A stochastic mathematical program with complementary constraints for market-wide power generation and transmission expansion planning

by

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ABSTRACT

Abstract—In the restructured electricity markets, the generators and the Independent System Operator (ISO) play important roles in the balance of electricity supply and demand. We consider a mixed integer bi-level model reformulated as a mathematical program with complementary constraints (MPCC) in which a single conceptual leader decides the transmission line expansion plan and generators plan for generation capacity expansion in the upper level. The overall objective is to maximize the total social welfare, which consists of buyer surplus, producer surplus and transmission rents. In the lower level, generators will maximize their operational profits by interaction with the ISO to decide their generation amounts. Meanwhile, the lower-level objective of the ISO is to maximize the social welfare by dispatching the electricity to satisfy demand and set the locational marginal prices (LMPs). Reformulating the complementarity constraints with binary variables results in a mixed integer program that can be solved to global optimality. However in reality, the demand and fuel cost will fluctuate with uncertainties such as climate change or natural resource limitations. A moment matching method for scenario generation can capture the uncertainties by producing a scenario tree. Then we combine the scenario tree with the mixed integer program to obtain a two-stage stochastic program where the first stage corresponds to the upper level investment decisions and the second stage represents the lower level operations. The extensive form of the stochastic program cannot be solved in our numerical example within a reasonable time limit. To reduce the computation time, a scenario reduction algorithm is applied to select fewer scenarios with properties similar to the original scenarios. Finally we solve the stochastic mixed-integer program with the Progressive Hedging Algorithm (PHA), which is a scenario-

based decomposition heuristic. We compare the results of the stochastic program and a deterministic optimization using expected values. The capacity expansion plan obtained with the stochastic program has higher expected social welfare than the expected value solution. The stochastic program yields a solution that hedges against uncertainty by lower generation expansion levels and fewer transmission lines to be built.

CHAPTER 1 OVERVIEW

1.1 Introduction

A reliable, reasonably priced supply of electricity is essential to the quality of life for residents and industries. The supply of electricity is also the basis of a region's economy. Without it, factories and business cannot function normally. Electricity is not only a basic necessity, but it is also regarded as a product that can be produced, sold, and transported for the profits of the generation companies. Similar to most commodities, electricity is sold at both wholesale and retail levels. The main differences from usual commodities are its lack of economical storage and physical constraints that govern its transmission.

Unlike other common energy sources such as fossil fuels, electricity must be used as it is being generated, or converted immediately into another form of energy. Although energy-storage technologies are being developed for offering wide ranges of power density and energy density, no single energy-storage technology has the capability to support enormous demand currently. In the future, the systems may be comprised of technologies such as electrochemical super capacitors, flow batteries, lithium-ion batteries, superconducting magnetic energy storage and kinetic energy storage [1]. Moreover, delocalized electricity production and different energy resources increase the difficulty of stabilizing the power network. Hence, electricity is difficult to store in the bulky and costly equipment [2]. Under these circumstances, how to regulate the power system over time is crucial in our modern society.

Further, the long lead-time required to expand generation and transmission capacity requires long-term planning that takes uncertainties into account. In addition to providing a sustainable power network, long-term capacity expansion planning significantly influences the development of market operations in short-term decision making. The decisions of expansion planning will determine our behavior of utilizing electricity for decades. Therefore generation and transmission expansion planning should be carefully designed to satisfy future demand.

In the early 1990s, most electric utilities in the U.S. owned the transmission lines and generation resources at the same time. They made all decisions concerning electricity production and distribution. However, the wholesale electricity market restructuring changed the organizational structure of the power provider from vertically integrated into different organizations, each organization with a separate function to maintain the balance of the market. The motivation of market design was to create an environment for competition in the electric power industry. Competition decreases the market power of each generation company. However the electricity market still needs coordination in another way to increase the social welfare as a new perspective. The independent system operator (ISO) has resulted for this purpose [3]. The ISO coordinates, controls and monitors the operation of the power system to maintain the reliability and economic benefits of the electricity network. But the ISO cannot build transmission lines or power plants on its own. The ISO is a non-profit organization.

The supply network for an electricity market includes the ISO and individual generation companies. The task of taking generation capacity investment and transmission expansion decisions has become an even more complex problem for the liberalized market because of the uncertainty of the competition. One of the methods to analyze the strategic behavior of generation competitors is game theory [4]. Game theory describes the simultaneous behavior of each generation company, whose goal is to maximize its own profit. The competition can be formulated as an equilibrium problem with equilibrium constraints. Under the framework of game theory, we can integrate generation and transmission expansion decisions with operational decisions among competitive generation companies. In economics we focus on the equilibrium behavior only. A market's equilibrium is a useful guide for its behavior [5].

Long-term planning is subject to uncertainties in the electricity network. To develop an expansion plan that can be applied in the changed electric power industry environment is important and practical. Two important uncertain factors in the planning procedure are demand and fuel costs [6].

The forecast of electrical demand is one of the important factors in a generation system analysis. The U.S. Energy Information Administration (EIA) projects the total electricity demand in the U.S. to grow by 28 percent (0.9 percent per year), from 3,839 billion kilowatthours in 2011 to 4,930 billion kilowatthours in 2040 [7]. Electricity suppliers must invest in new generation units and transmission lines to ensure the reliability of the electricity network. New power plants will be constructed to keep up with the increasing demand, and will require transmission lines to convey power to the areas where the energy is required. The ISO is responsible for transmission line planning [8].

For generation companies, the production cost mainly consists of fuel cost such as coal, natural gas and nuclear. The price of fuel is critical to decide the price of electricity and it fluctuates with uncertainties such as limitation of natural resources, economy and weather. For example, the fuel cost of coal-fired generation accounts for 45 percent of total levelized cost at a 5 percent discount rate. The fuel cost of gas-fired generation accounts for nearly 80 percent [9] of levelized cost on average. However those fuel costs fluctuate with some factors of uncertainty, the electricity wholesale price changes accordingly. Meanwhile, transmission congestion will affect the balance of supply and demand in the electricity market. The ISO will dispatch the power flow to maintain the reliability of the electricity market. Also the responsibility for the ISO is to set the locational marginal price (LMP) in each area. LMPs, defined as the least cost to serve the next increment of demand with power system operation constraints, reflect the value of energy at different locations.

1.2 Problem Statement

Considering both the investment and operational decision making, a mixed integer bi-level program model for capacity expansion in the integrated supply network for an electricity market was developed [10]. The upper level leader decides how to expand the capacity of generation and transmission expansion. Once the capacity expansion decisions are made, lower level decision makers make their optimal operating decisions toward their objectives.

However, the model does not consider uncertainty for the lower level problems in the decision making. In this thesis, we consider the problem of how to incorporate the uncertainty in the form of a two-stage stochastic program. Because the deterministic optimization problem is formulated with known parameters, it almost invariably includes some unknown parameters in the real world. We are intrigued by investigating the decisions obtained by a stochastic program compared with those from the deterministic model. We also address the problems of how to apply an appropriate method to generate scenarios for stochastic program. The future uncertainties are represented by different future scenarios. If the number of scenarios is too large, we need to apply an appropriate scenario reduction method to decrease the number of scenarios to become solvable. Due to the large problem that results, we adopt a scenario-based decomposition method to solve the stochastic program.

1.3 Thesis Structure

In Chapter 2, we review the literature on methodologies to solve the generation and transmission expansion problem. Also we introduce the scenario generation method and scenario reduction algorithm for capturing the uncertainty. The methodologies for solving stochastic programs and stochastic MPECs are discussed at the end. Chapter 3 contains the process of building a two-stage stochastic program as well as notation used in our model. Then scenario generation and reduction methods are introduced. PHA is also described in this chapter. In Chapter 4, our model is applied to a case study based on the New England electric power system. In Chapter 5, a comprehensive summary of the thesis is made and limitations of the model and case study are mentioned.

CHAPTER 2 LITERATURE REVIEW

2.1 Methodologies for Generation and Transmission Expansion Planning

Generation expansion and transmission planning have been discussed extensively in the past few years. As the structure of the electricity market was reformed, mathematical programs have been developed greatly for model formulation [11]. In the restructured electricity market, generation companies submit bids to supply electricity at prices based on the fuel cost. The ISO manages the electricity transmission and sets the Locational Marginal Prices (LMP) to match supply with demand. Uncertainties become a key factor in generation and expansion planning. To capture the uncertainty of demand, reference [6] introduces the application of stochastic models in the generation expansion. A scenario-based multi-objective transmission line expansion planning model is introduced by [12]. Fuel and carbon price risk will impact the long-term investment decisions. However, the expansion planning procedure does not account for the behavior of competition among generators in the electricity market.

Equilibrium models are more suitable for describing competitive behaviors in long-term planning [13]. In addition to the uncertainty of cost and demand, the behavior of the electricity producers and consumers must be taken into account in the competitive market. Game theory is generally applied to describe the competitive environment for strategic decision making firms. All firms compete to offer generation services at a price set by the ISO, as a result of the interaction of all of them. The goal of market design is to create an efficient electricity market. Efficiency means the output is produced by the cheapest supplier and is consumed by the consumer most willing to pay . The ideal electricity generation amount is optimal for both supply and demand [5]. Under a particular market design, we can derive a competitive equilibrium which is efficient in the electricity market [14]. The planner seeks for overall benefits to the electricity producers and buyers. In [15], the author provides a stochastic framework for evaluating the investment decisions and integrating scenarios into a single model with security criteria and illustrates how the results from stochastic models differ from the deterministic model perspective.

Game theory is a mathematical way to describe such strategic decision making behavior. The mathematical model is formulated with equilibrium constraints. One common economic model, Cournot competition, is applied to describe the competition among generation companies. It is a necessary step beyond the monopoly model and explains the role of market share in the determination of market power [5] and also it allows for convenient calculation. A consensus seems to have emerged that considering generators as Cournot competitors is appropriate in the restructured electricity market. However, Yao et al. [16] assume the generation companies do not anticipate the impact of their production decisions on congestion charges. A collection of models which incorporated game theory is discussed in [17] and one of them is a Cournot model that includes investments in new generation capacity. The competition concerning generation capacity is formulated by its own Mixed Linear Complementarity Problem (LCP). Mathematical Programs with Equilibrium Constraints (MPEC) is used in solving expansion planning for electricity markets. A bi-level formulation is introduced for long-term generation capacity investment decisions considering uncertainty of the investments of other generation companies. The bi-level model is formulated as an MPEC and transformed into a Mixed Integer Linear Program (MILP) [18].

Transportation of fuel to the integrated supply electricity market is considered by [10, 19, 20]. The authors construct a mixed integer bi-level programming model for fuel supplier, the ISO and individual generation companies. The fuel supplier delivers the fuel to the generation company considering the transportation cost. The ISO sets the LMP and allocates the power flow in the transmission network. As for the generation companies, they purchase the fuel from the fuel supplier and decide the amount of generation [19]. The deterministic model is a Mathematical Program with Complementary Constraints (MPCC). The authors of [10] provide a reformulation of the problem to obtain the global optimal solution with binary variables.. In this thesis, we will modify this model to consider uncertainty under the framework of the integrated electricity market.

2.2 Scenario Generation and Reduction Methodologies

As mentioned before, uncertainty is a principal factor in the generation and transmission planning. A stochastic program results from capturing the uncertain parameters of deterministic model as probabilistic scenarios.. The most common approach to scenario generation is to directly sample from a specific distribution. But the large scale of sampling could generate time and cost consuming issues. By using a specified marginal distribution and correlation matrix one can approximate the original distribution for sampling [21]. A method of estimating a scenario tree approximation to a stochastic process is presented by minimizing the distance of objective function [22]. From the statistical perspective, [23] formualtes a decision model for generating scenarios with internal sampling or finding a simple discrete approximation of the given distribution that could be used as model input . The basic idea is to minimize the distance between computed and specified statistical specifications. This method is called moment matching. Scenario generation methods are illustrated by [24] including sampling from specified marginal and correlations, path-based method, optimal discretization and moment matching method.

In this thesis, we would like to explore an approximation for the probability distribution with discrete distribution. One of the heuristics is presented by [25] that generates a discrete joint distribution corresponding to specified statistical specifications such as the first four marginal moments and correlations. The advantage of the method is not to consider the exact probability distribution of the stochastic parameters.

A large number of scenarios might limit the tractability of solution. Several scenario reduction methods have been developed during the past years. Forward selection and backward reduction algorithms are two common scenario reduction methods. In power management, scenario reduction algorithm is applied for solving computational complexity and time limitation. A smaller number of scenarios is selected with redistributed probability which improves the efficiency of computation time without losing significant difference between reduced scenarios and original scenarios. Through a scenario tree construction algorithm and scenario reduction algorithm, the stochastic parameters can be well approximated [26]. We will adopt forward selection as our scenario reduction algorithm because the number of reduced scenarios is small enough to compute.

2.3 Methodologies for Solving Stochastic Programs

After we obtain the scenarios from a scenario generation method, we incorporate these uncertainties into deterministic model. The deterministic model becomes s stochastic program. Stochastic program are frequently applied in long-term planning. A common methodology for solving multi-stage stochastic programs for energy planning uses duality in Benders decomposition to derive a piecewise linear function to approximate the expected cost function [27].

Including a large number of realizations in the extensive form of a stochastic program might make it too large to solve in finite time or resources. Benders decomposition can improve the performance of solving stochastic program [28]. On the other hand, the Progressive Hedging

Algorithm (PHA) can be applied for solving mixed integer multistage stochastic program [29]. In PHA, decomposition is used to divide the problem into smaller and more manageable sub-problems. When the first stage decisions have converged to within an acceptable tolerance interval, the optimal solution is be obtained. Progressive Hedging aggregate the solutions of scenario problems with modified cost function to progressively cause the probability-weighted average solution to become feasible and optimal eventually.

2.4 Stochastic MPECs for Long-term Energy Planning

The electricity market modeling can be classified into optimization problem for one firm, market equilibrium for all firms and simulation model. A stochastic model is developed for a single firm optimization model in [13]. A bi-level model is built to assist a generation company for its long-term generation capacity investment decisions. In the upper level, the objective is to choose the investment decisions to maximize expected profits. The lower level is constructed by equilibrium through a conjectured-price response formulation. To include uncertainty in future demand and the investments decisions of other generation companies, the bi-level problem can be solved as a linear mixed integer bi-level program which is converted to an equivalent single-level mixed integer problem [30]. A similar approach is applied in solving the medium-term decision problem faced by power retailer. The uncertainty includes future pool price, client demand and rival retailer price [31]. A strategic producer making decisions on generation investment is represented through a bi-level model with market clearing. The large scale mixed integer linear program is solved by a branch-and-cut method considering with demand variations [17]. Wind energy is discussed extensively in recent years. The authors of [32] consider wind power investment and transmission planning with MPEC. The goal is to identify the optimal wind projects and network improvement. The stochastic MPEC can be reformulated as a mixed-integer linear program solved also by branch-and-cut method. The authors take the production variability and uncertainty of wind facilities, future decline in wind power investment cost, and financial risk into account. They propose a risk-constrained multi-stage stochastic program with MPCC in [33]. Hence the stochastic bi-level model can be solved by a reformulation technique.

2.5 Summary

Long–term capacity planning in restructured electricity markets has been addressed recently in terms of mixed integer bi-level programming models. The upper level decision maker will decide the investment in generation capacity and transmission expansion first. In the lower level, the generators will decide the optimal generation amount and the ISO will dispatch the supply to meet the demand under a game theoretical model. The equilibrium solutions are derived among the interactions with generation companies and the ISO. Fuel suppliers are considered in the integrated electricity market. With the transformation of MPCC for bi-level model, the model can be solved in either a nonlinear programming reformulation or a binary variable reformulation [10, 14, 20].

However, in [10], the uncertainty in the long-term capacity planning is not considered in the equilibrium model. Uncertainty will affect the outcome of planning over long time period. In [18, 30, 31], the stochastic MPCC is discussed for long-term investment planning. The uncertainty of price, demand and investment decisions of other generation company are included. The uncertainties in fixed-demand levels and fuel costs have not been considered previously in stochastic MPCC models. Our thesis is focused on the combination of bi-level programming and stochastic programming by binary variable reformulation to obtain the optimal solutions under uncertainty.

CHAPTER 3 METHODS AND PROCEDURES

In this chapter, we present the modified capacity expansion planning model in deterministic form based on [10]. Meanwhile we also adopt the binary variable reformulation of complementarity constraints from [10] in order to achieve global optimality. Then we incorporate uncertainty concerning fixed-demand levels and fuel costs to convert the deterministic model into stochastic program. According to the information about demand and fuel cost, we apply a scenario generation algorithm to generate possible scenario outcomes. In the case study, the number of scenarios is too large to allow the solution of the stochastic program in a reasonable amount of time. To achieve tractability, we implement a scenario reduction algorithm to decrease the number of scenarios so that we can compute a solution with reasonable resources. Finally, the progressive hedging algorithm (PHA), a scenario-based decomposition heuristic, is used for solving the mixed integer stochastic program. The model formulation and notation are summarized in Section 3.1. The deterministic model and its MPCC reformulation are illustrated in Section 3.2 and Section 3.3, respectively. The two-stage stochastic program is formed in Section 3.4. Also the scenario generation and reduction algorithms are introduced in Section 3.5. Finally the PHA we use to solve the model is presented in Section 3.6.

3.1 Model Formulation and Notation

3.1.1 Model Formulation

First, we build the deterministic model formulation based on [10] which is a bi-level model for a conceptual leader making capacity expansion decisions in the upper level while generation companies and the ISO search for their own optimal solutions in the wholesale market in the lower level. We also modified the model in [10] to add the fixed-demand level into consideration as in

[19] to ensure the reliability in the regional electricity market. Moreover, different types of generation technologies are considered in our model to help us understand how to allocate capacity in an appropriate portfolio. The time frame in our model is also different from [10]. The uncertainties in different seasons are included in our model to illustrate how the seasonal variations affect our decisions. For the convenience of calculation, the model is based on a weighted average hour across seasons. All of the expansion costs are estimated on an hourly basis.

Second, the bi-level model becomes a MPCC upon replacing the lower level optimization problem with its Karush-Kuhn-Tucker (KKT) conditions, and then the binary variable reformulation is introduced to convert the MPCC to a mixed integer program (MIP) that can be solved to global optimality [10].

Finally, uncertainties of fixed-demand level and fuel cost are included in the form of probabilistic scenarios, which converts the MIP to a two-stage stochastic program. In the first stage, the conceptual leader makes the generation and transmission expansion decisions. In the second stage, the ISO and generation companies will react to the expansion decisions by maximizing their own objective functions in different scenarios.

