Economic Decision Making of Renewable Power Producers Under Uncertainty

by

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ABSTRACT

The recent booming development of renewable power generation and government subsidies are constantly under scrutiny and various opinions exist regarding whether subsidies should be continued or not. Motivated by the controversies and debates, this dissertation attempted to address the investment decision making problem under uncertainties in the renewable power industry from the perspective of an individual power producer.

Given that independent power producers still dominate the renewable power production and that majority of their output are sold through long-term power purchase agreements, this study focused on two types of uncertainties that could represent most of their kinds: the operations & maintenance (O&M) cost and governmental subsidy's renewal/expiration. Three types of investment activities that covers the major part of any renewable power plant's economic life are thoroughly investigated in a chronological order: an initial entry, exit when the plant reaches its economic life, and repowering.

A real-options approach was adopted and improved to model the value of a power plant considering its future activities, while both cost and policy changes modeled as some stochastic processes. Significant policy implications and managerial insights were obtained as a result of extensive analytical modeling and statistical study of empirical evidence.

CHAPTER 1 . INTRODUCTION

In recent years, there have been tremendous increases in renewable power production across the U.S. For instance, the total capacity of the wind power alone was 24,651 MW in 2008, and it increased to 34,296 MW in 2009 (EIA, 2010a). At the same time, the production technologies for the renewable energy have improved substantially. The main drivers for this phenomenal growth have not only been the economic efficiency and technology breakthroughs in renewable power production, but also been the favorable government support due to environmental concerns as well as higher oil and natural gas prices (Wiser *et al*., 2007).

From the perspective of a single renewable power producer, however, the economic uncertainties are massive and consequential. Though long-term contracted sales to utilities remains the most common off-take arrangement, increasing sales to power marketers leads to more producers taking on some merchant risks and having a portion of their revenues tied to the spot market prices, which, are highly volatile (Wiser and Bolinger, 2008). The installation cost, and capital cost of a new renewable power project are also subject to many changing market factors, such as material and energy input prices.

Another significant economic variable is the operations and maintenance (O&M) cost, which, usually includes the costs of wages and materials associated with operating and maintaining the plant, as well as rent (i.e., land lease payments). Other ongoing expenses, including taxes, property insurance, and workers' compensation insurance, are generally not included (Wiser and Bolinger, 2008). Although market data on actual project-level O&M costs for renewable power plants are scarce, evidences have shown that they are increasing on average with respect to operating time with substantial volatility (Fraunhofer IWES, 2006). Many factors can contribute to the uncertain spike in the O&M cost, such as unpredicted turbine break-down and natural disaster. In this dissertation we focus only on the O&M cost among all possible economic variables as it applies to all renewable power plants regardless ownership and financing programs.

On the other hand, governmental subsidies have also played a key role in promoting power generation from renewable sources, and are often subject to frequent expiration/renewal cycles due to legislative and political reasons. One of the most illustrative examples comes from the controversial federal Production Tax Credit (PTC), which provides per-kilowatt-hour tax credit for electricity generated by qualified energy resources. Since first enacted in 1992, the PTC has been renewed or extended on several occasions. Currently the PTC has been expired since the first day of 2014, and the most recent renewal occurred at the beginning of 2013, which was made to extend to the end of 2013 (Silverstein, 2013). In this dissertation we specifically study the PTC to represent various existing policy schemes.

Under these uncertainties, it is highly desirable to model and analyze the economically rational behaviors of renewable power producers so that the government can develop appropriate economic policies and the industry can plan their business strategies accordingly.

Hence, the overall objective of this dissertation is to understand how economically rational renewable power producers make decisions under significant and pervasive uncertainties and derive useful policy implications. In particular, we consider the following critical problems, and each of these problems will be solved as a part of the dissertation.

1) The optimal entry and exit decision making under stochastic O&M cost.

2) The optimal repowering decision making under stochastic O&M cost.

3) The optimal repowering decision making under policy uncertainties.

4) Empirical evidences and quantitative study of O&M cost evolution and policy changes.

1.1 Optimal Entry and Exit Decision Making

One major recurring question about the massive development of renewable power plants is regarding the retirement of these plants. In 1990's there were numerous cases of abandonment of renewable power plants that had been in operation since 1970's and 1980's where the plant owners simply walked away (WebEcoist.com, 2011) as the economic and non-economic conditions deteriorated. For example, there are thousands of abandoned turbines still littering the landscape in the areas of Altamont Pass, Tehachapi, and San Gorgonio in California alone (Wilkinson, 2010). The situation for solar farms is quite similar, and certainly no better (The Center for Land Use Interpretation, 2011).

At this point in time, the "rebirth" of renewable power plants is of an order of magnitude larger than the previous manifestation in 1970's and 1980's, and a priori there is no basis that one can assume the abandonment in a truly massive scale will not happen in the near future. In fact, currently, in numerous regions, if a plant owner walks away (i.e., abandons the renewable power plant), there is no or few consequences or penalties (WebEcoist.com, 2011). Specifically, we observe that, as the operation and maintenance (O&M) cost increases with respect to time (Wiser and Bolinger, 2008), there is a substantial incentive to walk away even when there remains some "physical" life in the plant (see e.g., Myers and Majd, 1983). Furthermore, we note that the O&M cost is increasing on average with respect to time, but the exact amount at a given time is typically stochastic not only because numerous repairs are unpredictable but also because the current physical age is often quite young relative to the physical life estimated by the plant builders (see e.g., Martinze *et al*., 2009).

More recently, we do note that, in a few places, there has been some consideration or implementation of a simple and straightforward exit fee as an insurance to enable the proper disposal of the renewable plants in case of abandonment (New Hampshire Office of Energy and Planning, 2008). This observation has provided us with a major motivation for studying the relationship between exit decision making and policy intervention.

Under these circumstances, it is highly desirable to understand the economically rational decisions on exits and entries of renewable power plants, which include timely policy implications especially in the areas of government's incentives and fees.

1.2 Optimal Repowering Decision Making

Developments in renewable industries have also raised another important issue regarding the upgrading and replacement opportunities of the aging renewable power plants. As stated before in Section 1.1, these plants, by today's standard, are absolutely inefficient as the operations and maintenance (O&M) cost increases, obsolete to the point of almost ineffective, and occupying perhaps the best locations and largest areas for renewable power production.

For example, for wind power, Wagman (2008) reports that the old turbines operating since 1980's in California are very inefficient and underpowered, and wind resource could be much better exploited by replacing them with new turbines with a much larger capacity, building them higher off the ground, and spacing them apart. In wind power industry, this series of actions, including the partial decommissioning of the aging plants (i.e., some infrastructure and landscape need not be fully restored as in rigorous full decommissioning) and the installation of new wind towers, is called “repowering” and in my research I extended this term to other renewable power plants.

This problem is by no means unique to the wind power industry. Many aging power plants from other renewable resources such as solar and biomass also exhibit similar problems. For example, the Arco solar power plant in San Luis Obispo, California, was originally constructed in 1983, and used to be the largest photovoltaic array in the world. It was dismantled in the late 1990’s and has not been repowered yet (WebEcoist.com, 2011), implying perhaps one of the best and largest power plants for solar power production has not been utilized for more than a decade. Therefore, it becomes very important for us to learn how renewable power producers make repowering decisions regarding their aging plants, and how to design a better policy scheme to encourage repowering.

In other countries with large numbers of aging renewable plants, various government incentives have been utilized to encourage repowering. For example, Demark has been offering production based feed-in tariffs for certified wind repowering projects since 2001, and it now has more than 2/3 of its oldest turbines repowered (Wiser, 2007). Also, Germany has been offering similar feed-in tariffs for repowering projects since the Amendment of the Renewable Technology Law in 2004 (WWEA, 2006).

In the U.S., however, we observe that repowering activities have been very slow until now. For example, in California, wind repowering projects have been conducted since the enactment of the state’s renewable portfolio standard (RPS) in 2002. By the end of 2007, approximately 365 MW’s of wind capacity have been repowered, which merely accounts for about 20% of all capacity installed before 1994 (KEMA, Inc., 2008).

A key difference between the slow pace of repowering in the U.S. and the fast pace of repowering in certain Western European countries is the government incentive policies as the physical characteristics and the technological capabilities are similar. For example, in California, numerous existing old turbines are operating as Qualifying Facilities (QF's), which are profitable given very generous contracted selling prices while excluded from the federal Production Tax Credit (PTC) if repowered (Wiser, 2007). For these renewable power producers of the aging plants, the economically rational decision may be to keep old turbines running until they no longer function, and perhaps abandon them at that time point. Hence, it is not surprising to actually see thousands of such aging turbines abandoned across the U.S. (Wilkinson, 2010).

1.3 Repowering Under Policy Uncertainties

In studying the first two problems, we focus merely on the stochastic O&M cost which represents the internal uncertainties that grows endogenously within the power plant. However, from a broader perspective of the whole industry there exist many industry-level uncertainties that will not only affect one individual plant. Instead, every single player in the industry makes their investment decisions under the influence of these external uncertainties.

As one of the biggest drivers of the growth of wind capacity in U.S, the PTC helps to reduce the price of wind-generated electricity, which even makes wind power now economically attractive in some regions. Its impact on wind generation expansion can also be partially observed by the fluctuation of annual installed capacity of wind since the PTC was first enacted in 1992. Specifically, the PTC expired in 2000, 2002 and 2004, each time resulting in decreases of at least 50% in new installed capacity (DOE, 2006). This historical experience indicates that the frequent expiration/extension cycle of PTC might have negative consequences for the growth of wind power. This boom-and-bust cycle might has made the PTC less effective in stimulating low-cost wind development for the reason that potential investors self-select to avoid investment in wind power due to the uncertainty in PTC.

Motivated by the existence of policy uncertainties as well as their effects on renewable power investment, we will formulate and solve a repowering decision making problem under both O&M cost and external policy uncertainties.

1.4 Structure of Dissertation

This dissertation consists of four main chapters, in which each chapter addresses one of the three problems stated in previous Section 1.1 through 1.3 as well as the empirical study. Chapter 2 models and analyzes how an economically rational decision maker will enter and exit a renewable power market when the O&M cost is represented by a geometric Brownian motion (GBM) process. Based on our findings, we also conduct extensive sensitivity analyses with respect to various critical parameters with major policy implications.

Chapter 3 expands the scope of our original problem in Chapter 2 into the repowering area. Following the assumption of GBM based O&M cost, we investigate the repowering decision making problem for three specific types of renewable power producers. Specifically, the decision making process of each type is formulated as a multi-stage optimal stopping problem, which is solved with analytical solutions in terms of O&M cost thresholds.

In Chapter 4, we will focus on the combined effect of stochastic O&M cost and exogenous policy uncertainties. We start with a specific policy renewal model where we assume the policy is currently not in effect but will be renewed. This is followed by an expiration model where the policy is currently in effect but will expire in the future. We then proceed to present a generalized model where the random policy switches between two regimes (i.e., in effect or not). For each of the three models the optimal repowering strategy will be obtained in terms of two threshold values.

In Chapter 5 we will empirically validate two most important and distinct modeling assumptions that I use in this dissertation across the three chapters: the GBM-based O&M cost assumption and the Poisson process-based policy cycle.

CHAPTER 2 . AN EXIT AND ENTRY STUDY OF RENEWABLE POWER PRODUCERS: A REAL OPTIONS APPROACH

In this chapter we aim to 1) formulate and analyze mathematical models of exit and entry decisions of a single renewable power plant from a real options perspective with the assumption that the O&M cost follows a GBM process, 2) show the managerial insights from sensitivity analyses surrounding the exit and entry decisions that are made by renewable power plant decision makers, and 3) derive policy implications that are theoretically interesting and practically timely. From this study, we hope to provide specific insights for relevant decisions and policies, and stimulate the critical discussion among renewable power plant decision makers, government regulators, legislative policy makers, as well as academics.

The rest of the chapter is organized as follows. We first present a review on relevant literature in Section 2.1. In Section 2.2 we model the exit decision of a currently operating renewable power plant under the assumption of a GBM based O&M cost. This model leads to the derivation of the threshold O&M cost, above which the renewable power plant exits the market, as well as to the derivation of the expected remaining life of the renewable power plant. In Section 2.3, we conduct sensitivity analysis of the exit threshold O&M cost and the expected remaining life with respect to various critical parameters such as the exit fee. We then consider the entry decision of a new renewable power plant in Section 2.4, and derive the threshold O&M cost, below which such an investment will be made, as well as the expected life of the renewable power plant. This is also followed by the sensitivity analysis of the entry threshold O&M cost and the expected life in Section 2.5. Section 2.6 presents some of the key features of our model through an illustrative numerical example based on a wind farm, and in Section

2.7 we discuss the policy implications and the various assumptions defining the scope of our study. Concluding remarks and comment on future research are provided in Section 2.8.

2.1 Literature Review

For this study, there are several groups of relevant literature. First, in the area of financial and economic applications in the electric power industry, Wang and Min (2006) applies a real options approach to a case of inter-related generation projects. Also, in Wang and Min (2008), a financial portfolio consisting of electric power commodities was managed while, in Wang and Min (2010), financial hedging techniques were developed for electric power producers.

In the context of nuclear power plants, Takashima *et al*. (2007) investigates decommissioning by applying a real options approach. That paper differs from our approach as the primary driver of the exit was claimed to be the stochastic electricity price, and not the cost components, which were assumed to be deterministic. We caution that there are other reasons for an exit such as competitive or regulatory concerns, which are beyond the scope of this paper. As for solar power, Lorenz *et al*. (2008) advocates phasing economic incentives out, but only prudently and gradually.

For wind power management, Fleten and Maribu (2004) addresses the investment timing and capacity choice of wind power under uncertainty under a fixed quantity contract. Also, Fleten *et al.* (2007) examines the investment strategies when renewable power is generated in a decentralized manner. These papers are different from our approach because they would not be able to explain the exit behavior that can be theoretically predicted or empirically observed. Also, the stochastic components in these papers are not the O&M cost (e.g., electricity price).

As for the stochastic O&M costs, Khoub Bakht *et al.* (2008) investigates statistical repair and maintenance cost models for tractors while Leung and Lai (2003) statistically studied the quality and reliability aspects of bus engines. Almansour and Insley (2011), on the other hand, applies the stochastic cost model for the oil sands in Canada. In addition, in Costa Lima and Suslick (2006), both the price and operating costs are modeled as GBM processes for mining projects.

Meanwhile, in the area of equipment replacement, there are numerous examples of more conventional (i.e., non-real options approaches) papers (see e.g., Hartman and Murphy, 2006). In Ye (1990), the replacement strategy is derived under the assumption of a GBM O&M cost while in Zambujal-Oliveira and Duque (2011) considers both O&M cost and salvage value as GBM processes.

2.2 Modeling and Analysis of Exit Decision

In this section, let us consider a firm consisting of a single renewable power plant that is currently in operation. For the decision maker, we assume that there exists an option to abandon the renewable power plant, and to exit the electricity market. Furthermore, we make the following critical assumptions that enable us to model and analyze the exit decision that is economically rational as well as tractable. We note that the relaxation of the assumptions that would enhance the realism of the models in this paper will be addressed in Section 2.7 later.

Assumption 1: There already exists a power purchase contract between the renewable power plant and a utility company with the appropriate transmission connection at a fixed selling price of P (\$/MWh) at any time point (see Roques *et al*., 2008). We assume that the upper bound of the renewable power purchase quantity of the contract will not be reached at this power plant.

Assumption 2: In making the exit decision, the decision maker will rely on the fixed (expected) production quantity per unit time, which is equal to the quantity demanded by the utility (e.g., via a contract). By this assumption, we are not implying the actual production and demand quantities remain constant over time, but, for ***planning purposes*** of the producer, we are simplifying the dynamic aspects of renewable energy source such as wind speed or daylight availability, etc. (see e.g., Fleten and Maribu, 2004). Specifically, we utilize the fixed annual production quantity, K (MWh/year). We further assume that, this production quantity is equal to the nameplate capacity times the capacity factor, which is a standard procedure for the generation planning in the electric power industry, alternative procedures notwithstanding (see e.g., Wiser and Bolinger, 2007)

Assumption 3: The O&M cost (\$/MWh) at any time point, C, follows the geometric Brownian motion (GBM) process. Specifically,

$$dC = \alpha_C C dt + \sigma_C C dz \tag{2.1}$$

where α_C is the instantaneous growth rate of the O&M cost (% per year), σ_C is the instantaneous volatility of C (% per square root of year), and dz is the increment of a standard Wiener process z ($dz = \varepsilon_t \sqrt{dt}$ where $\varepsilon_t \sim N(0,1)$).

Typically, the O&M cost includes costs associated with repairs, spare parts, maintenance, and consumables necessary for O&M (see e.g., EWEA, 2004). Also, we note that modeling the O&M cost as a GBM process is not new as we explained in the introduction.

Assumption 4: There exists a fixed exit fee of W ($W \geq 0$) when the option to abandon is exercised, and the decision maker is aware of this *a priori*. Currently in the U.S., few local, state, and federal rules and regulations exist that require the removal and disposal of renewable power plants when the option to abandon is exercised, and some authors claim that the renewable power plant decision makers have strong incentive to abandon their renewable power plants even if it is physically viable to operate (Wilkinson, 2010). Our model will allow this current situation as a special case of $W = 0$.

Assumption 5: The remaining cost components are irrelevant to the decision maker regarding the option to abandon. i.e., the cost components such as non-specific overheads, taxes, etc. are negligible for our planning purposes. We also assume that, there is no salvage value at the time point of exit, and our model does not explicitly factor in specific government programs such as the production tax credit (see e.g., Union of Concerned Scientists, 2011).

Assumption 6: More sophisticated options such as a partial shut-down, mothballing, etc. are not available for the renewable power plant. These options are beyond the scope of this paper, and will not be considered.

Under these assumptions, our problem can be interpreted as an optimal stopping problem. We observe that, the higher the O&M cost, the stronger the incentive to abandon. Therefore, intuitively, the range of the O&M cost where the optimal decision is to abandon may be characterized by a single scalar threshold of C^*. That is, if C is between $[C^*, \infty)$, then the optimal decision is to abandon. And, otherwise, the optimal decision is to continue operation. It can be shown (see e.g., Dixit and Pindyck, 1994) that indeed there will be only one threshold C^* for this problem.

As long as it is not optimal to abandon the renewable power plant (i.e., C is in the continuation region), the value of the renewable power plant project, $V(C)$, must satisfy the following differential equation, which results from Bellman's principle of optimality (Dixit and Pindyck, 1994):

$$\rho V dt = \left(P - C\right) K dt + \mathrm{E}\left[dV\right] \tag{2.2}$$

where ρ is the annual discount rate (% per year), which is often called the expected rate of return (Dixit and Pindyck, 1994). The left-hand side of (2.2) is the total return on the value of the renewable power plant. The first term of the right-hand side is the immediate profit flow from keeping the renewable power plant operating while the last term is the expected capital appreciation of the renewable power plant value function.

By Ito's Lemma, dV could be expanded as

$$dV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial C}\left(\alpha_C C dt + \sigma_C C dz\right) + \frac{1}{2}\frac{\partial^2 V}{\partial C^2}\sigma_C^2 C^2 dt$$

We notice that we are considering an indefinite horizon problem here so $V(C)$ is time invariant. Hence, $\frac{\partial V}{\partial t} = 0$ and the above equation becomes

$$dV = \frac{\partial V}{\partial C}\left(\alpha_C C dt + \sigma_C C dz\right) + \frac{1}{2}\frac{\partial^2 V}{\partial C^2}\sigma_C^2 C^2 dt \tag{2.3}$$

Substituting (2.3) into (2.2), and after rearranging and simplifying terms, we have

$$\frac{1}{2}\frac{\partial^2 V}{\partial C^2}\sigma_C^2 C^2 + \frac{\partial V}{\partial C}\alpha_C C - \rho V + (P - C)K = 0 \tag{2.4}$$

We note that (2.4) is a differential equation. To guarantee its convergence, we impose that $\alpha_C < \rho$ (see e.g., Costa Lima and Suslick, 2006). We also note that the following boundary conditions between the operation and abandonment states are needed to obtain the optimal threshold C^*.

$$V(C^*) = -W \tag{2.5}$$

$$V'(C^*) = 0 \tag{2.6}$$

The first one is the value-matching condition and the second one is the smooth-pasting condition. The value-matching condition (2.5) requires that at the exit threshold, the value of the renewable power plant project equals the value of exit. The smooth-pasting condition (2.6) assures that C^* is the optimal exercise point by defining the continuance and smoothness of $V(C)$ at C^*.

By solving (2.4) with (2.5) and (2.6), we have the following proposition.

Proposition 2.1 Given $0 < \alpha_C < \rho$, the value of an operating renewable power plant is

$$V(C) = A_1 C^{\beta_1} - CK/(\rho - \alpha_C) + PK/\rho \tag{2.7}$$

where $\beta_1 = \left(1/2\,\sigma_C^2 - \alpha_C + \sqrt{\left(\alpha_C - 1/2\,\sigma_C^2\right)^2 + 2\sigma_C^2\rho}\right)\Big/\sigma_C^2$ (2.8)

$$C^* = \frac{(PK/\rho + W)(\rho - \alpha_C)\beta_1}{(\beta_1 - 1)K} \tag{2.9}$$

$$A_1 = \frac{K}{(\rho - \alpha_C)\beta_1 \left(C^*\right)^{\beta_1 - 1}} \tag{2.10}$$

The proof is given in Appendix A.1.1. The economically rational decision is as follows. As soon as the O&M cost is equal to or greater than C^*, the power producer will pay the exit fee and abandon the renewable power plant with the corresponding renewable power plant project value of $-W$. Otherwise, the producer continues to operate the renewable power plant with the corresponding value of $V(C)$.

We also note that the value function $V(C)$ given by (2.7) has the following interpretation. Before the decision maker chooses to abandon (i.e., the decision maker is still holding the option to abandon), the value of the renewable power plant consists of two parts: the value of the operating renewable power plant and the value of the option to abandon the renewable power plant. Hence, in (2.7), the first one term is the value of the option to abandon, while the last two terms represent the expected cost and revenue streams when the initial price and cost are observed as P and C.

Thus far, we have derived the value function of the renewable power plant project, and the threshold value of C^* in terms of aforementioned critical parameters. We now proceed to derive the expected remaining life of the renewable power plant under the technical assumption of $\alpha_C > 1/2\sigma_C^2$, as shown in the relevant literature (see e.g., Mauer and Ott, 1995). Let $F(C) = \ln C$. Then, $dF(C)$ can be expanded by Ito's Lemma as $dF(C) = \left(\alpha_C - 1/2\sigma_C^2\right)dt + \sigma_C dz$ and therefore for any finite time period T, the change in $F(C)$ is distributed with mean $\left(\alpha_C - 1/2\sigma_C^2\right)T$ and variance $\sigma_C^2 T$. Hence the expected first passage time of C^*, measured from C_c, the current level of the O&M cost, can be calculated as $\left(\ln C^* - \ln C_c\right)/\left(\alpha_C - 1/2\sigma_C^2\right)$.

