Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated Benders decomposition

by

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DEDICATION

I would like to dedicate my thesis to my beloved parents and also my dearest wife, Elnaz, for their endless love, support and encouragement.

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ABSTRACT

Abstract- Environmental, social and economic concerns motivate the operation of closed-loop supply chain networks (CLSCN) in many industries. We propose a novel profit maximization model for CLSCN design as a mixed-integer linear program in which there is flexibility in covering the proportions of demand satisfied and returns collected based on the firm's policies.

Our major contribution is to develop a novel hybrid robust-stochastic programming (HRSP) approach to simultaneously model two different types of uncertainties by including stochastic scenarios for transportation costs and polyhedral uncertainty sets for demands and returns. Transportation cost scenarios are generated using a Latin Hypercube Sampling method and scenario reduction is applied to consolidate them. An accelerated stochastic Benders decomposition algorithm is proposed for solving this model. To speed up the convergence of this algorithm, valid inequalities are introduced to improve the quality of lower bound, and also a Pareto-optimal cut generation scheme is used to strengthen the Benders optimality cuts. Numerical studies are performed to verify our mathematical formulation and also demonstrate the benefits of the HRSP approach. The performance improvements achieved by the valid inequalities and Pareto-optimal cuts are demonstrated in randomly generated instances.

CHAPTER I OVERVIEW

1.1 Introduction

The growing need for remanufacturing and recycling due to resource scarcity and environmental concerns requires firms to coordinate the forward and reverse material flows in their supply chains. This motivates the design of a closed-loop supply chain network (CLSCN) to avoid sub-optimality arising from separate design of forward and reverse networks. As pointed out by Klibi et al. (2010), the design of a supply chain network is a crucial strategic decision, the effects of which will persist for many years while the business environment may change. Thus, some important parameters such as demand and costs are significantly uncertain. In addition, since opening or closing a facility is time-consuming and costly, making any change in these decisions in response to parameter oscillations is impossible within a short time frame (Pishvaee et al., 2011). Uncertainties are intensified in the reverse supply chain network where the quality and quantity of returned products vary unpredictably and fast. Therefore, the design of CLSCN should be robust to the inherent uncertainty in the network parameters.

Of the few recent relevant papers that consider uncertainty in the CLSCN design problem, most estimate the probability distributions for the parameters and then apply scenario-based stochastic programming (SP) (e.g. Salema et al., 2007; Santoso et al., 2005). SP is a powerful modeling tool when an accurate probabilistic description of the random variables is known. However, it has three main drawbacks (Bertsimas and Thiele, 2006; Gülpınar et al., 2013). First, in many real-life applications not enough historical data are available to estimate distributions for uncertain parameters. For instance, predicting demand of a new product is challenging. Secondly, an accurate distribution approximation may require a large number of scenarios. But the more scenarios used for representing uncertainty, the harder it is to solve the problem to optimality. Conversely, if the number of scenarios is limited for computational reasons, the range of future states under which decisions are made and assessed is restricted. As a consequence, the obtained solution may be infeasible for some realizations of uncertain parameters. Even if this occurs with very small probability, it could result in high cost due to the large scale of the CLSCN. Finally, SP models in which the expected total cost is minimized assume that the decision maker worries about the average performance of the system. However, there are situations where the decision maker is concerned with the worst-case. We highlight this concern with respect to uncertain demand and return quantities.

To avoid these drawbacks, robust optimization (RO) has emerged as an alternative methodology to cope with uncertainty. RO handles uncertainty in the input data by solving the so-called robust counterpart over properly predefined uncertainty sets. The robust counterpart is a deterministic worst-case formulation of the original problem in which the worst-case is calculated over all possible values the input parameters may take within their uncertainty sets. As mentioned by Alem and Morabito (2012), two main advantages of RO compared with SP are: first, independently of the number of uncertain parameters, the robust counterpart can remain computationally tractable. For instance, with polyhedral uncertainty sets, the robust counterpart of a linear program is also a linear program. Second, rough historical data and decision makers' experiences can be used to derive the boundaries of uncertainty sets, without the need for precise estimates of probability distributions.

The uncertain parameters we consider in our CLSCN design problem differ qualitatively different. Historical data for transportation costs can be used to formulate probabilistic scenarios for them, but no such data for demand and return quantities of a new product exist. Because the purpose of the network is to supply products and collect the returns, we design it for the extreme quantities to ensure that its capacity and configuration will suffice in any event. The need to consider both types of uncertainty and an integrated network has been emphasized recently by Melo et al. (2009), Klibi and Martel (2012) and Gabrel et al. (2014).

This paper contributes to the CLSCN design literature by developing a novel hybrid robust-stochastic optimization approach and also devising an efficient solution procedure. Specifically, a mathematical model is developed for a multi-period, single-product and capacitated CLSCN. The strategic decisions including locations and capacities of facilities as well as the tactical decisions including inventory levels, production amounts, and shipments among the network entities are determined to maximize the expected worst-case profit. The major contributions can be summarized as follows:

- A novel CLSCN design model as a mixed-integer linear program (MILP) to integrate both strategic and tactical decisions with flexibility to cover varying proportions of demands and customer returns based on the firm’s policies.
- A novel hybrid robust-stochastic programming (HRSP) approach to simultaneously model two different types of uncertainties including stochastic scenarios for transportation costs and polyhedral uncertainty sets for demand and return quantities.
- A scenario generation approach using Latin Hypercube Sampling (LHS) followed by scenario reduction to obtain a small but representative set of transportation cost scenarios.
- An accelerated stochastic Benders decomposition (BD) algorithm with two sets of valid inequalities (VI) to strengthen the master problem and improve the quality of the lower

bound. Pareto-optimal cuts are also used to accelerate the convergence of the solution algorithm.

The remainder of this paper is organized as follows. In the next section, we briefly review the literature on the CLSCN design problem and the relevant solution methods. The problem and its stochastic formulation are defined in Section 3. Then, the HRSP approach is presented in Section 4. In Section 5, the scenario generation and reduction algorithm for transportation costs is presented. The stochastic BD algorithm with some acceleration techniques for improving its convergence is provided in Section 6. Section 7 describes computational experiments and sensitivity analyses that allow us to derive managerial insights about this CLSCN. Finally, Section 8 concludes this paper and offers some suggestions for future research.

1.2 Literature Review

The relevant literature follows two separate but complementary streams. We first review studies of the CLSCN design problem and then discuss solution algorithms.

1.2.1 Closed-loop supply chain network design problem

To avoid the sub-optimality from modeling and designing forward and reverse networks separately, many researchers have integrated them in the more complex CLSCN (Melo et al., 2009). Many CLSCN models are inspired by facility location theory. In this regard, Melo et al. (2009) and Klibi et al. (2010) presented comprehensive reviews on the facility location models in supply chain planning and on supply chain network design under uncertainty, respectively, to point out some missing aspects. Moreover, Pokharel and Mutha (2009) summarized the current developments of reverse supply chains, while Brandenburg et al. (2014) and Dekker et al. (2012) reviewed quantitative models that address environmental and social aspects in the supply chain.

Originally, Fleischmann et al. (2001) considered the integration of forward and reverse flows as a CLSCN using some case studies. They found that this integrated approach could provide a potential for a significant cost savings compared to a segregated approach. The research that followed was primarily carried out with simple facility location models (e.g. Aras et al., 2008). Then, more complex models were proposed especially by considering the real-life characteristics (e.g. Cruz-Rivera and Ertel, 2009). Recently, the field has experienced a strong development over the last decade (e.g. Klibi and Martel, 2012; Alumur et al., 2012; Cardoso et al., 2013; Baghalian et al., 2013; Keyvanshokooh et al., 2013; Gao and Ryan, 2014; De Giovanni and Zaccour, 2014; Niknejad and Petrovic, 2014).

Given that all activities in both forward and reverse supply chains are subject to considerable uncertainty, many works addressed the CLSCN design problem where some network parameters such as demand, return and costs are uncertain. In a pioneering step, Salema et al. (2007) extended the model of Fleischmann et al. (2001) to a multi-product and capacitated CLSCN considering uncertainty in demand and return. To summarize the literature, in Appendix A we have devised a coding system in Table A.1 and classified the most cited and recent papers based on problem features, supply chain stages, objective, modeling, uncertainty programming, uncertain parameters, decisions, and solution methods in Table A.2. SP is the most popular tool applied to the configuration of a CLSCN under uncertainty. However, a limited number of studies employed RO (Pishvaee et al., 2011; Vahdani et al., 2012; Hasani et al., 2011). These applied a worst-case robust formulation (Soyster, 1973) which may result in an overly conservative solution. Considering this research gap, we apply a more recent RO approach (Bertsimas and Sim, 2004), which allows a tradeoff between optimality and robustness. To our knowledge, no existing research combines probabilistic scenarios for some parameters with uncertainty sets for others in the area of CLSCN design problem. But, Fanzeres dos Santos et al. (2014) used another hybrid approach in a different context, power system markets.

From Table A.2, it is also clear that minimizing cost has been the primary objective in most CLSCN models. These models typically require that every customer's demand and return has to be satisfied. However, it may not always be optimal to satisfy all demands and returns. Sometimes, there is not much competition in target market, so the cost of losing customers will be very low. Hence, the firm may maximize its profit by losing some customers. On the other hand, sometimes profit is increased with better customer service. This paper includes flexibility to determine what percentage of customers to serve.

1.2.2 Solution algorithms

Because the CLSCN design problem is an NP-hard combinatorial optimization problem, a plethora of solution algorithms including metaheuristic, heuristic, and exact methods have been developed, as shown in the last column of Table A.2. Most solution methods employ standard commercial packages such as CPLEX to solve mixed-integer programming formulations. However, when the number of discrete variables is large, the resulting models can be solved only by using metaheuristic or heuristic methods to obtain a near optimal solution. But, because the CLSCN design involves large investment and greatly influences the operational and tactical costs as well as efficiency of service, developing efficient exact algorithms for solving larger and more realistic cases is worthwhile (de Sá et al., 2013). Among these exact solution approaches, the branch-and-bound algorithm has been a popular methodology combined with other heuristics or Lagrangian relaxation methods to obtain better bounds. As shown in Table A.2, there are few papers that develop an exact solution scheme, a shortage highlighted by Klibi and Martel (2012).

Furthermore, as a discrete facility location problem, CLSCN is an attractive candidate for decomposition. It involves both binary variables related to the strategic configuration, and continuous variables associated with tactical and operational decisions. Table A.2 includes just four papers in which decomposition schemes were applied. Among the decomposition techniques, the BD method (Benders, 1962) is a classical exact algorithm suitable for solving large-scale MILP problems having special structure in the constraint set; i.e., upon fixing the values of the complicating integer variables, the MILP problem reduces to an easy linear program.

However, the classical BD and also its stochastic version, called the L-shaped method, might not be efficient (Saharidis and Ierapetritou, 2010). The major issues resulting in its slow convergence are (1) solving the Relaxed Master problem (RMP) which is in fact an integer or sometime MILP program, and (2) the quality of Benders cuts. To overcome these concerns, different acceleration techniques have been proposed to speed up BD. To improve the quality of Benders cuts, Magnanti and Wong (1981) defined a cut as Pareto-optimal. By applying these cuts to a problem in which the sub-problem is degenerate, the results showed a significant improvement in convergence. Saharidis et al., (2010) introduced a covering cut bundle strategy by producing a bundle of cuts in each iteration to cover all decision variables of the MP. Some other modifications to this algorithm were presented by McDaniel and Devine (1977), Saharidis et al., (2013), Tang et al., (2013), Sherali and Lunday (2013) and Oliveira et al., (2014) in different applications.

CHAPTER 2 HYBRID ROBUST AND STOCHASTIC OPTIMIZATION APPROCH FOR CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN USING ACCELERATED BENDERS DECOMPOSITION

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2.1 Problem Definition

As illustrated by Fig. 1, we consider a single product, multi-period, and capacitated CLSCN consisting of manufacturing/remanufacturing, distribution, collection, and disposal centers as well as retailers under demand, return and transportation cost uncertainty. The end-of-use products are collected from retailers, transported to collection centers, and after a quality test, divided into two categories: recoverable products sent to manufacturing/remanufacturing centers and scrapped products shipped to disposal centers. In the forward network, the remanufactured products along with the new ones are supplied to retailers from manufacturing/remanufacturing centers through distribution centers to meet their demand. We also assume a periodic review inventory policy for distribution centers to find inventory levels and include base-stock levels as decision variables (Keyvanshokooh et al., 2013). Moreover, it is assumed that the product is perishable and hence the excess amount of product in the retailers in one period cannot be used to satisfy the demand of next period.

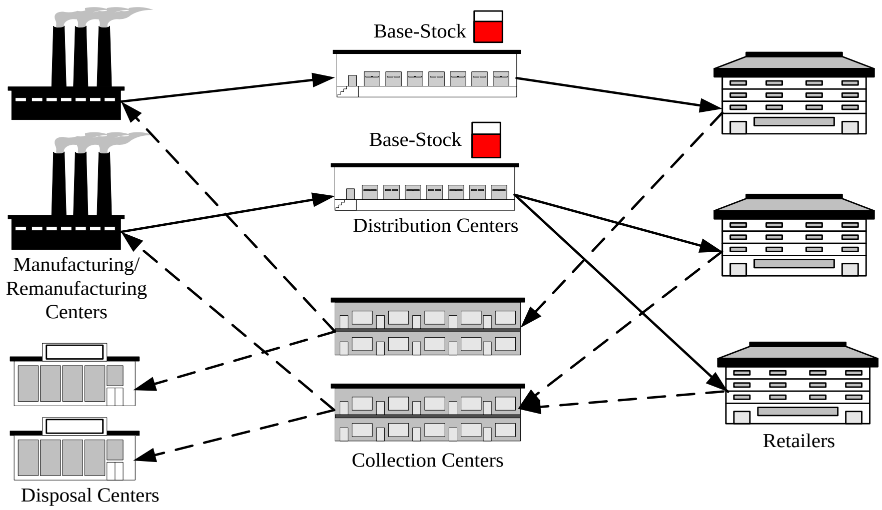


Figure 1. *The CLSCN structure*

This CLSCN model can apply to companies that are introducing new products to their target market consisting of their previous customers. For example, an exclusive company produces desktop and notebook computers. To improve customer satisfaction, it decides to also provide after-sales service and to this aim they want to produce some components such as modem and hard disk. On one hand, due to being an exclusive firm, these spare parts would appeal only to their customers who bought the desktop or notebook computers from this company before, so there is not much competition in the target market. Thus, the risk of losing customers will be very low. Then, if a small penalty is considered for not satisfying demand and return, profit may be maximized by covering just a portion of demand and return. On the other hand, if the company wants to emphasize satisfaction of customers, a high penalty cost should be considered for not satisfying demand and return. Our hybrid robust-stochastic formulation allows any condition between these two extremes. Most CLSCND models in the literature are trying to satisfy the whole demand and return quantities, or they just want to maximize their profit or minimize their costs without any attention to how much the demand and return they satisfy

(Keyvanshokooh et al., 2013; Amin and Zhang, 2013). However, one of our concerns in this research is to design this network considering different conditions of the target market to satisfy the demand and return of customers.