3.1.2 Notation

Sets

N : *Electricity nodes in the power network, indexed by i, j*

Tr : *Transmission lines from node i to node j, indexed by ij*

S: Scenarios, indexed by s

T: *Time periods in second stage, indexed by t*

G: The set of technologies for power plants, indexed by g

Upper level decision variables:

$nV_{i,g}$ $Generation\ capacity\ after\ expansion\ for\ generator\ i\ with\ technology\ g\ (MW)$

z_{ij} $Binary\ decision\ variable\ for\ new\ transmission\ line\ from\ node\ i\ to\ node\ j$

Lower level decision variables:

q_i^t $Demand\ satisified\ at\ electricity\ node\ i\ in\ period\ t\ (MWh)$

θ_i^t $Voltage\ angle\ at\ electricity\ node\ i\ in\ period\ t$

f_{ij}^t $Electricity\ flow\ on\ transmission\ line\ from\ node\ i\ to\ node\ j\ in\ period\ t\ (MW)$

$y_{i,g}^t$ $Energy\ generated\ by\ generator\ i\ with\ technology\ g\ in\ period\ t\ (MW)$

η^t $Price\ at\ the\ reference\ electricity\ node\ in\ period\ t\ (\$/MWh)$

ϕ_i^t $Nodal\ electricity\ price\ premium\ at\ electricity\ node\ i\ in\ period\ t\ (\$/MWh)$

Parameters

a_i $Intercept\ of\ electricity\ demand\ price\ at\ node\ i\ as\ a\ linear\ function\ of\ quantity$

b_i $Slope\ of\ electricity\ demand\ price\ at\ node\ i\ as\ a\ linear\ function\ of\ quantity$

p_i^t $The\ locational\ marginal\ price\ of\ node\ i\ in\ period\ t\ (\$/MWh)$

$gc_{i,g}$ $Investment\ cost\ for\ generation\ expansion\ in\ generator\ i\ (\$/MW/h)$

tc_{ij} $Investment\ cost\ for\ transmission\ line\ expansion\ for\ line\ ij\ (\$/MW/h)$

θ^{max} $\quad Maximum\ value\ for\ voltage\ angles$

θ^{min} $\quad Minimum\ value\ for\ voltage\ angles$

$V_{i,g}$ $\quad Generation\ capacity\ at\ generator\ i\ with\ technology\ g\ (MW)$

K_{ij} $\quad Capacity\ of\ transmission\ line\ ij\ (MW)$

B_{ij} $\quad Negative\ susceptance\ of\ transmission\ line\ ij\ (\Omega^{-1})$

r *Discount rate*

n_t *Weight parameter for period t in one scenario*

$U_{j,g}$ *Upper bound for generation level in state j with technology g*

Scenario Parameters

$c_{i,g}^{s,t}$ $\quad Fuel\ cost\ at\ generator\ i\ with\ technology\ g\ in\ period\ t\ (\$/MWh)$

$L_i^{s,t}$ $\quad Fixed\ demand\ level\ at\ node\ i\ in\ period\ t\ (MW)$

$prob^s$ $\quad Probability\ that\ scenario\ s\ occurs$

3.2 Deterministic Model

First we will introduce the deterministic model of the electricity supply network from [10] omitting the fuel supplier considered in that paper. In the upper level, a conceptual leader decides the expansions with different types of technology for generators and transmission line owner. The model provides an expansion plan guideline from a global perspective. The expansion of transmission line is determined by introducing binary variables.

At the lower level, the ISO and generators will seek for optimal solutions for each own objective function at the same time. The existence of an equilibrium is proved by [14] in the electricity network model. The ISO is responsible for maintaining the balance of the electric network that generators will satisfy at least the fixed-demand level. Meanwhile the goal for ISO is to maximize the total social welfare and for generators is to maximize its own profit.

The total social welfare is comprised of the total buyer surplus, producer surplus and transmission rents. Welfare measures are based on the prices and quantities of demand satisfied at each node [20]. Producer surplus at node *j* in period *t* is defined as the profit less the generation cost in (1).

$$PS_j^t = p_j \sum_g y_{j,g}^t - \sum_g c_{j,g}^t y_{j,g}^t \tag{1}$$

The classic tool for measuring welfare change is buyer's surplus. The buyer surplus *BS* is defined as the area to the left of the demand curve between prices associated with a price movement [34]. . The consumer demand curve measures how much the consumer is willing to pay. The difference between the maximum willingness to pay and what the consumer actually pays or a given quantity is the buyer's surplus [5]. Buyer surplus is an important criterion to measure market efficiency. If the sum of profit and buyer surplus is maximized, the market is efficiently operating. The buyer surplus is shown in Fig 1. Therefore for each buyer at node *j* in period *t* it is computed as in (2).

$$BS_j^t = \int_{L_j}^{q_j} (a_j + b_j s)ds - (a_j + b_j q^t{}_j)(q^t{}_j - L^t{}_j) + (a_j + b_j L^t{}_j - a_j - b_j q^t{}_j)L_j = -\frac{1}{2} b_j (q_j^{t^2} - L_j^{t^2}) \tag{2}$$

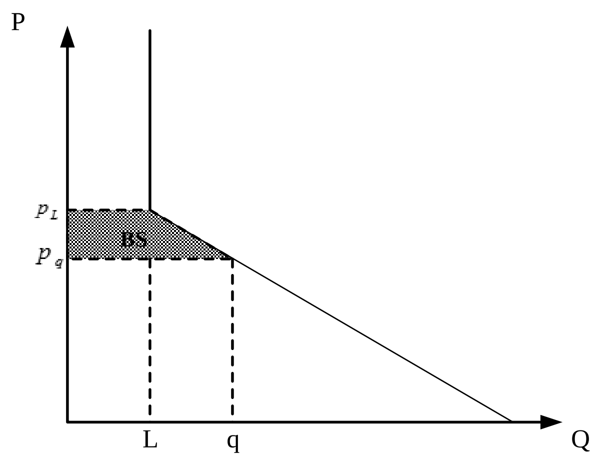


Fig.1. Buyer surplus

The amounts of transmission rents are the total transmission charges based on nodal price difference multiplied by the power flow on the line. The total transmission rents *TTr* in period *t* are defined in (3).

$$TTr_t = \sum_{ji \in L} f^t{}_{ji}(p^t{}_i - p^t{}_j) \tag{3}$$

The goal of the ISO is to maximize the welfare, which is the total consumer willingness-to-pay less the sum of all the generation costs. It is equivalent to the summation of consumers' surplus, producers' surplus, and transmission rents. Given $\sum_i f^t{}_{ji} = y^t{}_j - q^t{}_j$, the power flow from node *j* is equal to the generation amount less the demand. The total social welfare *SW* in period *t* can be represented as in equation (4).

$$SW_t = \sum_j (p^t{}_j \sum_g y^t{}_{j,g} - c^t{}_{j,g} \sum_g y^t{}_{j,g}) + \sum_j (-\frac{1}{2} b_j q_j^{t^2} + \frac{1}{2} b_j L_j^{t^2}) + \sum_i \sum_j (\sum_g y^t{}_{j,g} - q^t{}_j)(p^t{}_i - p^t{}_j) \tag{4}$$

Because the total demand to be satisfied must equal the total amount of generation for the balance of the electricity market, it implies $\sum_j q_j^t = \sum_{j,g} y_{j,g}^t$ and we use it to recalculate the transmission rent in (3) as (5).

$$TTr_t = \sum_j \sum_i (\sum_g y^t{}_{j,g} - q^t{}_j)(p^t{}_i - p^t{}_j) = \sum_j (p^t{}_j q^t{}_j - p^t{}_j \sum_g y^t{}_{j,g}) + \sum_j (\sum_g y^t{}_{j,g} - q^t{}_j) \sum_i p^t{}_i$$
$$= \sum_j (p^t{}_j q^t{}_j - p^t{}_j \sum_g y^t{}_{j,g})$$

(5)

Finally the total social welfare is (2) + (3) + (5) which is derived in equation (6).

$$SW_t = \sum_j (\frac{1}{2} b_j q_j^{t^2} + a_j q^t{}_j + \frac{1}{2} b_j L_j^{t^2} - c^t{}_{j,g} \sum_g y^t{}_{j,g}) \tag{6}$$

Upper level

The objective function (7) of the upper level includes the total social welfare less the generation and transmission expansion cost with the constraint (8) that the new generation capacity level is greater than or equal to the original capacity level. The total social welfare is computed by a weight parameter multiplying total social welfare in each time period t. For generation expansion and transmission expansion decisions, we assume the decisions are based on a single hour. The generation expansion decision variables are assumed to be continuous variables. The transmission line expansion decisions are assumed to be binary variables. It is consistent for us to compare the investment decisions in the first stage and operation decisions in the second level.

$$\max_{nV,z} \sum_{t} n_t \, [\sum_{j\in N} (\frac{1}{2} b_j {q^t_j}^2 + a_j \; q^t_j + \frac{1}{2} b_j {L^t_j}^2) - \sum_{g\in G} \sum_{j\in N} c^t_{j,g} y^t_{j,g}]$$

$$- \sum_{g\in G} \sum_{j\in N} gc_{j,g} (n V_{j,g} - V_{j,g})$$

$$- \sum_{ij\in Tr} tc_{ij} \, K_{ij} z_{ij} \tag{7}$$

$$V_{j,g} \leq nV_{j,g} \leq U_{j,g} \qquad \forall \, j \in N, \forall \; g \in G \tag{8}$$

Lower level

In the lower level, a Cournot model is adopted to formulate the strategic behavior of generation companies. Based on the fuel cost, generators decide the generation amount to inject into the electricity market to compete with each other. Because the total electricity generation amount would affect the LMP in different nodes, [10] assumes that the generators decide the LMP at the reference node. The ISO behaves similarly as in a Bertrand model to set price premia relative to the reference node [10]. The price premia are regarded as constants by each generator. In [14], the authors already proved an equilibrium exists in the restructured electricity market under these assumptions. Our model continues using these assumptions in the model formulation on an hourly basis.

<u>ISO's decision problem</u>

$$\operatorname*{Max}_{q,\theta,f} \sum\nolimits_{j\in N} (\tfrac{1}{2} b_j {q^t_j}^2 + a_j \; q^t_j + \tfrac{1}{2} b_j {L^t_j}^2) \tag{9}$$

$$s.t. \quad q^t_j + \sum\nolimits_i f^t_{ji} - \sum\nolimits_i f^t_{ij} = \; \sum\nolimits_g y^t_{g,j} \qquad \forall j \in N, t \in T \quad [p^t_j] \tag{10}$$

$$\theta_j^t \leq \theta^{max} \qquad \forall j \in N, t \in T \qquad [\alpha_j^{t+} \geq 0] \tag{11}$$

$$-\theta_j^t \leq -\theta^{min} \qquad \forall j \in N, t \in T \qquad [\alpha_j^{t-} \geq 0] \tag{12}$$

$$f_{ij}^t - B_{ij}\left(\theta_i^t - \theta_j^t\right) - (1 - z_{ij})M_o \ \leq 0 \ \ \forall ij \in Tr, t \in T \qquad [\gamma_{ij}^{t+} \geq 0] \tag{13}$$

$$B_{ij}\left(\theta_i^t - \theta_j^t\right) - f_{ij}^t - \left(1 - z_{ij}\right)M_o \ \leq 0 \ \ \forall ij \in Tr, t \in T \qquad [\gamma_{ij}^{t-} \geq 0] \tag{14}$$

$$f_{ij}^t \leq z_{ij}K_{ij} \qquad \forall ij \in Tr, t \in T \qquad [\lambda_{ij}^{t+} \geq 0] \tag{15}$$

$$-f_{ij}^t \leq z_{ij}K_{ij} \qquad \forall ij \in Tr, t \in T \qquad [\lambda_{ij}^{t-} \geq 0] \tag{16}$$

$$L_j^t \leq q_j^t \qquad \forall j \in N, t \in T \qquad \left[\zeta^t{}_j \geq 0\right] \tag{17}$$

For the ISO, the objective (9) is to maximize the total social welfare by dispatching the power flow in order to match the supply with demand [10]. We only consider the objective function related to the ISO's decision variables from total social welfare. The variable in brackets after each constraint represents the dual variable of the constraint. For each node, the sum of net injections and load will equal the generation amount. The constraint (10) is the flow balance equation. Locational marginal price (LMP) is defined as the least cost to serve the next increment of demand with power system operating constraints [35].The dual variable p_j is the LMP at node j. The voltage angle in Direct Current Optimal Power Flow (DCOPF) model has limitations [35]. The two constraints (11) and (12) are the upper and lower bounds on voltage angle. Constraints (13) and (14) represent the physical characteristics of transmission grids, in terms of a lossless linearized direct current approximation. For each transmission line, the thermal capacity is bounded for the power flow. The two constraints (15) and (16) are the limitations of capacity of each transmission line. Constraint (17) is the relationship between the fixed-demand level and satisfied demand. The satisfied demand must be greater than or equal to the fixed-demand level.

Generator i's decision problem

$$\min_{y_{i,g}^t \geq 0, \eta^t} \; \sum_g (\eta^t + \phi_i^t - c_{i,g}^t)\, y_{i,g}^t \tag{18}$$

$$s.t. \qquad \sum_g y_{i,g}^t - \eta^t \sum_j \frac{1}{b_j} = \sum_j \frac{\phi_j^t - \zeta_j^t - a_j}{b_j} - \sum_g \sum_{j \neq i} y_{j,g}^t \quad \forall i \in N, \quad [\beta^t{}_i] \tag{19}$$

$$y_{i,g}^t \leq nV_{i,g} \quad \forall\, i \in N, g \in G, t \in T \qquad [\mu_{i,g}^t \geq 0] \tag{20}$$

$$y_{i,g}^t \geq 0 \quad \forall\, i \in N, g \in G, t \in T \tag{21}$$

The objective function (18) is the LMP less the fuel cost times the generation amount which is defined as the profit of generator. The LMPs in other nodes are defined as the LMP of reference node plus the premia decided by ISO [10]. ISO is still the price setter by making decision on price premium. The reference bus price is the decision of all generation companies from the competition on production quantity [16]. The equation (19) represents the balance of total demand and total generation in terms of the residual demand. We can derive it from the ISO's KKT conditions [19]. The constraint (20) shows the relationship of the electricity generated amount less than or equal to the generation expansion level. The deterministic model is formulated from (1) – (21).

In the next section, we reformulate the deterministic model as a MPCC. It can be converted into MPCC-Nonlinear Program reformulation (MPCC-NLP), Single-Level Mixed Integer Quadratic Program (1-Level MIQP) or MPCC-Binary Variables Reformulated Mathematical Program (MPCC-BIN). We will adopt MPCC-BIN to reformulate the problem which guarantees global optimality [10].

3.3 Mathematical Program with Complementary Constraints

In the bi-level model, the lower level optimization can be reformulated equivalently in terms of complementarity constraints by applying the KKT conditions to each player's optimization problem [10]. The transformation can change the original deterministic mathematical program into an equivalent Mathematical Program with Complementarity Constraints (MPCC) with a mixed integer quadratic objective function [19].

The full set of constraints is as follows:

<u>ISO's problem</u>

$$a_j + b_j \; q_j^t - p_j^t + \zeta_j^t = 0 \qquad \forall j \in N, t \in T \tag{22}$$

$$\alpha_j^{t-} - \alpha_j^{t+} + \sum_{i,ji \in Tr} B_{ji} \left(\gamma_{ji}^{t+} - \gamma_{ji}^{t-} \right) - \sum_{i,ij \in Tr} B_{ij} \left(\gamma_{ij}^{t+} - \gamma_{ij}^{t-} \right) = 0 \quad \forall j \in N, t \in T \tag{23}$$

$$p_j^t - p_i^t - \gamma_{ij}^{t+} + \gamma_{ij}^{t-} - \lambda_{ij}^{t+} + \lambda_{ij}^{t-} = 0 \quad \forall ij \in Tr, t \in T \tag{24}$$

$$q_j^t + \sum_{ji \in L} f_{ji}^t - \sum_{ij \in L} f_{ij}^t = \sum_g y_{g,j}^t \;\; \forall j \in N, t \in T \tag{25}$$

$$0 \leq \theta_j^{max} - \theta_j^t \perp \alpha_j^{t+} \geq 0 \quad \forall j \in N, t \in T \tag{26}$$

$$0 \leq -\theta_j^{min} + \theta_j^t \perp \alpha_j^{t-} \geq 0 \quad \forall j \in N, t \in T \tag{27}$$

$$0 \leq B_{ij} \left(\theta_i^t - \theta_j^t \right) - f_{ij}^t + \left(1 - z_{ij} \right) M_o \perp \gamma_{ij}^{t+} \geq 0 \;\; \forall ij \in Tr, t \in T \tag{28}$$

$$0 \leq -B_{ij} \left(\theta_i^t - \theta_j^t \right) - f_{ij}^t + \left(1 - z_{ij} \right) M_o \perp \gamma_{ij}^{t-} \geq 0 \;\; \forall ij \in Tr, t \in T \tag{29}$$

$$0 \leq z_{ij} K_{ij} - f_{ij}^t \perp \lambda_{ij}^{t+} \geq 0 \quad \forall ij \in Tr, t \in T \tag{30}$$

$$0 \leq z_{ij} K_{ij} + f_{ij}^{t} \perp \lambda_{ij}^{t-} \geq 0 \quad \forall ij \in Tr, t \in T \tag{31}$$

$$0 \leq \zeta_{j}^{t} \perp -L_{j}^{t} + q_{j}^{t} \geq 0 \quad \forall j \in N, t \in T \tag{32}$$

<u>Generator's problem</u>

$$0 \leq \sum_{g} y_{j,g}^{t} \perp -\eta^{t} - \phi_{j}^{t} + c_{j,g}^{t} + \beta_{j}^{t} + \mu_{j,g}^{t} \geq 0 \quad \forall j \in N, g \in G, t \in T \tag{33}$$

$$\sum_{g} y_{j,g}^{t} - \sum_{i \in N} \frac{1}{b_{i}} \beta_{j}^{t} = 0 \quad \forall j \in N, t \in T \tag{34}$$

$$0 \leq \mu_{j,g}^{t} \perp nV_{j,g} - y_{j,g}^{t} \geq 0 \quad \forall j \in N, g \in G, t \in T \tag{35}$$

$$\mathrm{n}V_{j,g} \geq V_{j,g} \quad \forall j \in N, g \in G, t \in T \tag{36}$$

To solve the problem more efficiently and to obtain the global optimal solution, we converted MPCC into an equivalent mixed integer quadratic program by introducing binary variables κ and large parameters M [10]. Consider a generic complementary constraint r and e as follows:

$$0 \leq r \perp e \geq 0 \tag{37}$$

The reformulation of (37) is as follows:

$$0 \leq r \leq M\kappa \tag{38}$$

$$0 \leq e \leq M(1 - \kappa) \tag{39}$$

After the reformulation steps from (37)-(39), we establish a MPCC with binary variable reformulation model in (40)-(57) from previous constraints. The binary variable reformulation introduces integer variables in the lower level.