Hence, the expected remaining operating life is given by,

$$T_{EX}^* = \left(\ln C^* - \ln C_c\right)/\left(\alpha_C - 1/2\sigma_C^2\right) \tag{2.11}$$

where C^* is the exit threshold as in (2.9).

2.3 Sensitivity Analysis of the Exit Decision on α_C, σ_C, and W

Given the expression for C^* shown in (2.9), the sensitivity analysis can be performed in a straightforward manner with respect to the key parameters of α_C, σ_C, and W, and the results are summarized in the following proposition.

Proposition 2.2 Given $0 < \alpha_C < \rho$, $\dfrac{\partial C^*}{\partial \sigma_C} > 0$, $\dfrac{\partial C^*}{\partial \alpha_C} < 0$, $\dfrac{\partial C^*}{\partial W} > 0$, $\dfrac{\partial C^*}{\partial P} > 0$, and $\dfrac{\partial C^*}{\partial K} < 0$.

The outline of the proof is given in Appendix A.1.2, and the economic interpretation of the results is as follows. $\partial C^* / \partial \sigma_C > 0$ indicates that an increase in the volatility leads to the increase in the threshold value. This is because with increased volatility, there is a greater chance of a deeper reduction in the O&M cost in the near future, and it is beneficial to wait a little longer (hence, C^* becomes higher). As for $\partial C^* / \partial \alpha_C < 0$, we note that as the O&M cost growth rate increases, it is beneficial to exit earlier (hence, C^* becomes lower). $\partial C^* / \partial W > 0$ indicates that an increase in the exit fee leads to an increase in the threshold value. This is so because, with a higher level of the exit fee, the decision maker has an incentive to wait a little longer for a possible reduction in the O&M cost in the near future (hence, C^* becomes higher). Now, as for $\partial C^* / \partial P > 0$, if the contract power price is increasing, we note that the revenue and the value of the renewable site project should increase, and the decision maker has an incentive to continue to operate a little longer (hence, C^* becomes higher). Finally, $\partial C^* / \partial K < 0$ indicates that as the production quantity increases, the threshold value decreases. This is so because the total O&M cost increases even more as it is proportional to the production quantity.

Therefore, it is more beneficial to exit earlier (hence, C^* becomes lower). We note that, due to our Assumption 2 (this production quantity is equal to the nameplate capacity times the capacity factor), this sensitivity analysis can be applied to the capacity in place of production quantity interchangeably.

If we assume that preventing a premature exit (relative to the physical life) is environmentally desirable (so that we can delay exploiting new resources), increases in W is desirable. Hence, any government policy to increase the exit fee is desirable. For example, if a community is recruiting a single renewable power plant, the smaller capacity is more desirable than the larger capacity assuming that preventing a premature exit is the primary criterion. We caution that this interpretation is strictly focusing on the exit decision of the existing renewable power plants under the aforementioned criterion, and will be revisited numerous times in the succeeding sections of this paper.

Finally, we note that the sign of $\partial C^* / \partial \rho$ is ambiguous as some components of C^* increase while others decrease in a way that the total effect is unwieldy to interpret. We also note that, with the expected remaining operating life of (2.11), we have $\partial T_{EX}^* / \partial C_c < 0$, $\partial T_{EX}^* / \partial \alpha_C < 0$, $\partial T_{EX}^* / \partial \sigma_C > 0$, $\partial T_{EX}^* / \partial W > 0$, $\partial T_{EX}^* / \partial P > 0$, and $\partial T_{EX}^* / \partial K < 0$. The interpretation of these results is analogous to the case with respect to C^*.

2.4 Modeling and Analysis of the Entry Decision

In this section, we extend the previous model by considering the entry decision for the renewable power plant. For this extension, we assume that the aforementioned power purchase contract is available for a new renewable power plant. Two additional assumptions are made as follows.

Assumption 7: The construction period of the renewable power plant is assumed to be negligible in our model. This simplifying assumption is made so as to focus on the entry decision without diluting our attention on how best to make economic decisions during the construction period. This type of simplifying assumption can be found in numerous papers (see e.g., Fleten and Maribu, 2004).

Assumption 8: Once the construction occurs, then there is a lump sum investment cost of I ($), which includes the cost of materials, labor, land, etc. This cost is treated as an irreversible sunk cost, which cannot be recovered later.

We note that the firm makes an entry decision by evaluating the direct net revenue (i.e., revenue minus cost) from the renewable power plant plus the value of the option to exit. We recall that the option value critically depends on the O&M cost, and we will denote C_0 as the initial O&M cost at time point at which the renewable power plant starts to operate. We further note that there is no O&M cost prior to the start of the renewable power plant.

We also note that, under the additional assumption of the contract power price following a GBM process, the option value for waiting to enter can be incorporated. In our model construction, however, such an option is excluded by design because 1) there are numerous fixed price contracts already in practice (see aforementioned Fleten and Maribu, 2004, in the Introduction section) and 2) parallel GBM processes of the price and cost typically make analytical studies infeasible and often make numerical results difficult to sort out (see more in the Section 2.7).

Under our model framework, if the value of the potential renewable power plant project is greater than or equal to the irreversible investment I , the firm will enter the market. Therefore, the condition under which he decides to enter becomes:

$$V(C_0) = A_1 C_0^{\beta_1} - C_0 K / (\rho - \alpha_C) + PK/\rho \geq I \tag{2.12}$$

For the boundary "marginal" firm without a strictly positive net benefit, we define another type of the O&M cost threshold, $\overline{C}_0$. Namely,

$$A_1 \overline{C}_0^{\,\beta_1} - \overline{C}_0 K / (\rho - \alpha_C) + PK/\rho - I = 0 \tag{2.13}$$

This $\overline{C}_0$ of (2.13) can be viewed as the upper bound of the initial O&M cost at which the firm will decide to enter the market.

More formally, even though there is no explicit closed-form solution for $\overline{C}_0$ in (2.13), we have the following proposition that proves the existence and uniqueness of $\overline{C}_0$ under two fairly undemanding conditions.

Proposition 2.3 Let us assume that $\overline{C}_0 < C^*$ and $PK/\rho - I > 0$, then there exists a unique solution for $\overline{C}_0$ in $A_1 \overline{C}_0^{\,\beta_1} - \overline{C}_0 K / (\rho - \alpha_C) + PK/\rho - I = 0$.

The proof is given in Appendix A.1.3. The condition $\overline{C}_0 < C^*$ is not stringent as, if $\overline{C}_0 \geq C^*$, a power producer will enter and exit the market instantaneously, which leads to no practical use nor sense (not unlike assuming $\alpha_C \geq \rho$). The condition $PK/\rho - I > 0$ indicates that the cumulative revenue ($\int_0^\infty PKe^{-\rho s} ds = PK/\rho$) is greater than the initial investment I. It is a reasonable assumption that any firm considering the entry has a tangible revenue stream that will at least cover the initial investment.

From Proposition 2.3 and the monotonicity of $V(C_0)$ shown in the proof, we claim that any firm with $C_0 \leq \overline{C}_0$ will enter the market while any other one with $C_0 \geq \overline{C}_0$ will not.

Finally, we note that, as in the case of the exit decision, we derive the expected economic life of a new renewable power plant to be

$$T^* = \left(\ln C^* - \ln C_0\right) \Big/ \left(\alpha_C - 1/2\,\sigma_C^2\right) \tag{2.14}$$

and that of a marginal renewable power plant with $C_0 = \bar{C}_0$ to be

$$\bar{T} = \left(\ln C^* - \ln \bar{C}_0\right) \Big/ \left(\alpha_C - 1/2\,\sigma_C^2\right) \tag{2.15}$$

2.5 Sensitivity Analysis of the Entry Decision on α_C, σ_C, and W

Given the implicit function for $\bar{C}_0$ in (2.13), sensitivity analysis can be performed with respect to the key parameters and the results are summarized in the following proposition.

Proposition 2.4 Given $0 < \alpha_C < \rho$, $\dfrac{\partial \bar{C}_0}{\partial \sigma_C} > 0$, $\dfrac{\partial \bar{C}_0}{\partial \alpha_C} < 0$, $\dfrac{\partial \bar{C}_0}{\partial W} < 0$, $\dfrac{\partial \bar{C}_0}{\partial P} > 0$, and $\dfrac{\partial \bar{C}_0}{\partial I} < 0$.

The outline of the proof is given in Appendix A.1.4. The interpretation for $\partial \bar{C}_0 / \partial \alpha_C < 0$, $\partial \bar{C}_0 / \partial \sigma_C > 0$, and $\partial \bar{C}_0 / \partial P > 0$ is straightforward. i.e., as the volatility and the contract power price increase, the entry threshold O&M cost increases. i.e., more (marginal) firms will enter the market. On the other hand, as the growth rate in the O&M cost increases, less (marginal) firms will enter the market.

As for $\partial \bar{C}_0 / \partial W < 0$, an increase in the exit fee will lead to a decrease in the threshold O&M cost. This implies that less firms will enter, resulting in a lower power production quantity from renewable energy. At the same time, this will lead to a lower number of marginal firms, reducing the number of premature exits (relative to the physical life). Therefore, the economic and environmental consequences of a government policy for a higher exit fee on the entry (not exit) of renewable power plants are far from simple and straightforward.

As for $\partial \overline{C}_0 / \partial I < 0$, the increase in the initial investment will decrease the threshold O&M cost. This implies that any initial subsidy provided by the government will lead to more (marginal) firms entering the market resulting in a higher power production quantity from the renewable energy. On the other hand, this will lead to a higher number of marginal firms, increasing the number of premature exit (relative to the physical life). Once again, the consequence of a government policy for a higher level of initial subsidy on the entry (not exit) is complex.

In addition, we note that the signs of $\partial \overline{C}_0 / \partial \rho$ and $\partial \overline{C}_0 / \partial K$ are ambiguous with the reason that is similar to the one for the case of ρ in the exit sensitivity analysis.

Let us now turn our attention to the expected life of a new renewable power plant, T^*. As in the case of the exit decision, we can obtain $\partial T^* / \partial C_0 < 0$, $\partial T^* / \partial \alpha_C < 0$, $\partial T^* / \partial \sigma_C > 0$, $\partial T^* / \partial W > 0$, $\partial T^* / \partial P > 0$, and $\partial T^* / \partial K < 0$ with straightforward and intuitive interpretation.

As for the expected life of a new renewable power plant from a marginal producer, $\overline{T}$, generally, the signs from the sensitivity analysis are ambiguous and the corresponding interpretation unwieldy. The only two exceptions are $\partial \overline{T} / \partial W > 0$ and $\partial \overline{T} / \partial I > 0$, i.e., ., as the exit fee or the initial investment increases, the corresponding expected life increases. This is so because, as the exit fee or the initial investment increases, the threshold O&M cost to enter will decrease, which result in longer expected life.

2.6 Numerical Analysis: The Case of a Wind Farm

In this section, we numerically illustrate some of the key features of our models.

2.6.1. Parameter Values

Let us first present the parameter values used in this section. Even though these values are hypothetical, to be realistic numbers, we have consulted U.S. Energy Information Administration's *Updated Capital Costs Estimates for Electricity Generation Plants* (EIA, 2010b) as well as others (e.g., Kjarland, 2007; Takashima *et al*., 2007). They are summarized in Table 2.1.

Table 2.1 Parameters and corresponding values

Contract power price P	48 \$/MWh
Nameplate capacity/Capacity factor	3 MW/33.33%
Production quantity K	8,760 MWh/yr
Investment cost I	\$1,000,000
Exit fee W	\$300,000
Annual discount rate ρ	0.05
Annualized growth rate of O&M cost α_C	0.04
Annualized volatility of O&M cost σ_C	0.10

As explained in Assumption 2, the linkage between the capacity and production quantity is as follows. Let us assume that the total number of hours of operations per year is given by 8,760 hours. Then, as the production quantity per year as well as the nameplate capacity and capacity factor are assumed to be constant, $K = 3*0.3333*1*8,760 = 8,760$ (MWh/yr).

2.6.2. The Entry/Exit Decisions

By applying the parameter values to (2.7) - (2.10) and (13), the threshold values of C^* and $\overline{C}_0$ (\$/MWh) as well as the function of $V(C)$ can be calculated. At the same time with $\overline{C}_0$, we can use (2.15) to calculate $\overline{T}$ as well. The numerical results are summarized in Table 2.2.

Table 2.2 Numerical results of the entry/exit decisions

β_1	1.2170
A_1	300, 800
C^*	55.7623
$\overline{C}_0$	30.0549
$V(C)$	$300{,}800C^{1.2170} - 876{,}000C + 8{,}409{,}600$
T	17.6592

We do note that, the first term of $V(C)$ is the option value to exit, the second term is the O&M cost, and the third term is the revenue. Also, given $\overline{C}_0$ value of 30.0549, it can be verified that $V(\overline{C}_0)$ is 1,000,000, which is the initial investment. Furthermore, if a firm is considering entry, given the initial O&M cost, we can calculate the expected life of a new wind farm by (2.14). Likewise, if a firm is currently operating the wind farm, then given the current O&M cost, we can calculate the remaining life of the existing wind farm by (2.11).

Figure 2.1 shows the value of the project with respect to the O&M cost when the wind farm is in operation. As indicated in the graph, the value of the project decreases as the O&M cost increases until it reaches C^*. Once the cost reaches the exit threshold, the firm will pay the exit fee to abandon the wind farm and thus $V_{EX}(C) = -W$.

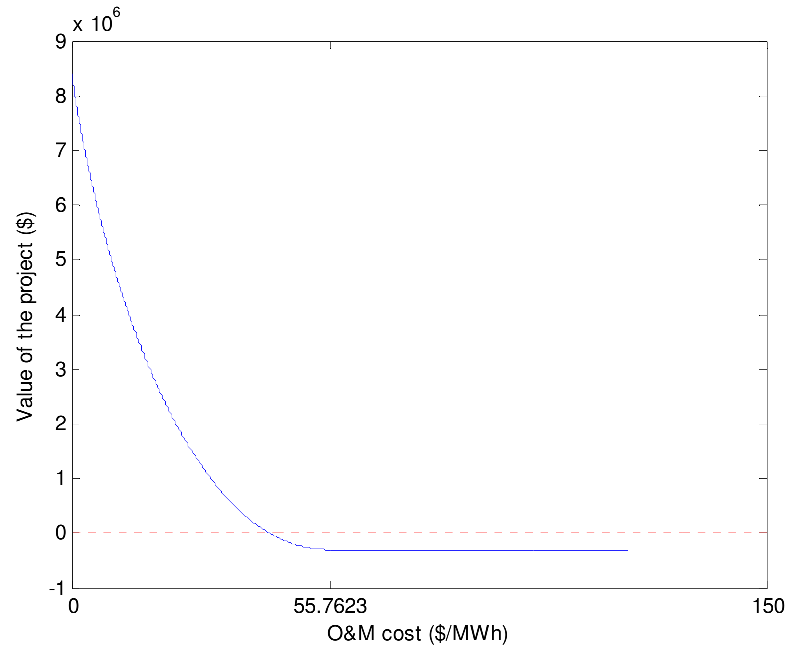

Figure 2.1 Value of the project vs. the O&M cost

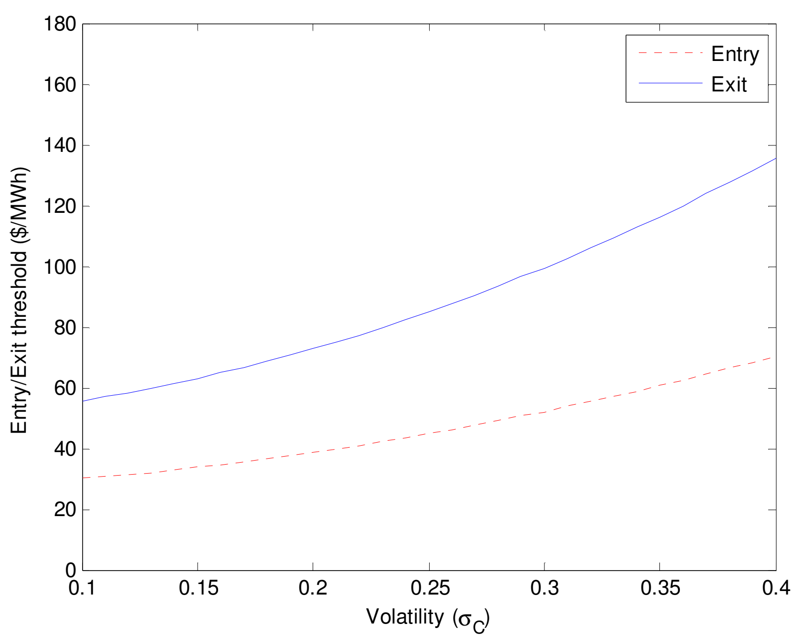

Figure 2.2 Entry/exit thresholds vs. the volatility of O&M cost

In Figure 2.2 above, the thresholds of the O&M cost C^* and $\overline{C}_0$ are depicted with respect to the O&M cost volatility. Both threshold values increase as the O&M cost becomes more volatile, which indicates that a higher degree of volatility will delay the exit and allow more (O&M cost-wise) marginal firms to enter. It is interesting to note that the slope is much steeper for the exit threshold than that for the entry threshold as the volatility increases. i.e., it seems that the exit threshold is more sensitive to volatility than the entry threshold is (Note that the entry cost is at time zero while the exit cost is some time in the future).

In Figure 2.3, the thresholds of the O&M cost C^* and $\overline{C}_0$ are depicted with respect to the O&M cost growth rate. Both threshold values decrease as the growth rate increases. This implies that a higher growth rate of the O&M cost induces an early exit and allows marginal firms to enter the market.

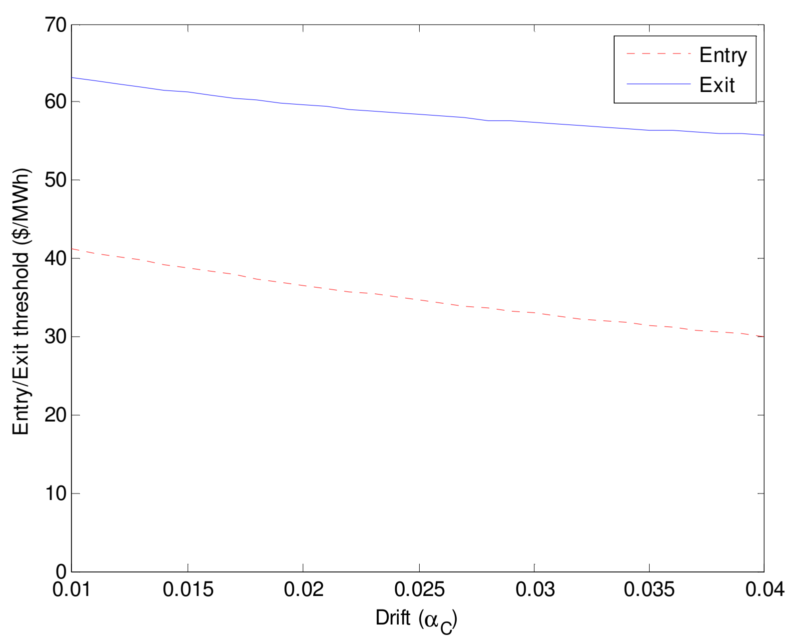

Figure 2.3 Entry/exit thresholds vs. the growth rate of O&M cost

In Figure 2.4, the thresholds of the O&M cost C^* and $\bar{C}_0$ are depicted with respect to the exit fee. As we can see from the graph, as the exit fee increases, the exit threshold increases, which leads the existing producer to delay abandoning. At the same time, it allows marginal firms to enter the market, reducing the total power production from wind energy.

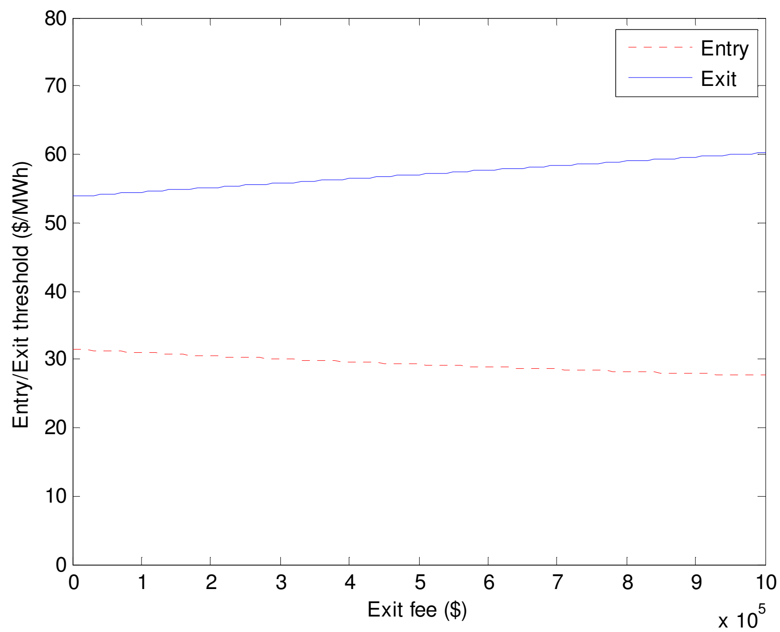

Figure 2.4 Entry/exit thresholds vs. the exit fee

We now proceed to examine the impact of bi-directional changes on the entry and exit thresholds. First in Figure 2.5, we are varying the volatility and growth rate, and observe that the entry threshold is behaving as predicted. Furthermore, the degree of changes seems linear, rather than nonlinear.

In Figure 2.6, we are varying the volatility and growth rate, and observe that the exit threshold is behaving as predicted. Furthermore, the degree of changes here also seems to be linear rather than nonlinear.

