

**Demand forecasting and decision making under uncertainty for long-term
production planning in aviation industry**

by

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DEDICATION

I would like to dedicate this thesis to my parents without whose support I would not have been able to complete this work.

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ABSTRACT

The aviation industry represents a complex system with low-volume high-value manufacturing, long lead times, large capital investments, and highly variable demand. Making important decisions with intensive capital investments requires accurate forecasting of future demand. However, this can be challenging because of significant variability in future scenarios. The purpose of this research is to develop an approach on making long-term production planning decision with appropriate demand forecasting model and decision-making theory.

The first study is focused on demand forecasting. Probabilistic models are evaluated based on the model assumptions and statistics test with historical data. Two forecasting models based on stochastic processes are used to forecast demand for commercial aircraft models. A modified Brownian motion model is developed to account for dependency between observations. Geometric Brownian motion at different starting points is used to accurately account for increasing variation. A comparison of the modified Brownian motion and Autoregressive Integrated Moving Average model is discussed.

The second study compared several popular decision-making methods: Expected Utility, Robust Decision Making and Information Gap. The comparison is conducted in the situation of deep uncertainty when probability distributions are difficult to ascertain. The purpose of this comparison is to explore under what circumstances and assumptions each method results in different recommended alternatives and what these results mean making good decisions with significant uncertainty in the long-term future.

CHAPTER 1. GENERAL INTRODUCTION

As the production rate of airliners increases dramatically following the nadir in 2009, major airplane manufactures such as Boeing and Airbus are benefiting from the extraordinary selling record. Figure 1.1 shows the historical order for global commercial airliners (Boeing, 2015; Airbus, 2015). In 2014, global orders were exceed 3,300 which is six times of orders in 2009. As the emerging of new market and positive outlook of global macroeconomics, the demand is likely to keep increasing in future. However, if the production rate approaches an upper limit, Boeing and other aircraft manufactures will encounter a tough capacity planning decision: whether they should expand their capacity? And if they should, how to make a long-term capacity planning strategy? It is a typical long-term capacity planning problem under high uncertainty. For such a complex problem, making decisions based on gut feelings is certainly not a good choice as it would make company in huge risk. A careful production planning can help a decision maker tackle this complex strategic problem.

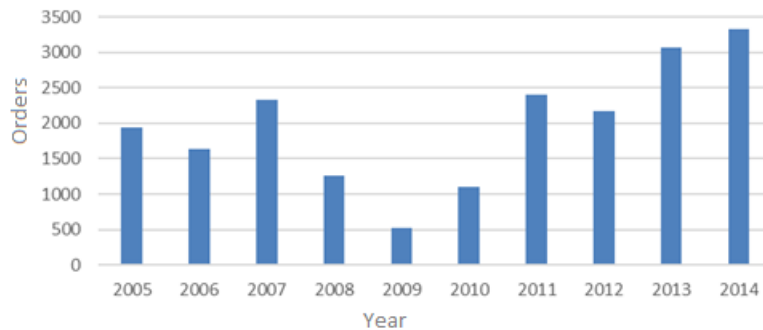


Figure 1.1 Historical order of global commercial airliners

Long-term capacity planning under the presence of uncertainty is a big challenge for many organizations. Because of the large uncertainties involved, most traditional short-term or medium-term capacity planning methods are not applicable. Thus it is essential to find new ways of dealing with this difficult problem. There are numerous literatures discussed capacity planning problems in various industries. Higgins et al. (2005) described how simulation model could be used for capacity planning under uncertainty in food industry. The application of simulation for capacity planning is also found in biomedicine which is used to support decision making (Groothuis et al., 2001). Several articles have discussed the capacity planning problem in industrial and manufacturing field. Eppen et al. (1989) developed a practical model using Mixed Integer Linear Programming to solve a capacity planning problem for General Motors. Nazzal et al. (2006) proposed a comprehensive capital investment decision framework by integrating simulation, statistics, and financial models to support decision making.