In the developed hybrid robust-stochastic programming approach, in the forward network, if the flow from distribution centers to retailers exceeds the retailers' demand, then a surplus cost is considered per unit of excess amounts of flow to retailers over their demands. On the other hand, if the retailers' demand is greater than the quantity delivered from distribution centers, then a penalty cost for unsatisfied demand is incurred. In the reverse network, if the flow from retailers to collection centers is greater than the potential returns, which is impossible in practice, then we apply a penalty cost per unit of excess flow from the retailers over returns collected. However, if the reverse flow from retailers to collection centers is less than the potential returns, then we impose a scrap cost per unit of uncollected returns. Defining the penalty and surplus costs in the forward network and the penalty and scrap costs in the reverse network balances the forward flows with the demands and the reverse flows with the return quantities as much as possible while ensuring complete recourse (Birge and Louveaux, 1997) in the stochastic program.

The main concern of this paper is to design the CLSCN in the presence of uncertainty. Two different types of uncertainty are present; one for transportation costs and the other for demand and return. During the last decade, the oscillations in fuel price have dramatically influenced transportation costs and it is quite likely that this uncertainty on fuel price will be sustained (Pishvaee et al., 2009). We assume the firm has historical data for transportation cost distributions from its previous sales and so model this uncertainty with probabilistic scenarios.

On the other hand, forecasting the precise distribution of future demands and returns of a new product is very difficult. Demand could be affected by unexpected events such as the appearance of a new competitor and return quantities depend on customer use patterns. Stationarity of probability distributions cannot be guaranteed especially in our multi-period planning horizon. Even if sufficient data are available to generate credible scenarios, considering many of them for both demands and returns will create computational challenges. Thus, we adopt a RO approach of formulating uncertainty sets instead of probability distributions for these quantities.

Considering these two types of uncertainty, our main contribution is to develop a novel hybrid robust-stochastic programming approach in which uncertain transportation costs are modeled using various scenarios while uncertain demand and return are modeled with polyhedral uncertainty sets. Because competition forces companies to try their best to completely satisfy the demands of their customers, even a small amount of unsatisfied demands or returns could result in unacceptable loss. Therefore, we consider the worst-case situation associated with not meeting demands or collecting returns and take the expectation of profit with respect to transportation costs. The CLSCN design problem is to concurrently determine the location of facilities, their capacities, and base-stock levels as the first-stage decisions in light of the recourse production amounts and network flows to meet the worst-case demand and return quantities in each transportation cost scenario.

2.2 Problem Formulation

We formulate a two-stage stochastic program with recourse for the CLSCN design problem assuming full knowledge of probability distributions for uncertain transportation costs. Regarding the uncertain demands and returns, we first consider their nominal values under each scenario for transportation cost. In the following subsection, we explain how uncertainty sets for these parameters are included in the HRSP approach. In the first-stage, strategic decisions such as facility locations, their capacity and base-stock levels are determined as the here-and-now decisions that should be made before realization of any uncertain parameters, and in the second-stage operational decisions such as network flows are made after realization of uncertain parameters. The following notations are used for in the formulation of the CLSCN:

Sets:

I Set of potential locations of manufacturing/remanufacturing centers,

J Set of potential locations available for distribution and collection centers,

K Set of fixed locations of retailers,

R Set of fixed locations of disposal centers,

S Set of scenarios for transportation costs,

T Set of time periods in the planning horizon, (t, p)

Parameters:

Demand and Return:

$\hat{D}_{ks}^{t}$ Nominal demand of retailer *k* at time period *t* in scenario *s*

$\hat{R}_{ks}^{t}$ Nominal return of used product from retailer *k* at time period *t* in scenario *s*

Fixed costs:

F_i^{MC} Fixed cost for opening manufacturing/remanufacturing center *i*

F_j^{DC} Fixed cost for opening distribution center *j*

F_j^{CC} Fixed cost for opening collection center *j*

Variable costs:

PR_k Revenue per unit of product sold by retailer *k* to customers

MC_i Unit manufacturing cost in manufacturing/remanufacturing center *i*

RC_i Unit remanufacturing cost in manufacturing/remanufacturing center *i*

IC_j^t Inventory cost per unit of product in time period *t* in distribution center *j*

CC_j Unit collection/inspection cost in collection center *j*

DC_r Unit disposal cost in disposal center *r*

PC^D Penalty cost per unit of non-satisfied demands of retailer

SC^R Scrap cost per unit of uncollected returns of retailer

SC^D Surplus cost per unit of excess amounts of flow over demands received by retailers

PC^R Penalty cost per unit of excess amounts of flow over returns collected from retailers

Transportation costs:

CIJ_{ijs}^t Transportation cost per unit of product transported from manufacturing/remanufacturing center *i* to distribution center *j* in time period *t* in scenario *s*

CJK_{jks}^t Transportation cost per unit of product transported from distribution center *j* to retailer *k* in time period *t* in scenario *s*

CKJ_{kjs}^t Transportation cost per unit of returned product transported from retailer *k* to collection center *j* in time period *t* in scenario *s*

CJI_{jis}^t Transportation cost per unit of recoverable product transported from collection center *j* to manufacturing/remanufacturing center *i* in time period *t* in scenario *s*

CJR_{jrs}^t Transportation cost per unit of scrapped product transported from collection center *j* to disposal center r in time period t in scenario s

Capacity costs:

c_i^{MC} Capacity cost of manufacturing/remanufacturing center i per unit of products per period

c_j^{DC} Capacity cost of distribution center j per unit of products per period

c_j^{CC} Capacity cost of collection center j per unit of products per period

Maximum available capacity of facilities:

CAP_i^{MC} Maximum available capacity of manufacturing/remanufacturing center i (units of products per period)

CAP_j^{DC} Maximum available capacity of distribution center j (units of products per period)

CAP_j^{CC} Maximum available capacity of collection center j (units of products per period)

Coefficients and ratios:

a Fraction of returned products that can be remanufactured

Pr_S Probability of transportation cost scenario s

Decision variables:

Binary variables (regarding opening network facilities):

X_i^{MC} Binary variable equal to 1 if a manufacturing/remanufacturing center is opened at location i, 0 otherwise

X_j^{DC} Binary variable equal to 1 if a distribution center is opened at location j, 0 otherwise

X_j^{CC} Binary variable equal to 1 if a collection center is opened at location j, 0 otherwise

Continuous variables (regarding determining the capacity for each facility):

W_i^{MC} Capacity of manufacturing/remanufacturing center *i* (units of products per period)

W_j^{DC} Capacity of distribution center *j* (units of products per period)

W_j^{CC} Capacity of collection center *j* (units of products per period)

Continuous variables (regarding the flows of network):

FIJ_{ijs}^t Quantity of products transported from manufacturing/remanufacturing center *i* to distribution center *j* in time period *t* in scenario *s*

FJK_{jks}^t Quantity of products transported from distribution center *j* to retailer *k* in time period *t* in scenario *s*

FKJ_{kjs}^t Quantity of returned products transported from retailer *k* to collection center *j* in time period *t* in scenario *s*

FIJ_{ijs}^t Quantity of recoverable products transported from collection center *j* to manufacturing/remanufacturing center *i* in time period *t* in scenario *s*

FJR_{jrs}^t Quantity of scrapped products transported from collection center *j* to disposal center *r* in time period *t* in scenario *s*

Other continuous variables:

PI_{is}^t Quantity of products produced by manufacturing/remanufacturing center *i* in time period *t* in scenario *s*

BS_j Base-stock level of product of distribution center *j* at the beginning of each period

Based on the above-mentioned notations, the two-stage stochastic CLSCN design problem can be formulated as follows:

$$Max\, Z = Q(X, W, BS) - \sum_{i \in I} F_i^{MC} X_i^{MC} - \sum_{i \in I} C_i^{MC} W_i^{MC} - \sum_{j \in J} F_j^{DC} X_j^{DC} - \sum_{j \in J} C_j^{DC} W_j^{DC} - \sum_{j \in J} F_j^{CC} X_j^{CC} - \sum_{j \in J} C_j^{CC} W_j^{CC} \tag{1}$$

S.t.

$$W_i^{MC} \le CAP_i^{MC} X_i^{MC} \qquad \forall j \in J \tag{2}$$

$$W_j^{DC} \le CAP_j^{DC} X_j^{DC} \qquad \forall j \in J \tag{3}$$

$$W_j^{CC} \le CAP_j^{CC} X_j^{CC} \qquad \forall j \in J \tag{4}$$

$$W_j^{DC} \ge BS_j \qquad \forall j \in J \tag{5}$$

$$X_j^{DC} + X_j^{CC} \le 1 \qquad \forall j \in J \tag{6}$$

$$X_i^{MC}, X_j^{DC}, X_j^{CC} \in \{0,1\}, W_i^{MC}, W_j^{DC}, W_j^{CC}, BS_j \ge 0 \qquad \forall i \in I, j \in J \tag{7}$$

where $Q(X,W,BS) = E_{\zeta}\left[\vartheta\left(X,W,BS,\zeta_s\right)\right] = \sum_s Pr_s \times \vartheta\left(X,W,BS,\zeta_s\right)$. The term $E_{\zeta}\left(\vartheta\left(X,W,BS,\zeta_s\right)\right)$ denotes the recourse function. For a given scenario ζ_s, $\vartheta\left(X,W,BS,\zeta_s\right)$ is the optimal objective function value of the second-stage problem (8)-(24):

$$\begin{aligned}\vartheta\left(X,W,BS,\zeta_s\right) = Max\ & \sum_{j\in J}\sum_{k\in K}\sum_{t\in T} PR_k FJK_{jks}^t - \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} CIJ_{ijs}^t FIJ_{ijs}^t - \sum_{j\in J}\sum_{k\in K}\sum_{t\in T} CJK_{jks}^t FJK_{jks}^t - \sum_{j\in J}\sum_{k\in K}\sum_{t\in T} CKJ_{kjs}^t FKJ_{kjs}^t \\ & - \sum_{j\in J}\sum_{i\in I}\sum_{t\in T} CJI_{jis}^t FJI_{jis}^t - \sum_{j\in J}\sum_{r\in R}\sum_{t\in T} CJR_{jrs}^t FJR_{jrs}^t - \sum_{i\in I}\sum_{t\in T} MC_i PI_{is}^t - \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} RC_i FJI_{jis}^t - \sum_{k\in K}\sum_{j\in J}\sum_{t\in T} CC_j FKJ_{kjs}^t \\ & - \sum_{j\in J}\sum_{t\in T}\left(\sum_{i\in I}\sum_{p\le t} FIJ_{ijs}^p - \sum_{k\in K}\sum_{p\le t} FJK_{jks}^p\right) IC_j^t - \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} DC_r FJR_{jrs}^t\end{aligned} \tag{8}$$

S.t. (9)

$$\sum_{j\in J} FJK_{jks}^t = \hat{D}_{ks}^t \qquad \forall k \in K, t \in T$$

$$\sum_{j\in J} FKJ_{kjs}^t = \hat{R}_{ks}^t \qquad \forall k \in K, t \in T \tag{10}$$

$$\sum_{k\in K}\sum_{p\le t} FJK_{jks}^p - \sum_{i\in I}\sum_{p\le t} FIJ_{ijs}^p \le 0 \qquad \forall j \in J, t \in T \tag{11}$$

$$\sum_{i\in I}\sum_{p\le t} FIJ_{ijs}^{p} - \sum_{k\in K}\sum_{p<t} FJK_{jks}^{p} = BS_j \qquad \forall j\in J,\, t\in T \tag{12}$$

$$PI_{is}^{t} + \sum_{j\in J} FJI_{jis}^{t} = \sum_{j\in J} FIJ_{ijs}^{t} \qquad \forall i\in I,\, t\in T \tag{13}$$

$$a\sum_{k\in K} FKJ_{kjs}^{t} = \sum_{i\in I} FJI_{jis}^{t} \qquad \forall j\in J,\, t\in T \tag{14}$$

$$(1-a)\sum_{k\in K} FKJ_{kjs}^{t} = \sum_{r\in R} FJR_{jrs}^{t} \qquad \forall j\in J,\, t\in T \tag{15}$$

$$PI_{is}^{t} + \sum_{j\in J} FJI_{jis}^{t} \le W_i^{MC} \qquad \forall i\in I,\, t\in T \tag{16}$$

$$\sum_{i\in I} FIJ_{ijs}^{t} \le W_j^{DC} \qquad \forall j\in J,\, t\in T \tag{17}$$

$$\sum_{k\in K} FKJ_{kjs}^{t} \le W_j^{CC} \qquad \forall j\in J,\, t\in T \tag{18}$$

$$FIJ_{ijs}^{t} \le M.X_j^{DC} \qquad \forall i\in I,\ j\in J,\, t\in T \tag{19}$$

$$FJK_{jks}^{t} \le M.X_j^{DC} \qquad \forall k\in K,\ j\in J,\, t\in T \tag{20}$$

$$FKJ_{kjs}^{t} \le M.X_j^{CC} \qquad \forall k\in K,\ j\in J,\, t\in T \tag{21}$$

$$FJI_{jis}^{t} \le M.X_j^{CC} \qquad \forall i\in I,\ j\in J,\, t\in T \tag{22}$$

$$FJR_{jrs}^{t} \le M.X_j^{CC} \qquad \forall r\in R,\ j\in J,\, t\in T \tag{23}$$

$$FIJ_{ijs}^{t}, FJK_{jks}^{t}, FKJ_{kjs}^{t}, FJI_{jis}^{t}, FJR_{jrs}^{t}, PI_{is}^{t} \ge 0 \qquad \forall i\in I,\ j\in J,\, r\in R,\, k\in K,\, t\in T \tag{24}$$

The objective (1) is to maximize the expected total second-stage profit less the first-stage costs including fixed costs of opening facilities and capacity costs. The second-stage profit (8) includes the revenue from selling new products less transportation costs, inventory costs, manufacturing costs of new products and remanufacturing costs of used products, collection costs of used products, and disposal costs of scrapped product. Constraints (2)-(4) ensure capacity restrictions for manufacturing/remanufacturing, distribution and collection centers, respectively. Constraints (5) guarantee that the capacity of each distribution center is at least equal to its base-stock level. Constraints (6) ensure that at each location *j* just one of distribution or collection centers is opened. Constraints (9)-(10) assure that the demand of retailers are satisfied by the distribution centers and also the returns of used products from retailers are collected by the collection centers, respectively. Note that here we put the nominal values of demands and returns of retailers. In the next section, we will explain how we deal with the robust uncertainty in these parameters. Constraint (11) assures the flow balance for each distribution centers. Constraints (12) enforce base-stock levels for each distribution center in scenario *s* and period *t*. Constraints (13)-(15) ensure the flow balance for manufacturing/remanufacturing and collection centers. Constraints (16)-(18) express the capacity constraints for the manufacturing/remanufacturing, distribution and collection centers, respectively. Constraints (19)-(23) connect the binary variables for facility existence with the corresponding flows, where *M* is a large number. Finally, constraints (7) and (24) enforce the binary and non-negativity restrictions on decision variables.