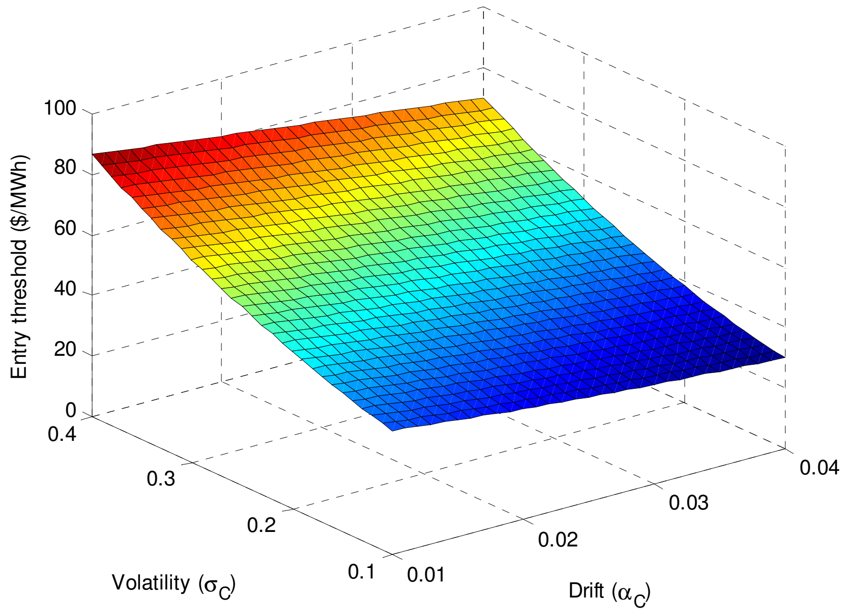

Figure 2.5 Entry threshold vs. the growth rate and volatility of O&M cost

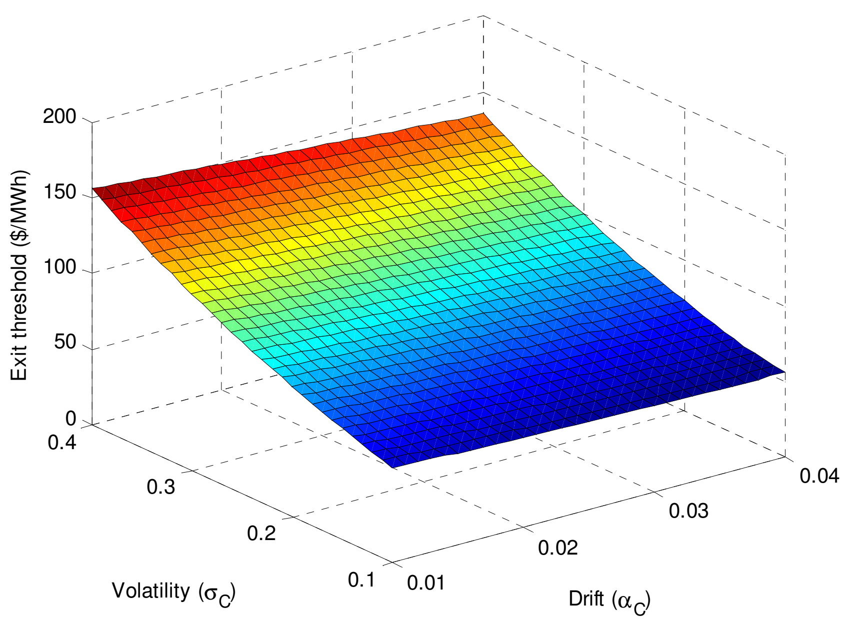

Figure 2.6 Exit threshold vs. the growth rate and volatility of O&M cost

We note that this type of bi-directional analyses can be conducted for the remaining parameters

2.6.3. Monte Carlo Simulation

In the previous subsections, the value of a wind farm project, the entry/exit thresholds of the O&M cost, and the expected economic life are calculated based on the fixed hypothetical data. In this section, in contrast, Monte Carlo simulation is used to simulate sample paths given $C_0 = \overline{C}_0$=30.0549 \$/MWh as the starting point and the GBM process as

$$dC = 0.04Cdt + 0.10Cdz$$

A typical sample path is shown below in Figure 2.7 with the horizontal axis as the time (year) and the vertical axis as the O&M cost (\$/MWh).

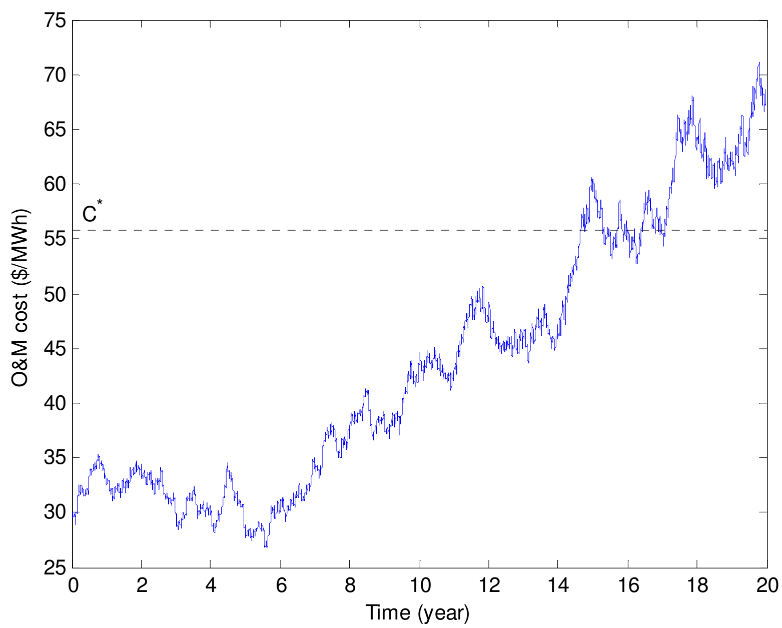

Figure 2.7 Sample path of a wind farm's O&M cost in 20 years

In our study, We generated such GBM paths for 1,000 times, given all parameters and time horizons as we indicated before. The algorithm of computing the expected marginal economic life T^* is simply presented as follows:

Initiation: Set $\text{index}_0 = \varnothing$ and $j = 1$.

Step 1: Generate a GBM process and find the first time it hits C^* as t_j. If it does not exit, let $t_j = 60$. $\text{index}_j = \{t_j, \text{index}_{j-1}\}$. Go to step 2.

Step 2: $j = j + 1$. If $j > 1{,}000$, go to step 3; otherwise go to step 1.

Step 3: Calculate the mean value of all elements in index_{j-1} then STOP.

By executing this simple algorithm, we compute the mean value of the economic life to be 17.4115 years, which, is fairly close to the analytical expected value of 17.6592 years that we obtained in the previous subsection. The difference becomes negligible once more simulations are run. Finally, we note that in this algorithm, there is one minor "adjustment" as follows. We implicitly assumed that the maximum physical life with all the repair and maintenance to be 60 years. Hence, any simulation run that was not reaching C^* in 60 years was terminated and we entered its age as 60 years. By running the Monte Carlo simulation with 60 years for 1,000 times, we have the result showing that the possibility of a wind farm not to retire within 60 years is less than 1%. Hence, we believe that this adjustment is indeed minor.

2.7 Discussion on Policy Implications and Assumptions

This section consists of two discussion items: policy implications and assumptions. Let us first address the policy implications. As shown in the previous two sensitivity analysis sections on exit and entry decisions, as the exit fee W increases, the exit threshold C^* increases. This implies that the government could increase the length of the economic life of the renewable site by imposing or increasing the exit fee. At the same time, as W increases, the entry threshold $\overline{C}_0$ decreases. That is, the number of firms that would enter into the renewable power market will decrease as the government imposes or increases the exit fee. Interestingly, the firms that would be able to enter the renewable power market will have a lower level of $\overline{C}_0$, which will increase the economic life of the renewable site.

Hence, the government's exit fee can be viewed as a policy tool to increase the economic life of a renewable site in two different ways. First, it achieves this objective by increasing the exit threshold C^*. Concurrently, it reduces the entry threshold $\overline{C}_0$, contributing positively to the same objective.

Moreover, we recall that, as the initial investment I decreases, the entry threshold $\overline{C}_0$ increases. Hence, if the government provides an initial subsidy, it can be interpreted that more firms will be able to enter. On the other hand, the marginal firms that enter will have a higher level of the initial O&M cost, which will only increase expected value-wise. In this sense, the initial subsidy encourages the premature exit of the (marginal) renewable sites. In view of this observation, let us consider the following scenario.

If the government subsidy is provided as a constant matching fund to C throughout the duration of the renewable site operation, this will directly extend the economic life of many renewable sites through an increase in pre-subsidy C^*. At the same time, more marginal renewable sites will enter through an increase in pre-subsidy $\overline{C}_0$. In this case, both expected economic life and production of electric power from the renewable energy increase.

From our observation, it is certainly worthwhile to investigate the conversion of the initial subsidy to the O&M subsidy. We note that, the O&M cost subsidy is different from the production tax credit (PTC) currently in use in the U.S. because the PTC is front-loaded for early years only, and is under the assumption that a sufficient amount of tax has already been paid to the government (see e.g., Wiser *et al*., 2007a).

Finally, we caution that, for a thorough investigation, the total benefit vs. cost will have to be quantified, which will critically depend on the distribution of firms over key parameters such as $\bar{C}_0$. For example, even in a simpler case of imposing a scalar *W*, it is recommended that the government strike a careful balance between one environmentally desirable goal of preventing premature exit (relative to the physical life of the renewable site) and another environmentally desirable goal of encouraging the entry of firms to the renewable power market, and increase the production of electric power from the renewable energy as the distribution of such firms over $\bar{C}_0$ values may be far from certain.

So far, we have discussed the policy implications. Now we proceed to discuss the assumptions. The most fundamental assumption in our model is that the O&M cost ($/MWh) follows GBM process and some cases have been referred earlier in this paper to support this assumption. We note that, in these referred cases, the cost is random over a period, and the expected cost as well as the volatility of the cost are increasing.

In the case of the wind farm, we can make similar observations on the O&M cost. In EWEA (2004, pp. 101), some of the O&M cost paths over time of wind turbines meet these characteristics. Qualitatively, Wiser and Bolinger (2008) reported that the average O&M cost of a wind turbine ($/MWh) ‘appears to increase with project age’, while some other authors claimed that the operating and maintenance costs escalate over time (Wilkinson, 2010).

In practice, we envision that first the raw data on the daily production as well as O&M cost of the renewable site will be recorded over a period. From these data, we will obtain the O&M cost per MWh data. We note that once enough of these O&M data become available in the future, more rigorous tests to see the degree of fit can be administered using various statistical tools (e.g. Postali and Picchetti, 2006).

In terms of the relaxation of simplifying assumptions, the power production quantity, capacity, and contract power price seem all worthwhile. In the case of power production quantity, a simulation model can accommodate the daily (perhaps hourly) fluctuation of the power production, taking the specific technological characteristics (wind speed, daylight availability, etc.) into account. Such a simulation will add numerical and computational insights to more tactical decisions of temporary contraction or expansion such as partial shut-down due to seasonality.

In the case of capacity, the granularity will be a central question. As the renewable site is subdivided into smaller groups (until an individual renewable generator such as a wind turbine or a solar panel), there will be an explosive number of options for partial shut-downs and gradual entries. We acknowledge that such features will add realism to the models studied here. However, the analytic tractability of any such extension remains to be seen.

Finally, for the contract power price, as mentioned previously, certainly a power price following a separate GBM will allow us to model the option to wait before any entry. With this parallel GBM process, whether the numerical and computational analyses (analytic solutions are not likely) can exploit the interaction of the price and O&M cost GBM's and yield unambiguous insights will be a significant future challenge.

2.8 Concluding Remarks

In this study, we modeled and analyzed how economically rational decision maker of a renewable site will exit and enter when the O&M cost is represented by a GBM process where the renewable site is without any input fuel cost. For such a site, we obtained the threshold level of the O&M cost above which a currently operating renewable site will exit. We also obtained the threshold level of the O&M cost below which a new renewable site will enter.

Based on these two findings, we conducted extensive sensitivity analyses with respect to various critical parameters with major policy implications. For example, the exit fee by the government will help in preventing premature exit relative to the physical life. At the same time, such a fee will prevent O&M cost-wise marginal firms from entering the market, which reduces the total production amount of electric power from the renewable energy. Moreover, the government subsidy for the initial investment is shown to allow the O&M cost-wise marginal firms to enter the market, of which renewable sites have a shorter expected economic life relative to the physical life. At the same time, such entries will increase the total production amount of electric power from the renewable energy.

As an alternative, it is desirable to investigate diverting the subsidy on the initial investment to the subsidy on the O&M cost as the O&M cost subsidy will extend the expected economic life. At the same time, more O&M cost-wise marginal firms will enter the market, which will increase the total production amount of electric power from the renewable energy.

We note that for conclusive results over competing policies, the total benefit and cost must be quantified, which is beyond the scope of this paper. However, this paper has discovered a much plausible alternative subsidy to the current government policy of front-loading grants and incentives in the initial and early years of renewable site operations.

As this paper can be seen as an initial exploration, there are numerous worthwhile future studies. Specifically, it may be worthwhile to relax each simplifying assumption, and examine the ramifications of such relaxation. For example, by assuming no prior power purchase contract, the electric power price can be modeled as a separate GBM process. This type of endeavor will enhance the realism of our study, and widen the applicability of our models.

In addition, as the data on O&M costs across renewable sites accumulate, it is worthwhile to measure the degree of the fitness for the GBM assumption. As such a degree is typically far from being binary, we do anticipate differing degree of fitness across renewable sites. However, such an examination will enhance our ability to fine-tune the exact GBM process (out of so many GBM inspired processes) and their corresponding parameter values.

Moreover, one could consider a single firm with multiple sites of the same or different renewable energies whose O&M costs may be positively/negatively correlated. Other expansions could include a competitive model of single-site firms of the same or different renewable energies as well as the management of changing technology with respect to time.

Finally, we note that, the massive abandonment of 1990's has already happened in the renewable power industries including the wind and solar power producers. Given the current expansion of these industries across the U.S. and other countries, we believe that the questions raised and addressed (to a varying degree) in this paper are timely (if not urgent), and we hope that this paper contributes positively to the possible future resolution involving aging and retiring renewable sites as well as alternative energy facilities of similar economic and environmental characteristics (to a degree, biomass and waste energy).

CHAPTER 3 . REPOWERING AND EXIT DECISIONS FOR RENEWABLE POWER PRODUCERS

In this chapter, our objectives are to understand (1) how renewable power producers make economically rational decisions regarding their current (aging) plants such as repowering or exiting and (2) how they respond to various government incentives and fees. For example, what kinds of incentives would expedite or delay repowering?

In this study of renewable power producers, we focus on the following three types of decision makers. Type I: large utilities with multiple means of generation such as coal and gas as well as wind and solar energy. It is extremely rare that a company of this type would consider abandoning a single renewable power production site, and in modeling their behavior, we assume that there exists only the repowering option (i.e., simply walking away is not an option). Type II: merchant generators with a single means of renewable power generation – typically medium size, and Type III: QF's with a single means of renewable power generation – typically small size. Type II companies are independent power producers who get paid according to the prevailing market price and Type III companies are qualifying facilities that get paid according to heavily favorable standard offer contracts. In both cases, repowering as well as abandonment are entirely plausible options for the renewable power producers.

For all three types of renewable power producers, as a first step to achieve the aforementioned two objectives, we present in this paper the mathematical models for such renewable power producers who face uncertain but on average increasing O&M costs and view repowering or exiting as a real option.

The rest of this chapter is organized as follows. We first present a brief literature review on relevant publications in Section 3.1. In Section 3.2 we first formulate a repowering decision model for a Type I producer, and derive the repowering threshold in terms of the O&M cost. This is followed by sensitivity analysis in Section 3.3 with respect to critical parameters. In Section 3.4 we extend the basic model to the case of a Type II or Type III producer who now has an option to exit as well as to repower. This is followed by another round of sensitivity analysis in Section 3.5. We then discuss the policy ramifications of what we have learned in Section 3.6, and illustrate some of the key features via an extensive numerical example in Section 3.7. Finally, we make concluding remarks and comment on future research in Section 3.8.

3.1 Literature Review

For this study, there are several streams of relevant publications. First, for the uncertain and on average increasing O&M cost, Ye (1990) derived the replacement policies under a GBM O&M cost. In Costa Lima and Suslick (2006), for mining projects, both selling price and operating cost are modeled as GBM processes. Next, for the decommissioning and replacement analysis, Takashima et al. (2007) investigated a real options approach in the context of nuclear power plants. For a more classical approach of replacement analysis, Hartman and Murphy (2006) examined equipment replacement policies under finite planning horizon. As for the repowering and exit decisions and policies of renewable power producers, Rio et al. (2011) presented a qualitative analysis of repowering policies. In Min, Lou, and Wang (2012) a real options approach is used to explain economically rational entry and exit strategies for renewable power producers. Fleten and Maribu (2004) addressed the investment timing and capacity choice of wind power under uncertainty under a fixed quantity contract. Specifically they assumed the decision maker has a deferrable opportunity to invest in one turbine out of a set of candidates with different capacity.

In contrast to these papers, our paper's unique contributions are (1) the mathematically concrete modeling of the renewable power producers' decisions on repowering or exit options when the O&M cost follows a GBM process, and (2) explicit policy implications of various possible government subsidies on expediting or delaying repowering/exiting with respect to not only the direction, but also the magnitude.

3.2 Basic Model Formulation and Solution

In this section, we consider a Type I firm with a single renewable power plant that is currently in operation. We note that, throughout this chapter, this site will be referred to as a plant. For example, in the case of wind power, a plant may consist of hundreds of wind towers. We also note that, in this section, for the decision maker of this firm, we assume that there exists an option to upgrade the plant with typically much larger capacity due to technological advancement, and this upgrading will be referred to as repowering.

Prior to the model formulation, we make the following critical assumptions that enable us to focus on the study of economically rational repowering decisions and to maintain mathematical derivations tractable. We note that we define indicator variable i such that $i = 1$ designates the current plant while $i = 2$ designates the repowered plant.

Assumption 1: For both current and repowering plant, there exists a power purchase agreement between the plant and a utility company with an appropriate transmission connection at a fixed selling price of P_i (\$/MWh) at any time point. Examples of the fixed energy pricing can be found in *Fixed Energy Prices for Renewable* (PG&E, 2012a) and *E-SRG PPA* (PG&E, 2012b).

Assumption 2: In making the repowering decision, the decision maker will rely on the fixed (expected) production quantity per unit time. Specifically, we will utilize the fixed annual production quantity, K_i (MWh per year), which equals to the nameplate capacity times the capacity factor (see e.g., Wiser and Bolinger, 2007).

Assumption 3: For both plants, the operation and maintenance cost (O&M cost; $/MWh) at any time point, C_i, follows a Geometric Brownian Motion (GBM) processes. Specifically,

$$dC_i = \alpha_{C_i} C_i dt + \sigma_{C_i} C_i dz \tag{3.1}$$

where α_{C_i} is the instantaneous growth rate of the O&M cost (% per year), σ_{C_i} is the instantaneous volatility of C_i (% per square root of year), and dz is the increment of a standard Wiener process z ($dz = \varepsilon_t \sqrt{dt}$ where $\varepsilon_t \sim N(0,1)$).

Assumption 4: As for the planning horizon, repowering will be executed only once. i.e., the producer is assumed to be unable or unwilling to re-repower its repowered plant. In other words, the decision left to be made is with respect to when to terminate the repowered plant and exit the market. The reason to do so is as follows: theoretically the producer can keep repowering into an infinite horizon, which makes the problem analytically intractable. However, the forecast-horizon theory seriously questions the relevance and value of the information beyond a certain finite horizon (Chand, *et al.*, 2002) as the forecasted information many years into the future may have little chance of any meaningful accuracy. Appendix A.2 provides an analytical comparison of this same problem with assumption of repowering twice.

Assumption 5: The remaining cost components are irrelevant to the decision maker regarding the option to repower and then exit. Also, we are assuming that the rest of the cost components such as administrative costs, insurance, tax, etc. are negligible for our planning purposes.

Under these assumptions, the problem can be separated into two sequential stages, as illustrated in Figure 3.1, where to repower current plant occurs at τ_r and to terminate the repowered one occurs at τ_D .

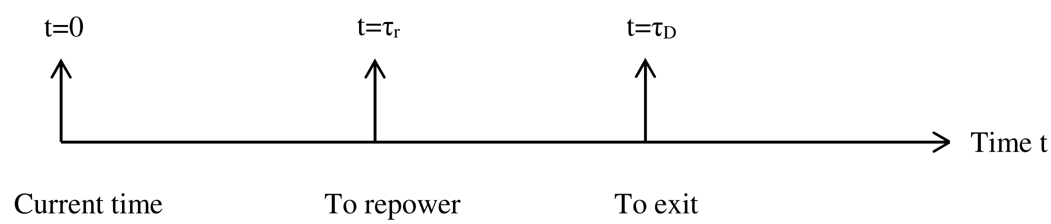

Figure 3.1 Basic model timeline

Under stochastic O&M cost, the value of the basic model from following an optimal policy can be written as (see e.g., Stokey (2008)):

$$F(c) = \sup_{C \geq c} \mathrm{E}_c \left[\int_0^{\tau_r} e^{-\rho t}(P_1 - C_1^t)K_1 dt - (W_r + I_2)e^{-\rho \tau_r} + \int_{\tau_r}^{\tau_D} e^{-\rho t}(P_2 - C_2^t)K_2 dt - W_2 e^{-\rho \tau_D} \right]$$

where $\mathrm{E}_c\left[\cdot\right]$ denotes an expected value conditional on the initial state.

We notice that each of the decision can be viewed as an individual optimal-stopping problem, and therefore we solve the problem backwards by starting with the final stage, where the repowering plant is operating and the time to exit needs to be determined. According to

Dixit and Pindyck (1994), there exist a single scalar threshold of C_2^* such that it is optimal to exit if C_2 is between $[C_2^*, \infty)$.

As long as it is not optimal to abandon the repowered plant (i.e., in the continuation region), the value function of the repowered plant $V_2(C_2)$ must satisfy the following differential equation, which results from Bellman's principle of optimality (Dixit and Pindyck, 1994):

$$\rho V_2 dt = \left(P_2 - C_2\right) K_2 dt + \mathrm{E}\left[dV_2\right] \tag{3.2}$$

where ρ is the annual discount rate (% per year).