But much of this previous research has focused on deterministic problems or short or medium-term planning. When handling uncertainty in capacity planning, multi-stage stochastic programming is a popular method(Chen et al., 2002; Ahmed et al., 2003; Geng et al., 2009). To understand the risks of capacity planning, Bonfill et al. (2004) considered three risk factors (financial risk, downside risk, and worst-case revenue) in a two-stage stochastic programming model. Incorporating game theory, utility theory, financial hedging, and operational hedging can provide a financial model for a capacity planning problem (Mieghem, 2003). However, understanding how to represent the risk and making good decision of long-term capacity planning problems that can be applied to real world problems in aviation industry largely remains an unanswered question.

A collaboration research is initialed in order to overcome this challenge. The goal of the collaboration research is to develop a flexible and practical airplane painting capacity planning tool for Boeing. It is designed to provide Boeing with sufficient information to support decision making for long-term capacity planning strategy. It consists four parts:

demand forecasting, mathematical modeling, simulation modeling, and decision making. This thesis shows partial work (demand forecasting and decision making) within the collaboration research.

The purpose of this thesis is to demonstrate several approaches for long-term production planning problem. A good decision making process about whether to increase capacity should require a reliable demand forecasting which provides sufficient information on plausible scenarios rather than single prediction. In the first study, different types of demand forecasting methods have been applied and evaluated based on the historical orders of the 737, 777 and other airplane types. A modified Brownian motion model to account for dependency between observations is purposed. We compare this new probabilistic model with Autoregressive Integrated Moving Average model. In the second study, a strategy level production planning model is developed. Under the framework of previous model, three decision making methods (Expected Utility, Robust Decision Making and Information Gap) are implemented and applied to the model. The results obtained from those decision-making theories are compared. Finally, we make suggestions and conclude the finding of this research.

CHAPTER 2. PROBABILISTIC METHODS FOR LONG-TERM DEMAND FORECASTING FOR AVIATION PRODUCTION PLANNING

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Abstract

The aviation industry represents a complex system with low-volume high-value manufacturing, long lead times, large capital investments, and highly variable demand. Decisions with intensive capital investments require accurate forecasting of future demand. However, this can be challenging because of the significant uncertainty in future demand. The use of probabilistic methods such as Brownian motion in forecasting has been well studied especially in the financial industry. Applying these probabilistic methods to forecast demand in the aviation industry can be problematic because of the independence assumptions of model and different characteristics of input data. This paper develops two models based on stochastic processes to forecast demand for commercial airliners: (1) a modified Brownian motion model to account for dependency between observations and (2) a geometric Brownian motion with different starting points. The paper compares the modified Brownian motion with an Autoregressive Integrated Moving Average model. These models are used to forecast demand for aircraft production in the next 20 years.

2.1 Introduction

As globalization increases, air travel increases as well, which in turn boosts the demand for new aircraft. In this environment, Boeing is analyzing its capacity to manufacture commercial aircraft to satisfy future demand. Given the substantial investment required to increase capacity, this analysis is predicated on a forecast model for demand for new aircraft for the next 20 years. There is no "best" method for long-term demand forecasting (McLeod et al., 1977). Regarding to the topic of production planning or capacity planning under demand uncertainty, most of previous literatures dedicated to finding optimal schedule or allocating resources while simply represented demand as a probability distribution (Johnson et al., 1974; Caldeira et al., 1983; Geng et al., 2009). Then existing tools in operations research such as stochastic programming could be applied. A comprehensive review for production planning under uncertainty was presented in the literature (Mula et al., 2006). Nonetheless, as rapid market change in modern era, the simplified version of demand uncertainty is rarely a good representer of reality. Airliner demand in aviation industry fluctuates greatly over long-term due to macroeconomics and market change. Therefore, a specific demand forecasting model which dedicates to long-term production planning problem is needed. The goal of this research is to develop a way to measure risk and uncertainty for future demand, so that it could provide adequate information for a decision maker in enterprise strategy planning.