2.3 Problem Formulation

First, we briefly review a RO approach presented by Bertsimas and Sim, 2003, 2004 as a prelude to describing our HRSP formulation. Consider the linear program (LP) where *C* is an *n*-vector, *A* is a $m \times n$ matrix, and *b* is an *m*-vector:

$$Min\ Cx\ s.t.\ Ax \le b,\ x \ge 0 \tag{25}$$

Assume uncertainty only affects the elements of matrix *A*. That is, consider a particular row *i* of *A* and let J_i represent the set of coefficients in row *i* of *A* subject to uncertainty. Each data element $\tilde{a}_{ij}, j \in J_i$ is modeled as a bounded and independent random variable taking value in an interval $\left[\hat{a}_{ij} - a_{ij}, \hat{a}_{ij} + a_{ij}\right]$ where $\hat{a}_{ij}$ is the nominal value and a_{ij} is the maximum deviation from this nominal value. With this assumption, LP (25) is reformulated as:

$$Min\ Cx\ s.t.\ \max_{\forall \tilde{a}_{ij} \in J_i} \left(\sum_j \tilde{a}_{ij} x_j \right) \le b_i \quad \forall i,\ x \ge 0 \tag{26}$$

Then, we define a scaled deviation $z_{ij} = \left(\tilde{a}_{ij} - \hat{a}_{ij}\right) / a_{ij}$ from its nominal value of $\hat{a}_{ij}$ that always belongs to the interval $\left[-1,1\right]$. Note that $\tilde{a}_{ij}$, $\hat{a}_{ij}$ and a_{ij} denote the uncertain value, its nominal value and its maximum deviation from the nominal value, respectively. It is unlikely that all of the uncertain input $\tilde{a}_{ij},\ j \in J_i$ will realize their worst-case values simultaneously. Thus, a maximum number of parameters that can deviate from their nominal values for each constraint *i* is considered as Γ_i, called the budget of uncertainty, where $\Gamma_i \in \left[0, \left|J_i\right|\right]$. The aggregated scaled deviation of uncertain parameters for constraint *i* is bounded as $\sum_{j \in J_i} \left|z_{ij}\right| \le \Gamma_i, \forall i$.

The budget of uncertainty plays a crucial role in adjusting the solution's level of conservatism of obtained solution against the robustness. If $\Gamma_i = 0$, it reduces to the nominal formulation where there is no protection against uncertainty . If $\Gamma_i = \left|J_i\right|$, the *i*th constraint is completely protected against the worst-case realization of uncertain parameters. Finally, if $\Gamma_i \in \left(0, \left|J_i\right|\right)$, the decision maker considers a trade-off between conservatism and cost of the solution against the level of protection against constraint violation. Based on this definition, the set J_i is defined as $J_i = \left\{ \tilde{a}_{ij} \middle| \tilde{a}_{ij} = \hat{a}_{ij} + a_{ij} z_{ij}, \forall i, j, z \in \Omega \right\}$ where $\Omega = \left\{ z \middle| \sum_{j=1}^{n} z_{ij} \le \Gamma_i, \left| z_{ij} \right| \le 1, \forall i \right\}$. Restating each constraint i as $\sum_j \tilde{a}_{ij} x_j = \sum_j \left(\hat{a}_{ij} + a_{ij} z_{ij} \right) x_j = \sum_j \hat{a}_{ij} x_j + \sum_j a_{ij} z_{ij} x_j$, LP (26) can be reformulated as:

$$Min\ Cx\ \ s.t.\ \sum_j \hat{a}_{ij} x_j + \underset{z_i \in \Omega_i}{Max} \sum_j a_{ij} z_{ij} x_j \le b_i\ \ \forall i,\ x \ge 0 \tag{27}$$

The lower level problem $\underset{z_i \in \Omega_i}{Max} \sum_j a_{ij} z_{ij} x_j$ for a given vector x^* is equivalent to LP (28):

$$Max \sum_j a_{ij} z_{ij} x_j^*\ \ s.t.\ \sum_j z_{ij} \le \Gamma_i\ \ \forall i,\ 0 \le z_{ij} \le 1\ \forall j \in J_i \tag{28}$$

Then by introducing the dual variables λ_i and μ_{ij}, the dual of LP (28) is:

$$Min\ \Gamma_i \lambda_i + \sum_{j \in J_i} \mu_{ij}\ \ s.t.\ \lambda_i + \mu_{ij} \ge a_{ij} x_j^*\ \ \forall i, j \in J_i, \mu_{ij} \ge 0\ \forall j \in J_i, \lambda_i \ge 0\ \forall i \tag{29}$$

The dual (29) is applied to LP (27) to obtain the robust counterpart of LP (25):

$$Min\ Cx\ \ s.t.\ \hat{a}_i x - \Gamma_i \lambda_i - \sum_{j \in J_i} \mu_{ij} \le b_i\ \forall i, \lambda_i + \mu_{ij} \ge a_{ij} x_j\ \forall i, j \in J_i,\ \mu_{ij}, \lambda_i \ge 0\ \forall i, j \in J_i \tag{30}$$

This RO approach provides an efficient way to determine bounds on the probability of violation of each constraint. Let x_j^* be the robust solution, then the violation probability of the ith constraint is calculated by:

$$Pr\left(\sum_j \tilde{a}_{ij} x_j^* < b_i \right) \le 1 - \Phi\left(\left(\Gamma_i - 1 \right) \Big/ \sqrt{\left| J_i \right|} \right) \tag{31}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. This upper bound provides a way of assigning a proper budget of uncertainty to each constraint when our uncertain parameter is independent and symmetrically distributed random variable in its associated uncertainty set.

In our HRSP approach for CLSCN, within each transportation cost scenario we define polyhedral uncertainty sets for demand and return in each period and for each retailer to apply the RO approach of Bertsimas and Sim, 2003, 2004. Fig. 2 illustrates the arrangement of the polyhedral uncertainty sets for demands of retailers in different periods for different transportation cost scenarios. The uncertain demands and returns are allowed to deviate from a nominal scenario toward a worst-case realization within a constrained polyhedral uncertainty set. For simplicity, Fig. 2 shows uncertainty sets for demand only.

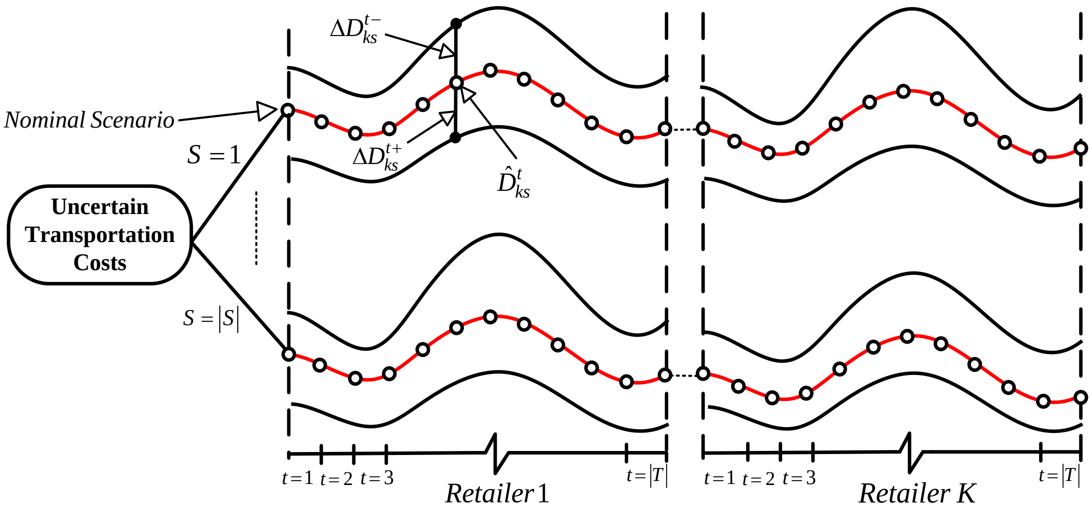


Figure 2. *Uncertainty characterization of the HRSP approach*

To develop the uncertainty sets, first we define the positive and negative deviation percentages from the nominal scenario for demands and returns, respectively, as follows:

$$\eta D_{ks}^{t^+} = \frac{D_{ks}^t - \hat{D}_{ks}^t}{\Delta D_{ks}^{t^+}} \quad \text{if} \quad D_{ks}^t > \hat{D}_{ks}^t, \qquad \eta D_{ks}^{t^-} = \frac{\hat{D}_{ks}^t - D_{ks}^t}{\Delta D_{ks}^{t^-}} \quad \text{if} \quad D_{ks}^t < \hat{D}_{ks}^t \tag{32}$$

$$\eta R_{ks}^{t^+} = \frac{R_{ks}^t - \hat{R}_{ks}^t}{\Delta R_{ks}^{t^+}} \quad \text{if} \quad R_{ks}^t > \hat{R}_{ks}^t, \qquad \eta R_{ks}^{t^-} = \frac{\hat{R}_{ks}^t - R_{ks}^t}{\Delta R_{ks}^{t^-}} \quad \text{if} \quad R_{ks}^t < \hat{R}_{ks}^t \tag{33}$$

Then, the uncertainty sets of demand and return for each scenario of transportation costs are:

$$J_s^D = \left\{ D_{ks}^t \,\middle|\, D_{ks}^t = \hat{D}_{ks}^t + \Delta D_{ks}^{t^+} \times \eta D_{ks}^{t^+} - \Delta D_{ks}^{t^-} \times \eta D_{ks}^{t^-}, \forall k, t, \forall \eta D_{ks}^{t^+}, \eta D_{ks}^{t^-} \in K^D \right\} \tag{34}$$

where

$$K^D = \{\eta D_{ks}^{t^+}, \eta D_{ks}^{t^-} \,\middle|\, 0 \le \eta D_{ks}^{t^+} \le 1,\ 0 \le \eta D_{ks}^{t^-} \le 1, \sum_{k\in K}\sum_{t\in T}\left(\eta D_{ks}^{t^+} + \eta D_{ks}^{t^-}\right) \le \Gamma_s^D\}$$

And

$$J_s^R = \left\{ R_{ks}^t \,\middle|\, R_{ks}^t = \hat{R}_{ks}^t + \Delta R_{ks}^{t^+} \times \eta R_{ks}^{t^+} - \Delta R_{ks}^{t^-} \times \eta R_{ks}^{t^-}, \forall k, t, \forall \eta R_{ks}^{t^+}, \eta R_{ks}^{t^-} \in K^R \right\} \tag{35}$$

where

$$K^R = \{\eta R_{ks}^{t^+}, \eta R_{ks}^{t^-} \,\middle|\, 0 \le \eta R_{ks}^{t^+} \le 1,\ 0 \le \eta R_{ks}^{t^-} \le 1, \sum_{k\in K}\sum_{t\in T}\left(\eta R_{ks}^{t^+} + \eta R_{ks}^{t^-}\right) \le \Gamma_s^R\}$$

The dimension of these sets is $|K|\times|T|$ for each transportation cost scenario. Recall that $\hat{D}_{ks}^t$ denotes the nominal scenario for demand of retailer *k* in period *t* for scenario *s* of transportation cost while $\Delta D_{ks}^{t^+}$ and $\Delta D_{ks}^{t^-}$ are the maximum positive and negative deviations from the nominal value, respectively, while $\eta D_{ks}^{t^+}$ and $\eta D_{ks}^{t^-}$ state the percentage by which the worst-case scenario deviates from the nominal value. The parameter Γ_s^D is the budget of uncertainty for demand in scenario *s* via which we can constrain the number of periods in which the worst-case scenario deviates from the nominal scenario. Similar definitions apply to the polyhedral uncertainty sets for returns in (35).

In the stochastic formulation (1)-(24), we just have uncertain demand and return parameters in constraint (9)-(10), which are assumed to belong to some polyhedral uncertainty sets. If we want to consider these constraints into our problem, then they cannot be satisfied certainly because we do not know the exact amount of uncertain demand and return quantities before obtaining the optimal flows from distribution centers to retailers and also the optimal flows from retailers to collection centers. Furthermore, even if we want to put these constraints into our problem formulation and solve it, then we will come up with the two situations: (1) the optimal flows from distribution centers to retailers may be less than the demand quantities of retailers, or (2) the optimal flows from distribution centers to retailers may be more than the demand quantities of retailers. In both cases, we actually violate this constraint and our problem becomes infeasible. We have the same situation in the reverse supply chain network as well.

To deal with this drawback, we remove these constraints from the stochastic formulation (1)-(24) and adding them to the objective function with some cost terms. As the cost terms, we define the penalty cost per unit of non-satisfied demands pc^D and the surplus cost per unit of excess amounts of flow over demands received by retailers sc^D in forward supply chain, and also the scrap cost per unit of uncollected returns sc^R and the penalty cost per unit of excess amounts of flow over returns collected from retailers pc^R in reverse supply chain. By doing so, we actually provide the decision maker with the opportunity of having flexibility to control each side of this issue. For example, if the company is in a very competitive market setting in such a way that it does not want to lose any customer, then we can increase the penalty cost for the amount of unsatisfied demand. On the other hand, if the manufacturing and remanufacturing resources of company are restricted or the cost price of product is high, and also if there is not much competition in the target market, then it can decrease its excess level of production over demands with the help of increasing the surplus cost per unit of excess amounts of flow over demands received by retailers. In Section 7.1.2, we investigate the effects of this concern in the CLSCN design problem.