The structure of (3.2) can be explained in the same fashion as for (2.2) in Chapter 2. By Ito's lemma, dV_2 could be expanded as

$$dV_2 = \frac{\partial V_2}{\partial t} + \frac{\partial V_2}{\partial C_2}\left(\alpha_{C_2} C_2 dt + \sigma_{C_2} C_2 dz\right) + \frac{1}{2}\frac{\partial^2 V_2}{\partial C_2{}^2}\sigma_{C_2}^2 C_2{}^2 dt$$

We note that we are considering an indefinite horizon problem here so $V_2(C_2)$ is time invariant. Hence, $\frac{\partial V_2}{\partial t} = 0$ and the above equation becomes

$$dV_2 = \frac{\partial V_2}{\partial C_2}\left(\alpha_{C_2} C_2 dt + \sigma_{C_2} C_2 dz\right) + \frac{1}{2}\frac{\partial^2 V_2}{\partial C_2{}^2}\sigma_{C_2}^2 C_2{}^2 dt \tag{3.3}$$

Substituting dV_2 in (3.3) into (3.2) we have

$$\rho V_2 dt = \left(P_2 - C_2\right) K_2 dt + \mathrm{E}\left[\frac{\partial V_2}{\partial C_2}\left(\alpha_{C_2} C_2 dt + \sigma_{C_2} C_2 dz\right) + \frac{1}{2}\frac{\partial^2 V_2}{\partial C_2{}^2}\sigma_{C_2}^2 C_2{}^2 dt\right]$$

Since $\mathrm{E}\left[dz\right] = 0$, we have $\mathrm{E}\left[\frac{\partial V_2}{\partial C_2}\sigma_{C_2} C_2 dz\right] = 0$, and the above equation can be simplified as

$$\rho V_2 dt = \left(P_2 - C_2\right) K_2 dt + \frac{\partial V_2}{\partial C_2}\alpha_{C_2} C_2 dt + \frac{1}{2}\frac{\partial^2 V_2}{\partial {C_2}^2}\sigma_{C_2}^2 {C_2}^2 dt$$

By dividing both sides by dt and grouping terms we have

$$\frac{1}{2}\frac{\partial^2 V_2}{\partial {C_2}^2}\sigma_{C_2}^2 {C_2}^2 + \frac{\partial V_2}{\partial C_2}\alpha_{C_2} C_2 - \rho V_2 + (P_2 - C_2) K_2 = 0 \quad (3.4)$$

Equation (3.4) is an ordinary differential equation (ODE). To guarantee its convergence as well as economic meaning, we impose that $\alpha_{C_2} < \rho$ (Costa Lima and Suslick, 2006).

We note that the annual discount rate ρ is also referred commonly as the expected rate of return if we resolve the same problem using the different approach of contingent claims analysis. In contingent claims analysis, this rate in practice has the interpretation as the opportunity cost of capital, and can be calculated by the CAPM model.

The following boundary conditions between the operating and exit states are needed to obtain the optimal threshold C_2^*.

$$V_2(C_2^*) = -W_2 \quad (3.5)$$

$$V_2\,'(C_2^*) = 0 \quad (3.6)$$

where W_2 represents the exit fee for the repowered plant (e.g., expense of land restoration, etc.).

The value-matching condition (3.5) requires that at the exit threshold, the value of the repowered plant project equals to the value of exit. The smooth-pasting condition (3.6) ensures that C_2^* is the optimal exercise point by requiring the continuance and smoothness of $V_2(C_2)$ at C_2^*.

The structure of the solution contains the general solution of the homogeneous part of (3.4) as well as a particular solution to the full equation, which is in the form of

$$V_2(C_2) = A_1 C_2^{\beta_1} + A_2 C_2^{\beta_2} - C_2 K_2 \Big/ \left(\rho - \alpha_{C_2}\right) + P_2 K_2 / \rho$$

where β_1 and β_2 are the roots of the characteristic quadratic equation as

$$\frac{1}{2}\sigma_{C_2}^2 \beta^2 + \left(\alpha_{C_2} - \frac{1}{2}\sigma_{C_2}^2\right)\beta - \rho = 0$$

Solving the quadratic equation we have

$$\beta_1 = \left(1/2\,\sigma_{C_2}^2 - \alpha_{C_2} + \sqrt{\left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)^2 + 2\sigma_{C_2}^2 \rho}\right) \Big/ \sigma_{C_2}^2 > 1$$

$$\beta_2 = \left(1/2\,\sigma_{C_2}^2 - \alpha_{C_2} - \sqrt{\left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)^2 + 2\sigma_{C_2}^2 \rho}\right) \Big/ \sigma_{C_2}^2 < 0$$

We also note that when $C_2 \to 0$, i.e., the O&M cost becomes negligible, the repowered plant will not be abandoned, which indicates that the value of the option to abandon approaches zero, therefore $A_2 = 0$. After eliminating this speculative bubble, the general solution then becomes

$$V_2(C_2) = A_1 C_2^{\beta_1} - C_2 K_2 \Big/ \left(\rho - \alpha_{C_2}\right) + P_2 K_2 / \rho \tag{3.7}$$

Solving (7) with boundary conditions, we have

$$C_2^* = \frac{\left(P_2 K_2/\rho + W_2\right)\left(\rho - \alpha_{C_2}\right)\beta_1}{\left(\beta_1 - 1\right) K_2} \tag{3.8}$$

$$A_1 = \frac{K_2}{\left(\rho - \alpha_{C_2}\right)\beta_1 \left(C_2^*\right)^{\beta_1 - 1}} \tag{3.9}$$

Before the decision maker decides to exit (i.e., the decision maker is still holding the option to exit), the value of the repowered plant consists two parts: the value of the operating plant and the value of the option to exit.

Based on this, (3.7) could be interpreted as follows: The first term is the value of the option to exit while the last two terms are the value of the operating repowered plant when the decision maker has to keep it operating despite any losses, given an infinite operating horizon. In other words, that last two terms represent the expected cost and revenue streams when the initial price and cost are observed as P_2 and C_2.

We note that C_2^0 will be the initial cost at any time point when (i.e., whenever) the repowered plant starts to operate. Thus the value of the repowering project will be

$$V_2(C_2^0) = A_1\left(C_2^0\right)^{\beta_1} - C_2^0 K_2 \Big/ \left(\rho - \alpha_{C_2}\right) + P_2 K_2 / \rho \tag{3.10}$$

The expected economic life of the repowered plant can be derived as

$$T_2^* = \left(\ln C_2^* - \ln C_2^0\right) \Big/ \left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right) \tag{3.11}$$

The derivation of (3.11) is briefly given as follows: Let $F(C_2) = \ln C_2$, $dF(C_2)$ can be expanded by Ito's lemma as $dF(C_2) = \left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)dt + \sigma_{C_2} dz$. Hence, for any finite time period T, the change in $F(C_2)$ is distributed with mean $\left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)T$ and variance $\sigma_{C_2}^2 T$. Hence the expected first passage time of C_2^* can be calculated as $\left(\ln C_2^* - \ln C_2^0\right) \Big/ \left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)$.

Now we proceed to move backward and examine stage one. That is, under what circumstance the decision maker will decide to repower the current (aging) plant? We define the repowering threshold of current plant's O&M cost as C_1^* so that the producer will repower when $C_1 \in [C_1^*, \infty]$. The optimality equation for the value of current plant $V_1(C_1)$ can be derived in a similar fashion. That is,

$$\rho V_1 dt = \left(P_1 - C_1\right) K_1 dt + \mathrm{E}\left[dV_1\right]$$

By applying Ito's lemma, we can obtain an ODE in a similar way as we did for stage two:

$$\frac{1}{2}\frac{\partial^2 V_1}{\partial C_1{}^2}\sigma_{C_1}^2 C_1{}^2 + \frac{\partial V_1}{\partial C_1}\alpha_{C_1} C_1 - \rho V_1 + (P_1 - C_1)K_1 = 0 \tag{3.12}$$

Also, similarly, the following boundary conditions need to be satisfied.

$$V_1(C_1^*) = V_2(C_2^0) - W_r - I_2$$

$$V_1{}'(C_1^*) = 0$$

where W_r represents the partial decommissioning cost of the current plant (total decommissioning typically is not necessary for repowering) and I_2 is the investment cost of repowering. We will assume that $V_2(C_2^0) - W_r - I_2 > 0$. That is, the partial decommissioning cost and the investment cost of the repowering are sufficiently low that the net value is positive.

Now the solution of (3.12) can be derived as follows:

$$V_1(C_1) = A_3 C_1^{\beta_3} - C_1 K_1 \Big/ \left(\rho - \alpha_{C_1}\right) + P_1 K_1 / \rho \tag{3.13}$$

where $\beta_3 = \left(1/2\,\sigma_{C_1}^2 - \alpha_{C_1} + \sqrt{\left(\alpha_{C_1} - 1/2\,\sigma_{C_1}^2\right)^2 + 2\sigma_{C_1}^2 \rho}\right) \Big/ \sigma_{C_1}^2$ (3.14)

$$C_1^* = \frac{\left[P_1 K_1/\rho - V_2\left(C_2^0\right) + W_r + I_2\right]\left(\rho - \alpha_{C_1}\right)\beta_3}{\left(\beta_3 - 1\right) K_1} \tag{3.15}$$

$$A_3 = \frac{K_1}{\left(\rho - \alpha_{C_1}\right)\beta_3 \left(C_1^*\right)^{\beta_3 - 1}} \tag{3.16}$$

If we denote the initial O&M cost of the current plant as C_1^0, the value of the current plant can be expressed as

$$V_1(C_1^0) = A_3\left(C_1^0\right)^{\beta_3} - C_1^0 K_1 \Big/ \left(\rho - \alpha_{C_1}\right) + P_1 K_1 / \rho \tag{3.17}$$

We can also derive the expected economic life of the current plant as

$$T_1^* = \left(\ln C_1^* - \ln C_1^0\right) \Big/ \left(\alpha_{C_1} - 1/2\,\sigma_{C_1}^2\right) \tag{3.18}$$

3.3 Basic Model Sensitivity Analysis

In this section, we further explore and analyze the properties of the optimal repowering strategy in the basic model by performing sensitivity analysis as follows: on the optimal exit threshold C_2^* with respect to repowered plant's critical parameters, the optimal repowering threshold C_1^* with respect to current plant's critical parameters, and C_1^* with respect to repowered plant's critical parameters.

Specifically, let us first examine C_2^* in (3.8) with respect to the repowered plant's critical parameters of α_{C_2}, σ_{C_2}, W_2, P_2, and K_2. It can be mathematically proved that $\partial C_2^* / \partial \alpha_{C_2} < 0$, $\partial C_2^* / \partial \sigma_{C_2} > 0$, $\partial C_2^* / \partial W_2 > 0$, $\partial C_2^* / \partial P_2 > 0$, and $\partial C_2^* / \partial K_2 < 0$, assuming that $0 < \alpha_{C_2} < \rho$. Recalling the expected economic life in (3.11), it can be shown that $\partial T_2^* / \partial \alpha_{C_2} < 0$, $\partial T_2^* / \partial \sigma_{C_2} > 0$, $\partial T_2^* / \partial W_2 > 0$, $\partial T_2^* / \partial P_2 > 0$, and $\partial T_2^* / \partial K_2 < 0$.

The result above implies that the volatility in the O&M cost may delay the exit of the repowered plant in the sense that the higher volatility leads to the higher threshold value for the exit. (i.e., with higher volatility, there is a greater chance that the O&M cost may actually decrease in the near future). The result also implies that a higher growth rate α_{C_2} as well as a higher level of repowering capacity K_2 encourages the producer to walk away earlier as it reduces the exit threshold. The greater capacity implies a higher level of the O&M cost impact on the project value as the O&M cost unit is $ per MWh. Moreover, a higher level of exit fee W_2 delays walking away for the reason that the exit option is less favorable.

Let us now examine C_1^* with respect to current plant's critical parameters of α_{C_1}, σ_{C_1}, W_r, P_1, and K_1. It can be verified that $\partial C_1^* / \partial \alpha_{C_1} < 0$, $\partial C_1^* / \partial \sigma_{C_1} > 0$, $\partial C_1^* / \partial W_r > 0$, $\partial C_1^* / \partial P_1 > 0$, and $\partial C_1^* / \partial K_1 > 0$, assuming that $0 < \alpha_{C_1} < \rho$. Recalling the expected economic life in (3.18), we also have $\partial T_1^* / \partial \alpha_{C_1} < 0$, $\partial T_1^* / \partial \sigma_{C_1} > 0$, $\partial T_1^* / \partial W_r > 0$, $\partial T_1^* / \partial P_1 > 0$, and $\partial T_1^* / \partial K_1 > 0$,. The corresponding interpretation can be made in a similar way. In particular, with respect to K_1 we note that a higher level of capacity K_1 delays repowering. This is because, with a higher level of capacity, the total revenue generated at P_1 dollars per MWh increases, and affords the decision maker to continue to operate the current plant a little longer.

Finally, in this section, we examine C_1^* with respect to the repowered plant's critical parameters K_2, I_2, and P_2. We note that the levels of the capacity, repowering investment cost, and selling price are often known or forecasted a priori for planning purposes.

First, for K_2, because $\dfrac{\partial C_1^*}{\partial K_2} = \dfrac{\partial\left[-V_2\left(C_2^0\right)+I_2\right]}{\partial K_2} \cdot \dfrac{\left(\rho-\alpha_{C_1}\right)\beta_3}{\left(\beta_3-1\right)K_1}$, the sign of $\partial C_1^* / \partial K_2$ is the same as $\partial\left[-V_2\left(C_2^0\right)+I_2\right] \big/ \partial K_2$. Furthermore, it can be verified that $\partial\left[-V_2\left(C_2^0\right)+I_2\right] \big/ \partial K_2 < 0 \;\Rightarrow\; \partial C_1^* / \partial K_2 < 0$. This has a significant implication. Namely, a larger capacity level of the repowered plant expedites repowering by reducing the repowering threshold. Intuitively, when the repowered plant's capacity increases, the value of the repowered plant increases as well. This increases the economic attractiveness of exercising this option, and the decision maker expedites repowering.

As for I_2 and P_2, it can be easily verified that $\partial C_1^* / \partial I_2 > 0$ and $\partial C_1^* / \partial P_2 < 0$. The interpretation is that as the repowering investment cost increases, repowering is delayed while as the repowered selling price increases, repowering is expedited.

3.4 Extension of Basic Model: Two Alternatives

In this section, we extend the basic model by considering other types of renewable power producers, namely, Types II and III in the Introduction section. In this extension, the decision maker has an option to repower as well as an option to simply terminate the current plant and walk away.

The decision timeline is shown in Figure 3.2, where the producer either exits at τ_d or repower at τ_r. And τ_D represents the terminal time of the repowered plant.

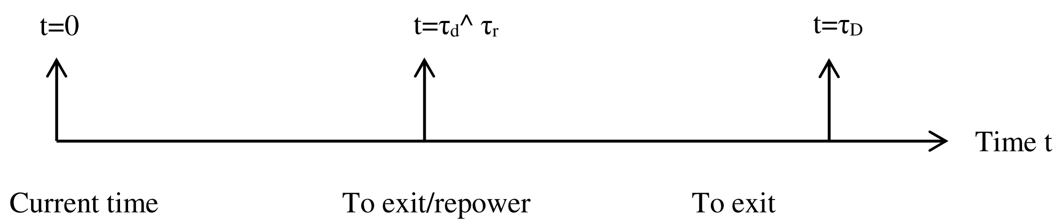

Figure 3.2 Extended model timeline

The value of the extended model from following an optimal policy can be written as

$$F(c) = \sup_{C \ge c} \mathrm{E}_c \left[\int_0^{\tau_r \wedge \tau_d} e^{-\rho t} (P_1 - C_1^t) K_1 dt + I_{\{\tau_r < \tau_d\}} \left(-(W_r + I_2) e^{-\rho \tau_r} + \int_{\tau_r}^{\tau_D} e^{-\rho t} (P_2 - C_2^t) K_2 dt - W_2 e^{-\rho \tau_D} \right) - I_{\{\tau_r \ge \tau_d\}} W_1 e^{-\rho \tau_d} \right]$$

where $I_{\{\}}$ is an indicator function.

Within this framework, the calculation of the repowering project's value (i.e., stage two if executed) is the same as before, and we will use the same mathematical expressions from the basic model.

However, at stage one, the options to repower and to exit coexist and thus each of these options has to be evaluated in the presence of the other one. As in Fleten and Maribu (2004), we treat these two options as two mutually exclusive alternatives and proceed as follows. First, we analyze the two options individually using Bellman's optimality principle and Ito's lemma. Because in each case the threshold is a single cut-off separating the continuation and stopping regions, they can both be treated as optimal-stopping problems and solved in a way similar to one in the basic model.

For clarity, we define C_1^r as the optimal repowering threshold of the current plant's O&M cost if only the repowering option is available and $V_1^r(C_1)$ as the corresponding current plant value function, given the O&M cost is C_1. It is easy to see that this case is equivalent to our basic model and the solution is as follows:

$$V_1^r(C_1) = A_3^r C_1^{\beta_3} - C_1 K_1 \Big/ \left(\rho - \alpha_{C_1}\right) + P_1 K_1 / \rho \tag{3.19}$$

$$C_1^r = \frac{\left[P_1 K_1/\rho - V_2\left(C_2^0\right) + W_r + I_2\right]\left(\rho - \alpha_{C_1}\right)\beta_3}{\left(\beta_3 - 1\right) K_1} \tag{3.20}$$

$$A_3^r = \frac{K_1}{\left(\rho - \alpha_{C_1}\right)\beta_3 \left(C_1^r\right)^{\beta_3 - 1}} \tag{3.21}$$

Unlike a Type I producer described in the basic model, for a Type II or Type III producer, the value of repowered plant is no longer assumed to be positive. For example, unlike the large, well-established firms that typically belong to Type I producers, some Type II or

Type III producers may face economically negative consequences due to financing problems, ownership changes, and market competition.

Next, we also define C_1^d as the optimal exit threshold of the current plant's O&M cost if only the exit option is available and $V_1^d(C_1)$ as the corresponding current plant value function, given the O&M cost is C_1. Following similar steps, we have

$$V_1^d(C_1) = A_3^d C_1^{\beta_3} - C_1 K_1 \big/ \left(\rho - \alpha_{C_1}\right) + P_1 K_1 / \rho \tag{3.22}$$

$$C_1^d = \frac{\left[P_1 K_1 / \rho + W_1\right]\left(\rho - \alpha_{C_1}\right)\beta_3}{\left(\beta_3 - 1\right) K_1} \tag{3.23}$$

$$A_3^d = \frac{K_1}{\left(\rho - \alpha_{C_1}\right)\beta_3 \left(C_1^d\right)^{\beta_3 - 1}} \tag{3.24}$$

We now proceed to put the two options together and re-evaluate the value of each option. According to Fleten and Maribu (2004), when there are two mutually exclusive alternatives available, it is now optimal to wait until the lowest C_1^i where the value to execute the alternative *i* is worth more than the value to execute any other alternatives. Following this rule, we examine C_1^r and C_1^d to find the optimal decision. For that, the following three cases can be mathematically derived:

i) When $C_1^r < C_1^d$, i.e., $V_2(C_2^0) - W_r - I_2 > -W_1$, it is optimal to repower when O&M cost hits C_1^r;

ii) When $C_1^r > C_1^d$, i.e., $V_2(C_2^0) - W_r - I_2 < -W_1$, it is optimal to exit when O&M cost hits C_1^d;

iii) When $C_1^r = C_1^d$, i.e., $V_2(C_2^0) - W_r - I_2 = -W_1$, it is optimal to do either.

The three cases above can be synthesized into one explicit decision rule: $C_1^* = C_1^r \wedge C_1^d$. We note that, compared to the basic model, the Type II or Type III producer may find it optimal to exit as the repowering option is simply not attractive enough for the producer to continue.

3.5 Extended Model Sensitivity Analysis

In this section, we conduct the sensitivity analysis as follows: on the repowered plant's optimal exit threshold C_2^* with respect to its critical parameters, the current plant's optimal repower/exit threshold C_1^* with respect to its critical parameters as well as with respect to some of repowered plant's critical parameters.

The sensitivity results of C_2^* are the same as in the basic model, i.e., $\partial C_2^* \big/ \partial \alpha_{C_2} < 0$, $\partial C_2^* \big/ \partial \sigma_{C_2} > 0$, $\partial C_2^* \big/ \partial W_2 > 0$, $\partial C_2^* \big/ \partial P_2 > 0$, and $\partial C_2^* \big/ \partial K_2 < 0$. For C_1^*, the sensitivity analysis is more complicated. We start with the parameters that do not affect the optimality criterion (i.e., $V_2(C_2^0) - W_r - I_2 > -W_1$ or not). These parameters are: α_{C_1}, σ_{C_1}, P_1, and K_1. The result is summarized as follows:

When Case i) holds, then $C_1^* = C_1^r$. For this case, $\partial C_1^* \big/ \partial \alpha_{C_1} < 0$, $\partial C_1^* \big/ \partial \sigma_{C_1} > 0$, $\partial C_1^* \big/ \partial P_1 > 0$. For $\partial C_1^* \big/ \partial K_1$, we have $\partial C_1^* \big/ \partial K_1 > 0$ if $V_2(C_2^0) - W_r - I_2 > 0$ and $\partial C_1^* \big/ \partial K_1 \le 0$ otherwise.

When Case ii) or Case iii) holds, then $C_1^* = C_1^d$. For such a case, $\partial C_1^* \big/ \partial \alpha_{C_1} < 0$, $\partial C_1^* \big/ \partial \sigma_{C_1} > 0$, $\partial C_1^* \big/ \partial P_1 > 0$, and $\partial C_1^* \big/ \partial K_1 < 0$.

We now proceed to analyze the parameters which affect the optimal criterion itself. These parameters are: W_r, I_2, K_2, P_2, and W_1. The results are summarized as follows:

1) Analysis of W_r

When $W_r < V_2(C_2^0) - I_2 + W_1$, Case i) holds and $C_1^* = C_1^r$, hence $\partial C_1^* / \partial W_r > 0$.

When $W_r > V_2(C_2^0) - I_2 + W_1$, Case ii) holds and $C_1^* = C_1^d$, hence $\partial C_1^* / \partial W_r = 0$.

When $W_r = V_2(C_2^0) - I_2 + W_1$, Case iii) holds at a kink and no straightforward expression of $\partial C_1^* / \partial W_r$ exists.

The interpretations are as follows. If the net value of the repowering project is greater than the exit fee of the current plant, then the repowering option will be used (not the exit option), and as the partial decommissioning cost increases the repowering threshold also increases. If the net value of the repowering project is smaller than the exit fee of the current plant, then the exit option will be used, and the increase in the partial decommissioning cost has no effect on the exit threshold. At the kink, $\lim_{\Delta W_r \to 0^-} \frac{\Delta C_1^*}{\Delta W_r} > 0$ while $\lim_{\Delta W_r \to 0^+} \frac{\Delta C_1^*}{\Delta W_r} = 0$. We note that for the remainder of the sensitivity analyses, similar interpretations can be made.