Forecasting future demand given historical data often applies traditional time series models (e.g., Autoregressive Integrated Moving Average). Autoregressive Integrated Moving Average (ARIMA) and Autoregressive Moving Average (ARMA) model have been studied for decades and have already been applied into many fields such as energy (Ediger et al., 2007), economics (Elliott et al., 2016), finance (Rounaghi et al., 2016), hydrology (Karthikeyan et al., 2013), transportation (Klepsch et al., 2017), etc.. However, demand in the aviation industry is influenced by numerous factors such as macroeco-

nomics, fuel price, globalization, and competitiveness. Historical demand for airplanes exhibits large variability. It may not be possible to accurately predict demand without establishing a multivariate causal model which requires enormous effort. This article from Boeing briefly discussed the complexity in developing this type of model (Peterson et al., 2013). Compared to traditional time series forecasting, probabilistic methods explicitly incorporate uncertainty and therefore show a range of plausible scenarios (Gneiting et al., 2014). This article provides a good review on methods in probabilistic forecasting (Zhang et al., 2014). For a production planning problem with high capital investment, probabilistic information for future states is more meaningful than a deterministic forecast of demand. This paper favors probabilistic models to quantify the uncertainty in future demand.

Brownian motion and geometric Brownian motion (GBM) are stochastic process which have been widely applied in various fields. Many literatures discussed the application of Brownian motion (Osborne, 1959, 1962; De Meyer et al., 2003; Azmoodeh et al., 2009), geometric Brownian motion (Voit, 2003; De Meyer et al., 2009) and the generalization model, fractional Brownian motion (Rogers, 1997; Sottinen, 2001), in economics and finance. Because of its nice properties and good interpretation, it was also applied to other fields: chemical engineering (Kramers, 1940), biology (Saffman et al., 1975), quantum mechanics (Caldeira et al., 1983), etc..

This paper focuses on Boeings future painting capacity planning for new airplanes and to determine whether additional painting capacity is needed. Forecasting demand is necessary to develop a reasonable production planning model. This paper uses probabilistic methods for long-term demand forecasting that is based on historical data of annual airplane orders and develops innovative methods to apply these models to forecasting aircraft demand. First, a Brownian motion model is developed to account for dependency between annual orders. Brownian motion assumes independence, but this paper proposes a unique method to dynamically adjust the forecast based on observed correla-

tion. We borrow from the autoregressive model to introduce correlation into Brownian motion. This approach is helpful when the historical data show a strong correlation. This model is applied to forecasting demand for Boeings 737. ARIMA model is also fitted to the data for comparison. Second, this paper constructs a model for geometric Brownian motion in which the starting point is shifted to forecast demand for Boeings 777. We demonstrate how these models account for both the trend (mean shift) and variation in annual demand.

2.2 Forecasting Models

2.2.1 Background on Brownian Motion and Geometric Brownian Motion

Brownian motion is a popular probabilistic model in forecasting. Brownian motion assumes that demand in one year is independent of demand in the other years. A Brownian motion model with drift assumes that annual demand follows a normal distribution with mean $\mu t + b$ and variance $t\sigma^2$, where μ is the mean shift in demand, t is the number of years after the current year, b is the current demand, and σ^2 is the variance of demand at time $t = 1$. Uncertainty (or variance) increases each year in this model. If the annual demand for airplanes follows a Brownian motion process, the demand at time t is:

$$X(t) = \sigma B(t) + \mu t + b \quad (2.1)$$

where $B(t) \sim N(0, t)$ is a standard Brownian motion (i.e., it is normally distributed with a mean 0 and variance t).