Our purpose is to minimize the maximum amounts of costs regarding violating these constraints (9)-(10), which are added to the objective function with some cost parameters as explained above. To apply the uncertainty sets (34)-(35) in the stochastic formulation (1)-(24), we isolate the term containing random demand and return parameters for scenario s as the following nonlinear term:

$$PC_s\left(FJK,FKJ\right)=\underset{D_{ks}^t\in J_s^D}{Max}\left[\sum_{k\in K}\sum_{t\in T}\left(D_{ks}^t-\sum_{j\in J}FJK_{jks}^t\right)\times PC^D,\sum_{k\in K}\sum_{t\in T}\left(\sum_{j\in J}FJK_{jks}^t-D_{ks}^t\right)\times SC^D\right]$$
$$+\underset{R_{ks}^t\in J_s^R}{Max}\left[\sum_{k\in K}\sum_{t\in T}\left(R_{ks}^t-\sum_{j\in J}FKJ_{kjs}^t\right)\times SC^R,\sum_{k\in K}\sum_{t\in T}\left(\sum_{j\in J}FKJ_{kjs}^t-R_{ks}^t\right)\times PC^R\right] \tag{36}$$

where D_{ks}^t and R_{ks}^t are the random demand and return which belong to sets (34) and (35), respectively. For each transportation cost scenario, this term represents the worst-case value for penalty, scrap and surplus costs. We reformulate this nonlinear optimization problem (36) as the following LP for each scenario s by defining auxiliary variables $Z1_s$ and $Z2_s$:

$$\underset{FJK,FKJ,Z1_s,Z2_s}{Min}\quad PC_s\left(FJK,FKJ\right)=Z1_s+Z2_s \tag{37}$$

S.t.

$$\sum_{k\in K}\sum_{t\in T}\left(D_{ks}^t-\sum_{j\in J}FJK_{jks}^t\right)\times PC^D\le Z1_s,\ \forall D_{ks}^t\in J_s^D \tag{38}$$

$$\sum_{k\in K}\sum_{t\in T}\left(\sum_{j\in J}FJK_{jks}^t-D_{ks}^t\right)\times SC^D\le Z1_s,\ \ \forall D_{ks}^t\in J_s^D \tag{39}$$

$$\sum_{k\in K}\sum_{t\in T}\left(R_{ks}^t-\sum_{j\in J}FKJ_{kjs}^t\right)\times SC^R\le Z2_s,\ \forall R_{ks}^t\in J_s^R \tag{40}$$

$$\sum_{k\in K}\sum_{t\in T}\left(\sum_{j\in J}FKJ_{kjs}^t-R_{ks}^t\right)\times PC^R\le Z2_s,\ \forall R_{ks}^t\in J_s^R \tag{41}$$

$$Z1_s, Z2_s \geq 0 \tag{42}$$

The constraints (38)-(42) should be satisfied for all realizations of the uncertain demands and returns in their polyhedral uncertainty sets. We find their robust counterparts, explained in detail for constraint (38). From the set definition (34), we can rewrite the constraints (38) as:

$$\underset{D_{ks}^t \in J_s^D}{\text{Max}} \left[\sum_{k \in K} \sum_{t \in T} \left(D_{ks}^t - \sum_{j \in J} FJK_{jks}^t \right) \times PC^D \right] \leq Z1_s \tag{43}$$

which can be transformed into:

$$\sum_{k \in K} \sum_{t \in T} \left(\hat{D}_{ks}^t - \sum_{j \in J} FJK_{jks}^t \right) \times PC^D + \underset{\eta D_{ks}^{t^+}, \eta D_{ks}^{t^-} \in K^D}{\text{Max}} \left[\sum_{k \in K} \sum_{t \in T} \left(\Delta D_{ks}^{t^+} \times \eta D_{ks}^{t^+} - \Delta D_{ks}^{t^-} \times \eta D_{ks}^{t^-} \right) \times PC^D \right] \leq Z1_s \tag{44}$$

In this constraint, we optimize over the positive and negative deviation percentages from nominal scenario for uncertain demand. We expand the maximization problem considering constraints from polyhedral uncertainty sets as follows:

$$- \underset{\eta D_{ks}^{t^+}, \eta D_{ks}^{t^-} \in K^D}{\text{Min}} \left[\sum_{k \in K} \sum_{t \in T} \left(-\Delta D_{ks}^{t^+} \times \eta D_{ks}^{t^+} + \Delta D_{ks}^{t^-} \times \eta D_{ks}^{t^-} \right) \times PC^D \right]$$

S.t.

$$\begin{aligned}
&-\eta D_{ks}^{t^+} \geq -1, \ \forall t \in T, k \in K && : \ \alpha 1_{ks}^t \\
&-\eta D_{ks}^{t^-} \geq -1, \ \forall t \in T, k \in K && : \ \alpha 2_{ks}^t \\
&\sum_{k \in K} \sum_{t \in T} \left(-\eta D_{ks}^{t^+} - \eta D_{ks}^{t^-} \right) \geq -\Gamma_s^D && : \ \beta 1^D \\
&\eta D_{ks}^{t^+}, \eta D_{ks}^{t^-} \geq 0
\end{aligned} \tag{45}$$

Then we take the dual as:

$$\underset{\alpha 1_{ks}^t, \alpha 2_{ks}^t, \beta 1^D}{\text{Max}} \left[-\Gamma_s^D \times \beta 1^D - \sum_{k \in K} \sum_{t \in T} \left(\alpha 1_{ks}^t + \alpha 2_{ks}^t \right) \right] \tag{46}$$

S.t.

$$-\alpha 1_{ks}^{t} - \beta^{D} \le -\Delta D_{ks}^{t^{+}}, \quad \forall t \in T, k \in K$$
$$-\alpha 2_{ks}^{t} - \beta^{D} \le \Delta D_{ks}^{t-}, \quad \forall t \in T, k \in K$$
$$\beta 1^{D}, \alpha 1_{ks}^{t}, \alpha 2_{ks}^{t} \ge 0, \forall t \in T, k \in K$$

In this LP (46), since the second constraint is actually redundant, we can remove $\alpha 2_{ks}^{t}$. According to strong duality theory, we then replace the objective function (46) without $\alpha 2_{ks}^{t}$ in the constraint (44) and, hence, the robust counterpart of constraint (38) is obtained as follows:

$$\left(\sum_{k\in K}\sum_{t\in T}\left(\hat{D}_{ks}^{t} + \alpha 1_{ks}^{t}\right) + \beta 1^{D} \times \Gamma_{s}^{D} - \sum_{k\in K}\sum_{t\in T}\sum_{j\in J} FJK_{jks}^{t}\right) \times PC^{D} \le Z1_{s}$$

$$\alpha 1_{ks}^{t} + \beta 1^{D} \ge \Delta D_{ks}^{t^{+}}, \quad \forall t \in T, k \in K \tag{47}$$

$$\alpha 1_{ks}^{t}, \beta 1^{D} \ge 0$$

The robust counterpart of the other constraints is found by the same procedure. Finally, our hybrid robust-stochastic formulation of this CLSCN design problem is:

$$\begin{aligned}
Max\, Z = \sum_{s\in S} \Pr_{s}.\Bigg(&\sum_{j\in J}\sum_{k\in K}\sum_{t\in T} PR_{k} FJK_{jks}^{t} - \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} CIJ_{ijs}^{t} FIJ_{ijs}^{t} - \sum_{j\in J}\sum_{k\in K}\sum_{t\in T} CJK_{jks}^{t} FJK_{jks}^{t} \\
&- \sum_{j\in J}\sum_{k\in K}\sum_{t\in T} CKJ_{kjs}^{t} FKJ_{kjs}^{t} - \sum_{j\in J}\sum_{i\in I}\sum_{t\in T} CJI_{jis}^{t} FJI_{jis}^{t} - \sum_{j\in J}\sum_{r\in R}\sum_{t\in T} CJR_{jrs}^{t} FJR_{jrs}^{t} - \sum_{i\in I}\sum_{t\in T} MC_{i} PI_{is}^{t} - \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} RC_{i} FJI_{jis}^{t} \\
&- \sum_{j\in J}\sum_{t\in T}\left(\sum_{i\in I}\sum_{p\le t} FIJ_{ijs}^{p} - \sum_{k\in K}\sum_{p\le t} FJK_{jks}^{p}\right) IC_{j}^{t} - \sum_{k\in K}\sum_{j\in J}\sum_{t\in T} CC_{j} FKJ_{kjs}^{t} - \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} DC_{r} FJR_{jrs}^{t} - Z1_{s} - Z2_{s}\Bigg) \\
&- \sum_{i\in I} F_{i}^{MC} X_{i}^{MC} - \sum_{i\in I} C_{i}^{MC} W_{i}^{MC} - \sum_{j\in J} F_{j}^{DC} X_{j}^{DC} - \sum_{j\in J} C_{j}^{DC} W_{j}^{DC} - \sum_{j\in J} F_{j}^{CC} X_{j}^{CC} - \sum_{j\in J} C_{j}^{CC} W_{j}^{CC}
\end{aligned} \tag{48}$$

S.t.

Constraints (2)–(7), (11)–(24)

$$\left(\sum_{k\in K}\sum_{t\in T}\left(\hat{D}_{ks}^{t} + \alpha 1_{ks}^{t}\right) + \beta 1^{D} \Gamma_{s}^{D} - \sum_{k\in K}\sum_{t\in T}\sum_{j\in J} FJK_{jks}^{t}\right) PC^{D} \le Z1_{s}, \forall s \in S \tag{49}$$

$$\left(\sum_{k\in K}\sum_{t\in T}\left(\hat{D}_{ks}^{t}-\alpha 2_{ks}^{t}\right)-\beta 2^{D}\Gamma_{s}^{D}-\sum_{k\in K}\sum_{t\in T}\sum_{j\in J}FJK_{jks}^{t}\right)SC^{D}\geq -Z1_{s},\forall s\in S \tag{50}$$

$$\left(\sum_{k\in K}\sum_{t\in T}\left(\hat{R}_{ks}^{t}+\gamma 1_{ks}^{t}\right)+\lambda 1^{R}\Gamma_{s}^{R}-\sum_{k\in K}\sum_{t\in T}\sum_{j\in J}FKJ_{kjs}^{t}\right)SC^{R}\leq Z2_{s},\forall s\in S \tag{51}$$

$$\left(\sum_{k\in K}\sum_{t\in T}\left(\hat{R}_{ks}^{t}-\gamma 2_{ks}^{t}\right)-\lambda 2^{R}\Gamma_{s}^{R}-\sum_{k\in K}\sum_{t\in T}\sum_{j\in J}FKJ_{kjs}^{t}\right)PC^{R}\geq -Z2_{s},\forall s\in S \tag{52}$$

$$\alpha 1_{ks}^{t}+\beta 1^{D}\geq \Delta D_{ks}^{t^{+}}, \qquad \forall t\in T,\ k\in K, s\in S \tag{53}$$

$$\alpha 2_{ks}^{t}+\beta 2^{D}\geq \Delta D_{ks}^{t^{-}}, \qquad \forall t\in T,\ k\in K, s\in S \tag{54}$$

$$\gamma 1_{ks}^{t}+\lambda 1^{R}\geq \Delta R_{ks}^{t^{+}}, \qquad \forall t\in T,\ k\in K, s\in S \tag{55}$$

$$\gamma 2_{ks}^{t}+\lambda 2^{R}\geq \Delta R_{ks}^{t^{-}}, \qquad \forall t\in T,\ k\in K,\ s\in S \tag{56}$$

$$Z1_{s}, Z2_{s}, \alpha 1_{ks}^{t}, \beta 1^{D}, \alpha 2_{ks}^{t}, \beta 2^{D}, \gamma 1_{ks}^{t}, \lambda 1^{R}, \gamma 2_{ks}^{t}, \lambda 2^{R}\geq 0, \quad \forall t\in T,\ k\in K, s\in S \tag{57}$$

In this formulation, the parameters Γ_s^D and Γ_s^R control the trade-off between the robustness and the level of conservativeness of the obtained solution at each scenario for transportation cost by restricting the number of times that demands and returns deviate from the nominal scenario in their associated uncertainty sets. As a result, higher values for the parameters Γ_s^D and Γ_s^R increase the level of robustness at the expense of a lower expected profit.

2.4 Scenario Generation and Reduction Algorithm for Transportation Costs

To obtain transportation cost scenarios over multiple periods, we combine forecast errors into a tree. As in Schütz et al. (2009) we use a deterministic *P*th order autoregressive process as the forecasting method, and add a realization of error term ε_{t+1}^{s}, to the predicted transportation cost at time *t+1* to obtain the transportation cost in scenario *s* denoted by c_{t+1}^{s}:

$$\hat{c}_{t+1}^{s} = \alpha + \sum_{i=1}^{P} \beta_i \, c_{t+1-i} + \varepsilon_{t+1}^{s} \tag{58}$$

where α is a constant parameter, β_i is an autoregressive parameter, and c_{t+1-i} is the historical transportation cost at period (t +1– i). Then, a transportation cost scenario is generated as:

$$\hat{c}_{t+j}^{s} = \begin{cases} \alpha + \sum_{i=1}^{P} \beta_i \, c_{t+j-i} + \varepsilon_{t+1}^{s} & j = 1 \\ \alpha + \sum_{i=1}^{j-1} \beta_i \, \hat{c}_{t+j-i}^{s} + \sum_{i=j}^{P} \beta_i \, c_{t+j-i} & 1 < j \le P \\ \alpha + \sum_{i=1}^{P} \beta_i \, \hat{c}_{t+j-i}^{s}, & j > P \end{cases} \tag{59}$$

The error terms are assumed to be normally distributed with mean zero and variance σ_{ε}^{2} . Fig. 3 illustrates the scenario tree for prediction error. Error terms are generated for each period independently.

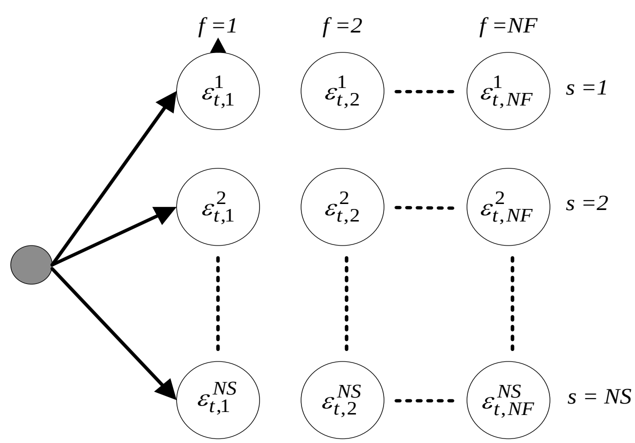


Figure 3. *Scenario tree for the prediction error term for each time period t*

For a pair of facilities $f = (i, j)$ let NF be the total number of flows from all facility type i to all facility type j and NS be the number of scenario values for period t in Fig 3. Based on this scenario tree for the prediction error terms, the procedure to combine forecasting and scenario generation is illustrated by Fig 4.

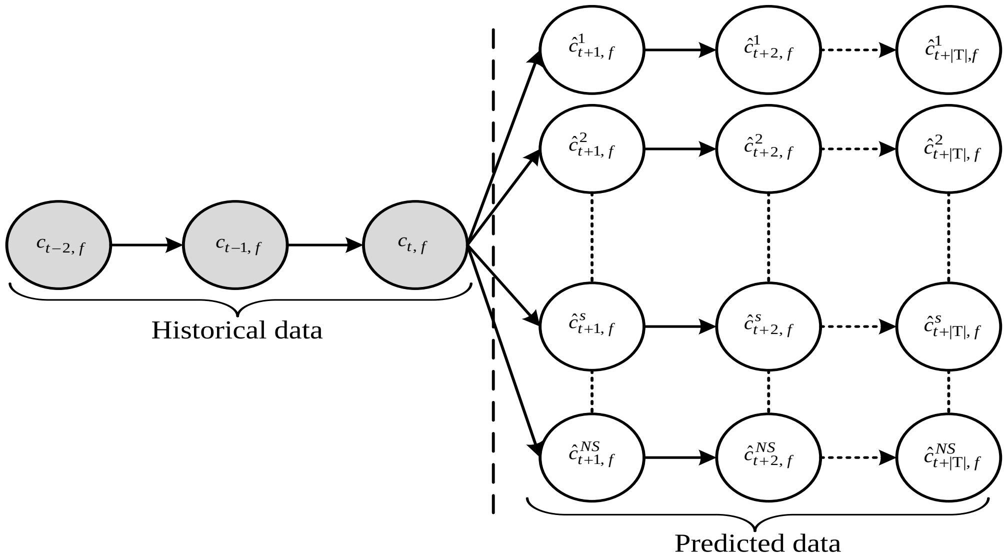


Figure 4. *Forecasting and scenario generation scheme for transportation costs*

To construct different scenarios for the error terms, most studies used Monte Carlo simulation (MCS). Instead, we apply Latin hypercube sampling (LHS) introduced by Olsson et al. (2003). MCS often requires a large sample size to approximate an input distribution, but LHS is designed to accurately approximate the input distribution through sampling in fewer iterations compared with MCS. Moreover, this method covers more of the domain of the random variables than MCS with the same sample size (Fattahi et al., 2014; Shi et al., 2013). To generate the scenario tree for $|T|$ periods, suppose that in each period the error terms are generated using LHS. Since the error terms are period-independent, using this procedure results in an exponentially increasing number of scenarios which makes CLSCN model hard to solve. To efficiently reduce the number of scenarios, a backward reduction technique (Dupačová et al., 2003) is used. A pseudo-code of the proposed scenario generation and reduction algorithm is presented in Appendix B.