2) Analysis of I_2

When $I_2 < V_2(C_2^0) - W_r + W_1$, Case i) holds and $C_1^* = C_1^r$, hence $\partial C_1^* / \partial I_2 > 0$;

When $I_2 > V_2(C_2^0) - W_r + W_1$, Case ii) holds and $C_1^* = C_1^d$, hence $\partial C_1^* / \partial I_2 = 0$;

When $I_2 = V_2(C_2^0) - W_r + W_1$, Case iii) holds at a kink and no straightforward expression of $\partial C_1^* / \partial I_2$ exists.

3) Analysis of K_2

When $K_2 > k_2$, Case i) holds and $C_1^* = C_1^r$, hence $\partial C_1^* / \partial K_2 < 0$;

When $K_2 < k_2$, Case ii) holds and $C_1^* = C_1^d$, hence $\partial C_1^* / \partial K_2 = 0$;

When $K_2 = k_2$, Case iii) holds at a kink and no straightforward expression of $\partial C_1^* / \partial K_2$ exists.

where $k_2 = \left[-A_1 \left(C_2^0 \right)^{\beta_1} + W_r + I_2 - W_1 \right] \Big/ \left[-C_2^0 / (\rho - \alpha_{C_2}) + P_2 / \rho \right]$

4) Analysis of P_2

When $P_2 > p_2$, Case i) holds and $C_1^* = C_1^r$, hence $\partial C_1^* / \partial P_2 < 0$;

When $P_2 < p_2$, Case ii) holds and $C_1^* = C_1^d$, hence $\partial C_1^* / \partial P_2 = 0$;

When $P_2 = p_2$, Case iii) holds at a kink and no straightforward expression of $\partial C_1^* / \partial P_2$ exists.

where $p_2 = \rho \left[-A_1 \left(C_2^0 \right)^{\beta_1} + C_2^0 K_2 / (\rho - \alpha_{C_2}) + W_r + I_2 - W_1 \right] \Big/ K_2$

5) Analysis of W_1

When $W_1 > -V_2(C_2^0) + W_r + I_2$, Case i) holds and $C_1^* = C_1^r$, hence $\partial C_1^* / \partial W_1 = 0$;

When $W_1 < -V_2(C_2^0) + W_r + I_2$, Case ii) holds and $C_1^* = C_1^d$, hence $\partial C_1^* / \partial W_1 > 0$;

When $W_1 = -V_2(C_2^0) + W_r + I_2$, Case iii) holds at a kink and no straightforward expression of $\partial C_1^* / \partial W_1$ exists.

3.6 Policy Implications

In this section, we discuss the policy implications we have learned from both the basic and extended models. We begin with the basic model, which represents Type I producers. As shown in the previous sensitivity analysis of the basic model, as the contracted selling price P_1 decreases, the current plant's repowering threshold C_1^* also decreases. This implies that reducing the contracted price to the current plant expedites repowering via the reduction of the repowering threshold.

In addition, it is also shown that a higher level of up-front repowering cost I_2 delays repowering. Therefore, an initial subsidy provided by the government to reduce upfront investment cost of the repowering plant can also be viewed as a policy tool to quicken repowering.

Furthermore, we recall that as the installed repowering capacity K_2 increases, the repowering threshold decrease. Hence, a larger repowering capacity can encourage quicker repowering. However, the larger repowering capacity also reduces the repowered plant's exit threshold, which causes an earlier exit of the repowered plant. This implies that there is a warning on any subsidy to encourage a larger capacity for the repowered plants. i.e., we need to strike a balance between faster repowering vs. longer repowered plant life

We now proceed to address policy implications on the extended model, which represents the Types II and III producers. We recall that, in Case i) when $V_2(C_2^0) - W_r - I_2 > -W_1$, repowering when the O&M cost hits C_1^r is more valuable than waiting and exiting at C_1^d. On the other hand, in Case ii) when $V_2(C_2^0) - W_r - I_2 < -W_1$, exiting when the O&M cost hits C_1^d is more valuable than waiting and repowering at C_1^r. This implies the necessity and importance of regulatory policies on repowering. That is, if the government provides an appropriate incentive in relation to repowering, more producers will upgrade the current aging plants. Examples of such incentives include reducing current contracted price, reducing the upfront repowering investment cost, as well as increasing the contracted selling price for the repowered plant.

This also indicates that the lack of relevant regulatory policies might have caused the current “slow repowering” situation observed in U.S wind power industry, for example. According to Wiser (2007), one of the biggest barriers to repowering in California is that current aged turbine owners’ lack economic interest in repowering for the reason that “existing projects are already profitable under standard offer contracts and many presumably more profitable than if repowered.” Our findings based on the mathematical models in this paper corroborate such an observation.

3.7 Numerical Example

In this section, we numerically illustrate the key features of the basic and extended models. The hypothetical parameters’ values we adapted in this section are calibrated from realistic data of U.S. Energy Information Administration’s relevant report (EIA, 2010b) as well as others (e.g., Kjarland, 2007, Takashima *et al*., 2007).

This section is organized as follows: we first provide a numerical example of the optimal decision making for Type I producers, followed by another example for Type II/III producers. Then we numerically demonstrate the significant policy implications we derived from the extended model’s sensitivity analysis results.

For the sake of concreteness, let us assume that we have a wind farm. Also, for simplicity we assume that the repowered plant has same parameters as the current one except the named capacity and investment cost. For the current plant, the annual production rate = 8760 MWh. For the repowered plant, we assume the annual production rate increases to 1.5*8760=13140 MWh. The partial decommissioning cost of the current plant is assumed to be $100,000. The complete list of parameter values are shown in Table 1.

Table 3.1 Parameters and corresponding values

	Current plant	Repowered plant
Contracted electricity selling price (including tax credit benefit)	48 $/MWh	48 $/MWh
Nameplate capacity	3 MW	4.5 MW
Capacity factor	33.33%	33.33%
Investment cost	1,000,000 $	1,500,000 $
Partial decommissioning fee W_r	100,000 $	N/A
Exit fee W	N/A	300,000 $
Annual discount rate ρ	0.05	0.05
Annualized growth rate of O&M cost	0.04	0.04
Annualized volatility of O&M cost	0.10	0.10
Initial O&M cost	25 $/MWh	25 $/MWh

By applying the parameter values, numerical results can be computed from the basic model solution as shown the in Table 3.2:

Table 3.2 Numerical results of the basic model

β_1	1.2170
A_1	452,330
C_2^*	55.1220
T_2^*	22.5907
β_3	1.2170
A_3	310,650
C_1^*	48.0692
T_1^*	18.6790

The numerical results shows that the current plant will be partially decommissioned and repowered when its O&M cost hits 48.0692 $/MWh, and its expected operating life is 18.6790 years; The repowered plant will retire when its O&M cost hits 55.1220 $/MWh, and its expected operating life is 22.5907 years.

Now we move to consider a Type II/III producer who also has the option to exit without repowering. For this extended case, we further assume the exit fee of current plant W_1 =

$300,000, and the repowering and exit thresholds are computed as follows (each if considered individually):

Table 3.3 Numerical results of the extended model

A_3^r	310,650
C_1^r	48.0692
A_3^d	300,800
C_1^d	55.7623
C_1^*	48.0692
T_1^*	18.6790

In this example, $C_1^r < C_1^d$, hence the optimal solution is to repower when O&M cost hits C_1^r. Figure 3.3 illustrates the two alternatives we considered.

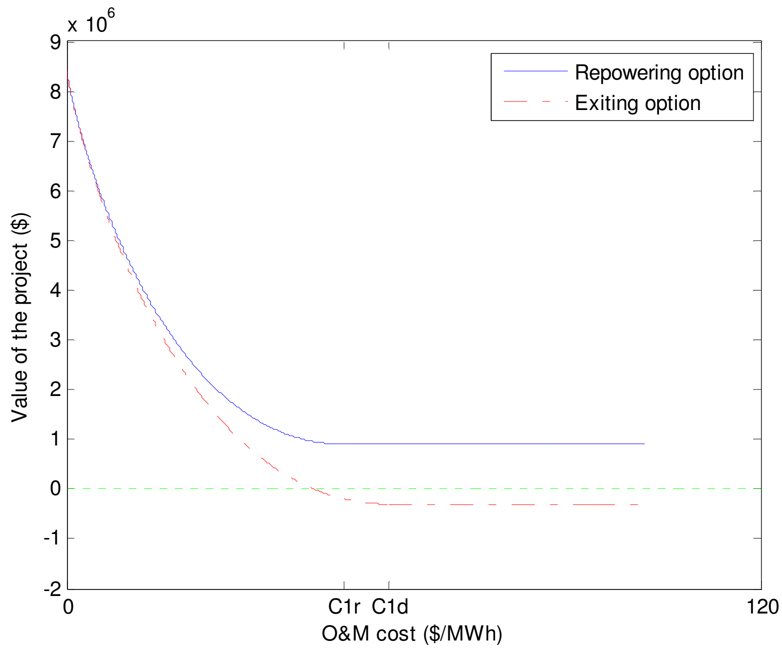

Figure 3.3 Comparison of project values in the extended model

Now we demonstrate the results of extended model sensitivity analysis by changing one critical parameter's value while keeping the others same as stated before to examine its impact on the whole model and crosscheck with the analytical results. The following tables summarize our results. We note that the parameters we tested (one at a time) are the up-front repowering investment cost I_2, repowering selling price P_2, repowering capacity K_2 and current selling price P_1.

Table 3.4 Numerical results of sensitivity analysis

	Change the value of I_2				Change the value of P_2			
	1,000,000	1,500,000	2,000,000	3,000,000	32	40	48	56
A_3^r	315,330	310,650	306,330	298,610	293,960	300,750	310,650	324,530
C_1^r	44.8680	48.0692	51.2704	57.6728	61.9997	55.8073	48.0692	39.2960
A_3^d	300,800	300,800	300,800	300,800	300,800	300,800	300,800	300,800
C_1^d	55.7623	55.7623	55.7623	55.7623	55.7623	55.7623	55.7623	55.7623
T_1^*	16.7100	18.6790	20.5211	22.9206	22.9206	22.9206	18.6790	12.9213
T_2^*	22.5907	22.5907	22.5907	N/A	N/A	N/A	22.5907	26.9000

Table 3.4 summarizes the numerical results of changing I_2 and P_2 respectively. As I_2 increases from \$1,000,000 to \$3,000,000, the repowering threshold increases from 44.8680 to 57.6728 (\$/MWh), while the exit threshold stays at 55.7623 (\$/MWh). The optimal decision thus switched from repowering to exiting with the expected life of the current plant increasing, which supports our sensitivity analysis results for the extended model. Similar observation can be made on P_2, too. As P_2 increases, the repowering threshold decreases and the exit threshold stays the same. Hence, the optimal decision will switch from exit to repowering. The increase in P_2 also makes the current plant's expected life shorter and the repowered plant's expected life longer. In addition, we find it interesting that the change in the repowering threshold becomes steeper as the selling price of the repowered plant increases.

Table 3.5 summarizes the numerical results of changing K_2 and P_1 respectively. As K_2 increases, the repowering threshold decreases and exit threshold stays the same. As P_1

increases, both repowering and exit thresholds increase, and so does the current plant's expected life. It is easy to see that these observations match the sensitivity analysis results.

Table 3.5 Numerical results of sensitivity analysis (continued)

	Change the value of K_2				Change the value of P_1			
	0.5	1.0	1.5	2.0	32	40	48	56
A_3^r	296,900	303,330	310,650	319,080	343,810	324,890	310,650	299,320
C_1^r	59.2225	53.6508	48.0692	42.4850	30.1220	39.0956	48.0692	57.0428
A_3^d	300,800	300,800	300,800	300,800	327,250	312,470	300,800	291,220
C_1^d	55.7623	55.7623	55.7623	55.7623	37.8151	46.7887	55.7623	64.7359
T_1^*	22.9206	21.8177	18.6790	15.1507	5.3252	12.7753	18.6790	23.5693
T_2^*	N/A	22.9206	22.5907	22.4243	22.5907	22.5907	22.5907	22.5907

3.8 Concluding Remarks

In this chapter, we modeled and analyzed how an economically rational renewable power producer makes decisions regarding repowering and/or exit of its current plant when the O&M cost is modeled as a GBM process. Based on producers' different sizes, generation resources, and market powers, we classify our study subject of such producers into Types I, II, and III. For a single producer of each type, we obtained the threshold levels of the O&M cost for the current plant's repowering/exit decisions as well as the repowered plant's exit decision.

Based on the optimal decisions, we conducted extensive sensitivity analyses with respect to various critical parameters with significant policy implications. In the basic model, a higher level of the repowered plant's contracted selling price expedites repowering while a higher level of the current plant's contracted selling price discourages early repowering. Also, a larger capacity of the current plant delays repowering and lengthens the life of the current plant. On the other hand, a larger capacity of the repowered plant expedites repowering, but shortens the life of the repowered plant.

In the extended model, we found that the optimal decision between exit vs. repower depends on the profitability of the repowering project as well as the cost of terminating the current plant, and we found corroborating evidence to the claim that the current producers' lack of economic interest in repowering in the case of wind power via mathematical models and sensitivity analysis.

This work can serve as a basis for studying the repowering policy for renewable power producers. So far, our analysis has emphasized the straightforward economic components of contracted price, upfront investment cost, and exit fee. For future studies, it is highly desirable to link our kind of investigation to more sophisticated programs of incentives and fees such as Production Tax Credit (PTC), Renewable Portfolio Standards (RPS's). Based on our findings thus far, it may be plausible that it is the environmental policies and programs that need stewardship perhaps more than the environment itself.

Moreover, it may also be worthwhile to relax some of our simplifying assumptions, and examine the ramifications of such relaxations. For example, instead of the power contracted selling price, one can consider market based selling price, which can be modeled as a separate GBM process, which will strongly expand the applicability of our approach. Furthermore, rigorous stochastic optimal control models can be utilized for policy and program design with explicit feedback control that would adapt to the positive/negative changes in the policy and program circumstances.

CHAPTER 4 . REPOWERING UNDER POLICY UNCERTAINTY

In this chapter we formulate a two-dimensional repowering model taking into consideration the exogenous policy uncertainty on top of the O&M cost using a real options approach. Specifically we study the typical PTC scheme and its uncertainty is modeled with Poisson processes and the O&M cost is formulated as a GBM process, as we did in previous two chapters. By doing so we aim to explore and understand the combined effect of policy uncertainty and stochastic O&M cost on the repowering decision making of renewable power producers.

Unlike O&M cost which evolves continuously as the plant operates, external uncertainties, such as technological progression and policy, are not likely to be well captured by a diffusion process; it is more likely to be a Poisson jump (Dixit and Pindyck, 1994). For example, the arrival of a new technology is uncertain and can be modeled as a Poisson arrival process. Once it becomes available, the initial investment cost of repowering will be significant reduced into a lower level. Government policies and subsidies, on the other hand, usually act in a more complicated way due to their frequent expiration-renewal cycle. The switches between policy in effect and not can also be formulated as Poisson jump processes.

In the rest of this section, we start with a comprehensive review of the related literature in investment study under the PTC uncertainty as well as methodologies. We then present a specific PTC renewal model where we assume the PTC is currently not in effect but will be renewed. This is followed by an expiration model where the PTC is currently in effect but will expire in the future. We then proceed to present a generalized model where the random PTC switches between two regimes (i.e., in effect or not). For each of the three models the optimal repowering policy will be obtained in terms of two threshold values.

4.1 Literature Review

One early study on investment decision and technology adoption is Balcer and Lippman (1984), in which firms anticipate a sequence of innovations of uncertain profitability and revise their expectations about the occurrence of the next innovation as time passes since the last innovation. It shows that it is optimal for a firm to adopt the current best technology if its technological lag exceeds a certain threshold.

The real options approach has been adopted widely to study the effects of various uncertainties on capital investment. For stochastic future technology advancement, works have been done under the assumption that the arrival of new technology as a Poisson process (Huisman and Kort, 2004). The state of technological progress has also been modelled as a GBM process so that the innovation arrives when the state rises to an upper boundary (Grenadier and Weiss, 1997). In Farzin *et al.* (1998), the decision maker faces uncertainties about not only the arrival speed, but also the degree of improvements of technologies.

The policy uncertainty modeling is to some extent similar to the one of technology. Their effects can both be reflected in terms of initial investment and revenue stream of a project and will both affect the decision maker's investment timing and scale. However, subsequent technological improvements would often not affect the values of projects that already exist, while a policy scheme like the PTC, is usually subject to renewal and expiration and will have long-term effect across the project's life span.

One of the earliest work to our knowledge that uses a real-options approach to model the investment behaviors under policy uncertainty is Hassett and Metcalf (1999), where the random discrete jump of investment tax credit is modeled in Poisson processes. Further applications have also been developed in Agliardi (2001), which assumes that investment is partially reversible and the investment tax incentive changes randomly so that the price of capital follows a combined Brownian and Poisson process. It shows that tax policy uncertainty delays investment and speeds up disinvestment. Lately there are also a few applications in power systems research that use similar approaches to investigate the optimal timing of power plant investment (see Himpler and Madlener (2011), Min *et al.* (2012), Takashima *et al.* (2007), and Fleten *et al.* (2007)). Batista *et al.* (2011) uses a real options approach to compute the incremental payoff from the sale of Certified Emission Reductions (CER) for Brazil's renewable power projects where the CER price is considered to be stochastic over time.

4.2 Repowering Under Policy Renewal Possibility

We consider the case in which the is currently not in effect and is perceived to have a probability of $\lambda_1 dt$ to be renewed in the next time interval of dt. The value of λ_1 represents the number of renewals per unit time (e.g., per year) and we use the term renewal for the future (not the present or past). The PTC level is s (\$/MWh) and we do not consider expiration once it is renewed in this section; this will be addressed in the generalized model later.

The following critical assumptions are made to enable us to focus on the study of economically rational repowering decisions and to maintain mathematical derivations tractable. We note that the subscript $i=1$ designates the current plant while $i=2$ designates the repowered plant.

Assumption 1: For both current and repowering plant, the electricity generated is sold at a fixed selling price of P_i (\$/MWh) via long term power purchase agreements (PPAs).

Assumption 2: In making the repowering decision, the decision maker will rely on the fixed (expected) annual production quantity, K_i (MWh), which equals to the nameplate capacity times annual capacity factor and number of hours in a year (see e.g., [8]).

Assumption 3: For both plants, the operation and maintenance cost (O&M cost; \$/MWh) at any time point, C_i, follows a geometric Brownian motion (GBM) processes. Specifically,

$$dC_i = \alpha_{C_i} C_i dt + \sigma_{C_i} C_i dz$$

where α_{C_i} is the instantaneous growth rate of the O&M cost (% per year), σ_{C_i} is the instantaneous volatility of C_i (% per square root of year), and dz is the increment of a standard Wiener process z ($dz = \varepsilon_t \sqrt{dt}$ where $\varepsilon_t \sim N(0,1)$).

We note that this assumption, along with the Possion assumption of PTC renewal/expiration, are two of the most important features that distinguish this work from others in close areas. Such assumptions allows us a much tractable analysis on the repowering decisions under cost and policy uncertainties. The empirical justification of these two assumption will be presented in Chapter 5.

Assumption 4: Any repowering plant built and starts operation when the PTC is in effect is eligible for the tax benefit of its entire operating life. The power producer will also have a sufficient level of tax liability to take full advantage of the PTC.

Assumption 5: We assume that the decision maker is currently considering the existing plant's repowering only once. That is, the decision maker is not currently considering any subsequent repowering decisions. The reason is that, given the typical plant's physical life of about 20 years, it may be impractical to plan what would happen during 20 to 40 years from now. We also note that the degenerate repowering decision of exiting the market is not considered.

Under these assumptions, our problem can be formulated as a two-stage optimal stopping problem under two types of uncertainty: the PTC renewal and O&M cost. To derive the optimal repowering strategy, we start backwardly from the repowered plant and first obtain the optimal time to terminate it without repowering again as stated in Assumption 5, which is analogous to the optimal stopping problem in Dixit and Pindyck (1994).

Given C_2^0 as the initial cost when the repowered plant starts to operate (the superscript 0 designates the initial point), we denote the value of the repowered plant when it is eligible for the PTC as V_2^H and that when it is ineligible for the PTC as V_2^L (the superscript H/L denotes that the PTC is in effect or not). Namely,

$$V_2^H(C_2^0) = A_1^H \left(C_2^0\right)^{\beta_1} - C_2^0 K_2 \Big/ \left(\rho - \alpha_{C_2}\right) + \left(P_2 + s\right) K_2 / \rho \tag{4.1}$$

$$V_2^L(C_2^0) = A_1^L \left(C_2^0\right)^{\beta_1} - C_2^0 K_2 \Big/ \left(\rho - \alpha_{C_2}\right) + P_2 K_2 / \rho \tag{4.2}$$

where

$$\beta_1 = \left(1/2\,\sigma_{C_2}^2 - \alpha_{C_2} + \sqrt{\left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)^2 + 2\sigma_{C_2}^2 \rho}\right) \Big/ \sigma_{C_2}^2 > 1 \tag{4.3}$$

$$C_2^H = \frac{\left(\left(P_2 + s\right) K_2/\rho + W_2\right)\left(\rho - \alpha_{C_2}\right)\beta_1}{\left(\beta_1 - 1\right) K_2} \tag{4.4}$$

$$C_2^L = \frac{\left(P_2 K_2/\rho + W_2\right)\left(\rho - \alpha_{C_2}\right)\beta_1}{\left(\beta_1 - 1\right) K_2} \tag{4.5}$$

$$A_1^H = \frac{K_2}{\left(\rho - \alpha_{C_2}\right)\beta_1\left(C_2^H\right)^{\beta_1 - 1}} \tag{4.6}$$

$$A_1^L = \frac{K_2}{\left(\rho - \alpha_{C_2}\right)\beta_1\left(C_2^L\right)^{\beta_1 - 1}} \tag{4.7}$$

We note that C_2^H and C_2^L are the threshold value of O&M cost at which the repowered plant will be terminated, ρ is the annualized discount factor, and W_2 is the exit fee to be paid upon termination.