GBM is a stochastic process in which the annual percentage changes in demand are independent and identically distributed. GBM is commonly used to predict stock prices and oil prices (Postali, 2006). The annual demand in a GBM is $Y(t) = \exp(X(t))$ where the logarithm of ratio $\frac{Y(t+1)}{Y(t)}$ follows a normal distribution $N(\mu + b, \sigma^2)$ (Marathe & Ryan, 2005).

A standard probability plot or quantile-quantile (Q-Q) plot can be used to verify the normality assumptions of both Brownian motion and GBM. A Q-Q plot displays the residuals of the observed data minus the mean versus the quantiles of the normal (Gaussian) distribution. For the GBM, if $\frac{Y(t+1)}{Y(t)}$ is log-normal distributed, the points on both plots should approximate a straight line. A Shapiro-Wilk test can be conducted to further verify the assumption of normality. If the p-value from Shapiro-Wilk test is less than 0.05, it is reasonable to reject the null hypothesis: the original data follow a normal distribution. Brownian motion assumes independence between observations, and GBM assumes independence of the log ratio. The linear independence assumption can be tested by the autocorrelation function (ACF), which calculates the correlation between demand of different years. The difference in the years is the lag. For the GBM to be valid, the correlation between demand ratios $\frac{Y(t+1)}{Y(t)}$ and $\frac{Y(t+1+k)}{Y(t+k)}$ should be not significant where $k > 0$ represents the lag. The maximum likelihood estimation (MLE) method can be used to estimate model parameters such as mean (i.e., drift) and standard deviation for Brownian motion or GBM based on historical data.

2.2.2 Modified Brownian Motion and GBM

As will be discussed in the application section, the historical demand for airplanes does not always follow the independence assumption. We develop a unique approach to forecast demand based on Brownian motion when the demand observations are dependent. The correlation between two adjacent years is defined as ρ . If N_1 and N_2 are random variables from a standard normal distribution with correlation ρ , we define N_{cor} to be a random variable where

$$N_{cor} = \rho N_1 + \sqrt{1 - \rho^2} N_2 \quad (2.2)$$

It can be shown that N_{cor} also follows a normal distribution and has a correlation of ρ with N_1 . At time t , if the demand are $X(t) = \rho\sqrt{t}N_1 + \mu t + b$ and $X(t + 1) =$

$\rho\sqrt{t+1}N_{cor} + \mu(t+1) + b$, then the correlation between $X(t)$ and $X(t+1)$ equals ρ . Thus, we can use the idea of Brownian motion but induce correlation between annual demand in consecutive years. A separate challenge to using the GBM to forecast demand is the increase in variance. In the Brownian motion model, the standard deviation increases by a factor of $\sqrt{1 + \frac{1}{t}}$ each year. In the GBM model, the standard deviation (or variance) increases by approximate a factor of $e^\mu\sigma$. Such a large variance with the GBM may result in an unrealistically large level of uncertainty. We propose a modified GBM based on the lag variable k . The lag $k = 1$ in the traditional GBM, which means that the ratio between two adjacent years $R(1) = \frac{Y(k+1)}{Y(k)}$ follows a lognormal distribution with mean μ and variance σ^2 . In the alternative method, lag $k = t$. For each year t , $R(t) = \frac{Y(t)}{Y(0)}$ has a lognormal distribution with mean μt and variance $\sigma^2 t$. A modeling challenge is to determine the time $t = 0$ at which the GBM begins. If $t = 0$ is set too far back in the past, the variance in the forecasted demand will be very large. If $t = 0$ is set as the last observed demand point in the historical data, the variance in the forecasted demand will be relatively small, which indicates more certainty than is probably warranted for the future.

2.2.3 Autoregressive Integrated Moving Average

Autoregressive Moving Average is one of the classical forecasting models (Brockwell et al., 2016). Unlike Brownian motion, which requires independent observations, ARMA performs well when time-series data exhibits strong dependence. Autoregressive Integrated Moving Average model extends the ARMA model by adding more parameters to handle the non-stationarity (trend and seasonality) in the data.