$$0 \le \theta_j^{max} - \theta_j^t \le M\kappa_{\theta_{\max}}^t \quad \forall j \in N, t \in T \tag{40}$$

$$0 \le \alpha_j^{t+} \le M\left(1 - \kappa_{\theta_{\max}}^t\right) \quad \forall j \in N, t \in T \tag{41}$$

$$0 \le -\theta_j^{min} + \theta_j^t \le M\kappa_{\theta_{\min}}^t \quad \forall j \in N, t \in T \tag{42}$$

$$0 \le \alpha_j^{t-} \le M\left(1 - \kappa_{\theta_{\min}}^t\right) \quad \forall j \in N, t \in T \tag{43}$$

$$0 \le B_{ij}\left(\theta_i^t - \theta_j^t\right) - f_{ij}^t + \left(1 - z_{ij}\right)M_o \le M\kappa_{\gamma_+}^t \quad \forall ij \in Tr, t \in T \tag{44}$$

$$0 \le \gamma_{ij}^{t+} \le M\left(1 - \kappa_{\gamma_+}^t\right) \quad \forall ij \in Tr, t \in T \tag{45}$$

$$0 \le -B_{ij}\left(\theta_i^t - \theta_j^t\right) - f_{ij}^t + \left(1 - z_{ij}\right)M_o \le M\kappa_{\gamma_-}^t \quad \forall ij \in Tr, t \in T \tag{46}$$

$$0 \le \gamma_{ij}^{t-} \le M\left(1 - \kappa_{\gamma_-}^t\right) \quad \forall ij \in Tr, t \in T \tag{47}$$

$$0 \le z_{ij} K_{ij} - f_{ij}^t \le M\kappa_{\lambda_+}^t \quad \forall ij \in Tr, t \in T \tag{48}$$

$$0 \le \lambda_{ij}^{t+} \le M(1 - \kappa_{\lambda_+}^t) \quad \forall ij \in Tr, t \in T \tag{49}$$

$$0 \le z_{ij} K_{ij} + f_{ij}^t \le M\kappa_{\lambda_-}^t \quad \forall ij \in Tr, t \in T \tag{50}$$

$$0 \le \lambda_{ij}^{t-} \le M(1 - \kappa_{\lambda_-}^t) \quad \forall ij \in Tr, t \in T \tag{51}$$

$$0 \le -L_j^t + q_j^t \le M\kappa_{\delta}^t \quad \forall j \in N, t \in T \tag{52}$$

$$0 \leq \zeta^{t}{}_{j} \leq M(1-\kappa_{\delta}^{t}) \quad \forall j \in N, t \in T \tag{53}$$

$$0 \leq y_{j,g}^{t} \leq M\left(1-\kappa_{\beta,g}^{t}\right) \quad \forall j \in N, \forall g \in G, t \in T \tag{54}$$

$$0 \leq -\eta^{t} - \phi_{j}^{t} + c_{j,g}^{t} + \beta_{j}^{t} + \mu_{j,g}^{t} \leq M\kappa_{\beta,g}^{t} \quad \forall j \in N, g \in G, t \in T \tag{55}$$

$$0 \leq \mu_{j,g}^{t} \leq M(1-\kappa_{\mu,g}^{t}) \quad \forall j \in N, g \in G, t \in T \tag{56}$$

$$0 \leq \mathrm{n}V_{j,g} - y_{j,g}^{t} \leq M\kappa_{\mu,g}^{t} \quad \forall j \in N, g \in G, t \in T \tag{57}$$

We set the values for *M* in each inequality as follows. First roughly estimate the largest possible values for the upper bound of equilibrium constraints which is equivalent to estimating the upper bounds of the dual and primal variables. For estimating the value of dual variables, we use individual node without transmission line network in order to find the variation in social welfare for κ_{λ}^{t} . On the other hand, removing the fixed-demand level can help us find the variation of social welfare for κ_{δ}^{t}. If the generation cost is 0, we can find the variation of social welfare for κ_{β}^{t}. While there is no limitation for generation level, we can find the variation of social welfare for κ_{μ}^{t}. At the end, we examined the *M* if it is binding. When it is binding, we add certain value to solve the model again. As a result of trial and error, the range of *M* is from 5500~10000 in the case study.

3.4 Two-stage Stochastic Program

Before we generate the scenarios according to the historical data, we can reformulate the deterministic model into a two-stage stochastic model. The index *s* represents the scenario. Here we selected fixed-demand level and fuel cost as stochastic parameters. The deterministic model can be expanded as an extensive form of the two stage formulation. In the objective function, we consider the weighted time periods for each scenario. The second stage objective function is calculated by expected value of total social welfare in all scenarios. The model as follows:

Objective function

$$\max_{nV,z} E\{\sum_{t} n_t \,[\sum_{j\in N}(\frac{1}{2}b_j {q^{s,t}_j}^2 + a_j \; q^{s,t}_j + \frac{1}{2}b_j {L^{s,t}_j}^2) - \sum_{g\in G}\sum_{j\in N} c^s_{j,g} y^s_{j,g}]\}$$
$$- [\sum_{g\in G}\sum_{j\in N} gc_{j,g}(nV_{j,g} - V_{j,g}) + \sum_{ij\in L} tc_{ij}\,(K_{ij}\, z_{ij})] \tag{58}$$

Variables and parameter constraints (22)-(25), (34), (40)-(57) are included with scenario index *s* except first stage decision variables and parameters. Constraint (8) is also included in the two-stage stochastic program.

3.5 Scenario Generation and Reduction Algorithms

Now we would like to introduce how to generate the scenarios for our stochastic program. The uncertain parameters are fixed-demand level, $L^{s,t}_i$, and fuel cost, $c^{s,t}_{i,g}$, for natural gas because demand forecasting is the most unstable key factor in power system planning and generation cost is mostly driven by fluctuating fuel cost. A moment matching method is applied in this thesis, using the historical data to create the scenarios that approximate the distribution of uncertainties. Its advantage is in using statistical specifications to approximate the original distribution without exactly knowing the true probability distribution. If the number of scenarios is too large to allow solution of the stochastic program in a reasonable amount of time, we need to apply a scenario reduction algorithm. Here we use fast forward selection because it requires less computation time than other methods to identify a small number of outcomes to represent the original scenario set.

3.5.1 Scenario Generation Algorithm

A stochastic program is a mathematical program considering uncertain information. It is difficult to accurately describe the future event that will occur. To capture the characteristics of uncertain quantities we use statistical properties to describe the possible outcomes in the future. Therefore the continuous probability distributions that may contain potential original data can be approximated by a discrete distribution with a finite number of scenarios. The discretization procedure is called scenario generation [36].

Sampling directly from the distribution is the most intuitive way to generate scenarios. It only needs historical data without assumptions on the distribution. As long as the sample size is large enough, the distribution could be close to the real distribution. However, larger samples may result in computational issues and redundant costs wasted. A small sample may not correctly describe the true distribution.

In our thesis, we select moment matching method for scenario generation because it generates scenarios efficiently under limited time and cost by using statistical information. This method was introduced by [23, 25]. Given a set of statistical specifications such as mean, variance, skewness and correlation, it presents a method based on nonlinear programming which can be used to generate a limited number of discrete scenarios that satisfy the specified statistical properties. The objective function is to minimize the distance between the statistical properties of the generated outcomes and the specified properties. The general description of the model can be described as follows:

$$\min_{x,p} \sum_{i \in O} w_i \,.\big(f_i(x,\pi) - S_{VAL_i}\big)^2 \qquad (59)$$

$$s.t. \quad \sum \ \pi \,.\Lambda = 1 \qquad (60)$$

$$\pi \geq 0 \qquad (61)$$

The set O is the set of all specified statistical properties. S_{VAL_i} is the specified value of statistical property i in O. Let x be the possible values of random vector to be generated and π be the corresponding probability vector. The mathematical expression $f_i(x,\pi)$ computes statistical property i in O. We want to build x and π so that the statistical properties of the approximating distribution match the specified statistical properties. In the constraints, we enforce that the sum of probability equals one. The matrix Λ consists of zeros and ones whose number of rows equals the length of π and the number of columns equals to the number of nodes in the scenario tree. In Chapter 4 we will use real demand data and natural gas prices to generate scenarios by moment matching method in the numerical example.

Also, reference [23] proposed an approach to decide the number of branches from each node of the scenario tree according to the degrees of freedom. Assume *D* is the dimension of each scenario node vector. *D*+1 becomes the number of random variables at a node including the branch probability. (*D*+1)*ý* would be the total number of final degree of freedoms where *ý* is the number of branches. We would like to select *ý* such that (*D*+1) *ý* -1 is greater or equal to the number of statistical specifications. The smallest value of *ý* is the number of branches we choose [22].

3.5.2 Scenario Reduction Algorithm

When the outcomes of scenario generation are too many to control, how to reduce the scenarios without losing the characteristics of original scenarios is the purpose of a scenario reduction algorithm. Due to the large number of scenarios generated, we adopt the forward selection algorithm [26] which is appropriate when the number of preserved scenarios is small.

The reason we consider fast forward selection is to efficiently compute a smaller number of outcomes to represent the original scenario set. The idea of the algorithm is to compare the distances of scenario pairs then select the smallest distance between the scenario pairs. The probability is recalculated for the preserved scenarios.

For a two-stage stochastic program with uncertain right hand side parameters in the constraint and uncertain cost in the objective function, we define a distance function c between scenarios as in (62) from [37]. The parameter w_0 could be the mean of probability measure.

$$c(w, \widetilde{w}) \equiv \max\{1, \left||w - w_0|\right|, \left||\widetilde{w} - w_0|\right|\}||w - \widetilde{w}|| \tag{62}$$

Given original distribution $\{w_1, w_2, \dots w_N\}$ with probability p_d where d=1,…N, in forward selection, we optimally choose one scenario at a time, u, to retain, where u solves (63).

$$\min_{u \in \{1,\dots N\}} \sum_{d=1}^{N} p_d c(w_d, w_u) \tag{63}$$

The fast forward selection algorithm [38] is implemented by Python code [39].

3.6 Progressive Hedging Algorithm

Because the extensive form of the stochastic mixed integer program is too large to solve, we apply PHA. PHA has been successfully applied as a heuristic to solve stochastic programs with integer variables by decomposing the problem into scenario subproblems. PHA aggregates the solutions with modified cost in the objective function progressively obtaining optimal solutions. Here we define a solution for a scenario subproblem as admissible if it satisfies the constraints for that scenario. In a two-stage stochastic program, a solution is implementable if the first-stage decisions are the same for every scenario. A solution is feasible if it is both admissible and implementable. Under certain conditions, the average solution will be admissible in each scenario. The goal of PHA is to apply the cost function modification progressively to cause the average solution to be implementable and, thus, optimal eventually.

Here we are going to introduce the algorithm of PHA in [29]. Suppose we are solving a two-stage stochastic program with the following objective function and constraints (64)-(68). $f^1(\acute{x})$ represents the cost function in the first stage with constraints $g_i^1(\acute{x})$ and the first stage decision variable $\acute{x}$. $Q(\acute{x}, w)$ is the recourse function in the second stage with scenario w. The objective function consists of $f^2(\dot{y}(w), w)$ with constraint $g_i^2(\acute{x}, \dot{y}(w), w)$ and the second stage variable $\dot{y}$.

$$\min f^1(\acute{x}) + D(\acute{x}) \tag{64}$$

$$s.t. g_i^1(\acute{x}) \leq 0, \quad i = 1, \dots m_1 \tag{65}$$

$$D(\acute{x}) = E_w[Q(\acute{x}, w)] \tag{66}$$

$$Q(\acute{x}, w) = \min f^2(\dot{y}(w), w) \tag{67}$$

$$s.t. g_i^2(\acute{x},\dot{y}(w),w) \leq 0, \quad i = 1, \dots m_2 \tag{68}$$

Under this structure, the Progressive Hedging Algorithm follows the steps below:

Step 0. Suppose some implementable solutions $\acute{x}^0$, some initial multiplier ρ^0, and *r*>0.

Let *v*=0. Go to *Step 1*. Let $\hat{x}^0 = \acute{x}^0$.

Step 1. Let $(\acute{x}_w^{v+1}, y_w^{v+1})$ for *w*=1,…,*W* solve (69)-(71). Let $\hat{x}^{v+1} = \left(\hat{x}^{v+1,1}, \dots, \hat{x}^{v+1,w}\right)^T$ where $\hat{x}^{v+1,w} = \sum_{l=1}^{w} p_l \, x\acute{x}_w^{v+1,l} \; for \; all \; w = 1, \dots, W.$

$$\min z = \sum_{w=1}^{W} p_w [f^1(\acute{x}_w) + f^2(\dot{y}_w, k) + {\rho_w}^{v,T} (\acute{x}_w - \hat{x}^v) + r/2 \left|\left|\acute{x}_w - \hat{x}^v\right|\right|^2] \tag{69}$$

$$s.t. g_i^1(\acute{x}) \leq 0, \quad i = 1, \dots m_1, w = 1, \dots W \tag{70}$$

$$g_i^2(\acute{x},\dot{y}(w),w) \leq 0, \quad i = 1, \dots m_2, w = 1, \dots W \tag{71}$$

Step 2. Let $\rho^{v+1} = \rho^v + r(\acute{x}^{v+1,w} - \hat{x}^{v+1})$. If $\hat{x}^{v+1} = \hat{x}^v$ and $\hat{\rho}^{v+1} = \hat{\rho}^v$ then stop; $\hat{x}^v$ and $\hat{\rho}^v$ are optimal. Otherwise, let *v=v+1* and go to *Step 1*.

However, a variety of critical issues arise when implementing PH. The authors of [29] investigate these issues and describe algorithmic innovations in decision variables. The choice of the multiplier ρ is crucial. In [29] it is recommended to choose it as given in (72) where $\acute{x}^{max}$ is the largest solution among the scenarios and $\acute{x}^{min}$ is the smallest solution in the initial iteration:

$$\rho(i) = \frac{cost(i)}{(\acute{x}^{max} - \acute{x}^{min} + 1)} \tag{72}$$

The ρ to the continuous variable is defined in (73):

$$For\ continuous\ variable\ \rho(i) = \frac{cost(i)}{\max((\sum_{w \in W} \Pr(w)|\acute{x}_w - \bar{\acute{x}}^{(0)}|),1)} \tag{73}$$

This selection heuristic can achieve a satisfactory tradeoff between computation speed and solution quality [29].

In addition, PHA can measure a bound on the optimal objective function value in any iteration. According to [40], the following result (74)-(75) shows the implicit lower bound $D(\rho)$ for objective value z^* in a minimization problem. For the maximization problem, we consider the negative value of the lower bound in the minimization problem as the upper bound of objective value in maximization problem.

$$\min z(w) = f^1(\acute{x}) + f^2(\dot{y}_w, k) + \rho_w{}^v\ \acute{x} \tag{74}$$

$$D(\rho) = \sum_w p_w\ z(w) \leq z^* \tag{75}$$

CHAPTER 4 CASE STUDY

4.1 Introduction of ISO-New England

In this chapter, we will implement our model in a case study of the New England region. The Independent System Operator of New England (ISO-NE) divides its service area into eight zones. The eight zones are Maine (ME), New Hampshire (NH), Vermont (VT), Connecticut (CT), Rhode Island (RI), Southeastern Massachusetts (SEMA), West Central Massachusetts (WCMA) and Northeast Massachusetts (NEMA). In the case study, the eight zones are regarded as eight nodes, each having demand and electricity supply. The geographic map of New England is shown in Fig. 2.

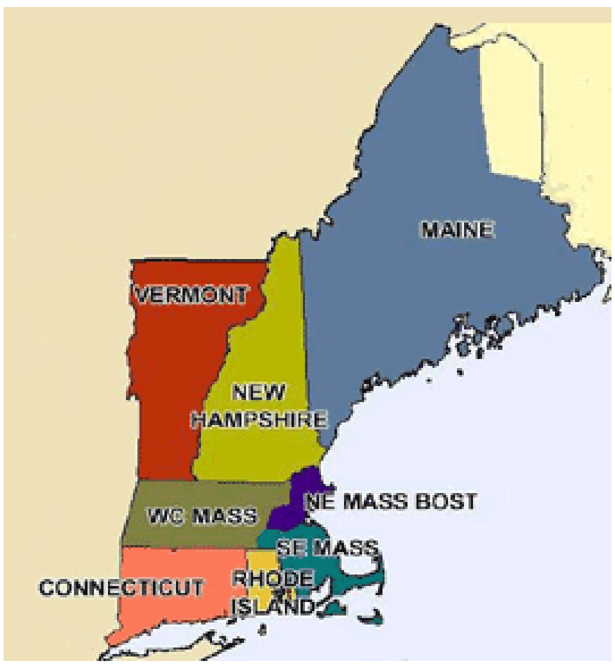


Fig.2. ISO-NE Electricity Regions [41]

ISO-New England is an independent and non-profit corporation. Its responsibility is to meet the electricity demands of the region's economy and oversee the day-to-day reliable operation of New England's power generation and transmission system. The goals of ISO-New England include designing, administering and monitoring the region's competitive wholesale electricity market and power system planning. Higher generation capacity and more transmission investment have made improvements in the reliability of electricity supply to each region in New England in the past years. ISO-New England has created substantial cost savings in these areas by transmission investment and new power plant projects. It saves over 40% of the value of the wholesale electric energy market from 2008 to 2012 [42]. The ISO does not own power plants or transmission lines but it has responsibility to develop the market incentives and operating rules for the electricity market.

The 2013 Regional Electricity Outlook [42] said that one of the challenges for ISO-New England is the potential for reduced operational performance due to increasing reliance on natural gas as a fuel source for power plants. The region's growth depends on the supply of natural gas, especially during the winter months when the priority for natural gas supply is to heat New England's homes and businesses. The limited supply and rising price of natural gas becomes a major challenge for managing the electric grid. Hence, generation expansion plan and transmission investments considering natural gas power plants are discussed in this chapter.

Now we introduce the electricity network in New England. In Fig. 3, we use node 1 as ME, node 2 as NH, node 3 as VT, node 4 as CT, node 5 as RI, node 6 as SEMA, node 7 as WCMA and node 8 as NEMA. The solid lines are the existing transmission lines according to the private communication from the ISO-NE. The dashed lines are candidate transmission lines for future transmission expansion chosen by the random selection of pairs of regions not already connected directly.

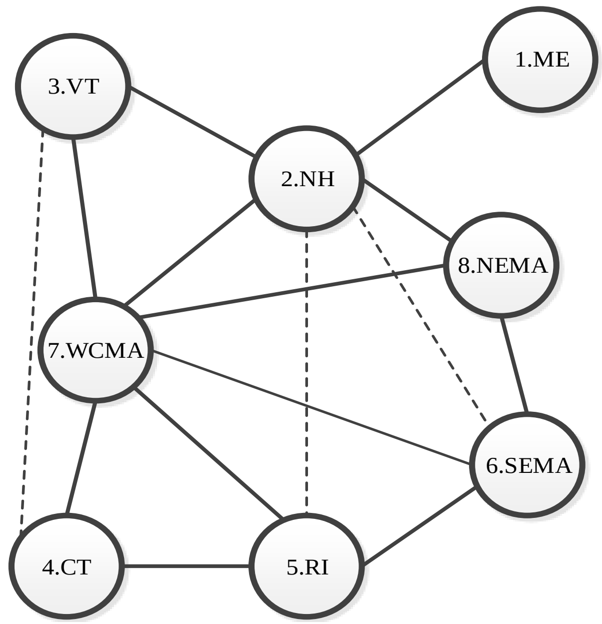


Fig.3. Transmission network in New England

Natural gas has become the dominant fuel for generating electricity in New England. In 2012, 52 percent of energy was produced by natural gas power plants [42]. One of the reasons for the dominance of natural gas is the relatively low cost compared to crude oil. Moreover, its clean burning nature is more environmentally friendly than coal or nuclear power plants. New technology of gas-fired power plants has also improved the efficiency of electricity production [43]. Therefore, we consider two types of natural gas power plants with uncertainty for natural gas price in the New England area.