2.5 Solution Algorithm

The L-shaped method introduced by Van Slyke and Wets (1969) is a BD method applied to two-stage recourse SP problems. The complex MILP is decomposed into a master problem (MP) including first-stage decision variables, and Benders sub-problems (BSP) including second-stage decision variables. The BSPs are scenario-specific and connected by the first-stage variables.

The BSP of this CLSCN can be formulated by fixing the first-stage variables to the given values $\{X_i^{MC} = \tilde{X}_{i,it}^{MC}, X_j^{DC} = \tilde{X}_{j,it}^{DC}, X_j^{CC} = \tilde{X}_{j,it}^{CC}, W_i^{MC} = \tilde{W}_{i,it}^{MC}, W_j^{DC} = \tilde{W}_{j,it}^{DC}, W_j^{CC} = \tilde{W}_{j,it}^{CC}, BS_j = \tilde{BS}_{j,it}\}$ at iteration *it*. The BSP actually includes the objective function (8) subject to the constraints (11)-(24) and (49)-(57) in which we fixed the first-stage variables to these given values and also the term $-Z1_s - Z2_s$ should be added to the objective function (8). If BSP is feasible for the given values of first-stage variables, then the dual of sub-problem (DSP) has a bounded solution as an extreme point of the dual polyhedron, and so an optimality cut (OC) is deduced. On the other hand, if BSP is infeasible, DSP is unbounded and an extreme ray of its dual polyhedron can be determined and thus a feasibility cut (FC) will be produced. However, it is straightforward to verify that our formulation possesses complete recourse; therefore, FCs are not needed (Birge and Louveaux, 2011). Thus, if *y* and *h* vectors represent the dual variables of the constraints (11)-(24) and (49)-(57) respectively, then the DSP which obtains a lower bound for the objective function of the original CLSND problem at each iteration *it* is formulated as follows:

$$\begin{aligned}
DSP: Min\, Z_{s,it}^{DSP} = & \sum_{j\in J}\sum_{t\in T}\left(\tilde{BS}_{j,it}\, y2_{js}^t + \tilde{W}_{j,it}^{DC}\, y7_{js}^t + \tilde{W}_{j,it}^{CC}\, y8_{js}^t\right) + \sum_{i\in I}\sum_{t\in T}\tilde{W}_{i,it}^{MC}\, y6_{is}^t + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} M.\tilde{X}_{j,it}^{CC}\, y13_{jrs}^t \\
& \sum_{i\in I}\sum_{j\in J}\sum_{t\in T}\left(M.\tilde{X}_{j,it}^{DC}\, y9_{ijs}^t + M.\tilde{X}_{j,it}^{CC}\, y12_{jis}^t\right) + \sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\left(M.\tilde{X}_{j,it}^{DC}\, y10_{jks}^t + M.\tilde{X}_{j,it}^{CC}\, y11_{kjs}^t\right) \\
& -\sum_{k\in K}\sum_{t\in T}\left(\Delta D_{ks}^{t^+} h5_{ks}^t + \Delta D_{ks}^{t^-} h6_{ks}^t + \Delta R_{ks}^{t^+} h7_{ks}^t + \Delta R_{ks}^{t^-} h8_{ks}^t\right)
\end{aligned} \qquad (60)$$

Subject to:

$$y1_{js}^{t} - y2_{js}^{t} + y10_{kjs}^{t} - PC^{D} h1_{s} + SC^{D} h2_{s} \geq PR_{k} - CJK_{jks}^{t} + IC_{j}^{t}, \;\; \forall j \in J, k \in K, t \in T \tag{61}$$

$$-y1_{js}^{t} + y2_{js}^{t} - y3_{is}^{t} + y7_{js}^{t} + y9_{ijs}^{t} \geq -CIJ_{ijs}^{t} - IC_{j}^{t}, \;\; \forall j \in J, i \in I, t \in T \tag{62}$$

$$y3_{is}^{t} - y4_{js}^{t} + y6_{is}^{t} + y12_{ijs}^{t} \geq -CIJ_{jis}^{t} - RC_{i}, \;\; \forall j \in J, i \in I, t \in T \tag{63}$$

$$a\, y4_{js}^{t} + (1-a)\, y5_{js}^{t} + y8_{js}^{t} + y11_{kjs}^{t} - SC^{R} h3_{s} + PC^{R} h4_{s} \geq -CKJ_{kjs}^{t} - CC_{j}^{t}, \;\; \forall j \in J, k \in K, t \in T \tag{64}$$

$$-y5_{js}^{t} + y13_{rjs}^{t} \geq -CJR_{jrs}^{t} - DC_{r}, \;\; \forall j \in J, r \in R, t \in T \tag{65}$$

$$y3_{is}^{t} + y6_{is}^{t} \geq -MC_{i}, \;\; \forall i \in I, t \in T \tag{66}$$

$$PC^{D}\, \Gamma_{s}^{D} h1_{s} - h5_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{67}$$

$$SC^{D}\, \Gamma_{s}^{D} h2_{s} - h6_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{68}$$

$$SC^{R}\, \Gamma_{s}^{R} h3_{s} - h7_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{69}$$

$$PC^{R}\, \Gamma_{s}^{R} h4_{s} - h8_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{70}$$

$$PC^{D}\, h1_{s} - h5_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{71}$$

$$SC^{D}\, h2_{s} - h6_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{72}$$

$$SC^{R}\, h3_{s} - h7_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{73}$$

$$PC^{R}\, h4_{s} - h8_{ks}^{t} \geq 0, \;\; \forall k \in K, \; t \in T \tag{74}$$

$$h1_{s} + h2_{s} \leq 1 \tag{75}$$

$$h3_{s} + h4_{s} \leq 1 \tag{76}$$

$$y1_{js}^{t}, y6_{is}^{t}, y7_{js}^{t}, y8_{js}^{t}, y9_{ijs}^{t}, y10_{jks}^{t}, y11_{kjs}^{t}, y12_{jis}^{t}, y13_{jrs}^{t}, h1_{s}, h2_{s}, h3_{s}, h4_{s}, h5_{ks}^{t}, h6_{ks}^{t}, h7_{ks}^{t}, h8_{ks}^{t} \geq 0 \quad \forall j \in J, i \in I, k \in K, t \in T \tag{77}$$

Then, based on the DSP's solution, the general MP which produces an upper bound for the objective function of original CLSND model at each iteration can be written as:

$$\text{MP: } Max\ Z^{MP} = \sum_{s} \Pr_s \times \theta_s - \sum_{i\in I} F_i^{MC} X_i^{MC} - \sum_{i\in I} C_i^{MC} W_i^{MC} - \sum_{j\in J} F_j^{DC} X_j^{DC} - \sum_{j\in J} C_j^{DC} W_j^{DC} - \sum_{j\in J} F_j^{CC} X_j^{CC} - \sum_{j\in J} C_j^{CC} W_j^{CC} \tag{78}$$

Subject to:

Constraints (2)-(7), $\theta_s \geq 0, \forall s \in S$

$$\begin{aligned}\theta_s \leq & \sum_{j\in J}\sum_{t\in T}\left(BS_{j,it}\tilde{y}2_{js}^t + W_{j,it}^{DC}\tilde{y}7_{js}^t + W_{j,it}^{CC}\tilde{y}8_{js}^t\right) + \sum_{i\in I}\sum_{t\in T} W_{i,it}^{MC}\tilde{y}6_{is}^t + \sum_{r\in R}\sum_{j\in J}\sum_{t\in T} M.X_{j,it}^{CC}\tilde{y}13_{jrs}^t + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T}\Big(M.X_{j,it}^{DC}\tilde{y}9_{ijs}^t \\ & + M.X_{j,it}^{CC}\tilde{y}12_{jis}^t\Big) + \sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\left(M.X_{j,it}^{DC}\tilde{y}10_{jks}^t + M.X_{j,it}^{CC}\tilde{y}11_{kjs}^t\right) - \sum_{k\in K}\sum_{t\in T}\left(\Delta D_{ks}^{t^+}\tilde{h}5_{ks}^t + \Delta D_{ks}^{t^-}\tilde{h}6_{ks}^t + \Delta R_{ks}^{t^+}\tilde{h}7_{ks}^t + \Delta R_{ks}^{t^-}\tilde{h}8_{ks}^t\right), \forall s \in S\end{aligned} \tag{79}$$

In this MP, the constraint (79) represents the optimality cut where $(\tilde{y}, \tilde{h})$ indicates the extreme point of the dual polyhedral resulted from solving the DSP. This reformulation has the drawback of involving a large number of OC constraints. At the optimal solution, not all of these constraints will be binding. Thus, one works with RMP by considering only a subset of these constraints. This RMP provides an upper bound to optimal solution of MP. At a given iteration of BD, RMP is first solved to obtain the values of first-stage decisions. Then, these values are used to solve DSP to obtain an extreme point and a new optimality cut (79) is included in the RMP. But this algorithm may require a large number of iterations to converge, especially for our complex MILP. To improve the slow convergence of the BD, there are some acceleration techniques in the literature such as generation of valid inequalities, disaggregation of Benders cut (Dogan and Goetschalckx, 1999), Pareto-optimal cut generations scheme (Magnanti and Wong, 1981; Papadakos, 2008), covering cut bundle strategy (Saharidis et al., 2010), local branching (Rei et al., 2009), generation of maximal non-dominated cuts (Sherali and Lunday, 2013), and dynamically updated near-maximal Benders cuts (Oliveira et al., 2014). However, due to the structure of our complex CLSCND problem, we employ the following acceleration strategies to improve the slow convergence of this stochastic BD.

2.5.1 Valid inequalities

One of the critical reasons for slow convergence of BD is the low quality of the RMP solutions at the primary iterations (Saharidis and Ierapetritou, 2010). To avoid this inefficiency, a series of valid inequalities (constraints) may be derived to be included into RMP to restrict the feasible region and so, produce high quality solutions. Consequently, the gap between the lower and upper bounds will be decreased and the algorithm will converge to an optimal solution faster. Based on the structure of problem, the following VIs are developed to narrow solution space of RMP and improve the lower bound:

(1) Force the capacity of established facilities to be at least equal to summation of maximum nominal retailers' demand and return:

$$\sum_{i\in I} W_i^{MC} \geq \sum_{j\in J} BS_j \tag{80}$$

$$\sum_{j\in J} W_j^{DC} \geq \max_{s\in S, t\in T} \sum_{k\in K} \hat{D}_{ks}^t \tag{81}$$

$$\sum_{j\in J} W_j^{CC} \geq \max_{s\in S, t\in T} \sum_{k\in K} \hat{R}_{ks}^t \tag{82}$$

Constraints (80)-(82) apply this idea to manufacturing/remanufacturing, distribution, and collection centers, respectively. By adding them to RMP, we improve the quality of RMP solutions, especially in early iterations.

(2) Force the opening of at least one facility of each type:

$$\sum_{i\in I} X_i^{MC} \geq 1 \tag{83}$$

$$\sum_{j\in J} X_j^{DC} \geq 1 \tag{84}$$

$$\sum_{j \in J} X_j^{CC} \geq 1 \tag{85}$$

It is easy to verify that they preserve complete recourse. Moreover, at the beginning of the L-shaped algorithm, we need an initial feasible solution of first-stage variables. We can obtain a good initial feasible solution by solving the optimization problem with the objective (78) without its first term subject to constraints (2)-(7) and the VIs (80)-(85).

2.5.2 Pareto-optimal cuts generation scheme

Magnanti and Wong (1981) proposed a procedure for generating Pareto-optimal cuts to strengthen the optimality cuts. A cut is called Pareto-optimal if no other cut makes it redundant and similarly, the point corresponding to that cut is called Pareto-optimal. Such a cut exists whenever a DSP has multiple optimal solutions, and it is the strongest among all the alternative cuts in the same iteration. Because our BSP has a network structure, it typically has multiple dual solutions that generate alternative optimality cuts. To generate a Pareto-optimal cut, consider our CLSCND problem as the MILP problem $Max\ c_1^T x + c_2^T y \ \ s.t. \ \ Ax + By \leq b, \ \ x \geq 0, \ \ y \in \{0,1\}$. Fixing integer variables $y = \tilde{y}$, the general form of the SP is as $Max\ c_1^T x \ \ s.t. \ \ Ax \leq b - B\tilde{y}, \ \ x \geq 0$ and then its DSP can be written as $Min\ (b - B\tilde{y})^T u \ \ s.t.\ A^T u \geq c_1^T, u \geq 0$. Let u^* be the optimal solution of the DSP and y^c be a core point of the solution space of RMP. A Pareto-optimal cut can be obtained by solving the following problem, which is also called Magnanti-Wong problem:

$$Min\ \left(b - By^c\right)^T u \ \ s.t.\ A^T u \geq c_1^T, \left(b - B\tilde{y}\right)^T u \geq \left(b - B\tilde{y}\right)^T u^*, u \geq 0 \tag{86}$$

The challenge at each iteration is to identify and update a core point, which is required to lie inside the relative interior of convex hull of the sub-region defined in terms of MP variables. To deal with this problem, Papadakos (2008) proved that it is not necessary to use a core point y^c to obtain a Pareto-optimal. As an update strategy, it is demonstrated that the convex combination of the current MP solution and the previous used core point suffices for obtaining a new core point at each iteration as $y_{it}^{c} \leftarrow \lambda y_{it-1}^{c} + (1-\lambda) y_{it}^{MP}$, $0 < \lambda < 1$. For the first iteration, y_0^c is set to equal to the solution of MP. Fig. 5 depicts the pseudo-code for the accelerated multi-cut L-shaped algorithm with the Pareto-optimal cut scheme. In step *iii*, the corresponding Magnanti-Wong problem (86) is solved to obtain Pareto-optimal cut.