Typically, there are two popular approaches to deal with heteroscedasticity in time-series data (Box et al., 2015), which indicates that variance increases with time. The first option is to develop a specific variance stabilizing transformation. According to Ziegel (2003), a variance-stabilizing transformation is a data transformation that is "specifically

chosen either to simplify considerations in graphical exploratory data analysis or to allow the application of simple regression-based or analysis of variance techniques". The second option is the Box-Cox transformation. One-parameter Box-Cox transformation is defined as the following: (Box & Cox, 1964):

$$S(\lambda) = \begin{cases} \frac{X^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \log(X) & \text{if } \lambda = 0 \end{cases} \quad (2.3)$$

where $S(\lambda)$ is the transformed data and λ is the transformation parameter. As the discussion mentioned the risk of using logarithmic transformation blindly to heteroscedastic time series (Box & Jenkins, 1973), Box-Cox transformation was applied to test the hypothesis of using logarithmic transformation (Chatfield & Prothero, 1973).

Autoregressive Integrated Moving Average model can be expressed as the following:

$$\phi(B)(1 - B)^d S_t = \theta(B) Z_t \quad (2.4)$$

where S_t is time series, $\{Z(t)\} \sim WN(0, \sigma_0^2)$. WN is white noise. B is backward shift operator which is defined as $S_{t-j} = B^j S_t, j = 0, \pm 1, \dots$. $\phi(B)$ and $\theta(B)$ are AR and MA polynomials respectively.

$$\phi(B) = 1 + \phi_1 B - \dots - \phi_p B^p \quad (2.5)$$

$$\theta(B) = 1 + \theta_1 B + \dots + \theta_q B^q \quad (2.6)$$

So, there are three parameters in ARIMA(p,d,q) process. Typically, we want to find a ARIMA process with non-negative p , d and q estimated by MLE which has lowest Akaike information criterion (AIC).

2.3 Application: Forecasting Demand for Airplanes

The demand for two models of airplanes were forecasted, each for a span of 20 years. The Boeing 737 is a short- to medium-range twinjet narrow-body airliner. It has been

continuously manufactured since 1967 and still remains popular in nowadays. The Boeing 777 is a family of long-range wide-body twin-engine jet airliners and it was designed to replace older wide-body airliners and bridge the capacity gap between Boeing's 767 and 747. It enters the market in 1990. Historical annual orders for each model are available from Boeing's website (Boeing, 2015).

2.3.1 737 Airplane

Figure 2.1 shows annual orders for the 737 from 1965 to 2015, and Figure 2.2 depicts the difference in orders between two adjacent years. These figures show an increasing trend in orders. The increasing differences between adjacent demands suggest that the variance in annual orders increases with time. It matches the assumption of Brownian motion with positive drift.