4.2 Assumptions

In our case study, we made the following parameter assumptions. The first stage decision variable of generation expansion is assumed to be continuous for the convenience of calculation. We consider four different types of power plants in the future investment plan. The four types of power plants are Advanced Combined Cycle (Advanced CC), Advanced Combustion Turbine (Advanced CT), Nuclear, and Onshore wind, where the energy resources of Advanced CC and Advanced CT are both natural gas. Advanced CT is usually reserved for peak hours. Because we believe natural gas will become the main energy resource of the future, we focus on these two power plant types in our case study. As nuclear power still remains the second largest supplier of electricity, the nuclear power plant should be taken into account. Also wind energy is considered in our case study. However, wind energy does have limitations regarding its transmission line and location. We assume wind energy in period 1 of scenario 1 generates 2% of the total demand. Therefore we set the upper bound of wind energy capacity at 400MW in all scenarios.

One reason that natural gas fueled power plants have become more and more popular is their lower carbon emissions compared to coal fueled power plants. The natural gas fueled power plant is the largest source of power supply in New England. Nuclear energy is also an essential source of electricity in New England area. The safety of the operation for nuclear power plays a vital role. It also addresses the political and environmental issues. But the nuclear power supply is still the second largest source in New England. Recently renewable energy has been promoted by government energy policy. Wind power is a clean electricity resource to be developed. Onshore wind farms can be built close to the electrical grid and the cost of building is lower than nuclear power plant.

The costs of power generating technologies can be divided into investment and generation costs. The investment is the amount of money required to build the power plant, and the generation cost is the cost of operating and maintaining the power plant as well as fuel costs.

Finally, we consider only the peak hours in three seasons because peak hours have the most significant effect on reliability of electricity market in each season: Summer, Winter, and Spring/Fall. For each time period, we generate equally likely scenarios for demand and natural gas price. The data of demand and natural gas price are collected in year 2011. We assumed the investment decisions are made in 2011 and the operational decisions are made in 2021.

4.3 Investment Cost

The generation investment cost for the power plant is based on the capital expenditure profile in [44]. The investment cost is calculated by using overnight build cost to multiply the capital expenditure percentage for each year. Then we apply the discount rate to achieve the present value. Finally we sum the cost of each year to transform the investment cost for generation expansion into equivalent annual payments. The overnight cost and capital expenditure profile are illustrated in Table 1 and Table 2 respectively.

For instance, to compute the annualized investment costs for Advanced CC, we multiply the overnight capital cost by the capital expenditure percentage for each year. For each year, the discount rate is considered to calculate the present value. Then we sum the present value for every year. The present value of the investment cost is shown as follows:

$$1023\times10^{6}\times0.25+\frac{1023\times10^{6}\times0.5}{(1+0.05)}+\frac{1023\times10^{6}\times0.25}{(1+0.05)^{2}}=974865.65\ \ (\$/MW) \tag{76}$$

Then we obtain equivalent annual costs over a 10-year horizon using the capital recovery factor.

$$974865.65 \times \frac{0.05 \times (1+0.05)^{10}}{(1+0.05)^{10} - 1} = 63,416.41 \ (\$ / MW) \tag{77}$$

Table 1.Overnight Cost of Power Plants [45]

Power plant	**Overnight Capital Cost ($/MW)**
Advanced CC	$1,023,000
Advanced CT	$676,000
Nuclear	$5,530,000
Onshore Wind	$2,213,000

Table 2.Capital Expenditure Profile [44]

Year	**Advanced CC**	**Advanced CT**	**Nuclear**	**Wind**
1	0.25	0.50	0.01	0.50
2	0.50	0.50	0.01	0.50
3	0.25		0.01	
4			0.01	
5			0.01	
6			0.02	
7			0.03	
8			0.20	
9			0.30	
10			0.30	
11			0.10	

We then divide the results by 8760 hours to obtain an equivalent hourly cost in $/MW/h. The result of generation investment cost is in Table 3.

Table 3. Generation Investment Cost

Power plant	**Generation Investment Cost ($/MW/h)**
Advanced CC	$7.23
Advanced CT	$4.90
Nuclear	$22.00
Onshore Wind	$17.49

For the candidate transmission lines, NH to RI, NH to SEMA, VT to CT, and CT to SEMA, were randomly selected. The conceptual leader will decide whether to expand transmission lines among these candidates. We represent the connection point in each zone by assuming they fall in the following cities: Portland (ME), Concord (NH), Burlington (VT), Hartford (CT), Providence (RI), Plymouth (SEMA), Worcester (WCMA) and Boston (NEMA). These cities were selected from private communication with ISO-NE.

Table 4. Locations for transmission line

Zone	NH	VT	CT	RI	SEMA
City	Concord	Burlington	Hartford	Providence	Plymouth

The type of candidate transmission line is 500kV. The unit investment cost of the transmission line is 1,854,000 ($/mile) [46] . We consider the life of transmission line as infinite, and the annualized investment cost is calculated as the distance times the unit investment cost and the interest rate.

Table 5. Investment cost of candidate transmission line

Candidate	NH-RI	NH-SEMASS	VT-CT
Distance(miles)	116.00	107.00	236.00
Cost($/MW/h)	122.75	113.22	249.73

The total generating capacity of each zone is obtained by the private communication from ISO-New England. Also, the slope and intercepts of demand curves are assumed by roughly estimating the maximum value of demand according to the inverse demand function in Table 6.

Table 6. Data for capacity, slope and intercept

Electricity Nodes	Total Capacity V_j (MW)	Slope of demand price b_j ($/MWh/MWh)	Intercept of demand price a_j ($/MWh)
ME	407.50	-0.08	200.00
NH	2249.50	-0.07	210.00
VT	630.00	-0.095	190.00
CT	2208.40	-0.045	360.00
RI	3640.50	-0.095	237.50
SEMA	1986.00	-0.09	315.00
WCMA	1277.50	-0.09	324.00
NEMA	1603.70	-0.07	322.00

The thermal capacities of all transmission lines are assumed to be 650 MW. The related parameters are shown in Table 7.

Table 7. Data for Transmission Lines

Transmission Line	Transmission Capacity $K_{i,j}$ (MW)	Negative Susceptance $B_{ij}(\Omega^{-1})$	z_{ij}
(1,2)	650	40	1
(2,3)	650	40	1
(2,7)	650	40	1
(2,8)	650	40	1
(3,7)	650	40	1
(4,5)	650	40	1
(4,7)	650	40	1
(5,6)	650	40	1
(5,7)	650	40	1
(6,7)	650	40	1
(6,8)	650	40	1
(7,8)	650	40	1
(2,5)	400	40	Candidate
(2,6)	650	40	Candidate
(3,4)	650	40	Candidate

4.4 Uncertainties

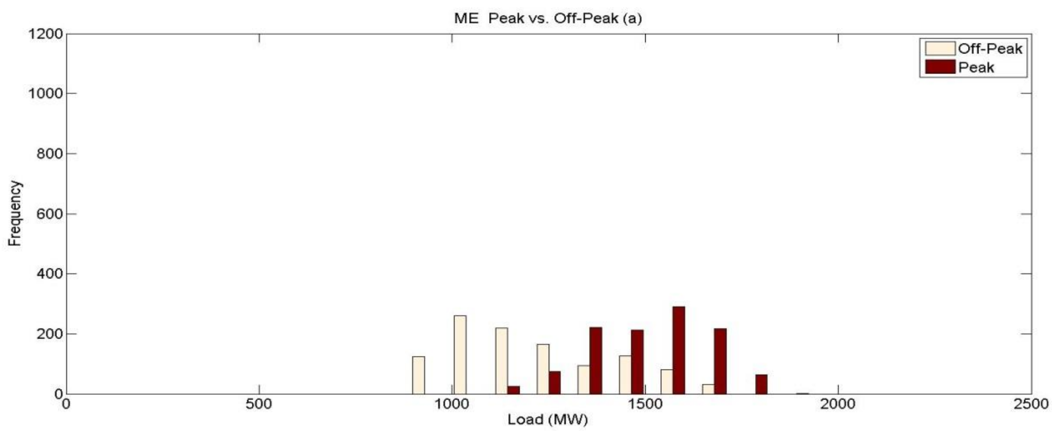
4.4.1 Demand

For every year from 2002 through 2012, ISO-NE provides the hourly loads in each zone [47]. We adopt the load data in 2011 as our data set. We separated this year's hours into three parts: Summer, Winter and Spring/Fall. The Summer season contains the months of July to September, and the Winter season includes December, January and February. The rest of the months are classified as the Spring/Fall season. For each part of the year, hours can be classified as peak hours and off-peak hours. ISO-NE defines peak hours as 7:00 am through 11:00 pm on all non-holiday weekdays. The off-peak hours are defined as the weekday hours between 11:00 pm and 7:00 am and all of Saturdays, Sundays and Holidays [48]. The percentage of hours in each time period in a year is illustrated in Table 8.

Table 8. Time periods in a year

	Summer Peak	Summer Off-Peak	Winter Peak	Winter Off-Peak	Spring & Fall Peak	Spring & Fall Off-Peak
Hour (hr)	1,105	1,103	1,020	1,164	2,159	2,233
Percent (%)	12.6	12.6	11.6	13.2	24.5	25.5

For each zone, we compare the load in peak hours and off-peak hours in each season. For example, Fig. 4 represents peak hour load versus off-peak hour of ME in three time periods. Similar figures for the rest of the zones are collected in the Appendix. According to the figures, in each season the peak hours have higher average load than off-peak hours. The highest load in the Summer occurs in Connecticut and the lowest load is in Vermont. Moreover, the loads in Summer and Winter are higher than in Spring/Fall.



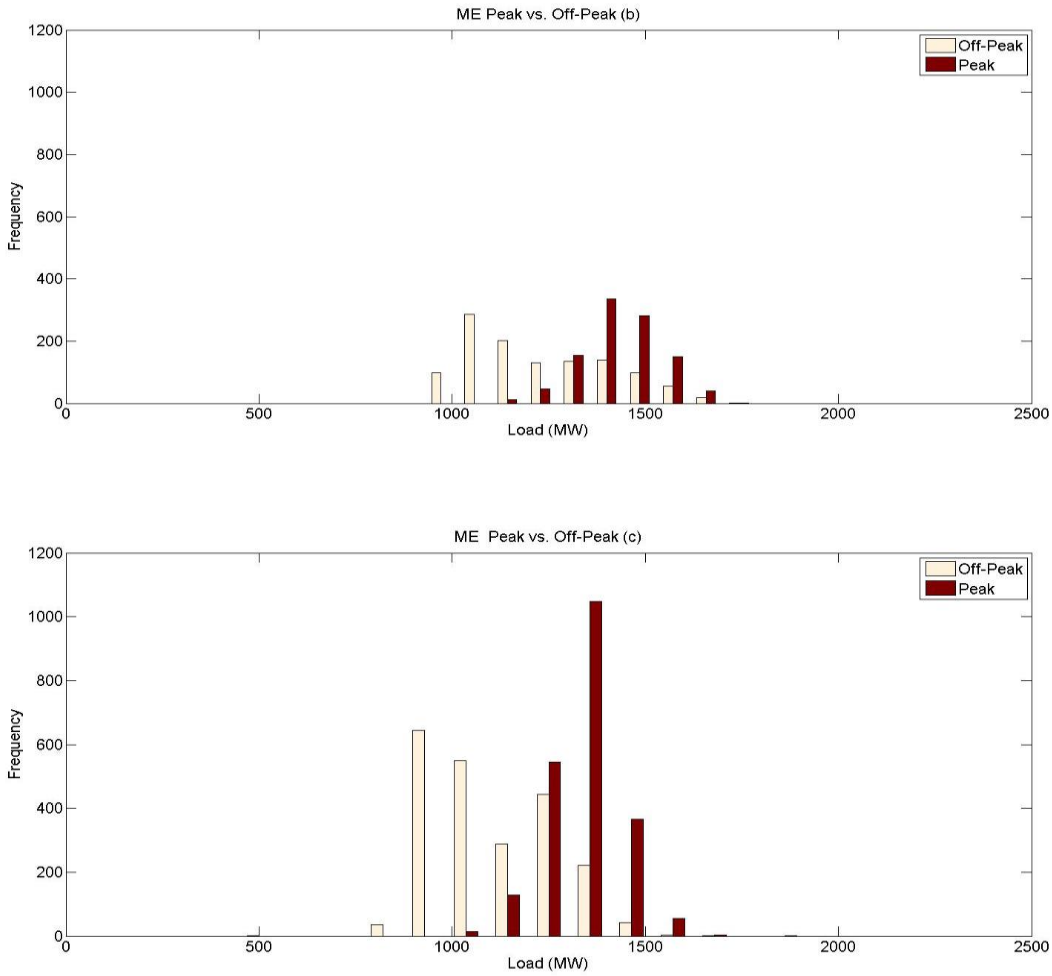


Fig.4. ME Peak load vs. Off Peak load in (a) Summer, (b) Winter, (c) Spring/Fall

The statistical specifications for Summer peak hours in 2011 of demands in each zone such as mean, variance and skewness are illustrated in Table 9. The maximum value of mean and variance is in Connecticut. The minimum value of mean and variance is in Vermont. The moment matching method is based on this information, as well as the corresponding data for peak and off-peak hours in each season, to generate scenarios. We have eight random demands and three statistical specifications for moment matching method.

Table 9. The statistical specifications of demand for Summer Peak in each zone

Statistical Specifications	ME	NH	VT	CT	RI	SE	WC	NE
Mean	1510.15	1649.74	748.55	4597.57	1235.32	2296.84	2493.45	3731.32
Variance	20.65	54.15	4.59	602.65	46.91	172.20	129.15	312.89
Skewness	-0.18	0.25	0.09	0.31	0.24	0.25	0.36	0.30

The growth of electricity demand has slowed since the 1950s in the U.S. The reason for the relatively slow growth is technological efficiency gains to offset increasing demand. According to [42], the total electricity demand is projected to grow by 28 percent by 2040 with a growth rate of 0.9 percent per year. Therefore, we assume 0.9 percent as our annual demand growth rate to find our fixed-demand levels in the operational year 2021. The rest of the demand data are shown in the Appendix.

4.4.2 Natural Gas Price

Natural gas price fluctuates according to economic growth or advanced drilling technology because technology improvements reduce the drilling cost and operation cost while achieving similar output [49]. Both factors are hard to predict. Moreover, fuel cost uncertainty for natural gas is significantly higher than uranium and cleaner for the environment in the long-term. Coal has had more stable price variability than natural gas but coal is also the largest contributor to greenhouse gas emissions. Therefore we select the price of natural gas as our stochastic parameter and main energy resource for our model in the New England area.

In 2012, natural gas prices for electric power reached a new record low since 2002 with the spot price at Louisiana's Henry Hub averaging $2.81/MMBtu. Low natural gas prices resulted in greater reliance on natural gas for power generation while more older coal-fired power generations retired in the past few years [50]. Natural gas has become an attractive energy source in New England area. The processes of unit conversions to generation cost and calculation of generation cost is shown in (78)-(80). Generation cost involves the variable operation and maintenance cost and fuel cost. Therefore the price of natural gas accounts for most of the generation cost.

$$\frac{p_{natural\ gas}\ (\$/thousand\ cubic\ feet)}{1.023} = p_{natural\ gas}\ (\$/MMBtu) \tag{78}$$

$$\frac{Heatrate_{natural\ gas}(Btu/kwh)}{1000} = Heatrate_{natural\ gas}(MMBtu/MWh) \tag{79}$$

$$gc_{natural\ gas} = p_{natural\ gas} * Heatrate_{natural\ gas} + Variable\ O\&M_{natural\ gas} \tag{80}$$

The wholesale natural gas price in New England is the sum of the Henry Hub price and a basis differential. This is similar for all locations in the U.S. The basis differential can be defined as the difference between the Henry Hub price and the corresponding spot price for natural gas in a specific location [51]. The basis differential variation depends on the distance between different destinations. For the absence of data in EIA concerning the price of natural gas to generate electricity [45] in Maine and New Hampshire, we assume the basis difference is the same as in Vermont. The Henry Hub price only provides daily data to the EIA. We assume the Henry Hub hourly price is the same as the corresponding daily data. Therefore we can derive the basis differential by calculating the monthly natural gas price and subtracting the monthly Henry Hub price. The hourly natural gas price can be obtained by adding the assumed hourly Henry Hub price to the averaged basis differential in 2011. Additionally, the inflation rate is assumed to be 5% per year from 2011 to 2021.

The statistical specifications for Summer peak hours of natural gas price in each zone such as mean, variance, skewness and the correlation with demands are illustrated in Table 10. The maximum value of mean is in Maine, New Hampshire and Vermont. The minimum value of mean is in Connecticut. The natural gas prices are similar in each zone.

Table 10.The statistical specifications of natural gas price for Summer Peak in each zone

Statistical Specifications	ME	NH	VT	CT	RI	SE	WC	NE
Mean	3.90	3.90	3.90	3.54	3.73	3.63	3.63	3.63
Variance	0.05	0.05	0.05	0.06	0.04	0.07	0.07	0.07
Skewness	-0.10	-0.10	-0.10	-0.01	0.26	0.38	0.38	0.38
Correlation with demand	0.40	0.26	0.27	0.34	0.26	0.24	0.26	0.21

4.5 Scenario Generation and Reduction Application

We have eight zones and each zone has two random variables: demand and natural gas price. The total number of random variables is therefore sixteen, as shown in (81). The statistical properties we consider here are mean, variance, skewness and the correlations between demand and price in each zone. The total number of specified statistical properties is 56 as shown in (82). According to (59)-(61), we obtain the number of outcomes is 4 in each combination of season and hour type.

I =16 (8 zones, two random variables) (81)

|O|={16*mean,16*variance,16*skewness, 8 correlations}=56 (82)

There are three periods in one scenario, which consists of fixed-demand and natural gas price for each zone in peak hours of Summer, Winter and Spring/Fall. The total number of scenarios is $4^3 = 64$ in the stochastic model, each having probability 1/64. The number of scenarios is too large for solution of the stochastic program to be tractable. Here we adopt fast forward selection to select 5 scenarios as our preserved scenario sets and redistribute the probability.

The preserved scenario probabilities are illustrated in Table 11. The expected fixed demand levels and generation costs are illustrated in Table 12 and Table 13. The fixed demand levels and fuel costs are detailed in the Appendix.