Proposed Solution Algorithm based on accelerated multi-cut L-shaped algorithm

Step 0. Initialization

i. $Z_0^{Upper} = +\infty$, $Z_0^{Lower} = -\infty$

ii. Solve the model with objective function (78) subject to the constraints (2)-(7), and VIs (80)-(85) to obtain an initial feasible solution $\{\tilde{X}_{i0}^{MC}, \tilde{X}_{j0}^{DC}, \tilde{X}_{j0}^{CC}, \tilde{W}_{i0}^{MC}, \tilde{W}_{j0}^{DC}, \tilde{W}_{j0}^{CC}, \tilde{BS}_{j0}\}$

iii. Find a core point (Set it as the solution of MP)

iv. it = 0

While ($Z_{it}^{Upper} - Z_{it}^{Lower} > \varepsilon$) **do**

Step 1. Solving DSPs for each scenario $s \in S$ using

$\{\tilde{X}_{i,it}^{MC}, \tilde{X}_{j,it}^{DC}, \tilde{X}_{j,it}^{CC}, \tilde{W}_{i,it}^{MC}, \tilde{W}_{j,it}^{DC}, \tilde{W}_{j,it}^{CC}, \tilde{BS}_{j,it}\}$

If *solved to optimality*

i. Solve the corresponding Magnanti-Wong problem (86) to obtain a Pareto-optimal cut

ii. Update Z_{it}^{Lower}

End if

Step 2. Add generated cuts to RMP

Step 3. Solving the RMP with the new cuts

i. Update Z_{it}^{Upper}

ii. it = it + 1

iii. Update the core point $\left(y_{it}^{c} \leftarrow \lambda y_{it-1}^{c} + (1-\lambda) y_{it}^{MP},\ 0 < \lambda < 1\right)$

End while

Figure 5. *The pseudo-code of the accelerated L-shaped algorithm*

2.6 Computational Results

In Section 2.6.1, the mathematical formulation is verified by performing sensitivity analysis on some important parameters in small instances. Then in Section 2.6.2, stability analysis is done to verify that our developed scenario generation and reduction algorithm generates appropriate scenario trees. Finally in Section 2.6.3, we describe computational experiments using the proposed stochastic BD algorithm for solving large-scale CLSCN design problems.

2.6.1 Sensitivity analysis of the hybrid robust-stochastic CLSCN design formulation

To assess the model performance, two test instances described in Table 1 are considered. We generate scenarios for uncertain transportation costs. Then, for each scenario uncertainty sets of demand and return are developed. To do so, we sample nominal demands from a uniform distribution specified in Table 1. Then, we determine maximum positive and negative deviations from the nominal scenario such that the deviation interval of uncertain demand is a subset of the interval defined in Table 1. The same procedure is used to obtain uncertainty sets for returns.

Table 1

Characteristics and transportation costs scenarios in the test instances 1 and 2.

Instance Size $\lvert I\rvert*\lvert J\rvert*\lvert K\rvert*\lvert R\rvert*\lvert T\rvert$	Scenarios (S)	Scenario Probability	Transportation costs	Nominal Demands	Nominal Returns
3*8*10*2*2	1	0.5	Unif(5,10)	Unif(2100,2850)	Unif(450,1050)
	2	0.2	Unif(10,15)	Unif(2350,2950)	Unif(580,1200)
	3	0.3	Unif(15,20)	Unif(2150,2650)	Unif(460,1150)
8*12*20*5*6	1	0.1	Unif(5,9)	Unif(1500,2000)	Unif(350,850)
	2	0.3	Unif(7,12)	Unif(1900,2450)	Unif(450,1050)
	3	0.1	Unif(6,11)	Unif(2500,3100)	Unif(850,1350)
	4	0.3	Unif(5,10)	Unif(2100,2850)	Unif(450,1050)
	5	0.2	Unif(10,15)	Unif(2350,2950)	Unif(680,1200)

Other parameters are generated randomly according to the uniform distributions specified in Table 2. The instances are solved by GAMS 23.5 using ILOG-CPLEX 11.0. To explore the effects of the main parameters on solutions, sensitivity analysis is performed on operational costs, penalty costs analysis, selling price and uncertainty budgets.

Table 2

The distributions from which the parameters used in the test instances are generated.

Parameter	Range	Parameter	Range	Parameter	Range
F_i^{MC}	Unif(2100000, 3100000)	MC_i	Unif (120,160)	CAP_i^{MC}	Unif(12000, 22000)
F_j^{DC}	Unif(831500, 1000000)	RC_i	Unif (20,40)	CAP_j^{DC}	Unif(12000, 20000)
F_j^{CC}	Unif(831500, 1000000)	IC_j^t	Unif (5,10)	CAP_j^{CC}	Unif(4800, 8100)
PC^D, PC^R	Unif (150,600)	CC_j	Unif (60,80)	C_i^{MC}	Unif(50, 100)
SC^D, SC^R	Unif (50,150)	DC_r	Unif (1,5)	C_j^{DC}	Unif(30, 50)
a	Unif (0.7,1)	PR_k	Unif (160,230)	C_j^{CC}	Unif(30, 50)

2.6.1.1 Operational costs

First, we examine solution sensitivity to some essential costs, such as remanufacturing, collection, and manufacturing costs. Changing these operational costs affects the amount of demands satisfied and the amount of returns collected. To investigate these effects, one cost at a time is multiplied by some constant coefficients. Then we examine the sensitivity of expected coverage of demand and returns, as well as profit, over the scenarios.

Table 3

Expected coverage of return and profit tor different remanufacturing costs.

Test instance 1			Test instance 2		
Change coefficient	Profit	Expected coverage of return	Change coefficient	Profit	Expected coverage of return
0.5	3095177.8	96.88%	0.5	607672.3	93.90%
1	2934364.2	96.88%	1	570964.2	93.90%
2	2621310.2	88.76%	2	-1277780	91.60%
10	276568	78.66%	10	-3351230	85.23%
30	-5072070	0.00%	50	-56133500	33.70%

Table 4

Expected coverage of return and profit for different collection costs.

Test instance 1			Test instance 2		
Change coefficient	Profit	Expected coverage of return	Change coefficient	Profit	Expected coverage of return
0	4264758	99.98%	0	751722.4	99.87%
0.6	3454197	98.93%	0.2	715681.2	95.78%
0.8	3187979	96.75%	0.6	607672.5	93.90%
0.9	3061048	96.23%	0.8	383767.6	93.90%
1	2934364	89.88%	1	570964.2	93.90%
2	1754915	75.76%	2	-2487887	87.65%
8	-5072070	54.34%	8	-4468432	66.42%

The fluctuation of the optimal expected profit and the expected coverage of return and demand over scenarios are demonstrated in Tables 3 and 4 for different values of remanufacturing and collection costs, respectively. Increasing these costs results in decreasing the expected coverage of return as well as the profit. In fact, with extreme increase in these operational costs, the expected coverage of return decreases to zero because collecting and acquiring end-of-use products is no longer economical. However, changing the collection and remanufacturing costs has no effect on the expected coverage of demand.

Table 5

Expected coverage of demand and return and profit for different manufacturing costs.

Test instance 1				Test instance 2			
Change coefficient	Profit	Expected coverage of demand	Expected coverage of return	Change coefficient	Profit	Expected coverage of demand	Expected coverage of return
0.5	5955323.7	99.38%	88.76%	0.5	9659102.6	97.60%	93.90%
0.8	4129472.5	97.53%	88.76%	0.8	5630294.3	96.40%	93.90%
1	2934364.2	97.53%	96.88%	1	570964.2	95.90%	93.90%
2	2383885	85.44%	96.88%	2	-10001500	95.30%	93.90%
3	-6554420	76.19%	99.01%	3	-21934000	83.40%	99.60%
4	-8825490	59.30%	99.01%	4	-30067200	67.10%	99.60%
5	-8872220	24.94%	99.21%	5	-34444100	34.30%	99.60%

Table 5 shows that when the manufacturing cost increases, the expected coverage of demand and profit will decrease, but the expected coverage of returns will increase. When the manufacturing cost increases in the forward network, the system tries to satisfy demand by remanufacturing used products collected from retailers. Thus, with increase in manufacturing cost, the expected coverage of returns increases and, since the remanufactured products are not sufficient to meet the demand, the expected coverage of demand decreases.

2.6.1.2 Penalty and other costs related to retailers

Next we investigate the impact of penalty costs for unsatisfied demand and scrap costs for uncollected returns on the expected coverage of demand and return, respectively, followed by the relation between surplus cost and the expected coverage of demand and also the relation between penalty cost and the expected coverage of return. There is an inverse relation between the surplus and penalty costs in the forward network and also between the scrap and penalty costs in the reverse network. In the forward network, the CLSCN seeks a trade-off between the penalty and surplus costs such that their total is minimized. Likewise, in the reverse network, the optimization achieves a trade-off between the penalty and scrap costs such that their total is also minimized. These costs serve to balance the forward flows with the demand and the reverse flows with the return quantities as much as possible.

Table 6

Expected coverage of demand and profit for different penalty costs for unsatisfied demands.

Test instance 1			Test instance 2		
Penalty cost of unsatisfied demand	Profit	Expected coverage of demand	Penalty cost of unsatisfied demand	Profit	Expected coverage of demand
0	6881686.461	78.23%	0	8011126.061	74.56%
155	5756598.493	85.43%	50	5458363.293	83.22%
300	5696535.978	89.68%	100	4747939.381	93.34%

800	5535406.573	92.56%	350	3558672.964	96.81%
1200	5427877.293	99.48%	450	3264315.516	98.57%
2400	5134039.509	99.96%	750	2973672.746	99.78%

Table 7

Expected coverage of return and profit for different scrap costs for uncollected returns.

Test instance 1			Test instance 2		
Scrap costs of uncollected return	Profit	Expected coverage of return	Scrap costs of uncollected return	Profit	Expected coverage of return
0	6881686.462	0	0	8011126.061	0
100	5089580.689	65.43%	50	6407165.786	58.22%
300	4660283.369	88.76%	150	5937110.786	81.78%
500	4304763.446	98.74%	450	5413307.469	96.57%
800	4302200.405	99.84%	750	4315328.712	99.78%

As the results in Table 6 show, increasing the penalty cost for unsatisfied demands results in higher expected coverage of demand and lower total profit. A similar sensitivity of the expected coverage of return to its corresponding scrap cost of uncollected returns is also seen in Table 7.

Table 8

Expected coverage of demand and profit for different surplus costs of excess amount of flows over demand.

Test instance 1			Test instance 2		
Surplus cost for demand	Profit	Expected coverage of demand	Surplus cost for demand	Profit	Expected coverage of demand
0	3084707.559	99.98%	0	1028633.229	98.75%
60	2976581.403	98.59%	75	462961.162	98.42%
100	2915489.545	97.69%	100	360710.162	97.24%
800	2313456.622	95.52%	900	-653970.454	96.41%
1200	2236937.507	85.48%	1200	-773171.213	95.27%
2000	1931361.174	75.66%	5000	-1138770	86.78%

Table 9

Expected coverage of return and profit for different penalty costs of excess amount of flows over return.

Test instance 1	Test instance 2

Penalty cost of return	Profit	Expected coverage of return	Penalty cost of return	Profit	Expected coverage of return
0	3082146.106	99.14%	0	1028633	99.71%
20	3044299.389	98.39%	25	838454.6	96.52%
50	2993617.919	97.66%	55	635474.7	94.21%
100	2985514.982	88.76%	100	407039.1	93.83%
1000	2941578.337	86.28%	1000	-1350870	89.22%
2000	2853703.669	85.36%	5000	-2085200	87.78%

Furthermore, as Table 8 illustrates, increasing the unit surplus cost for excess amount of flows over demands in retailers lowers the expected coverage of demand and the total profit. A similar result for penalty costs on the excess amount of flows over returns is shown in Table 9.

2.6.1.3 Selling price

The selling price has an important influence on the total profit and the expected coverage of demand. The sensitivity is explored by multiplying the price by a constant coefficient.

Table 10

Expected coverage of return and profit for different selling prices.

Test instance 1			Test instance 2		
Change coefficient	Profit	Expected coverage of demand	Change coefficient	Profit	Expected coverage of demand
0.2	-15235300	21.92%	0.2	-41582100	83.40%
0.5	-10527400	59.31%	0.4	-31680300	92.30%
0.7	-5258650	85.44%	0.7	-15665600	94.80%
1	2934364	97.53%	1	570964	95.90%
1.5	12190600	98.43%	2.5	84338690	98.70%
2.5	49782290	99.48%	5	225508800	99.80%
5	128181300	99.86	10	508778800	100.00%

From Table 10, increasing the selling price causes the expected coverage of demand and profit to increase. The reason for this system behavior is that the cost prices of the product for different customers are different and so the system tries to satisfy the demand of customers whose cost price is less than the selling price. Increasing the selling price raises the number of such customers and so the expected coverage of demands will increase.

2.6.1.4 Budget of uncertainty

The effect of uncertainty is studied by changing the budget of uncertainty parameters for uncertain demands and returns. We define ρ as the level of variability of the uncertain parameter respect to its nominal value and consider the values $\rho = 5\%, 10\%, 20\%$ and 30%. With the help of this parameter we can change the radius of the polyhedral uncertainty sets. In test instance 1, for each level of variability of uncertain demand, we vary Γ_s^D in each scenario s from *0* (the nominal formulation) to its maximum value $|K| \times |T| = 20$ (the worst-case formulation) by 1, while maintaining $\Gamma_s^R = 0$ to investigate just the uncertainty in demand. In addition, a bound on the probability of constraint violation is computed according to equation (31) under the assumption of symmetric distributions for independent demand and return quantities. The percent decrease in the optimal profit value and the constraint violation probability bound for each scenario *s* independently are plotted in Figures 6 and 7, respectively, as functions of Γ_s^D and Γ_s^R. Here, the relative decrease in optimal profit is calculated as $\left(z^N - z^R\right) / z^N$ where z^N and z^R are the nominal and robust optimal profits, respectively.