We then move to the current stage where the aging plant is still operating and the producer holds the option to repower it. In this case, the producer has two possible options for repowering: to wait and repower when the PTC arrives (Case 1.1), or to repower even if the PTC has not been renewed yet (Case 1.2). We first denote the O&M cost threshold at which the producer will repower with the presence of the PTC as C_{11}^*. The first subscript represents the current plant and the second represents Case 1.1. Since the producer perceives that the PTC will not expire once it gets renewed, C_{11}^* can be solved at which the payoff of the repowering project equals to the value of the repowered plant minus associated cost terms, i.e., $\chi = V_2^H(C_2^0) - W_r - I_2$, where W_r is the partial decommissioning cost and I_2 is the upfront investment cost of the repowered plant. For simplicity we denote the current project value considering both the current and repowered plants after the PTC turns on as $\Phi_1(C_1)$ and it can be expressed as

$$\Phi_1(C_1) = \begin{cases} A_3 C_1^{\beta_3} - \dfrac{C_1 K_1}{\rho - \alpha_{C_1}} + P_1 K_1/\rho & \text{if } C_1 \in (0, C_{11}^*) \\ \chi & \text{if } C_1 \in [C_{11}^*, \infty) \end{cases} \tag{4.8}$$

where

$$\beta_3 = \left(1/2\,\sigma_{C_1}^2 - \alpha_{C_1} + \sqrt{\left(\alpha_{C_1} - 1/2\,\sigma_{C_1}^2\right)^2 + 2\sigma_{C_1}^2 \rho}\right) \Big/ \sigma_{C_1}^2 \tag{4.9}$$

$$C_{11}^* = \frac{[P_1 K_1/\rho - \chi]\left(\rho - \alpha_{C_1}\right)\beta_3}{\left(\beta_3 - 1\right) K_1} \tag{4.10}$$

$$A_3 = \frac{K_1}{\left(\rho - \alpha_{C_1}\right)\beta_3 \left(C_{11}^*\right)^{\beta_3 - 1}} \tag{4.11}$$

Given the fact that the PTC is currently not renewed yet and may not be renewed soon, there exists another threshold C_{12}^* (the second subscript represents case 1.2) such that $C_{12}^* > C_{11}^*$ and the producer will repower anyway even if the PTC has not arrived yet when the O&M cost hits for C_{12}^*. This is also an optimal stopping problem, in which stopping means to repower without the PTC. When $C_1 \geq C_{12}^*$, the value of the current plant V_{12} equals to $\omega = V_2^L(C_2^0) - W_r - I_2$.

In the continuation region (i.e., $C_1 < C_{12}^*$) where to wait is the optimal strategy, the value of the producer when the PTC is yet to be in effect denoted by $V_{11}(C_1)$ must satisfy the following Bellman equation:

$$\rho V_{11} = \left(P_1 - C_1\right) K_1 + \frac{1}{dt}\mathrm{E}\left[dV_{11}\right] \tag{4.12}$$

By using Ito's lemma and retaining leading terms, dV_{11} can be expanded as

$$dV_{11} = \left(1 - \lambda_1 dt\right)\left[\frac{\partial V_{11}}{\partial C_1}\left(\alpha_{C_1} C_1 dt + \sigma_{C_1} C_1 dz\right) + \frac{1}{2}\frac{\partial^2 V_{11}}{\partial C_1^{\,2}}\sigma_{C_1}^2 C_1^{\,2} dt\right] + \lambda_1 dt\left(\Phi_1 - V_{11}\right) \tag{4.13}$$

Using (4.13) in (4.12) and letting dt go to zero gives

$$\frac{1}{2}\frac{\partial^2 V_{11}}{\partial C_1{}^2}\sigma_{C_1}^2 C_1{}^2+\frac{\partial V_{11}}{\partial C_1}\alpha_{C_1}C_1-\left(\rho+\lambda_1\right)V_{11}+\left(P_1-C_1\right)K_1+\lambda_1\Phi_1=0 \tag{4.14}$$

Using the two possible expressions for $\Phi_1(C_1)$, the solution of $V_{11}(C_1)$ above is given by

$$V_{11}(C_1)=\begin{cases}\delta_1 C_1^{\beta_5}+A_3 C_1^{\beta_3}-\dfrac{C_1K_1}{\rho-\alpha_{C_1}}+P_1K_1/\rho & \text{if } C_1\in(0,C_{11}^*)\\ \delta_2 C_1^{\beta_5}+\delta_3 C_1^{\beta_6}-\dfrac{C_1K_1}{\rho+\lambda_1-\alpha_{C_1}}+\dfrac{P_1K_1+\lambda_1\chi}{\rho+\lambda_1} & \text{if } C_1\in[C_{11}^*,C_{12}^*)\\ \omega & \text{if } C_1\in[C_{12}^*,\infty)\end{cases} \tag{4.15}$$

where β_5 (β_6) is the positive (negative) root of the characteristic quadratic equation as

$$1/2\,\sigma_{C_1}^2\beta^2+\left(\alpha_{C_1}-1/2\,\sigma_{C_1}^2\right)\beta-\left(\rho+\lambda_1\right)=0 \tag{4.16}$$

The following value-matching and smooth-pasting conditions of $V_{11}(C_1)$ must satisfy at $C_1=C_{11}^*$,

$$\delta_1\left(C_{11}^*\right)^{\beta_5}+\chi=\delta_2\left(C_{11}^*\right)^{\beta_5}+\delta_3\left(C_{11}^*\right)^{\beta_6}-\frac{C_{11}^*K_1}{\rho+\lambda-\alpha_{C_1}}+\frac{P_1K_1+\lambda\chi}{\rho+\lambda} \tag{4.17.1}$$

$$\delta_1\beta_5\left(C_{11}^*\right)^{\beta_5-1}=\delta_2\beta_5\left(C_{11}^*\right)^{\beta_5-1}+\delta_3\beta_6\left(C_{11}^*\right)^{\beta_6-1}-\frac{K_1}{\rho+\lambda-\alpha_{C_1}} \tag{4.17.2}$$

The following value-matching and smooth-pasting conditions of $V_{11}(C_1)$ must satisfy at $C_1=C_{12}^*$,

$$\delta_2\left(C_{12}^*\right)^{\beta_5}+\delta_3\left(C_{12}^*\right)^{\beta_6}-\frac{C_{12}^*K_1}{\rho+\lambda-\alpha_{C_1}}+\frac{P_1K_1+\lambda\chi}{\rho+\lambda}=\omega \tag{4.18.1}$$

$$\delta_2\beta_5\left(C_{12}^*\right)^{\beta_5-1}+\delta_3\beta_6\left(C_{12}^*\right)^{\beta_6-1}-\frac{K_1}{\rho+\lambda-\alpha_{C_1}}=0 \tag{4.18.2}$$

δ_1, δ_2, δ_3 and C_{12}^* can be obtained by simultaneously solving the four value-matching and smooth-pasting conditions at C_{11}^* and C_{12}^*. It turns out that there is no closed-form solution for C_{12}^* and it is implicitly determined by the following equation:

$$(\beta_5-\beta_6)\delta_3\left(C_{12}^*\right)^{\beta_6}-(\beta_5-1)\frac{C_{12}^*K_1}{\rho+\lambda_1-\alpha_{C_1}}+\beta_5\left(\frac{P_1K_1+\lambda_1\chi}{\rho+\lambda_1}-\omega\right)=0 \tag{4.19}$$

Coefficient δ_1, δ_2 and δ_3 are equal to:

$$\delta_1=\delta_2+\frac{\left(\rho+\lambda_1-\alpha_{C_1}\right)\delta_3\beta_6\left(C_{11}^*\right)^{\beta_6-1}-K_1}{\left(\rho+\lambda_1-\alpha_{C_1}\right)\beta_5\left(C_{11}^*\right)^{\beta_5-1}} \tag{4.20}$$

$$\delta_2=\frac{(1-\beta_6)(\rho+\lambda_1)\delta_3\left(C_{12}^*\right)^{\beta_6}+P_1K_1+\lambda_1\chi-(\rho+\lambda_1)\omega}{(\beta_5-1)(\rho+\lambda_1)\left(C_{12}^*\right)^{\beta_5}} \tag{4.21}$$

$$\delta_3=\frac{\beta_3(\beta_5-1)(\rho+\lambda_1)\left(\rho-\alpha_{C_1}\right)-(\beta_3-1)\beta_5\rho\left(\rho+\lambda_1-\alpha_{C_1}\right)}{\left(\rho+\lambda_1-\alpha_{C_1}\right)(\rho+\lambda_1)(\beta_5-\beta_6)\beta_3\left(\rho-\alpha_{C_1}\right)}\left(C_{11}^*\right)^{-\beta_6+1}K_1 \tag{4.22}$$

The following proposition proves the existence and uniqueness of the optimal repowering strategy represented by the two thresholds of (C_{11}^*, C_{12}^*).

Proposition 4.1 There exists a unique $C_{12}^* \in (C_{11}^*, \infty)$ such that it satisfies (4.19).

Proof: To show the existence and uniqueness of C_{12}^*, we first define a new function Θ such that $\Theta(C_1)=(\beta_5-\beta_6)\delta_3(C_1)^{\beta_6}-(\beta_5-1)\dfrac{C_1K_1}{\rho+\lambda-\alpha_{C_1}}$, and C_{12}^* is the solution of $\Theta(C_1)=\beta_5\left(\dfrac{P_1K_1+\lambda\chi}{\rho+\lambda}-\omega\right)$. Taking the first order derivative of C_1 gives $\dfrac{\partial\Theta(C_1)}{\partial C_1}=\beta_6(\beta_5-\beta_6)\delta_3(C_1)^{\beta_6-1}-(\beta_5-1)\dfrac{K_1}{\rho+\lambda-\alpha_{C_1}}$. Given $\beta_5>1$ and $\beta_6<0$, it is easy to see $\frac{\partial \Theta(C_1)}{\partial C_1} < 0$. By rearranging and simplifying (4.17.1) and (4.17.2) we also have C_{11}^* satisfies $\Theta(C_1) = \beta_5 \left(\frac{P_1 K_1 + \lambda \chi}{\rho + \lambda} - \chi \right)$. Because the payoff of the repowering project with PTC active is higher than when there is no PTC, i.e., $\chi > \omega$, we have $\beta_5 \left(\frac{P_1 K_1 + \lambda \chi}{\rho + \lambda} - \omega \right) > \beta_5 \left(\frac{P_1 K_1 + \lambda \chi}{\rho + \lambda} - \chi \right) = \beta_5 \frac{P_1 K_1 - \rho \chi}{\rho + \lambda} > 0$. Combining this with the negative slope of $\Theta(C_1)$ leads $C_{12}^* > C_{11}^*$. □

Proposition 4.1 actually tells us that to repower without PTC always has its value no matter how high the probability of PTC being renewed soon is. In other words, the producer will not wait for PTC forever; he will repower anyway if PTC is not available when the O&M cost hits C_{12}^* .

The following proposition states the sign of the coefficients in the current project's value function.

Proposition 4.2 The coefficients δ_1, δ_2, and δ_3 have the following signs: $\delta_1 < 0$, $\delta_2 > 0$, and $\delta_3 > 0$.

Proof: The sign of δ_2 follows immediately from (4.21). The necessary and sufficient condition of $\delta_3 > 0$ is that the numerator of (4.22)'s fraction part to be positive, i.e.,

$$\beta_3 (\beta_5 - 1)(\rho + \lambda)\left(\rho - \alpha_{C_1}\right) - (\beta_3 - 1)\beta_5 \rho \left(\rho + \lambda - \alpha_{C_1}\right) > 0 \qquad (4.23)$$

We know that $\beta_5 \geq \beta_3$, where the equality sign only holds for $\sigma_{C_1} \to \infty$ for which we have $\beta_5 = \beta_3 = 1$. Rewrite $\beta_5 = \xi \beta_3$ and substitute in the left-hand side of (4.23):

$$\varphi(\xi) = \beta_3\left(\xi\beta_3 - 1\right)\left(\rho+\lambda\right)\left(\rho-\alpha_{C_1}\right) - \xi\left(\beta_3 - 1\right)\beta_3\rho\left(\rho+\lambda-\alpha_{C_1}\right)$$

Then $\varphi(1) = 0$ and $\partial\varphi/\partial\xi = \beta_3\left(\rho - \beta_3\alpha_{C1}\right)\lambda + \rho\left(\rho-\alpha_{C_1}\right) > 0$, implying $\varphi > 0$ for any $\xi > 1$, which leads to (4.23).

For the sign of δ_1, we first define the following two functions:

$$Y(C_1) = \frac{(1-\beta_6)\left(\rho+\lambda\right)\gamma_2\left(C_1\right)^{\beta_6} + P_1K_1 + \lambda\chi - \left(\rho+\lambda\right)\omega}{\left(\beta_5 - 1\right)\left(\rho+\lambda\right)\left(C_1\right)^{\beta_5}} > 0$$

$$Z(C_1) = \frac{\left(\rho+\lambda-\alpha_{C_1}\right)\gamma_2\beta_6\left(C_1\right)^{\beta_6 - 1} - K_1}{\left(\rho+\lambda-\alpha_{C_1}\right)\beta_5\left(C_1\right)^{\beta_5 - 1}} < 0$$

Thus $\delta_3 = Y(C_{12}^*) + Z(C_{11}^*)$. Let $C_1 = C_{12}^*$, we have $Y(C_{12}^*) + Z(C_{12}^*) = 0$ from (4.19). Take the first derivative of $Z(C_1)$ we have

$$\frac{\partial Z(C_1)}{\partial C_1} = \frac{C_1^{-\beta_5}}{\beta_5}\left[\gamma_2\beta_6\left(\beta_6 - \beta_5\right)C_1^{\beta_6 - 1} - \frac{K_1\left(1-\beta_5\right)}{\rho+\lambda-\alpha_{C_1}}\right] = \frac{C_1^{-\beta_5}}{\beta_5}\psi(C_1)$$

$\left.\frac{\partial Z(C_1)}{\partial C_1}\right|_{C_1 = C_{12}^*}$ can be verified to be positive since $\psi(C_1) > 0$ from (4.19). Combining with $\partial\psi(C_1)/\partial C_1 < 0$ yields $\partial Z(C_1)/\partial C_1 > 0$ for any $C_1 \in [C_{11}^*, C_{12}^*]$, which, eventually leads to $\delta_1 = Y(C_{12}^*) + Z(C_{11}^*) < Y(C_{12}^*) + Z(C_{12}^*) < 0$. □

We notice that in (4.15) when $C_1 < C_{11}^*$, the first term $\delta_1 C_1^{\beta_5}$ actually consists of two parts. The first part, $\left(\delta_1 - \delta_2\right)C_1^{\beta_5}$, is a negative correction term due to the fact that PTC has not been renewed yet; the second part, $\delta_2 C_1^{\beta_5}$ is the value of the option to repower without the PTC. It turns out the correction factor always dominates the option value and thus $\delta_1 < 0$.

So far we solved the producer's repowering problem by considering the two possible moves when the PTC is not yet renewed. We establish the following decision making rule:

Consider the repowering problem when the PTC has not yet renewed, the producer will adopt the following strategy: (1) wait and repower when the current plant's O&M cost hits C_{11}^* if the PTC has been renewed by then; (2) if the PTC is not available at C_{11}^*, keep waiting and repower when it gets renewed; If the PTC is still not available when the current plant's O&M cost hits C_{12}^*, repower anyway.

4.3 Repowering Under Policy Expiration Possibility

In this section we consider an opposite case where the PTC is currently in effect but has a probability of $\lambda_0 dt$ to be permanently removed in the next time interval of dt, where λ_0 represents the number of expirations per unit time. Under this circumstance there are also two possible repowering moves for the producer: to repower after the PTC expires or before. Similar to the previous model, the repowering threshold C_{13}^* and corresponding value function after the PTC permanently expires can be derived as follows

$$\Phi_2(C_1) = \begin{cases} A_5 C_1^{\beta_3} - \dfrac{C_1 K_1}{\rho - \alpha_{C_1}} + P_1 K_1/\rho & \text{if } C_1 \in (0, C_{13}^*) \\ \omega & \text{if } C_1 \in [C_{13}^*, \infty) \end{cases} \tag{4.24}$$

$$C_{13}^* = \frac{[P_1 K_1/\rho - \omega]\left(\rho - \alpha_{C_1}\right)\beta_3}{\left(\beta_3 - 1\right) K_1} \tag{4.25}$$

$$A_5 = \frac{K_1}{\left(\rho - \alpha_{C_1}\right)\beta_3 \left(C_{13}^*\right)^{\beta_3 - 1}} \tag{4.26}$$

Given the fact that the PTC is still in effect for now, there exists another threshold C_{14}^*

($C_{14}^* < C_{13}^*$) such that the producer will repower before the PTC expires, which gives payoff χ. The value function of the producer in $(0, C_{14}^*)$ must satisfy the following ODE

$$\frac{1}{2}\frac{\partial^2 V_{12}}{\partial C_1^{\ 2}}\sigma_{C_1}^2 C_1^{\ 2} + \frac{\partial V_{12}}{\partial C_1}\alpha_{C_1} C_1 - (\rho + \lambda_0)V_{12} + (P_1 - C_1)K_1 + \lambda_0\Phi_2 = 0 \tag{4.27}$$

Using the first expression in (21), the whole solution of $V_{12}(C_1)$ is given by

$$V_{12}(C_1) = \begin{cases} \delta_4 C_1^{\beta_7} + A_5 C_1^{\beta_3} - \dfrac{C_1 K_1}{\rho - \alpha_{C_1}} + P_1 K_1/\rho & \text{if } C_1 \in (0, C_{14}^*) \\ \chi & \text{if } C_1 \in [C_{14}^*, \infty) \end{cases} \tag{4.28}$$

where β_7 is the positive root of the characteristic quadratic equation as

$$1/2\,\sigma_{C_1}^2 \beta^2 + \left(\alpha_{C_1} - 1/2\,\sigma_{C_1}^2\right)\beta - (\rho + \lambda_0) = 0 \tag{4.29}$$

C_{14}^* and δ_4 can be solved by applying boundary conditions between $V_{12}(C_1)$ and the repowering payoff χ, which leads to the following

$$(\beta_7 - \beta_3)A_5\left(C_{14}^*\right)^{\beta_3} - (\beta_7 - 1)\frac{C_{14}^* K_1}{\rho - \alpha_{C_1}} + \beta_7\frac{P_1 K_1}{\rho} - \beta_7\chi = 0 \tag{4.30}$$

$$\delta_4 = \left[\chi - A_5\left(C_{14}^*\right)^{\beta_3} + \frac{C_{14}^* K_1}{\rho - \alpha_{C_1}} - P_1 K_1/\rho\right]\left(C_{14}^*\right)^{-\beta_7} \tag{4.31}$$

The following proposition states the existence and uniqueness of the optimal repowering policy as well as the sign of the coefficient in the producer's value function.

Proposition 4.3 There exists a unique $C_{14}^* \in (0, C_{13}^*)$ such that it satisfies (4.30). The coefficient $\delta_4 > 0$.

This proposition can be proved using the slope negativity of (4.30) in $(0, C_{14}^*)$. We establish the following decision making rule for this PTC expiration case:

Consider the repowering problem when the PTC is in effect but will expire in the future, the producer will adopt the following strategy: (1) wait until the O&M cost hits C_{14}^{*} and repower if the PTC is still in effect by then; (2) if the PTC has expired, keep waiting till the O&M cost hits C_{13}^{*} and repower.

In this section, we generalize the previous two models by accounting simultaneously the cases of the PTC being in effect and not.

We assume the uncertainty PTC follows a Poisson process randomly switching between two regimes: in effect at a fixed level s or zero. Starting with a state when it is not in effect, the probability for it to be renewed in the next short interval of time dt is $\lambda_1 dt$. When it is in effect, the corresponding probability for it to expire is $\lambda_0 dt$.

To value the repowering option, we start from the repowered plant as before. The value function of the repowered plant as well as the exit threshold values with/without the PTC are the same as in (4.1) through (4.7).

We then move to the current stage where the producer has the option to repower the current plant. Based on the solution structures of the two previous cases we suggest that there exist two threshold values $(\underline{C_1}, \overline{C_1})$ for the optimal repowering policy. When the current plant's O&M cost is in the region $(0, \underline{C_1})$, no repowering project will be undertaken irrespective of whether the PTC is in effect. Over an interval $(\underline{C_1}, \overline{C_1})$, the producer will repower only if the credit is in effect and will still wait if not. Once the O&M cost hits the upper threshold $\overline{C_1}$, the producer will repower anyway regardless the PTC is in effect or not.

To determine the thresholds $\underline{C_1}$ and $\overline{C_1}$, we first denote the value function when the

PTC is in effect and not as $V_1^1(C_1)$ and $V_1^0(C_1)$ respectively. For each of these three regions, we obtain expressions for these, as well as the conditions they must satisfy at the thresholds.

Over the region $(\overline{C_1},\infty)$, the producer will always repower the current plant, so we have

$$V_1^1(C_1)=\chi \tag{4.32}$$

$$V_1^0(C_1)=\omega \tag{4.33}$$

where the expressions of χ and ω are as the same as before.