Table 11. Probability for preserved scenarios

	Probability
Scenario 1	0.07
Scenario 2	0.14
Scenario 3	0.30
Scenario 4	0.33
Scenario 5	0.16

Table 12. Expected fixed demand level

Node	Summer	Winter	Spring/Fall
1	1414.53	1576.10	1157.23
2	1633.40	1682.66	1155.53
3	726.02	795.34	581.63
4	4789.73	4565.29	2982.55
5	1186.43	1174.12	802.36
6	2233.03	2220.15	1413.35
7	2636.60	2643.22	1774.70
8	3634.05	3494.95	2558.10

Table 13. Expected generation costs of Combined Cycle and Combustion Turbine

	Summer	Winter	Spring/Fall
1.cc	34.88	40.83	33.59
1.ct	58.31	67.33	56.35
2.cc	32.8	42.05	32.92
2.ct	55.15	69.18	55.33
3.cc	36.35	42.47	34.77
3.ct	60.53	69.82	58.14
4.cc	33.65	44.31	42.55
4.ct	56.44	72.62	69.93
5.cc	32.96	47.79	30.74
5.ct	55.39	77.89	52.02
6.cc	33.85	42.34	28.66
6.ct	56.74	69.63	48.88
7.cc	31.96	39.87	29.81
7.ct	53.88	65.88	50.61
8.cc	33.85	38.73	35.49
8.ct	56.75	64.14	59.23

4.6 Generation Cost

Generation cost includes the variable operation and maintenance (O&M) cost and fuel cost. We assume the inflation rate is 5%. For example, the nuclear fuel cost is $7.01/MWh [52]. The generation cost is calculated by multiplying the fuel cost by the heat rate and adding the variable O&M cost. Therefore we convert the nuclear cost into dollars per MMBtu by dividing by 3.413 and multiply by the heat rate in Table 7. Finally we divide by 1000 to change the units into MWh. The generation cost for nuclear power plant is $28.88/MWh. The process is illustrated as follows:

$$\frac{7.01\times(1+0.03)^{10}}{3.413}\times 10464\times\frac{1}{1000}=28.88\ \ (\$/MWh) \tag{83}$$

Wind power does not incur fuel cost or variable O&M cost. The generation cost for wind power is zero. The heat rate and variable O&M cost of each type of power plant is illustrated in Table 14 from [53].

Table 14.The Heat Rates and Variable O&M Costs

	Heat Rate (Btu/KWh)	Variable O&M ($/MWh)
CC	6430	$3.27
CT	9750	$10.37
Nuclear	10464	$2.14
Wind	N/A	$0.00

Finally the generation cost is estimated by the sum of fuel cost and variable O&M cost. The generation cost for Advanced CT is the highest among these four types of power plants. Wind power has zero generation cost from [67]. The generation cost and demand of reduced scenarios are presented in the Appendix.

4.7 Framework of Stochastic Program

Here we only consider peak hours in each season because we want to ensure our planning can result in the most reliable power network. After the first stage decisions are revealed, the random outcomes are generated with Summer Peak period, Winter Peak period and Spring/Fall Peak period. The scenario framework is illustrated in Fig.5. We assume the proportion of hours represented during each period, n_t , is 0.33 for convenience. But in reality we should change the proportion according to the length of each season.

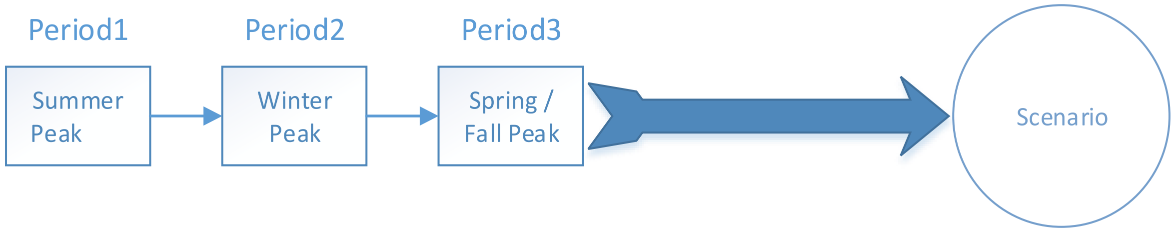


Fig.5.The scenario framework

Since the total number of scenario outcomes is 64, we adopt the fast forward scenario reduction algorithm to reduce the number of scenarios. The stochastic program structure is shown in Fig.6.

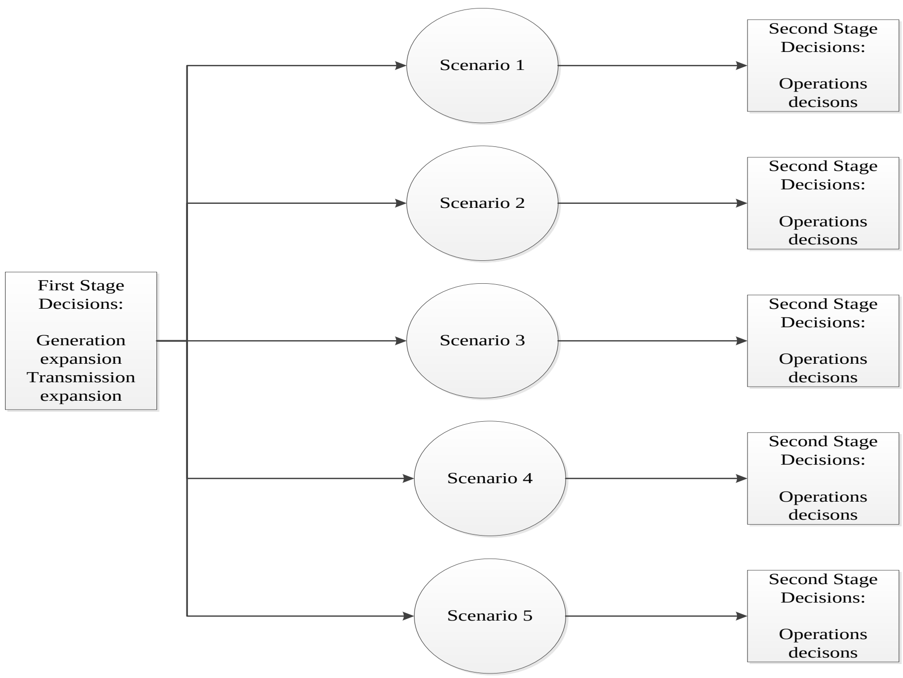


Fig.6. The framework of stochastic programming

4.8 Numerical Results

In our case study, we consider the demand and natural gas price in 2011 on a single hour. And we generate the scenarios in 2021 by modifying the scenarios generated for 2011 with future assumptions. The investment decisions are made in 2011 and operational decisions are made in 2021. One hour represents a weighted average over the three seasons of Summer, Winter and Spring/Fall in peak hours.

4.8.1 Progressive Hedging Algorithm Application

In this case study, we implemented the moment matching method in GAMS 23.4 using CONOPT as NLP solver and PHA in GAMS 23.4 using CPLEX as MIQCP solver. The fast forward selection is implemented in Python 3.4 [39]. Computational experiments are executed on a desktop with Intel Pentium 4 CPU 3.40 GHZ and 4 GB RAM. The time for solving a scenario subproblem in GAMS ranges from 48 seconds to 2160 seconds. For each scenario the MIP has 1,583 constraints and 1,186 variables, including 447 binary variables, and can take as much as one hour to solve.

However, our model includes integer solutions, so PHA is not guaranteed to converge to optimality. Still, we apply PHA in a certain number of iterations and then we use the aggregated solutions as our first stage solution. We then fix the aggregated first stage solution to solve for the second stage decisions in each scenario. The resulting objective value is our lower bound of the optimal (maximum) objective value. Also, in each PHA iteration we calculate an upper bound by calculating the objective value for individual scenarios considering the dual prices [40]. After deriving the objective value for each scenario, the average objective value is our upper bound in maximization problem. By comparing lower bound and upper bound on the optimal objective value, we can understand how close we are to the true optimal solution.

The aggregated solutions of generation expansion after 8 iterations are shown in Table 15. As for the transmission line expansion, all scenarios are consistent in deciding to build a transmission line from VT to CT. The minimum difference between upper bound and lower bound is from from the upper bound computed in iteration six and illustrated in Table 16 . The lower bound is only 1.1% different from the upper bound.

Table 15. Aggregated generation expansion (MWh)

Node	CC	CT	NU	WI
1	266.11	0	0	329.80
2	0.00	0	0	333.00
3	1333.48	0	0	380.00
4	2035.02	0	0	400.00
5	0.00	0	0	400.00
6	0.00	0	0	400.00
7	152.02	0	0	393.50
8	0.00	0	0	398.60

Table 16. Objective value for the sixth iteration of upper bound and lower bound

	Objective Value
Upper Bound	1944887
Lower Bound	1922483

The bounds on the optimal objective function value from each iteration are shown in Table 17. In iteration two and four, one scenario subproblem could not be solved within two hours. The difference between the lower bound and upper bound fluctuates because the upper bound does not monotonically decrease. We only fixed the transmission line at iteration seven and stop because of large amount of computation time.

Table 17. Bounds on objective value with different penalties

Iteration	Upper bound	Lower bound	Difference bounds	Gap (%)
1	1959957	1920278	39679	2.0
2	1959099	-inf	N/A	N/A
3	1963817	1923348	35751	1.8
4	1994252	-inf	N/A	N/A
5	1955621	1923358	32263	1.6
6	1944887	1923028	21859	1.1
7	1976226	1922483	53742	2.7

For examining the scenario reduction result, we use the first stage average solutions at the last iteration fixed and optimize the second stage decisions in each of the 64 scenarios generated. The total expected objective value is $1,810,356. The difference from our lower bound objective value is 5.8 %. It shows that the scenario reduction with forward selection can represents most of the 64 scenarios.

The second stage decisions of demand to be satisfied are shown in Fig. 7 for scenario one. The demand to be satisfied in period one (Summer) is higher than period two (Winter) and period three (Spring/Fall). Connecticut has the highest demand in our model. The results of the rest of the demand to be satisfied are illustrated in the Appendix.

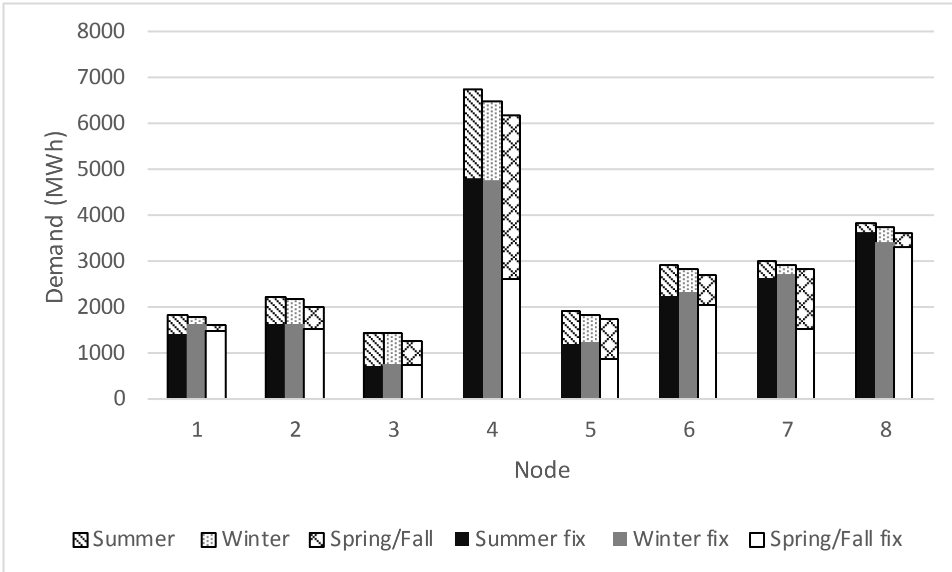


Fig 7. Demand to be satisfied in second stage of scenario one from SP solution

The LMPs in scenario one are shown in Fig. 8. It shows the price distributed without significant difference between each state. It ranges from \$50/MWh to \$81/MWh. The rest of the LMPs are also shown in the Appendix.

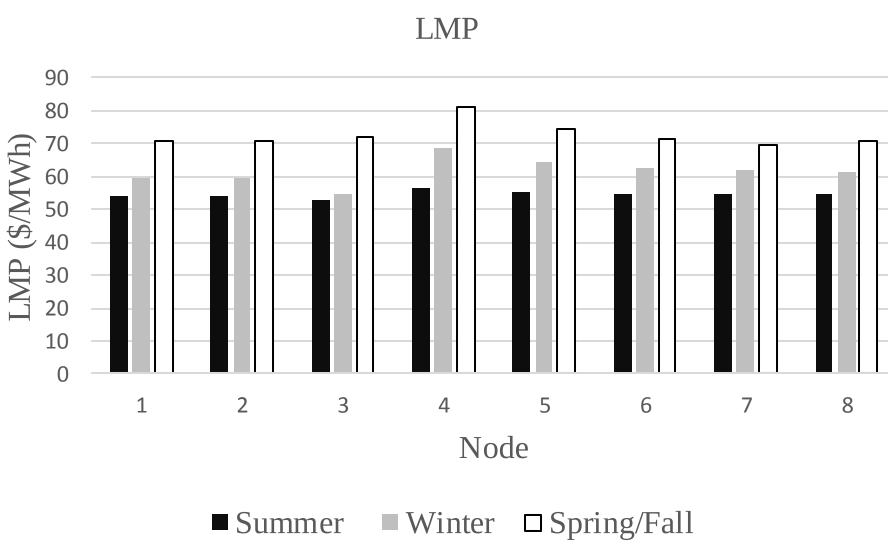


Fig 8. LMPs in second stage of scenario one from SP solution

4.8.2 Expected Value Solution

In order to assess the value of planning for uncertainty, we consider solution of the deterministic expected value model. First we calculate the expected values of the random parameters. Then we solve the deterministic model with the expected value of the random parameters. The first stage solutions are obtained and we fix the first stage decisions in the stochastic program model. Then we solve for the optimal second-stage solution in reduced scenarios. The resulting objective value represents the Expectation of the Expected Value Solution (EEV).

The first stage decisions of generation expansion are shown in Table 18. The generation expansion is higher than in the stochastic program solution. For the transmission line expansions decisions, in the EV solution we build only two transmission lines: NH-SEMASS and VT-CT.

Table 18. EV solutions of generation expansion (MWh)

Node	CC	CT	NU	WI
1	1632.10	0.00	0.00	364.90
2	1488.02	0.00	0.00	366.50
3	1656.50	0.00	0.00	390.00
4	1957.84	0.00	550.37	400.00
5	0.00	0.00	0.00	400.00
6	1062.79	0.00	0.00	400.00
7	1592.33	0.00	0.00	396.50
8	1041.89	0.00	0.00	399.30

The second stage decisions of demand to be satisfied from expected value solution are shown in Fig. 9 for scenario one. The demand to be satisfied in period one (Summer) is higher than period two (Winter) and period three (Spring/Fall). Connecticut has the highest demand in our model.

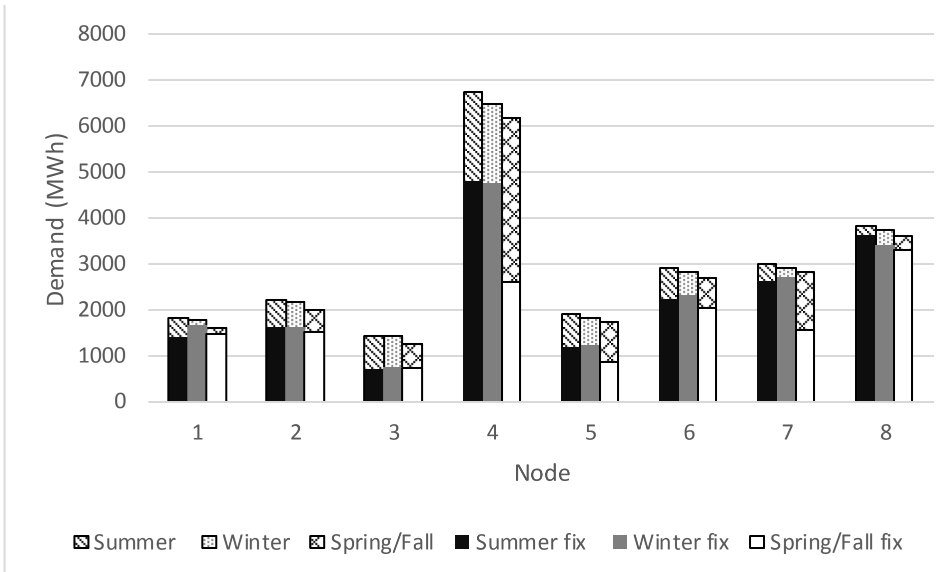


Fig 9. Demand to be satisfied in second stage of scenario one from EV solution

The LMPs from expected value solution are also derived in scenario one in the Fig.10. It shows the price distributed without significant difference between each state. The LMPs range from \$56/MWh to \$81/MWh. The average LMP in the EV solution for all nodes and seasons is 3% higher than in the stochastic program solution.

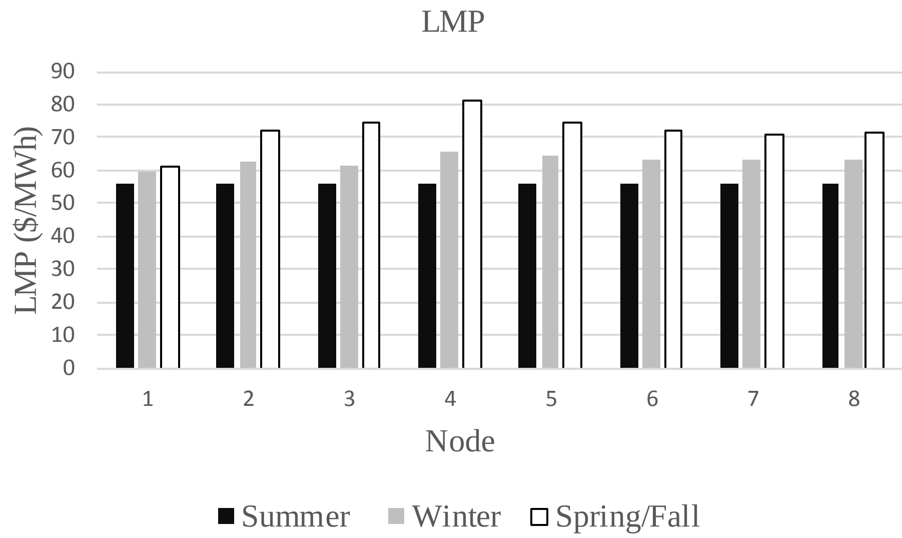


Fig 10. LMP in second stage of scenario one from EV solution

We calculate the expected buyer surplus, producer surplus and transmission rents along with investment costs in Table 19. The buyer surplus in stochastic program solution is higher than in the expected value solution by $55,906. Because there is a 51% probability that fixed demand will exceed the expected value, the buyer surplus in the EV solution is lower. The average price for all nodes and periods in the EV solution is larger by 3.3% than in the stochastic program solution. The expected producer surplus increases in the EV solution to compensate for the loss of buyer surplus. The expected producer surplus in the EV solution is $20,562 higher than in the SP solution. Because an additional transmission line is built in the EV solution, for some scenarios there is no congestion. Therefore the expected transmission rents are lower than in the stochastic program solution by $7,465.