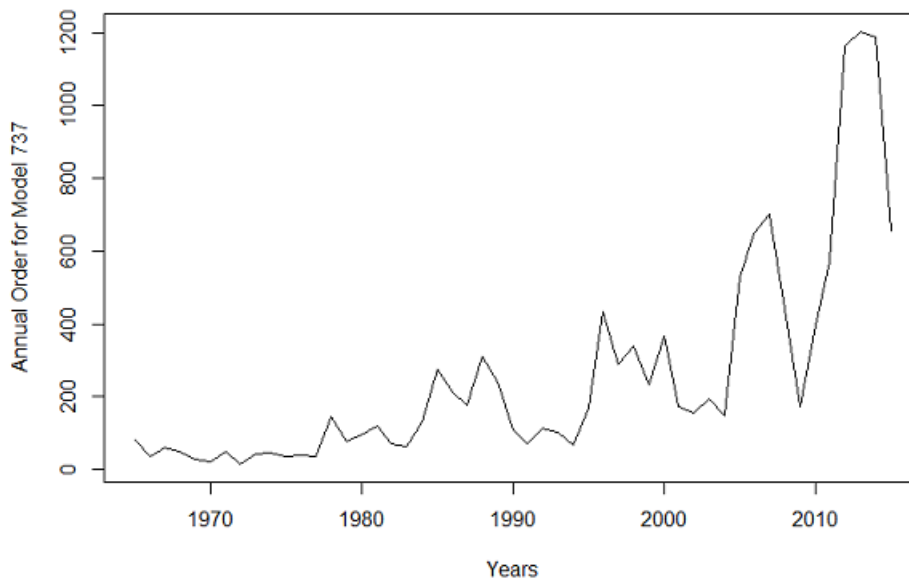


Figure 2.1 Annual orders for the 737

Plots of the ACF and partial autocorrelation function (PACF) examine linear dependence in the data. The ACF (Figure 2.3) decays slowly and shows that the annual order data exhibits strong dependence or correlation. The PACF measures the correlation

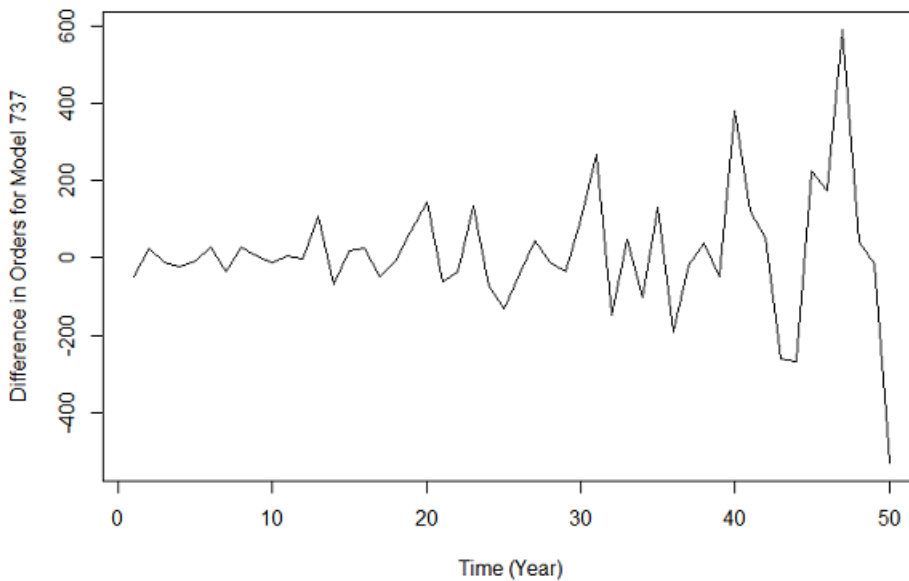


Figure 2.2 Difference in orders between adjacent years for the 737

between demand in years t and $t + k$ given the correlation between demand in year t and $t + \delta$ for all $\delta < k$. The PACF in Figure 2.4 demonstrates very little correlation for $k > 2$ given correlation between adjacent years. These plots suggest that the annual orders for the 737 are dependent between adjacent years. This dependence violates the independence assumption required for Brownian motion, and simple Brownian motion is not a good model for this data.

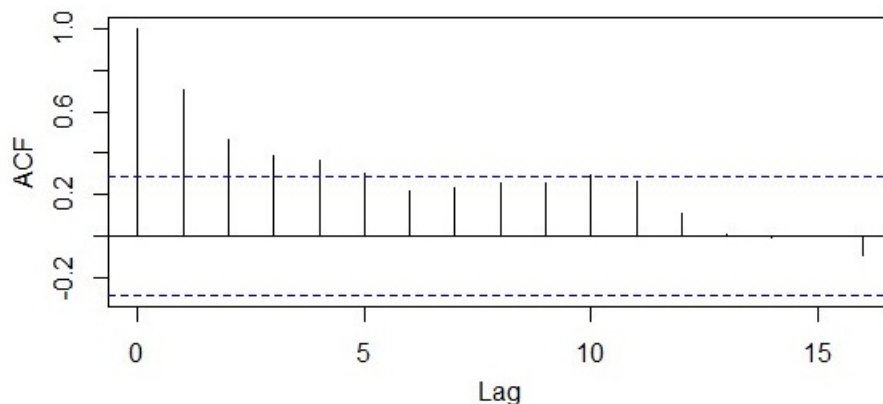


Figure 2.3 Autocorrelation of annual orders for the 737

The modified Brownian motion as described in Subsection 2.2.2 is used because the