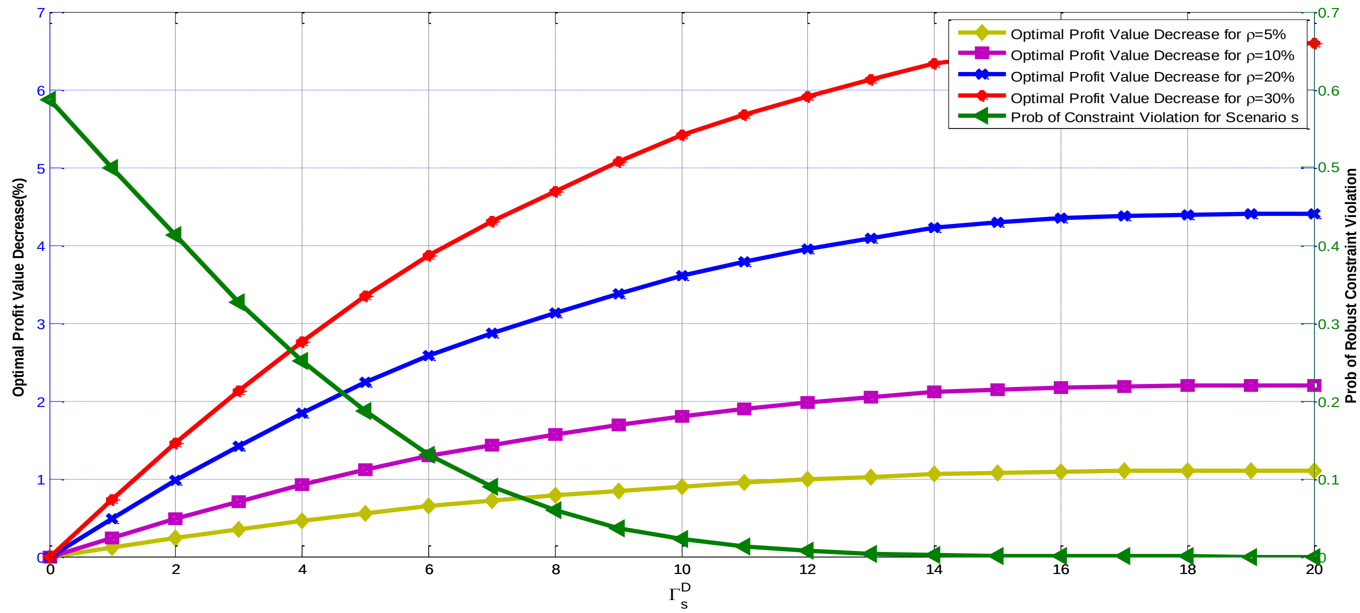
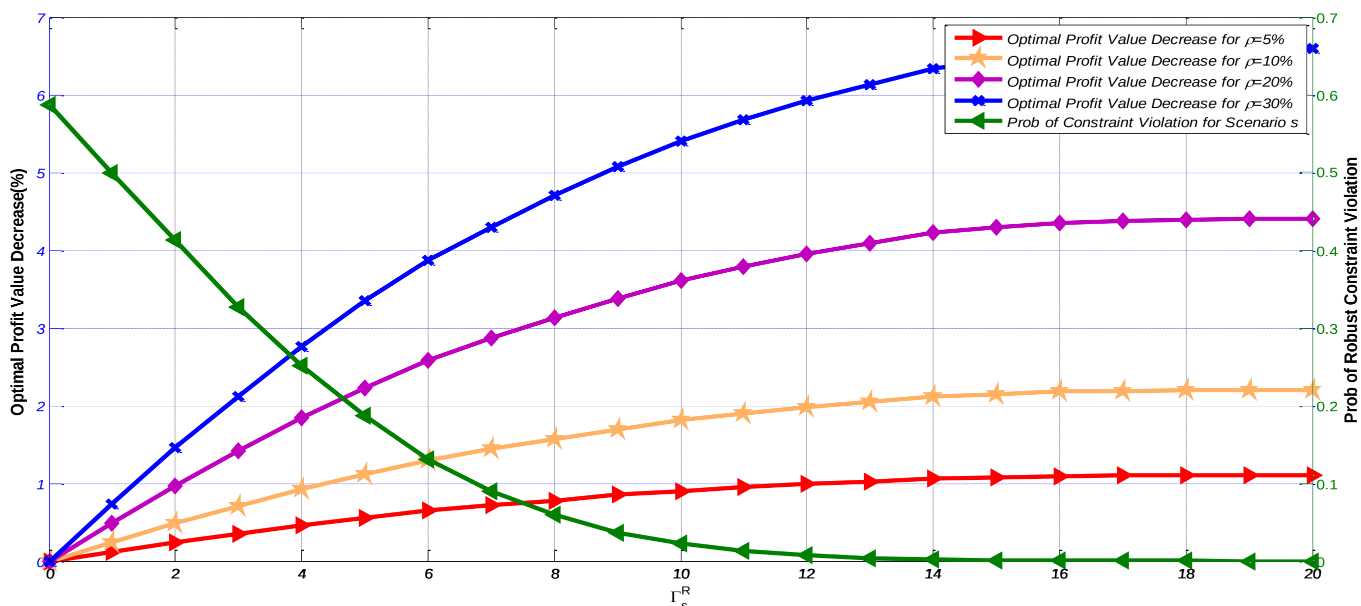


Figure 6. *Optimal profit decrease and probability of robust constraint violation as a function of*



Γ_s^D

Figure 7. *Optimal profit decrease and probability of robust constraint violation as a function of*

Γ_s^R

As expected, we observe from Figures 6 and 7 that for each level of variability, the magnitude of reduction in profit increases while the constraint violation probability decreases with as the uncertainty budget increases.

However, the bound on the probability of constraint violation computed according to equation (31) is in fact for a single robust constraint. To our knowledge, no bounds have been developed for probability of violating multiple robust constraints together. To investigate this, we compute an empirical frequency of constraint violation in a simulation. To do so, the test instance 1 is solved for different values of Γ_s^D and Γ_s^R which are increased from 0 to 20 in increments of one. Then, for each solved test instance 1 with different values of Γ_s^D and Γ_s^R, we generate random values a thousand times for the collection of uncertain demands and returns from their associated polyhedral uncertainty sets. Next, based on the optimal values for the decision variables and these sampled values of demand and return quantities, we check to see whether the robust constraints are feasible or not, and so obtain the violation frequency of our hybrid robust-stochastic problem. To compare with each other for test instance 1, these frequencies as well as the constraint violation probability bound for each scenario *s*, which is calculated based on equation (31), are plotted together in Figure 8 as functions of Γ_s^D and Γ_s^R.

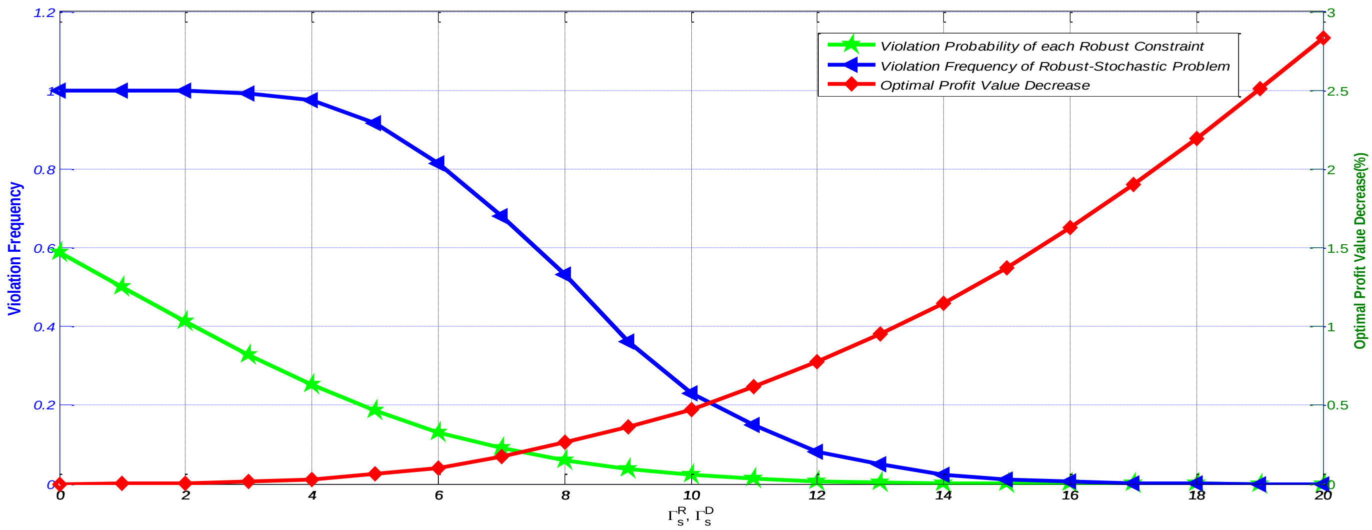


Figure 8. *Violation frequency and optimal profit value decrease as a function of* Γ_s^D *and* Γ_s^R

In Figure 8, the red curve with diamond markers shows the deviation from optimal profit for different values of Γ_s^D and Γ_s^R. Also, the green plot with triangle markers shows the violation probability of a single constraint computed using equation (31). However, according to the approximate violation frequencies illustrated by the blue plot, if we choose $\Gamma_s^D, \Gamma_s^R \leq 4$, then the overall violation probability is approximately 1, which means that when the highest objective value is obtained it is not robust with respect to changing the values of our uncertain parameters. For $4 < \Gamma_s^D, \Gamma_s^R \leq 14$, by increasing the values of Γ_s^D and Γ_s^R, the value of robustness or being feasible for our problem increases sharply. Finally, if we choose our budget of uncertainty parameters from $\Gamma_s^D, \Gamma_s^R > 14$, then there is not much difference in terms of the robustness and in fact it is close to 100%, but the objective value may be unacceptably high.

2.6.2 Stability analysis of scenario generation and reduction algorithm

One of the main criteria which should be satisfied by a scenario generation and reduction method is stability. If several scenario trees with the same input data are generated and we solve our problem with these scenario trees, then we should obtain (approximately) the same optimal value of the objective function. That is, if we generate |L| scenario trees $\xi_l, l=1,...,L$ using our scenario generation and reduction algorithm, solve the CLSCN design problem with each one of these scenario trees, and obtain the optimal solution $x_l^*, l=1,...,L$, then stability means that we should have $f\left(\hat{x}_l^*, \xi_l\right) \approx f\left(\hat{x}_u^*, \xi_u\right), \forall l, u \in 1,...,L$ where $f\left(\hat{x}_l^*, \xi_l\right)$ is the optimal objective function value with respect to the scenario tree l. This type of stability means that the real performance of the optimal solution x_l^* is stable, i.e. it is not dependent upon which scenario tree we choose (Kaut and Wallace, 2007). To carry out this stability analysis, we generate 8 scenario trees for test instances 1 and 2, solve the hybrid robust-stochastic CLSCN design problem with the other input data held constant and then the optimal objective function values are reported in Table 11. The lack of any substantial difference between the optimal objective values indicates stability.

Table 11

Stability analysis of the scenario generation and reduction algorithm

Test instance 1		Test instance 2	
Scenario Tree	Optimal objective function value	Scenario Tree	Optimal objective function value
1	2934364.2	1	570964.2
2	2945120.7	2	572356.1
3	2975218.2	3	570852.3
4	3012312.5	4	575123.7
5	3024521.1	5	569978.9
6	2898959.8	6	572123.5
7	2955412.9	7	570768.1
8	3003252.1	8	572325.4

2.6.3 Computational efficiency of the accelerated L-shaped algorithm

We measure the computational efficiencies achieved by adding VIs and employing Pareto-optimal cuts in terms of computing times, number of Benders iterations and the quality of lower bounds. The characteristics of the test instances are described in Table 12. The transportation cost scenarios are based on an *AR (1)* process with α = 30, β_1 = 0.15, and error terms normally distributed with a mean 0 and a variance. The accelerated BD algorithm is coded in MATLAB and tested on a computer with CPU Intel Core i7, 2.5 GHz and 8 GB RAM. CPLEX 11.0 is used to solve MP and BSPs. We solve each instance both with/without VIs and Pareto-optimal cuts.

Table 12

Characteristics and size of the generated test instances.

Instance	$\|I\|$	$\|J\|$	$\|K\|$	$\|R\|$	$\|T\|$	Number of scenarios	Number of variables	Number of constraints
1	8	12	20	5	3	5	8.75×10^3	9.76×10^3
2	10	14	20	8	6	10	6.2634×10^4	6.827×10^4
3	12	16	25	10	8	15	1.7485×10^5	1.8782×10^5
4	15	18	30	12	9	20	3.5494×10^5	3.7709×10^5
5	18	20	35	16	12	24	7.4841×10^5	7.7816×10^5
6	20	24	40	18	12	28	1.1735×10^6	1.2286×10^6
7	24	26	43	20	14	32	1.8818×10^6	1.9624×10^6
8	26	30	45	25	16	35	2.7386×10^6	2.8468×10^6

2.6.3.1 Effectiveness of the valid inequalities

Table 13 compares the effects of different combinations of VIs on the lower bounds, optimality gap, and the number of BD iterations (Iters). Here, BD denotes the BD algorithm without any VI, and BDVI1, BDVI2, and BDVI12 denote that VIs (80)-(82), (83)-(86), and (80)-(86), respectively, are added to the MP. The stopping criteria are (1) optimality gap below a threshold value 0.009 or (2) a maximum of 70 Benders iterations is reached.

Table 13
Lower bounds, optimality gap and number of Benders iterations for different combinations of VIs.

	Test NO.	BD	BDVI1	BDVI2	BDVI12	Test NO.	BD	BDVI1	BDVI2	BDVI12
Z_{lb}		20489040	20497213	20489040	20503960		164097472	164097472	164097472	164136565
Gap (%)	1	0.315	0.275	0.315	0.242	5	0.767	0.767	0.767	0.743
Iters		6	6	6	5		18	17	18	17
Z_{lb}		76750546	76750546	76750546	76814729		156060139	156094286	156060139	156105154
Gap (%)	2	0.532	0.532	0.532	0.448	6	0.567	0.545	0.567	0.538
Iters		15	13	14	11		10	8	10	8
Z_{lb}		140005839	140317678	140014137	140345464		180275935	180281299	180275935	180281299
Gap (%)	3	1.243	1.018	1.237	0.998	7	0.821	0.818	0.821	0.818
Iters		70	70	70	70		21	21	21	20
Z_{lb}		177735807	177825740	177825740	177908700		173109952	173118493	173109952	173243278
Gap (%)	4	0.843	0.792	0.792	0.745	8	1.353	1.348	1.353	1.275
Iters		25	24	25	23		70	70	70	70

In Table 13, we see that VI2 itself does not make any significant impact on either the number of Benders iterations or the optimality gap. However, BDVI12 consistently improves the lower bound as compared with the classical BD algorithm, and so increases the convergence rate. Moreover, when the maximum number of iterations is reached, for example in instances 3 and 8, the gaps provided including both VIs are better than those provided with either VI alone. Of the two sets of VIs alone, the first (VI1) improves the lower bound more and so more efficient.

2.6.3.2 Effectiveness of Pareto-optimal cuts

In Table 14, the lower bound, the optimality gap and the number of Benders iterations are displayed for the BD variants with Pareto-optimal cuts and also with a hybrid strategy that combines the VIs with the Pareto-optimal cuts. Note that in our computational experiments, a core point approximation $\bar{y}_0^c$ is initialized with a feasible solution to our RMP and then we update the approximation at each successive iteration by setting $\bar{y}_{it}^{c} \leftarrow \lambda \bar{y}_{it-1}^{c} + (1-\lambda)\hat{y}_{it}^{MP}$. We set $\lambda = 0.5$ because, according to empirical observations of Papadakos (2008) and Oliveira et al., (2014). The BDVI12 variant has lower optimality gaps than the BD variant with Pareto-optimal cuts. The hybrid strategy achieves even better results in terms of optimality gap and the iteration count compared with other variants of BD algorithm.

Table 14

Lower bounds, optimality gap and number of Benders cycles for different BD algorithms.