Table 19.Comparison of objective value components in Stochastic Program (SP) and Expected Value (EV) solutions

	Expected buyer surplus	Expected producer surplus	Expected transmission rents	Generation expansion cost	Transmission expansion cost
SP	1,368,852	661,777	13,263	121,160	249
EV	1,312,946	682,340	5,797	142,047	362

Comparing our generation expansion decisions the generation expansion in the SP solution is lower than in the EV solution. The total social welfare is higher in some scenarios for stochastic program solution.

The expectation of the expected value solution (EEV) is obtained as $1,858,671. Finally the Value of the Stochastic Solution (VSS) is calculated as optimal objective function value minus EEV. However we do not have the optimal solution from limited iterations. The lower bound helps us evaluate the VSS. The VSS is at least $63,810 which is 3.4% of the EEV.

CHAPTER 5 CONCLUSION

5.1 Summary

We formulated a stochastic program to identify welfare-maximizing generation and transmission expansion plans in a restructured electricity network. The scenarios are generated by the moment matching method. The advantage of the moment matching method is that it does not require complete knowledge of the distribution of the random variables. Using historical data captures the statistical specifications to create a similar sample with simulated statistical properties. By including uncertainties in the lower level, the MIP is converted into a stochastic MIP (SMIP). Moreover, we also investigate generating scenarios for different time periods. Totally we consider 64 scenarios at one time on a single hour basis. But because the size of the model is still too large to solve all scenarios simultaneously, we adopt the fast forward selection algorithm to reduce the scenarios into five scenarios with redistributed probability. Solving the model by PHA still requires a large amount of time but we can derive an upper bound on the maximum expected social welfare less investment cost in any iteration. This information provides a bound on how far from optimality our solutions are. The generation expansion level decisions in the stochastic program solution are lower than the corresponding levels in the expected value solution. Fewer transmission lines are built in the stochastic program solution. Because of the variations in demand and fuel cost, the total expected social welfare from the stochastic program solution is approximately two percent larger with stochastic program solution and the investment cost is lower.

5.2 Limitations

In our thesis, the fixed O & M cost is not considered along with generation investment cost. Overnight cost is the only factor we considered in the generation investment cost. If we consider the fixed O & M cost, the model is more close to the reality and our decision might change at the end. In particular, including the fixed O&M cost associated with wind power might reduce the amount of wind generation expansion.

As it is, we assume the wind power capacity expansion is constrained by assumptions. We may want to use as much wind energy as possible to meet Renewable Portfolio Standard (RPS). But because the wind energy is relatively cheap in our case study, we had to set an artificial upper bound on the wind energy capacity. Properly accounting for the fixed O&M cost might eliminate the need for this capacity cap. However, we also do not include the production tax credit for wind power, which may actually result in a negative generation cost for wind power and encourage its use.

The model does not consider the temporal constraints such as ramping constraints and the actual structure of supply function bid. But in the long term, the equilibrium model has been shown to approximate the behavior of generators.

5.3 Future Research

During the process of generating scenarios, the data set plays an important role. The quality of the information gathered will affect the performance of the model. If the data are more reliable and complete, the scenarios are more useful. While the advantages in stochastic program are usually clear, constructing stochastic programs usually requires information that has not been routinely collected. Distributions and basic parameter values might not be known. Approximations that deal with these difficulties by constructing models that use whatever information is known could be the only way to implement a stochastic program. The performance of different scenario generation and reduction methods could be tested in our model. Selecting a different number of scenarios selected might change our solution.

Different uncertainties could be considered in our model such as the weather variation for the wind energy production. The capacity factor for wind power should be included in the future research. The perspective of uncertainty results in a different expansion plan. Carbon emission is another popular issue currently. The production tax credits would affect the planner's investment decisions. Thus a more realistic model needs to be developed.

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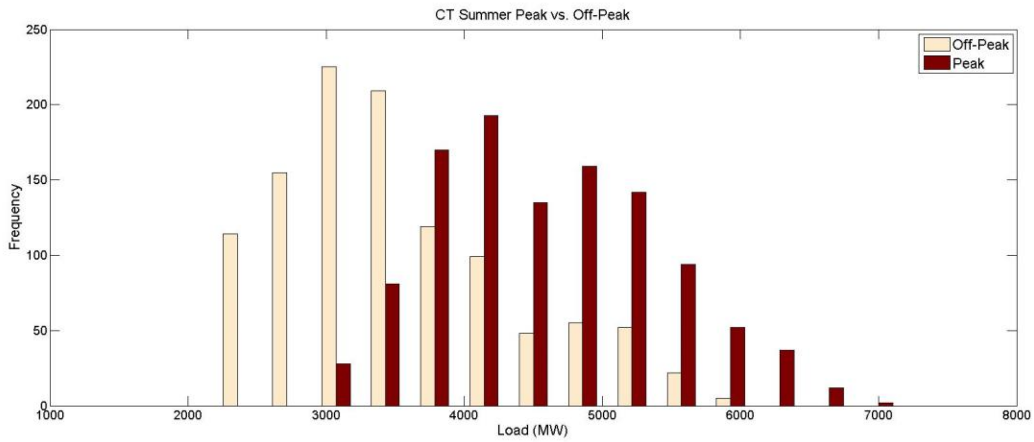
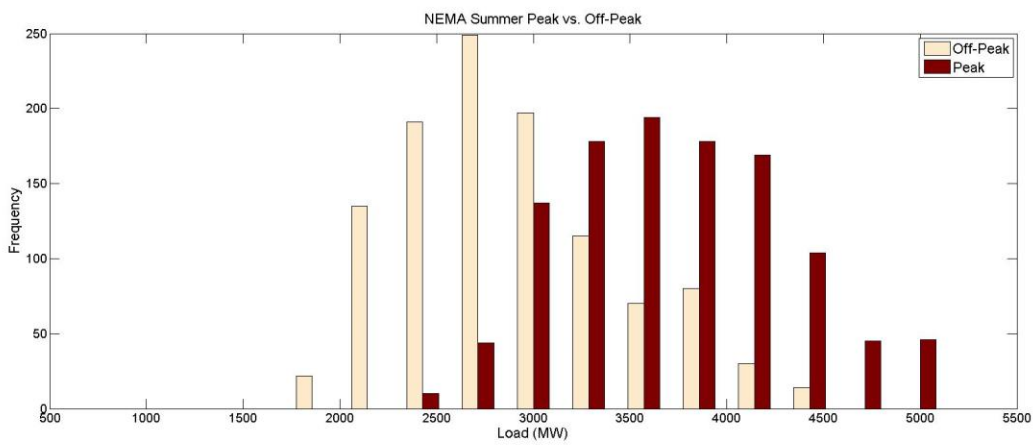
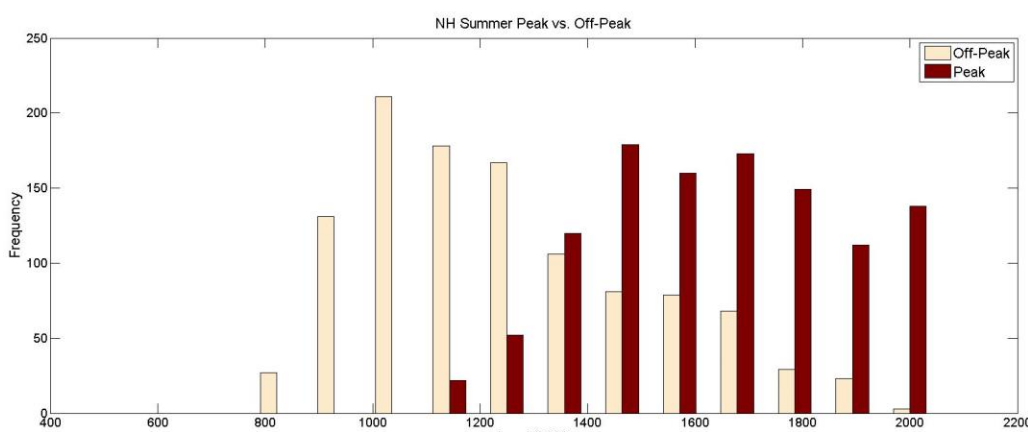
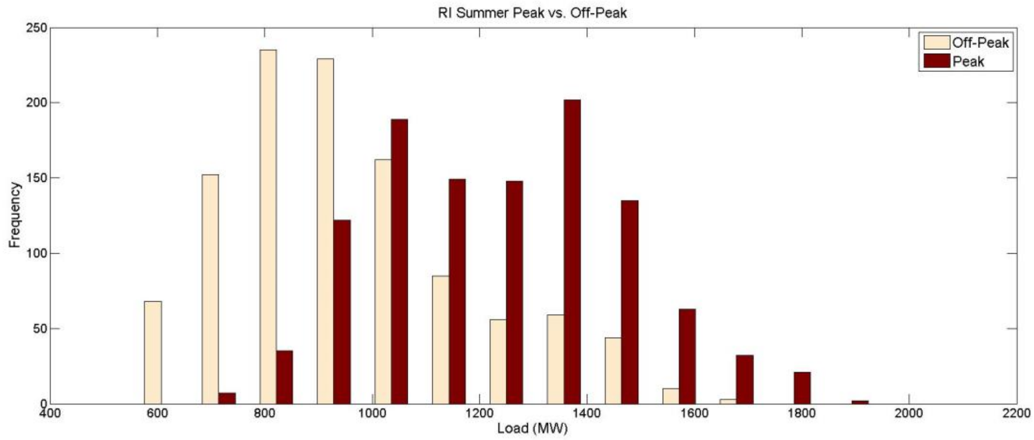
APPENDIX

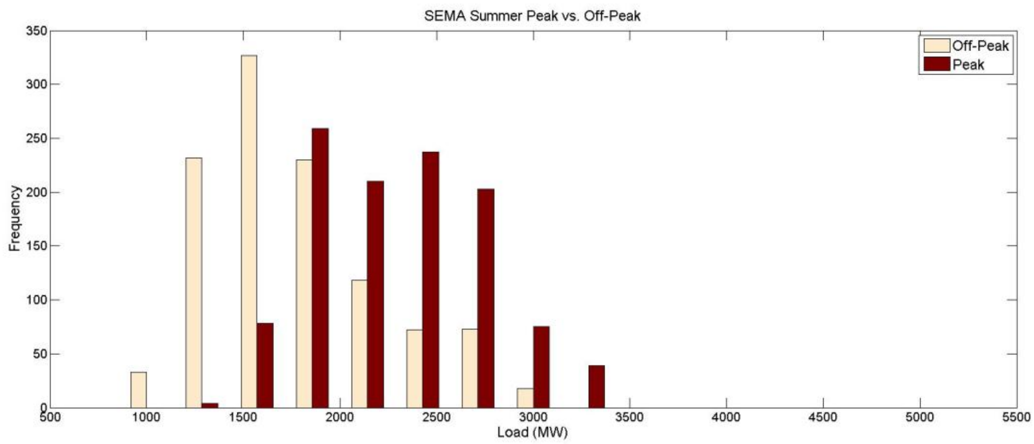
A. Peak hour vs. Off-peak hour demand








RI Summer Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
50
100
150
200
250
400
600
800
1000
1200
1400
1600
1800
2000
2200


SEMA Summer Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
50
100
150
200
250
300
350
500
1000
1500
2000
2500
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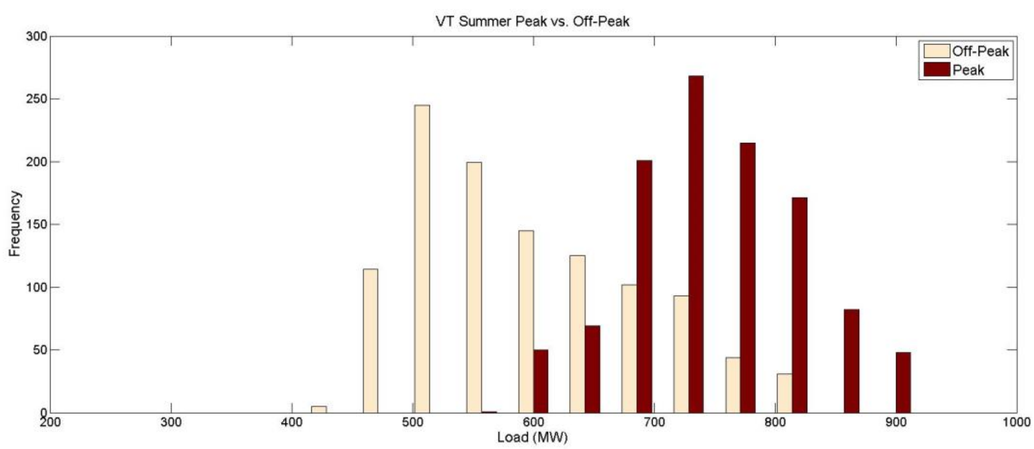
3500
4000
4500
5000
5500


VT Summer Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
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100
150
200
250
300
200
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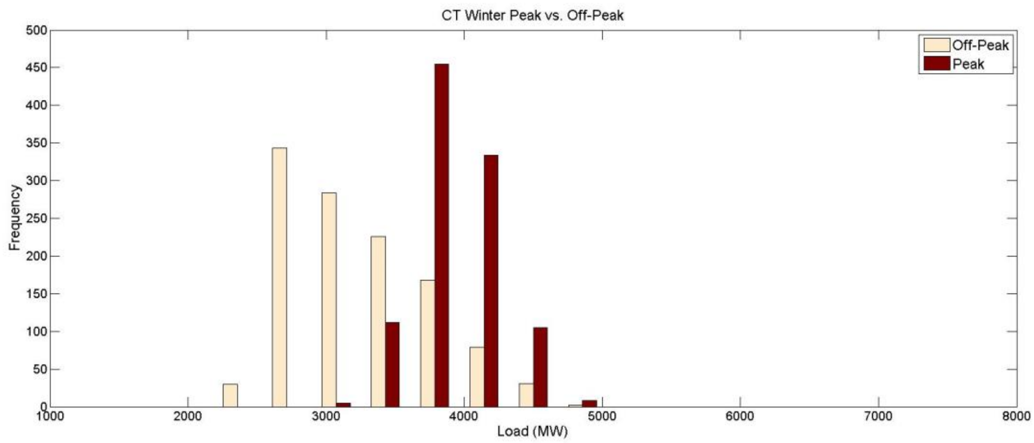
600
700
800
900
1000


CT Winter Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
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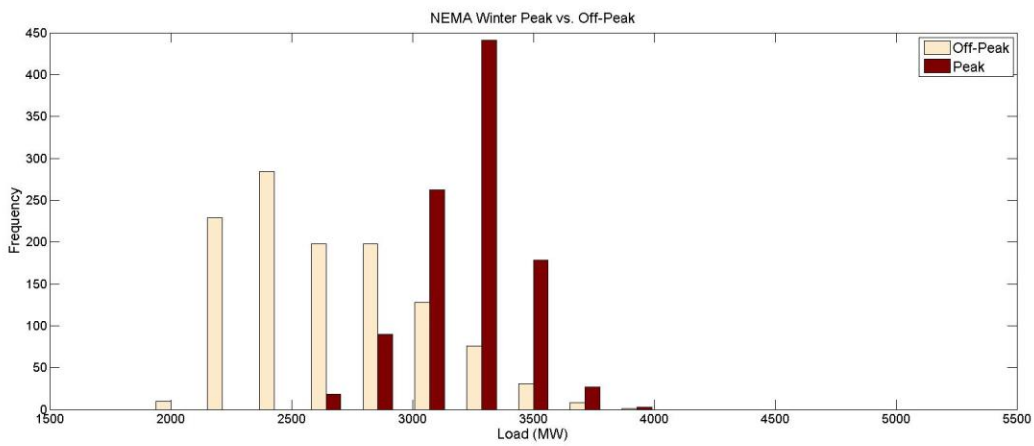
100
150
200
250
300
350
400
450
500
1000
2000
3000
4000
5000
6000
7000
8000


NEMA Winter Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
50
100
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300
350
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2500
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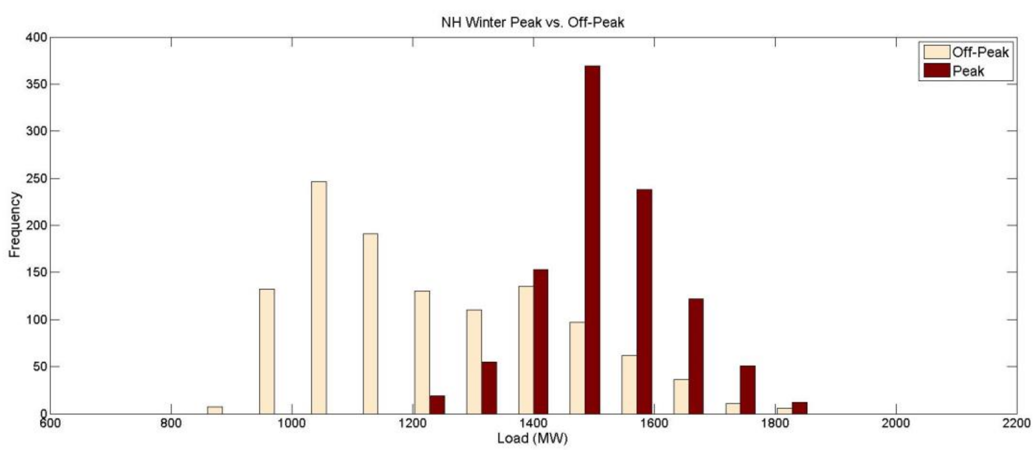
3500
4000
4500
5000
5500


NH Winter Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
50
100
150
200
250
300
350
400
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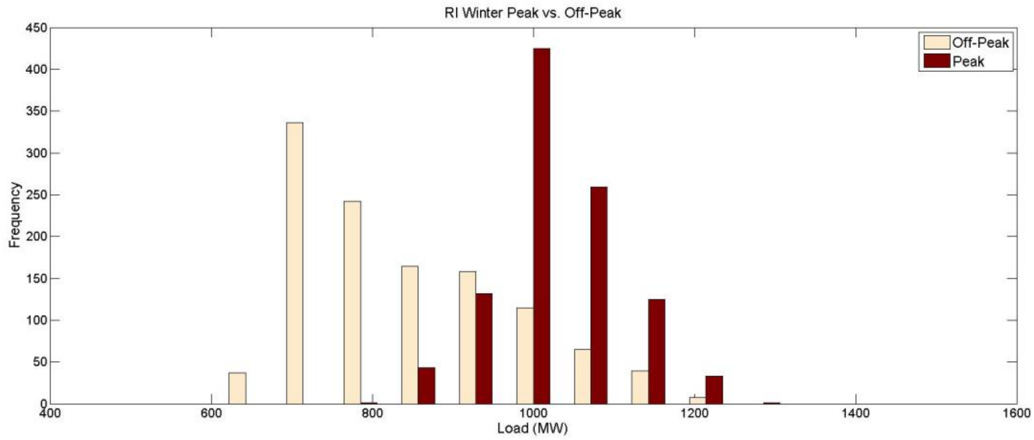
800
1000
1200
1400
1600
1800
2000
2200


