<th>Resilience</th>
<th>Robust allocation</th>
<th>Resilience</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
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<td>None</td>
<td>0.784</td>
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<tr>
<td>Linear</td>
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<td>Linear</td>
<td>0.788</td>
</tr>
<tr>
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</tr>
<tr>
<td>Quadratic</td>
<td>0.985</td>
<td>Quadratic</td>
<td>0.786</td>
</tr>
<tr>
<td>Logarith</td>
<td>0.987</td>
<td>Logarith</td>
<td>0.832</td>
</tr>
</tbody>
</table>

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SafeLife-X
Resilience in terms of customers and duration

<table>
<thead>
<tr>
<th>Allocation function</th>
<th>Allocation to increase hardness ($ million)</th>
<th>Allocation to improve recovery ($ million)</th>
<th>Resilience</th>
<th>Average number of customers without power</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No efforts</td>
<td></td>
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<td>0.963</td>
<td>232,000</td>
<td>13</td>
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<tr>
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<td>1000</td>
<td>0.986</td>
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<td>5.1</td>
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<td>1.000</td>
<td>36</td>
<td>13</td>
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<tr>
<td>Quadratic</td>
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<td>238</td>
<td>0.986</td>
<td>114,000</td>
<td>10.4</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>648</td>
<td>352</td>
<td>0.989</td>
<td>101,000</td>
<td>8.7</td>
</tr>
</tbody>
</table>
Best fit for allocation functions

Allocation functions to reduce average impacts

Allocation functions to reduce recovery time

- Data
- Linear
- Exponential
- Quadratic
- Logarithmic
What if logarithmic is wrong?

<table>
<thead>
<tr>
<th>Allocation function</th>
<th>Resilience</th>
<th>Average customers without power</th>
<th>Duration (days)</th>
<th>Optimal resilience</th>
<th>Average customers without power</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logarithm</td>
<td>0.989</td>
<td>101,000</td>
<td>8.7</td>
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<tr>
<td>Linear</td>
<td>0.980</td>
<td>159,000</td>
<td>10.2</td>
<td>0.986</td>
<td>232,000</td>
<td>5.1</td>
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<td>Expon</td>
<td>1.000</td>
<td>36</td>
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<td>1.000</td>
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Recommendations for ConEdison

- Logarithmic allocation function seems most appropriate
  - Approximates data well
  - Allocation performs well even if another function is correct
- Allocate between 50 and 65% of budget to reduce number of customers who lose power and 35 to 50% to improve recovery
Conclusions

• Assumptions impact optimal allocation
  – Linear or exponential allocation function with certainty \( \rightarrow \) allocate entire budget to reduce one factor
  – Quadratic or logarithmic \( \rightarrow \) may allocate to reduce both factors

• Heuristics
  – Focus resources on small initial value and large effectiveness
  – Uncertainty: divide resources approximately equal manner if marginal benefits decrease rapidly

• Future work
  – Apply allocation model to specific projects
  – Resources can improve both factors simultaneously

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