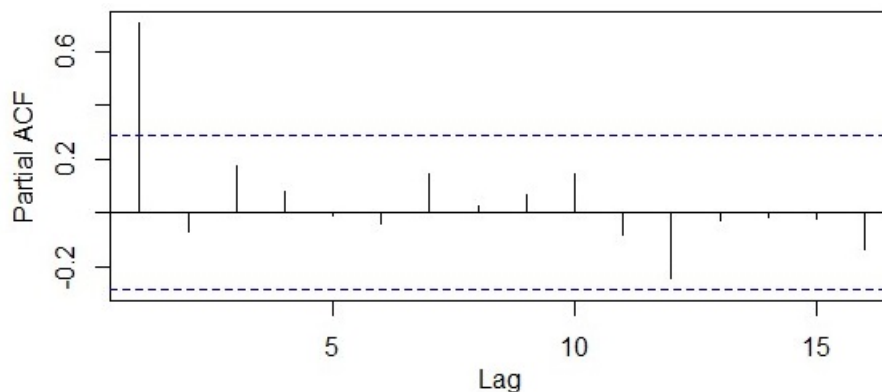


Figure 2.4 Partial autocorrelation of annual orders for the 737

modified Brownian motion model accounts for correlation between demand in adjacent years. The correlation between orders of adjacent years is $\rho = 0.84$, and the standard deviation is $\sigma = 294.8$. Since the ACF in Figure 2.3 suggests correlation for a period of 12 years, we calculate the baseline or current demand b as the weighted sum of the annual orders of the previous 12 years:

$$b = \sum_{t=1}^{12} [X(-t) * w_t] \quad (2.7)$$

where $X(-t)$ represents the annual orders t years prior to the baseline and w_t is the normalized correlation for lag t . Compared to standard methods (averaging or choosing nearest data) in estimating baseline, this approach takes consideration of linear correlation in historical data. It is similar to the idea of weighted average but emphasizes on statistical correlation rather than time.

The drift μ for the Brownian motion is estimated by assuming a linear increasing trend for the 737. However, only data after 1989 was used for two reasons. First, it usually takes a while before the market adopts a new product. Second, by observing the time series in Figure 2.1, the years 1965 through 1988 served as a transition period for the 737, and the year 1989 seems to represent the beginning of relatively constant upward drift. Regression analysis on the data from 1989 to 2015 generated an estimate that $\mu = 31.7$. Table 2.1 depicts the parameters for this modified model.

Table 2.1 Modified Brownian motion parameters for the 737

Drift (μ)	Sigma (σ)	Baseline (b)	Correlation (ρ)
31.7	294.8	483	0.83

We simulate this modified Brownian motion model and ran 100,000 replications over a 20-year period. Figure 2.5 depicts the simulation results with the median (red circles) and the 95% and 5% quantiles (blue triangles). For comparison, Boeings 20-year outlook (Boeing, 2015) for a single-aisle airplane demand in the next 20 years is 26,730 airplanes. Assuming that Boeing captures 50% of the market, the demand for the 737 airplane is 13,365 planes. Based on our modified Brownian motion model, the forecast median demand for the 737 is 20,072 planes. We show median instead of mean because the distribution of prediction from simulation is skew. Median is more robust when extremely high demand presents in the simulation output. Because the modified Brownian motion model assumes increasing trend in future, the median prediction is increasing steady with time. It starts with 530 in year 2016 and ends with 1,548 in year 2035. In standard Brownian motion, the corresponding drift would be 50 which means the median prediction from modified Brownian motion is higher than standard Brownian motion. It is due to the high correlation in the model. The combination effect of both drift and correlation makes the interpretation of increasing trend in the model a little tricky. However, the model is now able to go beyond linear increasing assumption with the ability to predict non-linear smooth curve. If there is no major change happens in future, the median prediction seems plausible. The 90% probability interval which defines by the 95% and 5% quantiles from simulation outputs is the other major interest. The 5% lower curve shows small variability and ranges from 87 to 242. It means company may still have risk with low demand in future. On the contrary, there is a large variability for the 95% upper curve with minimum 1,008 at year 2016 to maximum 3,479 at year 2035. Based on