Then we move to the region $(\underline{C_1},\overline{C_1})$, where the producer will repower only if the PTC is in effect. Because repowering will be undertaken when the PTC is in effect, $V_1^1(C_1)$ is given by (29) as above. For $V_1^0(C_1)$, the following ODE can be obtained in the same fashion as in Section 2.

$$\frac{1}{2}\frac{\partial^2 V_1^0}{\partial C_1^{\,2}}\sigma_{C_1}^2 C_1^{\,2}+\frac{\partial V_1^0}{\partial C_1}\alpha_{C_1}C_1-\left(\rho+\lambda_1\right)V_1^0+\left(P_1-C_1\right)K_1+\lambda_1\chi=0 \tag{4.34}$$

The solution to (4.34) consists of a general solution of the homogeneous part and a particular solution to the whole equation, which is

$$V_1^0(C_1)=B_1C_1^{\beta_5}+B_2C_1^{\beta_6}-\frac{C_1K_1}{\rho+\lambda_1-\alpha_{C_1}}+\frac{P_1K_1+\lambda_1\chi}{\rho+\lambda_1} \tag{4.35}$$

Finally we address the region $(0,\underline{C_1})$. In this region, the producer waits in both policy regimes, and each regime can switch to the other in the next short interval of time. Following the same steps we have a pair of differential equations:

$$\frac{1}{2}\frac{\partial^2 V_1^0}{\partial C_1^{\,2}}\sigma_{C_1}^2 C_1^{\,2}+\frac{\partial V_1^0}{\partial C_1}\alpha_{C_1}C_1-\rho V_1^0+\left(P_1-C_1\right)K_1+\lambda_1\left(V_1^1-V_1^0\right)=0 \tag{4.36}$$

$$\frac{1}{2}\frac{\partial^2 V_1^1}{\partial C_1^{\ 2}}\sigma_{C_1}^2 C_1^{\ 2}+\frac{\partial V_1^1}{\partial C_1}\alpha_{C_1}C_1-\rho V_1^1+\left(P_1-C_1\right)K_1+\lambda_0\left(V_1^0-V_1^1\right)=0 \tag{4.37}$$

To solve the two ODE's, we manipulate (4.36) and (4.37) by multiplying (4.36) by $1/\lambda_1$ plus (4.37) multiplied by $1/\lambda_0$,

$$\begin{aligned}&\frac{1}{2}\left(\frac{1}{\lambda_1}\frac{\partial^2 V_1^0}{\partial C_1^{\ 2}}+\frac{1}{\lambda_0}\frac{\partial^2 V_1^1}{\partial C_1^{\ 2}}\right)\sigma_{C_1}^2 C_1^{\ 2}+\left(\frac{1}{\lambda_1}\frac{\partial V_1^0}{\partial C_1}+\frac{1}{\lambda_0}\frac{\partial V_1^1}{\partial C_1}\right)\alpha_{C_1}C_1\\&-\rho\left(\frac{1}{\lambda_1}V_1^0+\frac{1}{\lambda_0}V_1^1\right)+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\left(P_1-C_1\right)K_1=0\end{aligned} \tag{4.38}$$

We then subtract (4.36) from (3.37)

$$\frac{1}{2}\left(\frac{\partial^2 V_1^1}{\partial C_1^{\ 2}}-\frac{\partial^2 V_1^0}{\partial C_1^{\ 2}}\right)\sigma_{C_1}^2 C_1^{\ 2}+\left(\frac{\partial V_1^1}{\partial C_1}-\frac{\partial V_1^0}{\partial C_1}\right)\alpha_{C_1}C_1-\left(\rho+\lambda_0+\lambda_1\right)\left(V_1^1-V_1^0\right)=0 \tag{4.39}$$

Define the following two new functions

$V_1^a=V_1^0/\lambda_1+V_1^1/\lambda_0$, $V_1^b=V_1^1-V_1^0$

Then (4.38) and (4.39) can be simplified as

$$\frac{1}{2}\frac{\partial^2 V_1^a}{\partial C_1^{\ 2}}\sigma_{C_1}^2 C_1^{\ 2}+\frac{\partial V_1^a}{\partial C_1}\alpha_{C_1}C_1-\rho V_1^a+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\left(P_1-C_1\right)K_1=0 \tag{4.40}$$

$$\frac{1}{2}\frac{\partial^2 V_1^b}{\partial C_1^{\ 2}}\sigma_{C_1}^2 C_1^{\ 2}+\frac{\partial V_1^b}{\partial C_1}\alpha_{C_1}C_1-\left(\rho+\lambda_0+\lambda_1\right)V_1^b=0 \tag{4.41}$$

Each of these two equation yields a solution as follows,

$$V_1^a(C_1)=D_a C_1^{\beta_3}+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\left(\frac{P_1K_1}{\rho}-\frac{C_1K_1}{\rho-\alpha_{C_1}}\right) \tag{4.42}$$

$$V_1^b(C_1)=D_b C_1^{\beta_9} \tag{4.43}$$

where β_9 is the positive root of

$$1/2\,\sigma_{C_1}^2\beta^2+\left(\alpha_{C_1}-1/2\,\sigma_{C_1}^2\right)\beta-\left(\rho+\lambda_0+\lambda_1\right)=0 \tag{4.44}$$

Therefore, the expressions of $V_1^1(C_1)$ and $V_1^0(C_1)$ in this region can be written as

$$V_1^1(C_1)=\frac{\lambda_0\lambda_1\left[D_aC_1^{\beta_3}+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\left(\frac{P_1K_1}{\rho}-\frac{C_1K_1}{\rho-\alpha_{C_1}}\right)\right]+\lambda_0D_bC_1^{\beta_9}}{\lambda_0+\lambda_1} \tag{4.45}$$

$$V_1^0(C_1)=\frac{\lambda_0\lambda_1\left[D_aC_1^{\beta_3}+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\left(\frac{P_1K_1}{\rho}-\frac{C_1K_1}{\rho-\alpha_{C_1}}\right)\right]-\lambda_1D_bC_1^{\beta_9}}{\lambda_0+\lambda_1} \tag{4.46}$$

So far we have derived the current plant's value functions. By imposing boundary conditions on these functions we have the following six equations that can be used to solve for six unknowns: the two thresholds $\underline{C_1}$, $\overline{C_1}$, and the four coefficients B_1, B_2, D_a and D_b.

$$\left\{\lambda_0\lambda_1\left[D_a\underline{C_1}^{\beta_3}+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\left(\frac{P_1K_1}{\rho}-\frac{\underline{C_1}K_1}{\rho-\alpha_{C_1}}\right)\right]+\lambda_0D_b\underline{C_1}^{\beta_9}\right\}\Big/\left(\lambda_0+\lambda_1\right)=\chi \tag{4.47}$$

$$\lambda_0\lambda_1\left[D_a\beta_3\underline{C_1}^{\beta_3-1}-\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\frac{K_1}{\rho-\alpha_{C_1}}\right]+\lambda_0D_b\beta_9\underline{C_1}^{\beta_9-1}=0 \tag{4.48}$$

$$\begin{aligned}&\left\{\lambda_0\lambda_1\left[D_a\underline{C_1}^{\beta_3}+\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\left(\frac{P_1K_1}{\rho}-\frac{\underline{C_1}K_1}{\rho-\alpha_{C_1}}\right)\right]-\lambda_1D_b\underline{C_1}^{\beta_9}\right\}\Big/\left(\lambda_0+\lambda_1\right)\\&=B_1\underline{C_1}^{\beta_5}+B_2\underline{C_1}^{\beta_6}-\frac{\underline{C_1}K_1}{\rho+\lambda_1-\alpha_{C_1}}+\frac{P_1K_1+\lambda_1\chi}{\rho+\lambda_1}\end{aligned} \tag{4.49}$$

$$\begin{aligned}&\left\{\lambda_0\lambda_1\left[D_a\beta_3\underline{C_1}^{\beta_3-1}-\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_0}\right)\frac{K_1}{\rho-\alpha_{C_1}}\right]-\lambda_1D_b\beta_9\underline{C_1}^{\beta_9-1}\right\}\Big/\left(\lambda_0+\lambda_1\right)\\&=B_1\beta_5\underline{C_1}^{\beta_5-1}+B_2\beta_6\underline{C_1}^{\beta_6-1}-\frac{K_1}{\rho+\lambda_1-\alpha_{C_1}}\end{aligned} \tag{4.50}$$

$$B_1\overline{C_1}^{\beta_5} + B_2\overline{C_1}^{\beta_6} - \frac{\overline{C_1}K_1}{\rho+\lambda_1-\alpha_{C_1}} + \frac{P_1K_1+\lambda_1\chi}{\rho+\lambda_1} = \omega \tag{4.51}$$

$$B_1\beta_5\overline{C_1}^{\beta_5-1} + B_2\beta_6\overline{C_1}^{\beta_6-1} - \frac{K_1}{\rho+\lambda_1-\alpha_{C_1}} = 0 \tag{4.52}$$

Although a closed-form analytical solution is not possible for this non-linear equation system, it can be solved numerically using approximation algorithms such as the Newton-Raphson method. The next section provides a numerical example of the generalized model solution and compares it with the two previous cases. Together they capture the main managerial insights of this section.

4.4 Case Study: A Wind Farm Repowering Project

In this section we assume that the representative producer has a wind farm that is currently operating. The hypothetical parameters' values we adapted are calibrated from realistic data of EIA (2010) as well as others (e.g., Takashima *et al.* (2007) and Kjarland (2007)). The complete list of parameter values are shown in Table 4.1.

Table 4.1 Parameters and corresponding values

	Current plant	Repowered plant
Contracted electricity selling price	62 \$/MWh	40 \$/MWh
PTC level if applicable	N/A	22 \$/MWh
Nameplate capacity	3 MW	4.5 MW
Capacity factor	33.33%	33.33%
Investment cost	N/A	1,500,000 \$
Partial decommissioning fee W_r	100,000 \$	N/A
Exit fee W_i	N/A	300,000 \$
Annual discount rate ρ	0.05	0.05
Annualized growth rate of O&M cost	0.04	0.04
Annualized volatility of O&M cost	0.10	0.10
Initial O&M cost	N/A	25 \$/MWh

We first demonstrate the key features of the first two models. Table 4.2 summarizes the optimal repowering policy given the perceived PTC renewal/expiration rate ranging from

0.00 to 0.05. It shows that in the renewal model where the PTC is not in effect yet, the producer will wait longer to repower given a high probability of the PTC being effective in the near future. In the expiration model where the PTC is currently in effect, the producer will repower earlier given a higher probability of expiration.

Table 4.2 Numerical results of first two models

Renewal Possibility Only			Expiration Possibility Only		
λ_1	C_{11}^*	C_{12}^*	λ_0	C_{14}^*	C_{13}^*
0.00	47.9241	71.5111	0.00	47.9241	71.5111
0.01	47.9241	76.0274	0.01	45.0388	71.5111
0.02	47.9241	80.5347	0.02	42.7093	71.5111
0.03	47.9241	85.0349	0.03	40.8238	71.5111
0.04	47.9241	89.5292	0.04	39.2847	71.5111
0.05	47.9241	94.0179	0.05	38.0146	71.5111

For the generalized model, we use the Newton-Raphson method in MATLAB to find the approximate solution to the non-linear equation system (4.47) - (4.52). In order to converge to optimality with reasonable computing time, we scale down the equations by dividing the values of W_r, W_2, I_2, K_1 and K_2 by 8,760,000. This scale comes from the presumed number of hours in a year (8,760) multiples by 1,000, which in this case significantly reduced computing time. The following tables summarize the numerical solution with both λ_0 and λ_1 ranging from 0.01 to 0.05.

Table 4.3 indicates that λ_0 and λ_1 have opposite effects on the optimal repowering threshold with PTC. While $\underline{C_1}$ decreases as the probability of expiration λ_0 increases, it increases as the probability of being in effect increases. However, the effect of λ_1 is much less significant because in this case expiration only occurs after renewal and thus has a lower impact on the producer's current decision making. Table 4.4 shows that when the PTC is currently absent, an increase in λ_1 increases the repowering threshold. λ_0 has the opposite impact but is almost negligible.

Table 4.3 Numerical results of the generalized model

$\underline{C_1}$	λ_1				
λ_0	0.01	0.02	0.03	0.04	0.05
0.01	45.33369	45.60478	45.85113	46.07297	46.27133
0.02	43.23727	43.71871	44.15438	44.5462	44.89682
0.03	41.52914	42.17035	42.75013	43.27199	43.73991
0.04	40.123	40.88476	41.57423	42.19601	42.75499
0.05	38.95197	39.80481	40.57818	41.27732	41.90761

Table 4.4 Numerical results of the generalized model (continued)

$\overline{C_1}$	λ_1				
λ_0	0.01	0.02	0.03	0.04	0.05
0.01	76.0265	80.53376	85.03424	89.52871	94.01762
0.02	76.02598	80.53323	85.03384	89.52843	94.01744
0.03	76.02568	80.53291	85.03358	89.52825	94.01732
0.04	76.02548	80.5327	85.03341	89.52812	94.01723
0.05	76.02536	80.53256	85.03329	89.52803	94.01717

In Table 4.5 we take the difference between $\underline{C_1}$ and $\overline{C_1}$ from Table III and IV. The two thresholds intend to differ more as the uncertainty in the PTC increases, indicating the impact of uncertainty in the decision maker's repowering preference.

Table 4.5 Numerical difference in the generalized model

$\overline{C_1} - \underline{C_1}$	λ_1				
λ_0	0.01	0.02	0.03	0.04	0.05
0.01	30.6928	34.92897	39.18312	43.45574	47.74629
0.02	32.78872	36.81452	40.87945	44.98223	49.12063
0.03	34.49654	38.36256	42.28345	46.25626	50.27741
0.04	35.90248	39.64794	43.45918	47.33211	51.26224
0.05	37.07339	40.72774	44.45511	48.25071	52.10956

By comparing the numerical solution of the generalized model with the two special models, we also find that, given the same values of λ_0 and λ_1, the solution seems to always fall between the corresponding special solutions. That is, $C_{14}^* < \underline{C_1} < C_{11}^*$, $C_{13}^* < \overline{C_1} < C_{12}^*$. This implies that the combination of first two models might serve as a good approximation of the generalized repowering policy. This is important because solving the generalized system can be numerically challenging under certain circumstances especially when it is difficult to find an appropriate initial point.

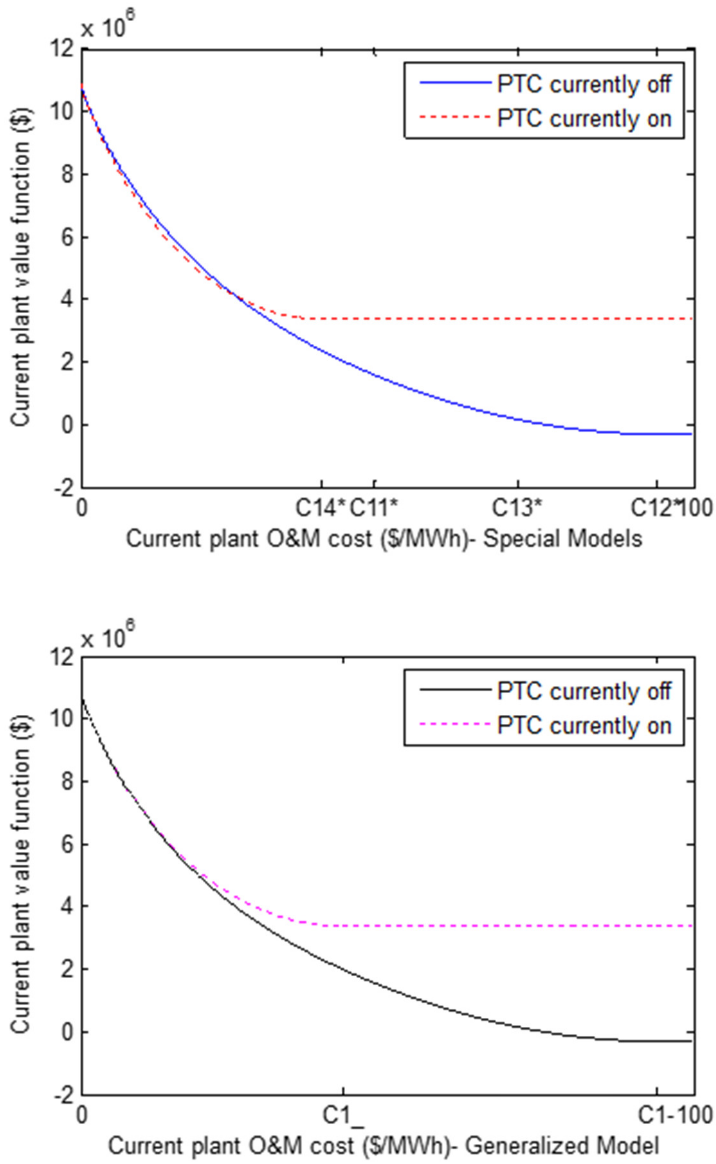

Figure 4.1 Current plant value functions given $\lambda_0 = 0.04,\ \lambda_1 = 0.05$

Figure 4.1 above presents and compares the value functions of the current plant under different model settings, given $\lambda_0 = 0.04$ and $\lambda_1 = 0.05$. The top figure demonstrates the first two model, where the solid line represented the scenario with renewal probability only.

If we compare the top solid line with the bottom solid line, it shows that in $(0, C_{14}^*)$, the top line yields higher value than the bottom one, which implies that the renewal model overestimates the value of the current plant by ignoring the probability of the PTC expires after being renewed. It can also be shown that the expiration model underestimates the value of the current plant.

4.5 Concluding Remarks

In this chapter we studied the optimal timing of repowering for a representative renewable power producer under stochastic O&M cost and uncertain PTC. While the O&M cost was modeled as a GBM process, we focused on how the uncertainty in the PTC's renewal/expiration affects the producer's repowering decision making. This paper, to our knowledge, is the first one studying the repowering problem of the current aging renewable power plants considering both O&M cost and policy uncertainties.

We first constructed two special models in which the producer perceives the PTC changing from being not in effect to in effect and from being in effect to not in effect. We solved each model analytically and obtained a corresponding two-threshold optimal repowering policy. We showed that when the PTC is currently not in effect, the probability of it being renewed will keep the producer wait longer comparing to the case with no chance of PTC and thus postpone repowering. On the other hand, when the PTC is currently in effect, the producer will expedite repowering if the expiration possibility increases.

We then presented a generalized model where the PTC uncertainty is formulated as a Poisson process switching between two regimes. We solved the model numerically and found that the generalized solution can be approximated by combining the corresponding solutions of the two special models. We also showed that the uncertainties in the PTC's renewal and expiration have significant impacts on the optimal repowering thresholds. The empirical observation of slow repowering progress in the U.S. is consistent with our model analyses. Thus a more appropriately designed policy scheme with less uncertainty might be necessary to encourage timely repowering.

CHAPTER 5 . EMPIRCAL STUDY OF THE WIND INDUSTRY

This empirical chapter is divided into two parts to address the validity and necessity of the two most significant modeling assumptions used in this dissertation.

We first use empirical evidence from German wind industry to address the validity of the assumption that the O&M cost follows a GBM process. We then examine the PTC renewal/expiration history to justify our use of Poisson processes.

5.1 On The GBM Assumption of O&M Cost

In this section we test the average repair & maintenance cost (euro/kW per year) of wind turbines under the Scientific Measurement and Evaluation Program (WMEP) within Germany's "250 MW Wind" project (Fraunhofer IWES, 2006).

The cost data we use is in courtesy of Fraunhofer Institute for Wind Energy and Energy System (IWES) and was depicted for three rated capacity classes: turbines with less than 500kW (A), with 500kW to 999kW (B) and turbines with more than 1000 kW (C). For each class, the average R&M cost was computed for all turbines that were installed in the same year and thus the data reflects the general trend of O&M cost development with respect to operating life.

Three statistical tests are conducted in order to check whether the data sets satisfy the two necessary properties of a GBM process: 1) normality of log-ratios with constant mean and variance; 2) log-ratios independent from previous data (Marathe and Ryan, 2005).

For normality we first use normal Q-Q plot to graphically depict the out data values against associated quantiles of the normal distribution. We then conduct Shapiro-Wilk W test to statistically measure whether the p-value falls into the region where the null hypothesis of normal distribution can be rejected. For independency we use the autocorrelation function plot to compare the autocorrelation versus lags.

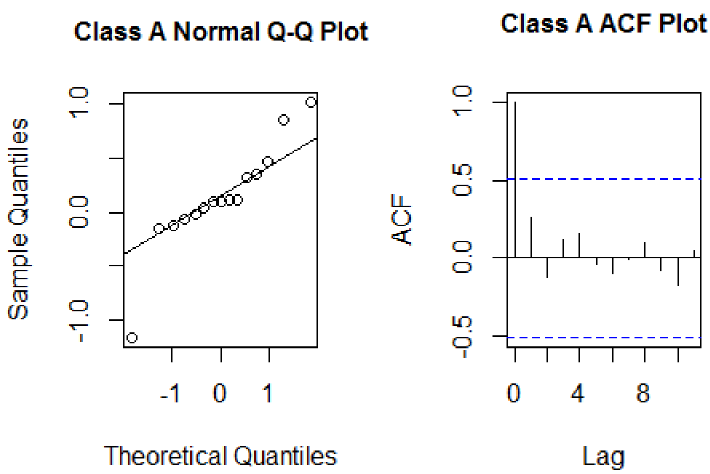

Figure 5.1 Normal Q-Q plot and ACF plot for class A

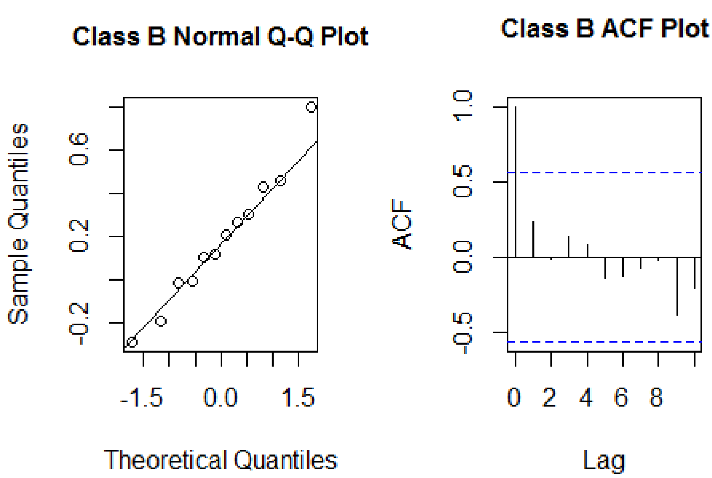

Figure 5.2 Normal Q-Q plot and ACF plot for class B

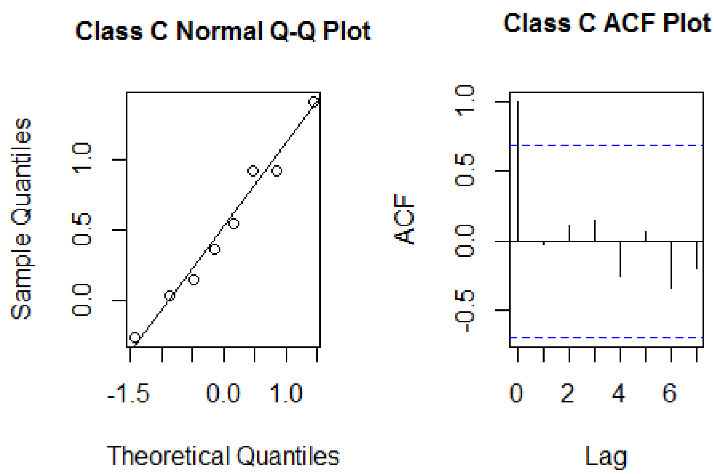

Figure 5.3 Normal Q-Q plot and ACF plot for class C

Figure 5.1 through Figure 5.3 above show that for each class, the data points in the Q-Q plot aligns close with a straight line, indicating that the log-ratios are very likely to be normally distributed.

The Shapiro-Wilk W test gives p-values as 0.05364 (class A), 0.9776 (class B) and 0.9201(class C). All p-values are greater than the significance level of 0.05 and it indicates that the null hypothesis of normal distribution cannot be rejected.

According to the ACF plots in Figure 5.1-5.3 we can also conclude that there exits very weak dependence in the cost log-ratios since the autocorrelation falls far within the 95% confidence interval lines.

Based on the results of the three statistical tests we conducted on Germany's O&M cost data, it is appropriate and justified to assume that the annual O&M cost follows a GBM process.

5.2 On The Poisson Arrival Assumption of the PTC

The other important modeling assumption that distinguishes this paper to the other policy studies is the Poisson behavior of the PTC's renewals and expirations. To validate this assumption, we summarize the history of PTC since its first debut in 1992 and conduct goodness of fit test to the inter-arrival times of policy renewal and expiration.