RI Winter Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
50
100
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400
600
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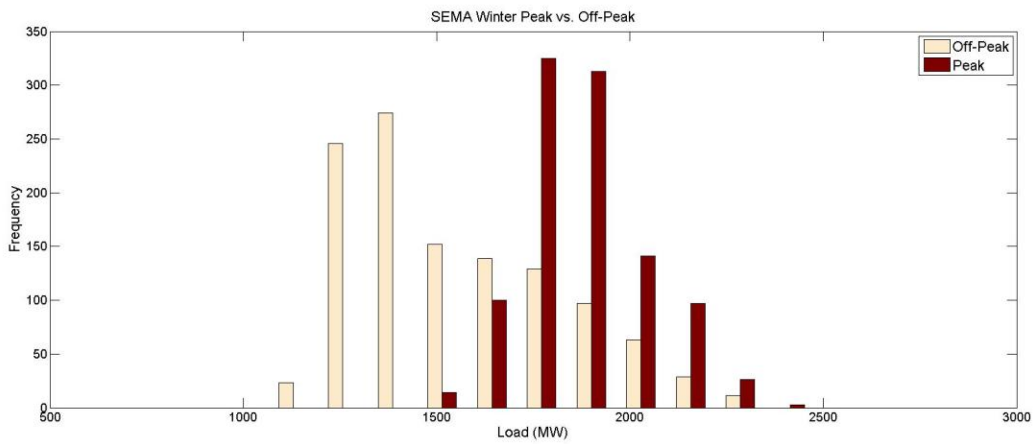
1000
1200
1400
1600


SEMA Winter Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
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100
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2500
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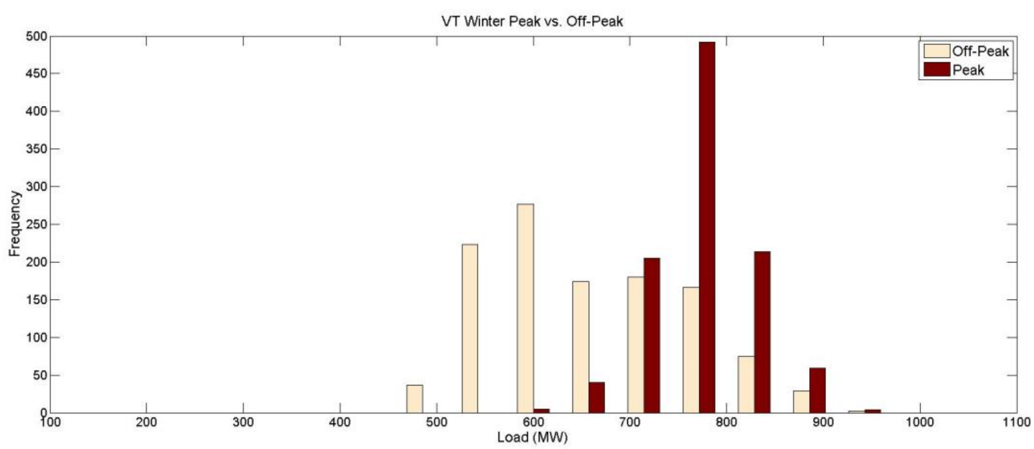
VT Winter Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
50
100
150
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300
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400
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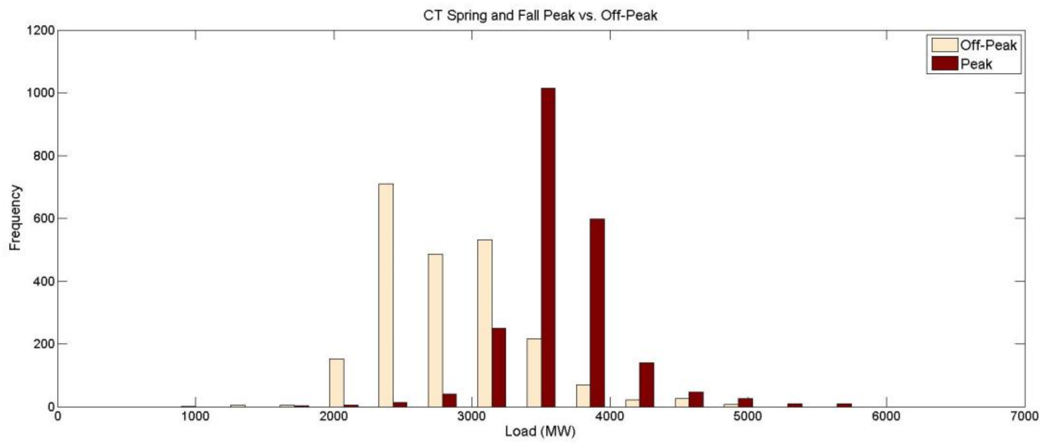
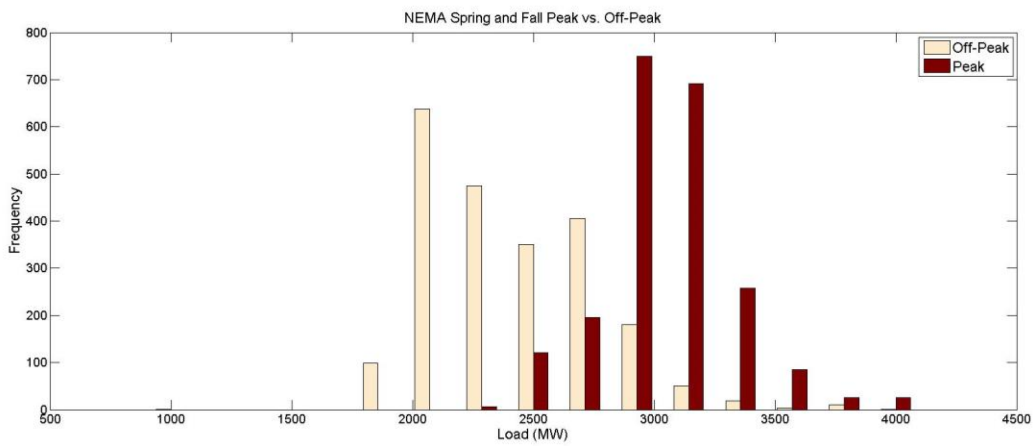
700
800
900
1000
1100


CT Spring and Fall Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
200
400
600
800
1000
1200
0
1000
2000
3000
4000
5000
6000
7000


NEMA Spring and Fall Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
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1500
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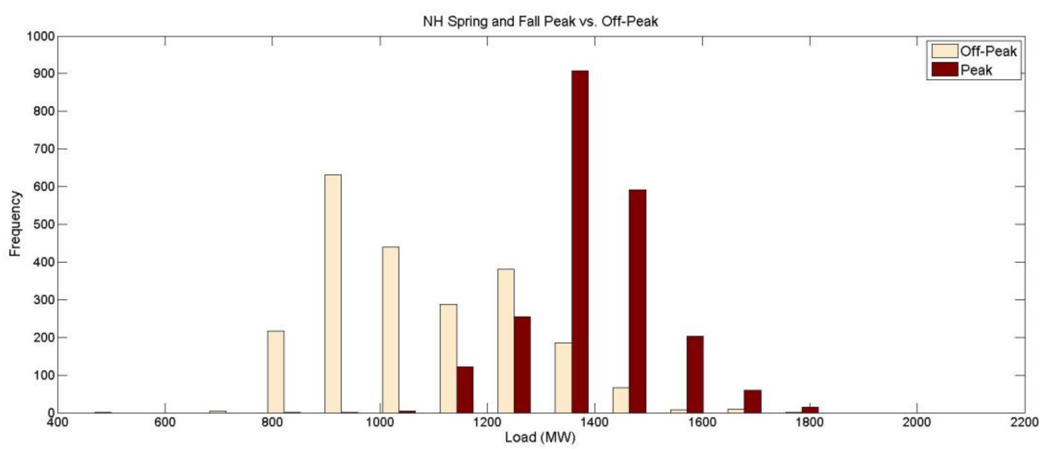
NH Spring and Fall Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
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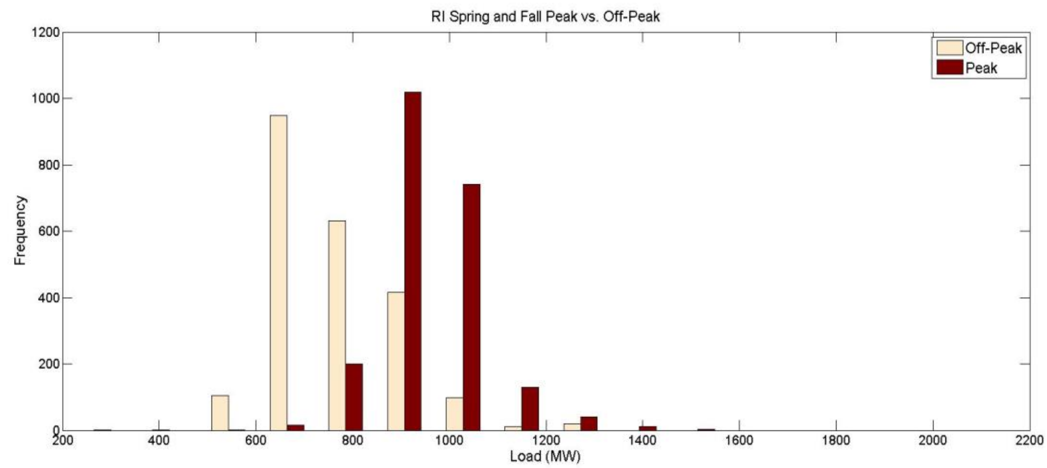
RI Spring and Fall Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
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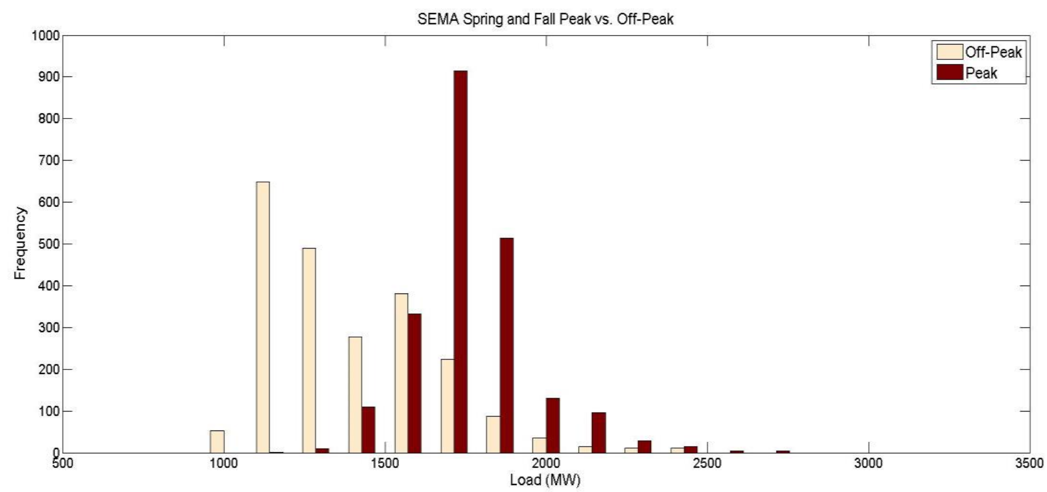
SEMA Spring and Fall Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
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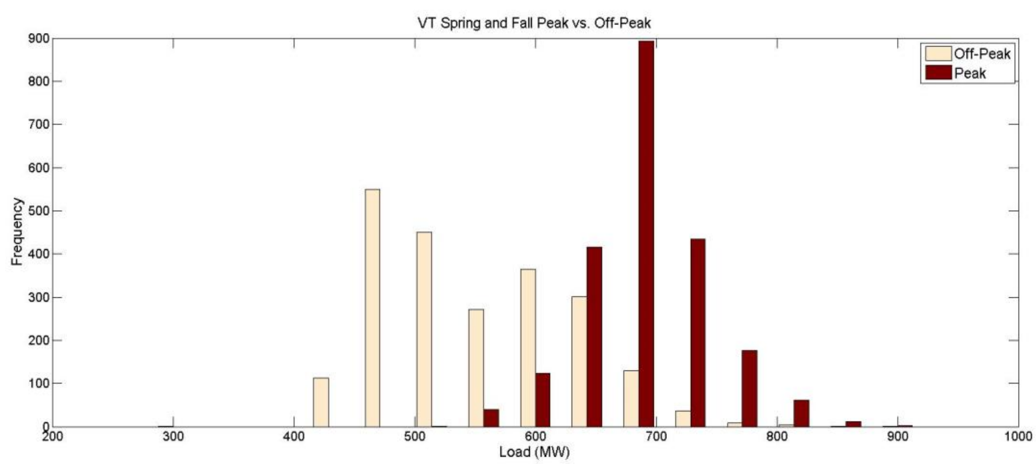
2000
2500
3000
3500


VT Spring and Fall Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
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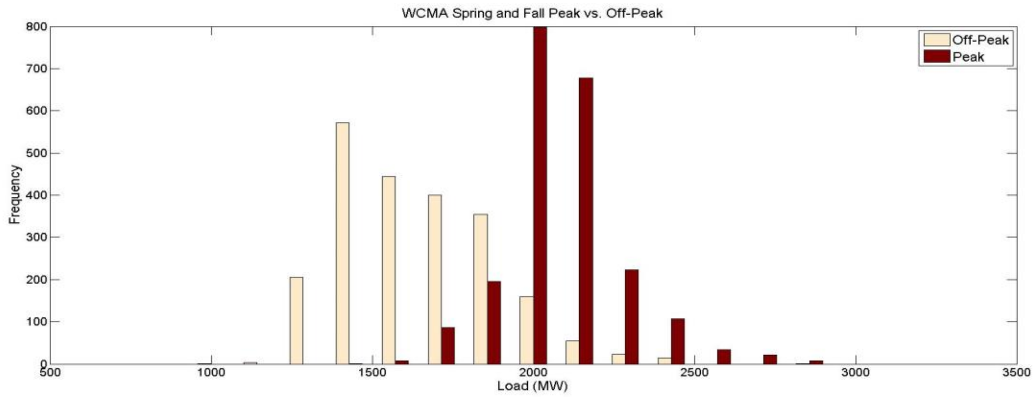
WCMA Spring and Fall Peak vs. Off-Peak
Off-Peak
Peak
Frequency
Load (MW)
0
100
200
300
400
500
600
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800
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1000
1500
2000
2500
3000
3500

B. Selected fixed demand level for 5 scenarios

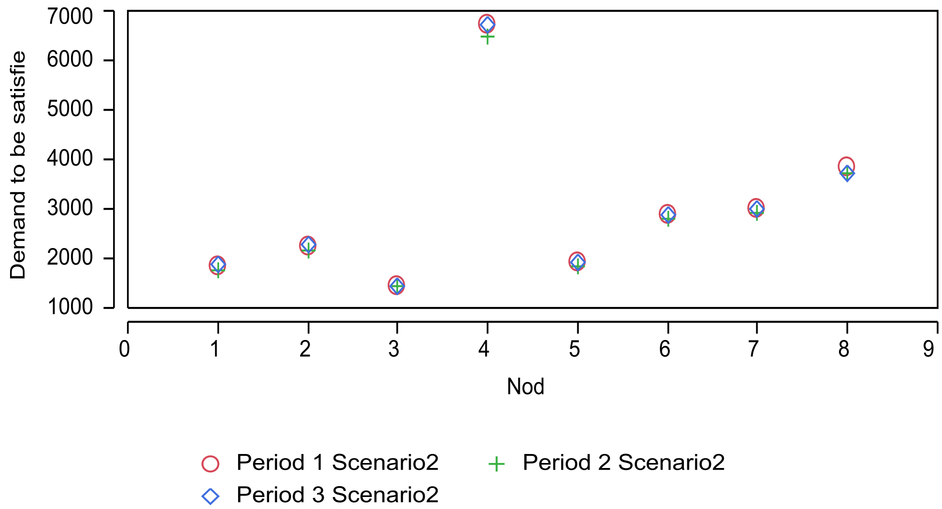
Node	Scenario	Summer	Winter	Spring/Fall
1	sc1	1414.53	1646.20	1482.61
2	sc1	1633.40	1619.04	1515.30
3	sc1	726.02	769.82	743.33
4	sc1	4789.73	4759.93	2616.84
5	sc1	1186.43	1229.33	862.53
6	sc1	2233.03	2312.46	2029.58
7	sc1	2636.60	2716.40	1535.18
8	sc1	3634.05	3425.82	3293.81
1	sc2	1414.53	1646.20	974.99
2	sc2	1633.40	1619.04	904.10
3	sc2	726.02	769.82	489.99
4	sc2	4789.73	4759.93	4086.84
5	sc2	1186.43	1229.33	1081.53
6	sc2	2233.03	2312.46	1660.52
7	sc2	2636.60	2716.40	1697.24
8	sc2	3634.05	3425.82	2664.46
1	sc3	1414.53	1646.20	1249.85
2	sc3	1633.40	1619.04	1151.30
3	sc3	726.02	769.82	625.49
4	sc3	4789.73	4759.93	3161.63
5	sc3	1186.43	1229.33	697.86
6	sc3	2233.03	2312.46	1306.53
7	sc3	2636.60	2716.40	2139.14
8	sc3	3634.05	3425.82	2222.96
1	sc4	1414.53	1433.77	1249.85
2	sc4	1633.40	1811.83	1151.30
3	sc4	726.02	847.15	625.49
4	sc4	4789.73	4170.09	3161.63
5	sc4	1186.43	1062.01	697.86
6	sc4	2233.03	2032.74	1306.53
7	sc4	2636.60	2494.64	2139.14
8	sc4	3634.05	3635.31	2222.96
1	sc5	1414.53	1646.20	1096.74
2	sc5	1633.40	1619.04	1180.35
3	sc5	726.02	769.82	553.44
4	sc5	4789.73	4759.93	2573.65
5	sc5	1186.43	1229.33	783.21
6	sc5	2233.03	2312.46	1320.98
7	sc5	2636.60	2716.40	1573.27
8	sc5	3634.05	3425.82	2654.20

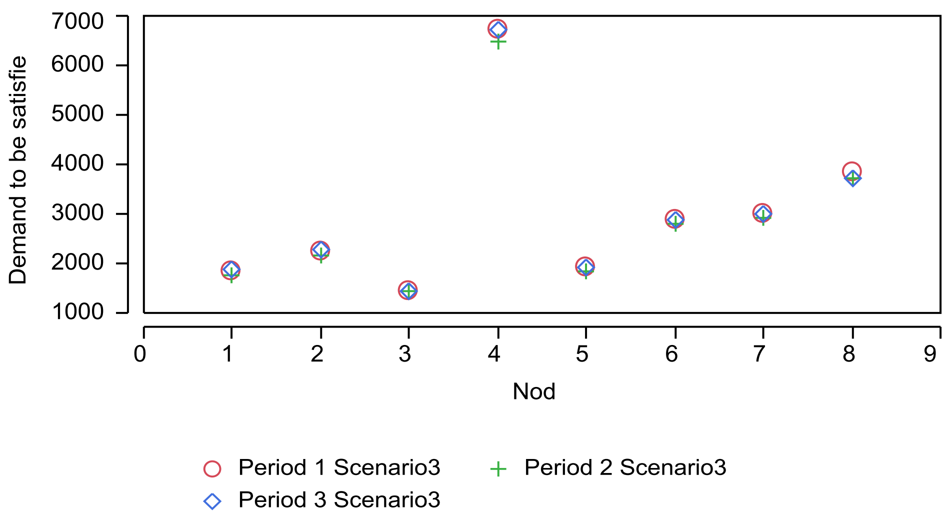
C. Generation costs for nodes in scenario with technology (node.scenario.technology) in the first and the fourth column