	Test NO.	BD with Pareto-optimal cut	BD with hybrid strategy	Test NO.	BD with Pareto-optimal cut	BD with hybrid strategy
Z_{lb}		20504983	20519053		164149600	164149600
Gap (%)	1	0.237	0.167	5	0.735	0.735
Cycles		5	3		15	15
Z_{lb}		76814729	76814528		156094500	156130215
Gap (%)	2	0.448	0.448	6	0.545	0.522
Cycles		11	11		7	5
Z_{lb}		140317678	140695119		180292029	180297394
Gap (%)	3	1.018	0.747	7	0. 812	0.809
Cycles		70	15		19	16
Z_{lb}		177938726	177966995		173109952	173109952
Gap (%)	4	0.728	0.712	8	1.335	1.156
Cycles		22	19		70	70

Table 15 displays the computational times for solving each test instance by each BD variant and also by directly solving the extensive form by CPLEX. The CPLEX computational times are smaller for the test instances with few scenarios. As the number of scenarios increases, the BD algorithms, especially with the hybrid strategy, outperforms CPLEX. From the results of Tables 13, we can observe that adding the VI1 is more efficient than VI2 in terms of number of iterations and gap. Moreover, we see that when we apply the BD algorithm with Pareto-optimal cut, the number of iterations and optimality gap are decreased for most test instances compared with BD, BDV1, BDV2, and BDV12. But, this algorithm has the highest computational time compared with all BD algorithms. The smaller numbers of Benders iterations when using Pareto-optimal cuts do not necessarily mean smaller computational times in fact, the computational times are increased as a result of the time spent to solve the Magnanti-Wong problem to obtain the Pareto-optimal cuts. Here, each iteration is more effective than each iteration in other BD algorithms and that is why we have less the number of iterations and optimality gap compared with BD, BDV1, BDV2, and BDV12, but each iteration takes longer. However, when we add both VI1 and VI2 to the BD algorithm with Pareto-optimal cut as the BD algorithm with hybrid strategy, it gives us the best performance in terms of both computational time and also optimality gaps and number of Benders cycles for large-scale instances such as instance 5, 6, 7, and 8. Therefore, the Pareto-optimal cuts generation scheme plus adding both valid inequalities demonstrates the best performance in general.

Table 15
Result summary of computational times (in seconds) for different solution algorithms.

Test NO.	BD	BDVI1	BDVI2	BD with Pareto-optimal cut	BD with hybrid strategy	CPLEX
1	25.19	19.08	21.57	27.08	24.01	**11.06**
2	88.08	85.12	82.21	92.43	90.34	**35.48**
3	121.56	119.32	121.34	145.21	122.67	**58.32**
4	162.78	**141.11**	155.62	189.34	157.17	159.12
5	287.54	271.37	285.09	298.32	**268.54**	332.34
6	399.91	375.76	388.23	467.78	**372.62**	465.12
7	684.23	655.34	675.54	699.12	**646.46**	-*
8	1018.45	899.21	986.24	1146.13	**888.32**	-*

* The dashes means that we were not able to solve these test instances.

CHAPTER 3 CONCLUSION AND FUTURE RESEARCH

In this paper, a mixed-integer linear programming model for a multi-period, single-product and capacitated CLSCN design problem is formulated to maximize the expected profit. As the major contribution, a hybrid robust-stochastic programming approach is developed to model qualitatively different uncertainties. We assume historical data exist for transportation costs and use them to generate probabilistic scenarios by a scenario generation and reduction algorithm. Then, in each scenario for transportation costs, polyhedral uncertainty sets are proposed for demand and return quantities in the absence of historical data for a new product. Some numerical instances are created to analyze and validate formulation. To solve this combinatorial problem, an accelerated stochastic BD algorithm is proposed. Two groups of valid inequalities are added to the master problem to efficiently improve the lower bound, and Pareto-optimal cuts are also applied to further accelerate convergence. The computational results demonstrate that the combination of all valid inequalities is most effective for improving the lower bound. Also, the Pareto-optimal cut generation scheme results in significant improvement for some instances where the number of Benders iterations is large. Overall, the combination of VIs and Pareto-optimal cuts demonstrates the best average performance.

As this paper introduces a novel combination of robust and stochastic optimization in the context of CLSCN design, there are some opportunities for future research such as applying other robust optimization approaches and even other uncertainty sets such as ellipsoidal ones, as well as investigating the management of disruption risk in the CLSCN design problem. Moreover, to solve this large-scale problem, other versions of the BD approach such as a Benders-based branch-and-cut approach, where a single branch-and-cut tree is constructed and then the Benders cuts are added during the exploration of this tree, can be applied for performance comparisons.

Appendix A. Literature review

Table A.1
Classification of closed-loop supply chain network design problems.

Category	Detail	Code	Category	Detail	Code
Problem features:	*Periods:*			Collection/inspection centers	CIC
	Multi-period	MP		Disassembly centers	DAC
	Single period	SP		Remanufacturing centers	RMC
	Product:			Repair centers	RPC
	Single-product	SPr		Redistribution centers	RDC
	Multi-product	MPr	**Objectives:**	Min cost	C
	Flow capacity:			Max profit	Pr
	Uncapacitated flow	UF		Others	O
	Capacitated flow	CF	**Modeling:**	Mixed-integer non-linear program	MINLP
	Facility capacity:			Mixed-integer linear program	MILP
	Uncapacitated	UC	**Outputs:**	Inventory	I
	Capacitated	CC		Location/allocation	LA
	Sourcing:			Facility capacity	FC
	Single Sourcing	SS		Transportation amount	TA
	Multiple Sourcing	MS		Price of products	P
	Capacity expansion:	CE		Production amount	PA
SC network Stages:	*Forward stage:*			Transportation mode	TM
	Distribution centers/Warehouses	DC		Demand/Return satisfaction	DRS
	Manufacturing centers	MC	**Uncertain Programming:**	Deterministic Programming	DP
	Supply centers	SC		Stochastic Programming	StP
	Customer zones/retailer	CZ		Robust Programming	RtP
	Reverse stages:			Fuzzy Programming	FuP
	Redistribution centers	RDC	**Uncertain Parameter:**	Demand/Supply	DS
	Disposal centers	DSC		Return	RE
	Recycling centers	RYC		Costs	CO
	Recovery centers	RCC		Others	Ot

Table A.2

Review of most cited and also recent articles in the CLSCN design problem

Articles	Problem features	SCN stages	Objective	Modeling	Uncertainty Parameters	Uncertainty Programming	Outputs	Solution Method
Fleischmann et al. (2001)	SP, SPr, UF, UC, MS	MC, DC, DAC, RMC, DSC, CZ	C	MILP	-	DP	LA, TA, DRS	CPLEX
Salema et al. (2007)	SP, MPr, UF, CC, MS	MC, DC, CIC, RMC, RCC, CZ	C	MILP	DS, RE	StP	LA, TA	Branch & Bound algorithm
Üster et al. (2007)	SP, MPr, UF, UC, SS	MC, DC, CIC, RMC, CZ	C	MILP	-	DP	LA, TA	Benders' Decomposition with multiple cuts
Ko and Evans (2007)	MP, MPr, UF, CC, MS,CE	MC, DC, RPC, CZ	C	MINLP	-	DP	LA, TA, FC	Genetic algorithm-based heuristic
Lu and Bostel (2007)	SP, SPr, UF, UC, MS	MC, CIC, RMC, DSC,CZ	C	MILP	-	DP	LA, TA	Algorithm based on Lagrangian heuristics
Listeş (2007)	SP, SPr, UF, CC, SS	MC, RCC, DSC, CZ	C	MILP	DS, RE	StP	LA, TA	L-Shaped algorithm
Min and Ko (2008)	MP, MPr, UF, UC, MS,CE	MC, DC,CIC,RPC, CZ	C	MINLP		DP	LA,TA,FC, DRS	Genetic algorithm
Lee and Dong (2008)	MP, MPr, UF, CC, MS	MC, DC,CIC,RDC,RMC,CZ	C	MINLP	DS, RE	StP	LA, TA	Heuristic algorithm based on Simulated Annealing & sample average approximation
Salema et al. (2009)	MP, MPr, UF, CC, MS	MC, DC, DAC, RMC, CZ	C	MILP	-	DP	LA, TA, I, PA, DRS	Branch & Bound algorithm
Easwaran and Üster (2009)	SP, MPr, UF, CC, MS	MC, DC, CIC, CZ	C	MILP	-	DP	LA, TA	Benders' Decomposition & Tabu search heuristics
Pishvaee et al. (2009)	SP, SPr, UF, CC, MS	MC, DC, CIC, DSC, CZ	C	MILP	DS, RE, CO	StP	LA, TA	LINGO
El-Sayed et al. (2010)	MP, SPr, UF, CC, MS	SC, MC, DC,CIC,RDC,RMC,DSC, CZ	P	MILP	DS	StP	LA, TA, I	Xpress-Mosel
Pishvaee et al. (2010)	SP, SPr, UF, CC, MS	MC, DC, CIC, RDC, DSC, CZ	C, O	MILP	-	DP	LA, TA, FC	Memetic algorithm
Wang and Hsu (2010)	SP, SPr, UF, CC, MS	SC, MC, DC,RDC,RMC,DAC,CZ	C	MINLP	-	DP	LA, TA, PA	Genetic algorithm
Salema et al. (2010)	MP, MPr, CF, CC, MS	MC, DC, DAC, DSC, CZ	C	MILP	-	DP	LA, TA, I, PA, DRS	CPLEX
Easwaran and Üster (2010)	SP, MPr, UF, CC, MS	MC, DC, CIC, RMC, CZ	C	MILP	-	DP	LA, TA	Benders' decomposition
Pishvaee and Torabi (2010)	MP, SPr, UF, CC, MS	MC, DC, CIC, RCC, RYC, CZ	C, O	MILP	DS, RE, Ot	FuP	LA, TA	Interactive fuzzy solution appro
Pishvaee et al. (2011)	SP, SPr, UF, CC, MS	RCC, RDC, CIC, DSC, CZ	C	MILP	DS, RE, CO	RtP	LA, TA, DRS	CPLEX
Hasani et al. (2011)	MP, MPr, UF, CC, MS	SC, MC, DC, CIC, RMC, CZ	C	MILP	DS, CO	RtP	LA, TA, I, DRS	LINGO
Vahdani et al. (2012)	SP, MPr, UF, CC, SS	SC, MC, DC, CIC, RYC ,CZ	C, O	MILP	CO	RtP, FuP	LA, TA	a hybrid solution method
Zeballos et al. (2012)	MP, MPr, CF, CC, MS	MC, DC, DAC, DSC, CZ	C	MILP	RE	StP	LA, TA, I, DRS	CPLEX
Pishvaee and Razmi (2012)	SP, SPr, UF, CC, MS	MC, CIC, DSC, RYC, CZ	C, O	MILP	DS, RE, CO, Ot	FuP	LA, TA	Interactive fuzzy solution approach
Ramezani et al. (2013)	SP, MPr, CF, CC, SS	SC, MC, DC, CIC, RMC, DSC, CZ	C, O	MILP	DS, RE, CO	StP	LA, TA, FC	CPLEX
Amin and Zhang (2013)	SP, MPr, UF, CC, MS	MC, CIC, DSC, CZ	C, O	MILP	DS, RE	StP	LA, TA	CPLEX
Keyvanshokooh et al.(2013)	MP, MPr, UF, CC, SS	MC, DC, CIC, RCC, DSC, CZ	C	MILP	-	DP	LA, TA, PA, FC, I, P	CPLEX
Devika et al. (2014)	SP, SPr, UF, CC, SS	SC,MC,DC,CIC,RMC,RYC,RCC, DSC	C, O	MILP	-	DP	LA,TA,PA	Imperialist competitive algorithm & variable neighborhood search algorithm
Soleimani and Govindan (2014)	SP, MPr, UF, CC, MS	MC, RDC, CIC, DSC, RYC	C	MILP	DS, RE, CO	StP	LA, TA	CPLEX
Faccio et al. (2014)	SP, MPr, UF, CC, MS	MC, DC, RDC, RMC, DSC, CZ	C	MILP	-	DP	LA, TA, I, TM	Commercial Solver
Zeballos et al. (2014)	MP, MPr, UF, CC, MS	SC,MC,DC, CIC, DAC, RPC, DSC, CZ	C	MILP	DS	StP	LA, TA, I, TM	CPLEX
This paper	MP, SPr, UF, CC, MS	MC, DC, CIC, RMC, DSC, CZ	P	MILP	DS,RE,CO	StP, RtP	LA, TA, I, PA, FC	Accelerated stochastic Benders Decomposition with Pareto-optimal cuts

Appendix B. Scenario generation and reduction algorithm

Scenario Generation & Reduction Algorithm

Nomenclature:

NS : The number of constructed scenarios in each time period

NS_{target} :The desired number of reduced scenarios

NF: The number of flows among each pair of facility types

Sc_t :The set of scenarios in time period *t*

$\Pr_{tj}$: The probability of generated scenario *j* in time period *t*

DSc_t : The set of scenarios that is deleted in time period *t*

Dis_{ijt} : The Euclidean distance between scenario *i* and *j* in time period *t*

Begin

For *each time period* $t \in T$

Step 1. Generate $\varepsilon_{t,f}^{s}\left(f=1,2,...,NF;\ s=1,2,...,NS\right)$ using the LHS method as follows:

i. P is an $NF \times NS$ matrix in which each row is a random permutation of *1,..,NS*.

ii. R is an $NF \times NS$ matrix generated randomly using uniform distribution *(0,1)*.

iii. $G = \frac{1}{NS}(P-R)$.

iv. $\hat{\varepsilon}_{t,f}^{s} = F_{\varepsilon}^{-1}(G_{fs})\ \left(f=1,2,...,NF;\ s=1,2,...,NS\right)$ where $F_{\varepsilon}^{-1}()$ is inverse cumulative distribution function of ε.

Step 2. Construct transportation cost scenarios.

If (t>1) then

a. Construct the set Sc_t by $NS \times NS_{\text{target}}$ scenarios with the help of equation (57) using $\varepsilon_{t,f}^{s}\left(f=1,2,...,NF;\ s=1,2,...,NS\right)$ and Sc_{t-1} as the available transportation costs from previous periods.

b. Calculate the probability of $\Pr_{tj} = \frac{\Pr_i}{NS},\ (i-1)\times NS+1 \le j \le NS\times i,\ i=1,..,NS_{\text{target}}$

Else

a. Construct *NS* transportation cost scenarios for the time period 1 using equation (56).

b. Calculate the probability of $\Pr_{1j} = \frac{1}{NS},\quad j=1,..,NS$

End if

Step 3. Backward scenario reduction method:

i. Define Sc_t as a set of all initial scenarios at time period *t* and DSc_t is a null set.

ii. **While** ($|Sc_t| > NS_{\text{target}}$)

a. $Dis_{ijt} = \sqrt{\left(\sum_{f=1}^{NF}\sum_{p=1}^{t}\left(c_{p,f}^{i} - c_{p,f}^{j}\right)^2\right)},\ i=1,..,|Sc_t|,\ j=1,..,|Sc_t|$

b. $MDis_{s,t} = \min_{s'=1,..,|Sc_t|} \{Dis_{ss't}\} \quad \forall s = 1,..,|Sc_t|, \forall t \in T$, and then find the scenario index *r* that has the minimum distance with scenario s.

c. Calculate $PS_{s,t} = \Pr_s \times MDis_{s,t} \quad \forall s = 1,..,|Sc_t|, \forall t \in T$

d. Find the scenario index *d* such that $PS_{d,t} = \min PS_{s,t} \quad \forall s = 1,..,|Sc_t|, \forall t \in T$

e. $Sc_t = Sc_t - \text{scenario}(d), DSc_t = DSc_t + \text{scenario}(d), \Pr_{t,r} = \Pr_{t,r} + \Pr_{t,d}$

End while

End for

End

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