As stated before, assuming a series of event arrivals follows a Poisson process, the expected time (i.e., inter-arrival time) between two events should be exponentially distributed. Following this we first compute the mean value and the estimated lambda of the distribution. We then divided the observations into equal-length bins and use Chi-Square test to compare the actual probability of each bin and the theoretical probability drawn from the distribution (see Table 5.1, time unit in year).

Table 5.1 Bins and frequencies for Chi-Square test

PTC Renewal Interval	Count		PTC Expiration Interval	Count
0 - 0.14	6		0 - 2	8
0.14 - 0.28	1		2 - 4	1
0.28 - 0.42	0		> 4	1
0.42 - 0.56	1			
0.56 - 0.70	0			
> 0.70	1			

The result shows that for renewal lambda is approximately 6.3654 and p-value is 0.9476; for expiration lambda is close to 0.5059 and p-value is 0.9113. Both p-values are greater than the significance level of 0.05 and thus we conclude that the assumption of the arrivals of PTC renewal and expiration following Poisson processes cannot be rejected.

CHAPTER 6 . CONLCUSION

Like any other major social economic issues involving environment and climate change, the promotion of renewable energy and corresponding government subsidies are almost constantly under scrutiny and various opinions exist (includes opposing ones) regarding whether subsidies should be continued or not. Motivated by the current existing controversies and 'seems-to-be' ever going debates, this dissertation attempted to provide a new point of view to address one of the practical issue in this area – the investment decision making from the perspective of independent producers under cost and policy uncertainties.

Given that independent power producers still dominate the renewable power production and that majority of their output are sold through long-term power purchase agreements, I focused on two types of uncertainties that could represent most of their kinds: the O&M cost and the PTC. Three types of investment activities that covers the major part of any renewable power plant's economic life are thoroughly investigated in a chronological order: a new investor's initial entry, exit when the plant reaches its economic life, and repowering.

In the first part, we modeled and analyzed how an economically rational decision maker will exit and enter a renewable power plant when the O&M cost is represented by a GBM process where the renewable power plant is without any input fuel cost. For such a power plant, we obtained the threshold level of the O&M cost above which a currently operating renewable power plant will exit. We also obtained the threshold level of the O&M cost below which a new renewable power plant will enter. The analytical results of this part provided some significant policy implications. For example, to implement some sort of exit fee by the government will leads to two diverse results: it will not only help in preventing premature exit relative to the physical life, but also prevent O&M cost-wise marginal firms from entering the market, which might be less favorable since it reduces the total production amount of electric power from the renewable energy.

In the second part, we moved our attention to the end of a single plant's economic life, and analyzed the effect of cost uncertainty on the producer's repowering decision making. Based on producers' different sizes, generation resources, and market powers, we classify our study subject of such producers into Types I, II, and III. For a single producer of each type, we obtained the threshold levels of the O&M cost for the current plant's repowering/exit decisions as well as the repowered plant's exit decision. Sensitivity analysis as well as numerical study were also done to support our significant implications. One significant implication of this part is the necessity and importance of regulatory policies on repowering.

In the third part, we took policy uncertainty into consideration in addition to the repowering model presented in the second part. We first constructed two special models in which the producer perceives the PTC changing from being not in effect to in effect and from being in effect to not in effect. We solved each model analytically and obtained a corresponding two-threshold optimal repowering policy.

We then presented a generalized model where the PTC uncertainty is formulated as a Poisson process switching between two regimes. This model has no closed form analytical solution and can only be solved numerically. We used a numerical study to illustrate how the algorithm works and the result firmly supports the argument that the uncertain policy expectation for repowering project might have played a significant role in the slow progress of repowering in the U.S.

Finally we used the empirical O&M cost data of German wind industry and PTC historical data to validate our assumptions regarding the stochastic O&M cost and policy renewal (and expiration). This quantitative study greatly strengthened out analytical results.

Although each problem has been solved separately in each chapter with appropriate assumptions, they are not stand alone. From a sustainable point of view, any decision regarding investment in the renewable industry has to be carefully calibrated because of the dynamic nature of the industry. A decision that is made for now will not only determine the producer's short-term profitability, but also impact his future options in the long run.

This dissertation, to my personal knowledge, is the first attempt in the renewable power investment area to systematically address the consequential decision making problem considering uncertainties from both internal and external sources. However, we do recognize its limitations and are planning to explore them as for future study. First, a more detailed setting of having other economic variables involved, such as other cost components, is very desirable. Second, computational algorithms also need to be developed so that a larger scale of such problem can be solved more efficiently. More updated empirical data of O&M cost from recent installed plants are also desirable.

APPENDIX

A.1 Proofs in Chapter 2

A.1.1 Proof of Proposition 2.1

The structure of (2.4)'s solution contains the general solution of the homogeneous part of it as well as a particular solution to the full equation, which is in the form of

$$V(C) = A_1 C^{\beta_1} + A_2 C^{\beta_2} - CK/(\rho - \alpha_C) + PK/\rho$$

where β_1 and β_2 are the roots of the characteristic quadratic equation as

$$1/2\,\sigma_C^2 \beta^2 + \left(\alpha_C - 1/2\,\sigma_C^2\right)\beta - \rho = 0$$

Solving the quadratic equation we have

$$\beta_1 = \left(1/2\,\sigma_C^2 - \alpha_C + \sqrt{\left(\alpha_C - 1/2\,\sigma_C^2\right)^2 + 2\sigma_C^2 \rho}\right) \Big/ \sigma_C^2 > 1$$

$$\beta_2 = \left(1/2\,\sigma_C^2 - \alpha_C - \sqrt{\left(\alpha_C - 1/2\,\sigma_C^2\right)^2 + 2\sigma_C^2 \rho}\right) \Big/ \sigma_C^2 < 0$$

We also notice that when $C \to 0$, i.e., the O&M cost becomes negligible, the renewable site will not be abandoned, which indicates that the value of the option to abandon approaches zero, therefore $A_2 = 0$. After eliminating this speculative bubble, the general solution then becomes

$$V(C) = A_1 C^{\beta_1} - CK/(\rho - \alpha_C) + PK/\rho$$

Solving the above solution with boundary conditions (2.5) and (2.6) leads to (2.9) and (2.10).

A.1.2 Proof of Proposition 2.2

To prove $\partial C^*/\partial\sigma_C > 0$, we transform it into an equivalent problem of proving $\partial C^*/\partial\sigma_C = \left(\partial C^*/\partial\beta_1\right)\cdot\left(\partial\beta_1/\partial\sigma_C\right) > 0$ by the chain rule.

For simplicity we denote $\sqrt{\left(\alpha_C - 1/2\,\sigma_C^2\right)^2 + 2\sigma_C^2\rho}$ as $\sqrt{\Delta}$, and

$$\begin{aligned}\frac{\partial\beta_1}{\partial\sigma_C} &= \left[\left(\sigma_C + \frac{\sigma_C^3 - 2\alpha_C\sigma_C + 4\rho\sigma_C}{2\sqrt{\Delta}}\right)\sigma_C^2 - 2\left(\frac{1}{2}\sigma_C^2 - \alpha_C + \sqrt{\Delta}\right)\sigma_C\right] \Big/ \left(\sigma_C^2\right)^2 \\ &= \left(\frac{\sigma_C^4 - 2\alpha_C\sigma_C^2 + 4\rho\sigma_C^2}{2\sqrt{\Delta}} + 2\alpha_C - 2\sqrt{\Delta}\right) \Big/ \sigma_C^2 \\ &= \frac{\sigma_C^4 - 2\alpha_C\sigma_C^2 + 4\rho\sigma_C^2 + 4\alpha_C\sqrt{\Delta} - 4\Delta}{2\sigma_C^2\sqrt{\Delta}}\end{aligned}$$

Since the denominator is positive, we only need to check the sign of the numerator to determine the sign of the whole equation. For the numerator, if $4\alpha_C\sqrt{\Delta} > -\sigma_C^4 + 2\alpha_C\sigma_C^2 - 4\rho\sigma_C^2 + 4\Delta$, it is positive. Thus we square both sides of the inequality and derive the difference between them as

$$\left(4\alpha_C\sqrt{\Delta}\right)^2 - \left(-\sigma_C^4 + 2\alpha_C\sigma_C^2 - 4\rho\sigma_C^2 + 4\Delta\right)^2 = -4\rho\sigma_C^4\left(\rho - \alpha_C\right) < 0$$

Therefore we have $\partial\beta_1/\partial\sigma_C < 0$. For the sign of $\partial C^*/\partial\beta_1$, $\dfrac{\partial C^*}{\partial\beta_1} = -\dfrac{\left(PK/\rho + W\right)\left(\rho - \alpha_C\right)}{\left(\beta_1 - 1\right)^2 K} < 0$.

The other remaining properties can be proved similarly.

A.1.3 Proof of Proposition 2.3

First we use (2.10) to reform (2.13) into a function of the entry threshold $\bar{C}_0$ as

$$F(\bar{C}_0) = \frac{\left(\bar{C}_0\right)^{\beta_1} K}{\left(\rho - \alpha_C\right)\beta_1\left(C^*\right)^{\beta_1 - 1}} - \bar{C}_0 K \big/ \left(\rho - \alpha_C\right) + PK/\rho - I$$

When $\bar{C}_0 = 0$ and C^*, the value of $F(\bar{C}_0)$ can be calculated as

$$F(\bar{C}_0 = 0) = PK/\rho - I > 0$$

$$F(\bar{C}_0 = C^*) = -W - I < 0$$

By taking the partial derivative of $F(\bar{C}_0)$ with respect to $\bar{C}_0$ we also have that $F(\bar{C}_0)$ is monotonically decreasing, i.e.,

$$\partial F(\bar{C}_0)/\partial \bar{C}_0 \, K\left[\left(\bar{C}_0/C^*\right)^{\beta_1 - 1} - 1\right]\Big/\left(\rho - \alpha_C\right) < 0 \text{ given } \bar{C}_0 < C^*$$

Hence, there exists a unique solution of $\bar{C}_0$.

A.1.4 Proof of Proposition 2.4

For Proposition 2.4 we briefly present the proof for $\partial \bar{C}_0/\partial W < 0$ here.

Recall $F(\bar{C}_0)$ in Appendix A.1.3, it can be further (fully) expanded by using (9) and get

$$F(\bar{C}_0) = \frac{\left(\bar{C}_0\right)^{\beta_1} K^{\beta_1} \left(\beta_1 - 1\right)^{\beta_1 - 1}}{\left(\rho - \alpha_C\right)^{\beta_1} \beta_1^{\beta_1} \left(PK/\rho + W\right)^{\beta_1 - 1}} - \bar{C}_0 K/\left(\rho - \alpha_C\right) + PK/\rho - I$$

As shown in A.1.3, $\partial F(\bar{C}_0)/\partial \bar{C}_0 \neq 0$ given $\bar{C}_0 < C^*$, which indicates that implicit function theorem can be applied to (2.13). We then differentiate (2.13) with respect to W into the following form

$$\frac{\partial A_1}{\partial W}\left(\bar{C}_0\right)^{\beta_1} + \left(A_1 \beta_1 \left(\bar{C}_0\right)^{\beta_1 - 1} - \frac{K}{\rho - \alpha_C}\right)\frac{\partial \bar{C}_0}{\partial W} = 0$$

$$\frac{\partial \bar{C}_0}{\partial W} = -\frac{\frac{\partial A_1}{\partial W}\left(\bar{C}_0\right)^{\beta_1}}{A_1 \beta_1 \left(\bar{C}_0\right)^{\beta_1 - 1} - K/\left(\rho - \alpha_C\right)}$$

As in (2.10), $A_1 = \dfrac{K}{(\rho - \alpha_C)\beta_1 \left(C^*\right)^{\beta_1 - 1}}$, and after substitution, we have

$$\frac{\partial \overline{C}_0}{\partial W} = -\frac{\dfrac{\partial A_1}{\partial W}\left(\overline{C}_0\right)^{\beta_1}}{\left[K/(\rho - \alpha_C)\right]\left[\left(\overline{C}_0/C^*\right)^{\beta_1 - 1} - 1\right]}$$

Since $\overline{C}_0 < C^*$, the denominator above as $\left[K/(\rho - \alpha_C)\right]\left[\left(\overline{C}_0/C^*\right)^{\beta_1 - 1} - 1\right] < 0$.

Also, it could be mathematically proved that the numerator, $\left(\partial A_1/\partial W\right)\left(\overline{C}_0\right)^{\beta_1} < 0$ because $\partial A_1/\partial W < 0$. Therefore, $\partial \overline{C}_0/\partial W < 0$.

The other remaining properties can be proved similarly.

A.2 Effect of Planning Horizon on Repowering Decisions

Renewable power generation facilities tend to have a highly variable lifespan depending on factors such as construction quality and operation conditions. In life-cycle analysis and levelized cost calculation of these renewable energy sources, generation units such as wind turbines and solar panels are often assumed to have an economic life of 20 years. We also note that, it is difficult to forecast the renewable power industry in 40 or 50 years and to predict what options the producer will have by then, due to the uncertainties from technology development, market and policy reformation. Based on this, we generate two different rolling-horizon scenarios as follows:

Scenario 1 (the baseline): the study horizon contains only one cycle of repowering. In other words, within the planning horizon, the current plant will be repowered once, and the producer is assumed to be unable or not willing to re-repower its repowered plant.

Scenario 2 (the extension): the study horizon contains only two cycles of repowering, that is, the current plant will be repowered twice and then terminated permanently.

We note that the baseline scenario is exactly the one we adopted in Chapter 3. Here we compare the results of the two scenarios to examine the effect of different length planning horizons on repowering decision making.

The optimal repowering policy in the baseline has already been revealed in Section 3.2. To facilitate the comparison we modified the notations and restate them as

$$V_2^b(C_2) = A_2^b C_2^{\beta_2} - C_2 K_2 \big/ \left(\rho - \alpha_{C_2}\right) + P_2 K_2 / \rho$$

$$C_2^b = \frac{\left(P_2 K_2/\rho + W_2\right)\left(\rho - \alpha_{C_2}\right)\beta_2}{\left(\beta_2 - 1\right) K_2} \tag{A.2.1}$$

$$A_2^b = \frac{K_2}{\left(\rho - \alpha_{C_2}\right)\beta_2 \left(C_2^b\right)^{\beta_2 - 1}}$$

$$V_1^b(C_1) = A_1^b C_1^{\beta_1} - C_1 K_1 \big/ \left(\rho - \alpha_{C_1}\right) + P_1 K_1 / \rho$$

$$C_1^b = \frac{\left[P_1 K_1/\rho - V_2^b\left(C_2^0\right) + W_1^r + I_2\right]\left(\rho - \alpha_{C_1}\right)\beta_1}{\left(\beta_1 - 1\right) K_1} \tag{A.2.2}$$

$$A_1^b = \frac{K_1}{\left(\rho - \alpha_{C_1}\right)\beta_1 \left(C_1^b\right)^{\beta_1 - 1}}$$

$$\beta_2 = \left(1/2\,\sigma_{C_2}^2 - \alpha_{C_2} + \sqrt{\left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)^2 + 2\sigma_{C_2}^2 \rho}\right) \Big/ \sigma_{C_2}^2 > 1$$

$$\beta_1 = \left(1/2\,\sigma_{C_1}^2 - \alpha_{C_1} + \sqrt{\left(\alpha_{C_1} - 1/2\,\sigma_{C_1}^2\right)^2 + 2\sigma_{C_1}^2 \rho}\right) \Big/ \sigma_{C_1}^2 > 1$$

For the extension scenario, the problem can be solved in the same fashion by starting backwardly from the final stage, where the second repowered plant is operating and the time to exit needs to be determined. The value function of the project $V_3^e(C_3)$ as well as the optimal exit threshold C_3^e can be derived as follows:

$$V_3^e(C_3) = A_3^e C_3^{\beta_3} - C_3 K_3 \Big/ \left(\rho - \alpha_{C_3}\right) + P_3 K_3 / \rho$$

$$C_3^e = \frac{\left(P_3 K_3/\rho + W_3\right)\left(\rho - \alpha_{C_3}\right)\beta_3}{\left(\beta_3 - 1\right) K_3} \tag{A.2.3}$$

$$A_3^e = \frac{K_3}{\left(\rho - \alpha_{C_3}\right)\beta_3 \left(C_3^e\right)^{\beta_3 - 1}}$$

where $\beta_3 = \left(1/2\,\sigma_{C_3}^2 - \alpha_{C_3} + \sqrt{\left(\alpha_{C_3} - 1/2\,\sigma_{C_3}^2\right)^2 + 2\sigma_{C_3}^2 \rho}\right) \Big/ \sigma_{C_3}^2$.

Then we move back to stage 2 where the first repowered plant is operating and the time to re-repower needs to be determined. The value function $V_2^e(C_2)$ and the re-repowering threshold C_2^e can be obtained as follows:

$$V_2^e(C_2) = A_2^e C_2^{\beta_2} - C_2 K_2 \Big/ \left(\rho - \alpha_{C_2}\right) + P_2 K_2 / \rho$$

$$C_2^e = \frac{\left(P_2 K_2/\rho - V_3^e\left(C_3^0\right) + W_2^r + I_3\right)\left(\rho - \alpha_{C_2}\right)\beta_2}{\left(\beta_2 - 1\right) K_2} \tag{A.2.4}$$

$$A_2^e = \frac{K_2}{\left(\rho - \alpha_{C_2}\right)\beta_2 \left(C_2^e\right)^{\beta_2 - 1}}$$

Following similar steps we move backwards to stage 1 to obtain the current plant's value function $V_1^e(C_1)$ and repowering threshold C_1^e as follows:

$$V_1^e(C_1) = A_1^e C_1^{\beta_1} - C_1 K_1 \Big/ \left(\rho - \alpha_{C_1}\right) + P_1 K_1 / \rho$$

$$C_1^e = \frac{\left[P_1 K_1/\rho - V_2^e\left(C_2^0\right) + W_1^r + I_2\right]\left(\rho - \alpha_{C_1}\right)\beta_1}{\left(\beta_1 - 1\right)K_1} \tag{A.2.5}$$

$$A_1^e = \frac{K_1}{\left(\rho - \alpha_{C_1}\right)\beta_1\left(C_1^e\right)^{\beta_1 - 1}}$$

To facilitate the comparison, we first assume as time goes by, the contracted selling price for future plants will increase, and the production size will also increase due to better design and technology, that is, $P_1 < P_2 < P_3$, $K_1 < K_2 < K_3$. We also assume that for each plant, the O&M cost will follow exactly the same evolution, that is, $C_1^0 = C_2^0 = C_3^0$, $\alpha_{C_1} = \alpha_{C_2} = \alpha_{C_3}$, $\sigma_{C_1} = \sigma_{C_2} = \sigma_{C_3}$, and hence $\beta_1 = \beta_2 = \beta_3$.

Moreover, we further assume all repowering projects, no matter when they get built, are profitable to the producer for planning purposes, that is, $V_3^e\left(C_3^0\right) - W_2^r - I_3 > 0$, $V_2^b\left(C_2^0\right) - W_1^r - I_2 > 0$ and $V_2^e\left(C_2^0\right) - W_1^r - I_2 > 0$, which can be justified in the sense that repowering would not have become an option if it had a negative return.

1) **Comparison on the First-repowered Plant**

We first compare the repowering/exit thresholds of the first-repowered plant $C_2^{b/e}$ in both scenarios. Recalling (A.2.1) and (A.2.4) we have $C_2^b > C_2^e$ holds if and only if $W_2 > -V_3^e\left(C_3^0\right) + W_2^r + I_3$. Because $V_3^e\left(C_3^0\right) - W_2^r - I_3$ is presumed to be positive, $-V_3^e\left(C_3^0\right) + W_2^r + I_3 < 0 < W_2$. Therefore the necessary and sufficient condition for $C_2^b > C_2^e$ always holds, which proves that for the first-repowered plant, the repowering threshold of the baseline scenario is greater than the exit threshold of the extension. This can be intuitively interpreted as the producer tends to be more tolerant to the loss from the operating asset when the available option has a lower expected return.

We also consider the expected economic life of the plant in both scenarios, which can be derived by using Ito's lemma and expressed as follows:

$T_2^b = \left(\ln C_2^b - \ln C_2^0\right) \big/ \left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)$ (in the baseline)

$T_2^e = \left(\ln C_2^e - \ln C_2^0\right) \big/ \left(\alpha_{C_2} - 1/2\,\sigma_{C_2}^2\right)$ (in the extension)

From what we derived before it is easy to see that $T_2^b > T_2^e$, the first-repowered plant is expected to have a longer economic life in the baseline scenario. Hereby we make a mathematical analogy that in our problem, a longer planning horizon with more cycles of repowering will result in a longer expected economic life for a single repowered plant.

2) Comparison on the Current Plant

In this sub-section, we compare the repowering thresholds of the current plant $C_1^{b/e}$ in both scenarios. Recalling (A.2.2) and (A.2.5) we have $C_1^b > C_1^e$ holds if and only if $V_2^b\left(C_2^0\right) < V_2^e\left(C_2^0\right)$. We also have that given the same C_2^0, $C_2^b > C_2^e$ is the necessary and sufficient condition of $V_2^b\left(C_2^0\right) < V_2^e\left(C_2^0\right)$. From the results we derived in 1), $C_2^b > C_2^e$ always hold under our assumptions, therefore $C_1^b > C_1^e$. This implies that the current plant's repowering threshold increases when the producer selects a longer planning horizon with multiple repowering cycles.

Analogous to 1), we also express the expected remaining life of the current plant in the term of $C_1^{b/e}$ defining C_1^c as the current observed O&M cost of the current plant:

$$T_1^b = \left(\ln C_1^b - \ln C_1^c\right) \big/ \left(\alpha_{C_1} - 1/2\,\sigma_{C_1}^2\right) \qquad \text{(in the baseline)}$$

$$T_1^e = \left(\ln C_1^e - \ln C_1^c\right) \big/ \left(\alpha_{C_1} - 1/2\,\sigma_{C_1}^2\right) \qquad \text{(in the extension)}$$

It is easy to see that $T_1^b > T_1^e$, the current plant is expected to have a longer remaining life in the baseline scenario. A similar mathematical analogy can be observed for the current plant that as the current plant will have a longer remaining life when the planning horizon consists of more cycles of repowering.

Through our comparison, we observed that, a longer planning horizon with more cycles of repowering leads to a longer expected economic life for each plant (no matter it is the currently operating one or a projected-to-be-repowered one). This implies that a long planning horizon might delays the repowering decision making for the current plant, which is unfavorable to the current renewable power generation market where a huge amount of obsolete generation units need to be repowered.

CHAPTER 7 BIBLIOGRAPHY

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