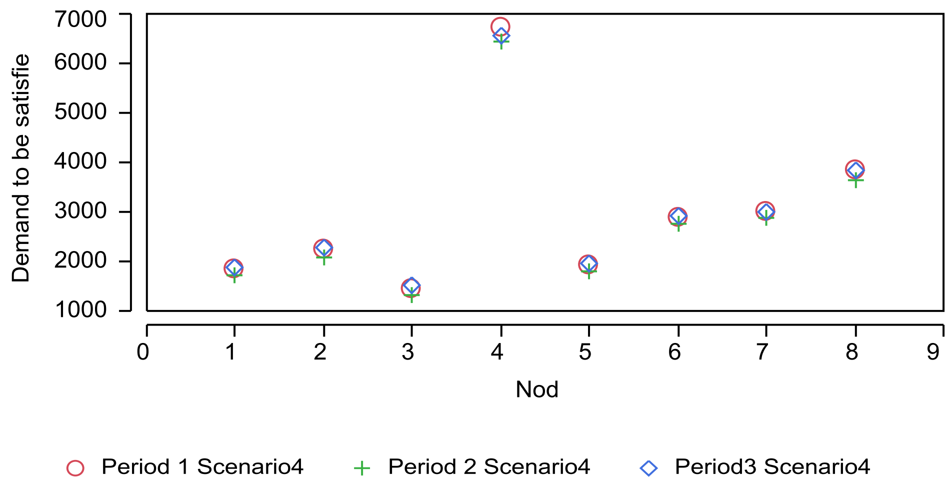
	Summer	Winter	Spring/Fall		Summer	Winter	Spring/Fall
1.sc1.cc	34.88	39.71	42.59	2.sc1.cc	32.80	42.57	41.79
1.sc1.ct	58.31	65.63	69.99	2.sc1.ct	55.15	69.97	68.78
1.sc1.nu	31.02	31.02	31.02	2.sc1.nu	31.02	31.02	31.02
1.sc1.wi	0.00	0.00	0.00	2.sc1.wi	0.00	0.00	0.00
1.sc2.cc	34.88	39.71	39.66	2.sc2.cc	32.80	42.57	40.80
1.sc2.ct	58.31	65.63	65.55	2.sc2.ct	55.15	69.97	67.27
1.sc2.nu	31.02	31.02	31.02	2.sc2.nu	31.02	31.02	31.02
1.sc2.wi	0.00	0.00	0.00	2.sc2.wi	0.00	0.00	0.00
1.sc3.cc	34.88	39.71	32.98	2.sc3.cc	32.80	42.57	29.98
1.sc3.ct	58.31	65.63	55.43	2.sc3.ct	55.15	69.97	50.87
1.sc3.nu	31.02	31.02	31.02	2.sc3.nu	31.02	31.02	31.02
1.sc3.wi	0.00	0.00	0.00	2.sc3.wi	0.00	0.00	0.00
1.sc4.cc	34.88	43.12	29.96	2.sc4.cc	32.8	41.01	31.81
1.sc4.ct	58.31	70.8	50.85	2.sc4.ct	55.15	67.6	53.65
1.sc4.nu	31.02	31.02	31.02	2.sc4.nu	31.02	31.02	31.02
1.sc4.wi	0.00	0.00	0.00	2.sc4.wi	0.00	0.00	0.00
1.sc5.cc	34.88	39.71	32.98	2.sc5.cc	32.8	42.57	29.98
1.sc5.ct	58.31	65.63	55.43	2.sc5.ct	55.15	69.97	50.87
1.sc5.nu	31.02	31.02	31.02	2.sc5.nu	31.02	31.02	31.02
1.sc5.wi	0.00	0.00	0.00	2.sc5.wi	0.00	0.00	0.00
3.sc1.cc	36.35	44.02	67.76	4.sc1.cc	33.65	41.23	67.76
3.sc1.ct	60.53	72.16	108.16	4.sc1.ct	56.44	67.94	108.16
3.sc1.nu	31.02	31.02	31.02	4.sc1.nu	31.02	31.02	31.02
3.sc1.wi	0.00	0.00	0.00	4.sc1.wi	0.00	0.00	0.00
3.sc2.cc	36.35	44.02	40.97	4.sc2.cc	33.65	41.23	47.78
3.sc2.ct	60.53	72.16	67.54	4.sc2.ct	56.44	67.94	77.86
3.sc2.nu	31.02	31.02	31.02	4.sc2.nu	31.02	31.02	31.02
3.sc2.wi	0.00	0.00	0.00	4.sc2.wi	0.00	0.00	0.00
3.sc3.cc	36.35	44.02	30.03	4.sc3.cc	33.65	41.23	35.05
3.sc3.ct	60.53	72.16	50.95	4.sc3.ct	56.44	67.94	58.57
3.sc3.nu	31.02	31.02	31.02	4.sc3.nu	31.02	31.02	31.02
3.sc3.wi	0.00	0.00	0.00	4.sc3.wi	0.00	0.00	0.00
3.sc4.cc	36.35	39.34	31.76	4.sc4.cc	33.65	50.59	45.44
3.sc4.ct	60.53	65.07	53.58	4.sc4.ct	56.44	82.13	74.32
3.sc4.nu	31.02	31.02	31.02	4.sc4.nu	31.02	31.02	31.02
3.sc4.wi	0.00	0.00	0.00	4.sc4.wi	0.00	0.00	0.00
3.sc5.cc	36.35	44.02	30.03	4.sc5.cc	33.65	41.23	35.05

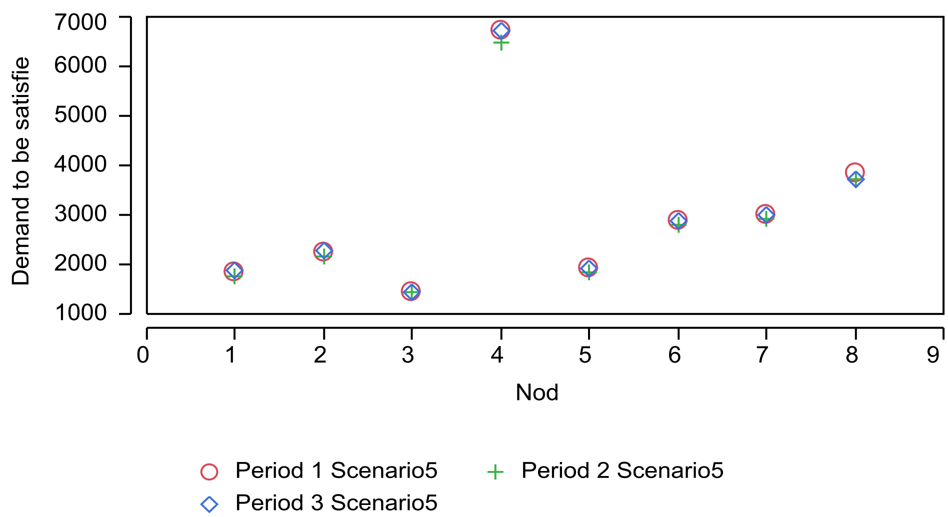
	Summer	Winter	Spring/Fall		Summer	Winter	Spring/Fall
3.sc5.ct	60.53	72.16	50.95	4.sc5.ct	56.44	67.94	58.57
3.sc5.nu	31.02	31.02	31.02	4.sc5.nu	31.02	31.02	31.02
3.sc5.wi	0.00	0.00	0.00	4.sc5.wi	0.00	0.00	0.00
5.sc1.cc	32.96	51.79	59.66	6.sc1.cc	33.85	38.81	24.77
5.sc1.ct	55.39	83.95	95.88	6.sc1.ct	56.74	64.27	42.97
5.sc1.nu	31.02	31.02	31.02	6.sc1.nu	31.02	31.02	31.02
5.sc1.wi	0.00	0.00	0.00	6.sc1.wi	0.00	0.00	0.00
5.sc2.cc	32.96	51.79	28.35	6.sc2.cc	33.85	38.81	28.35
5.sc2.ct	55.39	83.95	48.4	6.sc2.ct	56.74	64.27	48.4
5.sc2.nu	31.02	31.02	31.02	6.sc2.nu	31.02	31.02	31.02
5.sc2.wi	0.00	0.00	0.00	6.sc2.wi	0.00	0.00	0.00
5.sc3.cc	32.96	51.79	28.43	6.sc3.cc	33.85	38.81	31.07
5.sc3.ct	55.39	83.95	48.53	6.sc3.ct	56.74	64.27	52.53
5.sc3.nu	31.02	31.02	31.02	6.sc3.nu	31.02	31.02	31.02
5.sc3.wi	0.00	0.00	0.00	6.sc3.wi	0.00	0.00	0.00
5.sc4.cc	32.96	39.69	28.84	6.sc4.cc	33.85	49.53	26.28
5.sc4.ct	55.39	65.59	49.14	6.sc4.ct	56.74	80.52	45.26
5.sc4.nu	31.02	31.02	31.02	6.sc4.nu	31.02	31.02	31.02
5.sc4.wi	0.00	0.00	0.00	6.sc4.wi	0.00	0.00	0.00
5.sc5.cc	32.96	51.79	28.43	6.sc5.cc	33.85	38.81	31.07
5.sc5.ct	55.39	83.95	48.53	6.sc5.ct	56.74	64.27	52.53
5.sc5.nu	31.02	31.02	31.02	6.sc5.nu	31.02	31.02	31.02
5.sc5.wi	0.00	0.00	0.00	6.sc5.wi	0.00	0.00	0.00
7.sc1.cc	31.96	35.11	25.23	8.sc1.cc	33.85	33.47	26.78
7.sc1.ct	53.88	58.66	43.66	8.sc1.ct	56.75	56.16	46.02
7.sc1.nu	31.02	31.02	31.02	8.sc1.nu	31.02	31.02	31.02
7.sc1.wi	0.00	0.00	0.00	8.sc1.wi	0.00	0.00	0.00
7.sc2.cc	31.96	35.11	46.83	8.sc2.cc	33.85	33.47	26.76
7.sc2.ct	53.88	58.66	76.43	8.sc2.ct	56.75	56.16	46
7.sc2.nu	31.02	31.02	31.02	8.sc2.nu	31.02	31.02	31.02
7.sc2.wi	0.00	0.00	0.00	8.sc2.wi	0.00	0.00	0.00
7.sc3.cc	31.96	35.11	25.09	8.sc3.cc	33.85	33.47	45.79
7.sc3.ct	53.88	58.66	43.45	8.sc3.ct	56.75	56.16	74.84
7.sc3.nu	31.02	31.02	31.02	8.sc3.nu	31.02	31.02	31.02
7.sc3.wi	0.00	0.00	0.00	8.sc3.wi	0.00	0.00	0.00
7.sc4.cc	31.96	49.55	30.15	8.sc4.cc	33.85	49.43	26.71
7.sc4.ct	53.88	80.54	51.14	8.sc4.ct	56.75	80.37	45.91
7.sc4.nu	31.02	31.02	31.02	8.sc4.nu	31.02	31.02	31.02
7.sc4.wi	0.00	0.00	0.00	8.sc4.wi	0.00	0.00	0.00
7.sc5.cc	31.96	35.11	25.09	8.sc5.cc	33.85	33.47	45.79
7.sc5.ct	53.88	58.66	43.45	8.sc5.ct	56.75	56.16	74.84
7.sc5.nu	31.02	31.02	31.02	8.sc5.nu	31.02	31.02	31.02
7.sc5.wi	0.00	0.00	0.00	8.sc5.wi	0.00	0.00	0.00

D. Demand to be satisfied

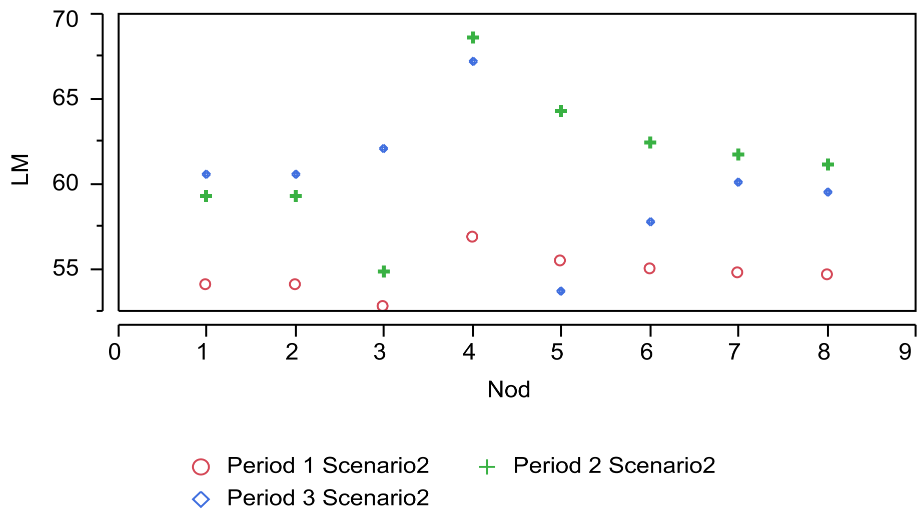


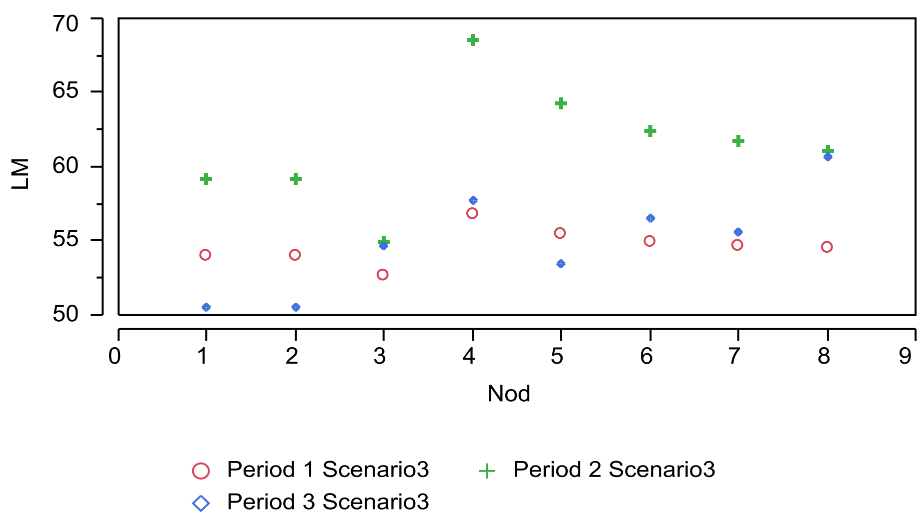


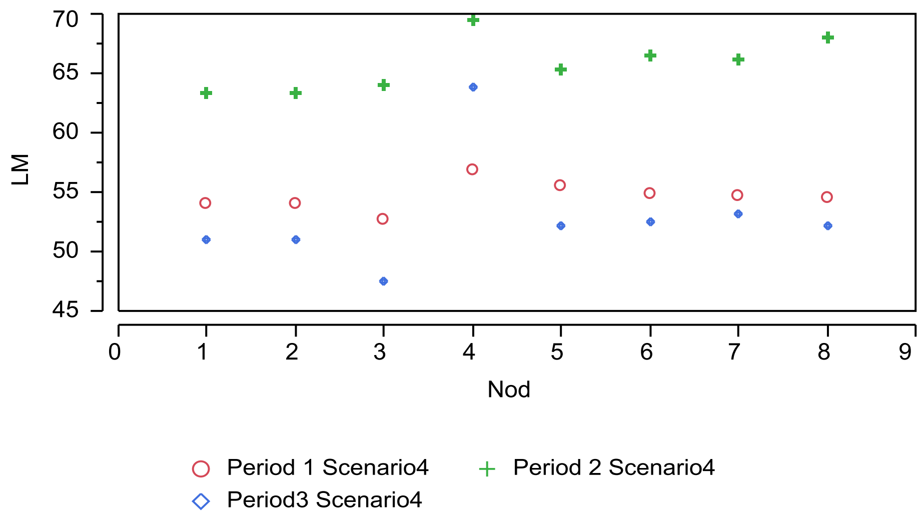

Demand to be satisfie
7000
6000
5000
4000
3000
2000
1000
0
1
2
3
4
5
6
7
8
9
Nod
Period 1 Scenario4
Period 2 Scenario4
Period3 Scenario4


Demand to be satisfie
7000
6000
5000
4000
3000
2000
1000
0
1
2
3
4
5
6
7
8
9
Nod
Period 1 Scenario5
Period 2 Scenario5
Period 3 Scenario5

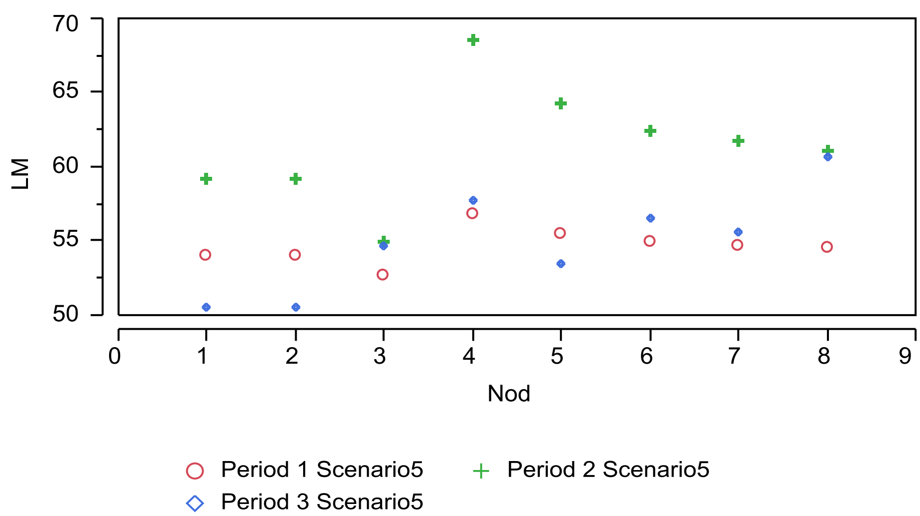
E.LMP in Second stage







70
65
60
55
50
45
LM
0
1
2
3
4
5
6
7
8
9
Nod
Period 1 Scenario4
Period 2 Scenario4
Period3 Scenario4



70
65
60
55
50
LM
0
1
2
3
4
5
6
7
8
9
Nod
Period 1 Scenario5
Period 2 Scenario5
Period 3 Scenario5