the forecasting result, the probability interval at year 2035 is pretty wide which means there is a lot of uncertainty.

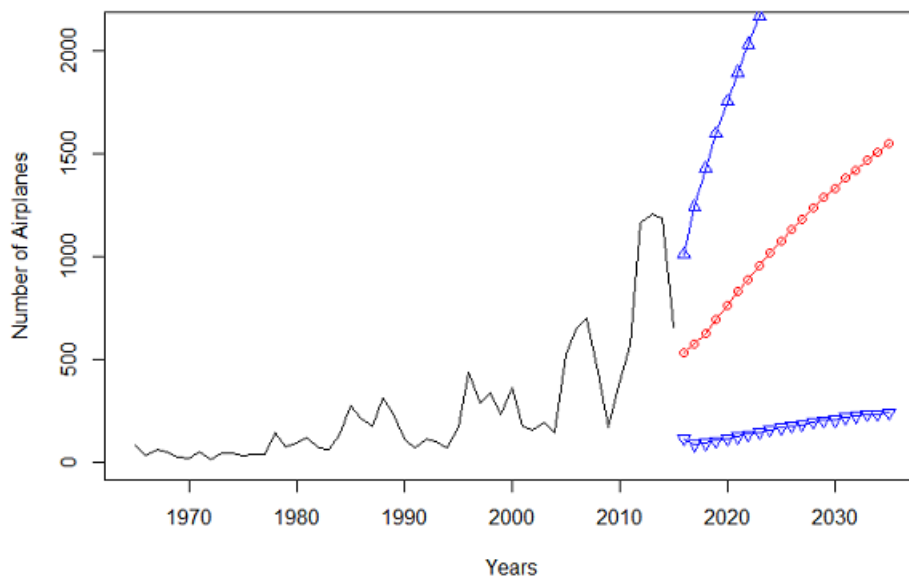


Figure 2.5 Twenty-year demand forecast for the 737 by Brownian motion

We also use the ARIMA model to forecast annual orders for the 737. Since the ARIMA model explicitly accounts for autocorrelation, it is likely to be a good model for the 737 which exhibits autocorrelation. Before applying ARIMA model, we need to make sure the input time series data is stationary. From Figure 2.1, it is clear that the expected annual orders for the 737 is a function of time and the variance is increasing with time. In other words, the original data is non-stationary with trend and changing covariance. First, Box-Cox transformation is applied to stabilize variance. MLE is used to estimate the most likely λ based on all observations. The optimum λ from MLE is -0.02 which is quite close to 0. Moreover, the 95% confidence interval of λ contains 0. Thus, it is reasonable to choose $\lambda = 0$, which is essentially the logarithmic transformation and $S(\lambda) = \log(X)$. Figure 2.6 shows the plot of transformed data. Compared to original data (Figure 2.1), variance is approximately constant after transformation. We use transformed data S_t as input to ARIMA model. Second, 1-lag differencing is applied to

transformed data in order to remove trend. Figure 2.7 presents the result of differencing which is no trend and small positive mean. The small positive mean indicates that there will be an upward drift once we reverse the differencing. It also suggests the parameter d in ARIMA should be chosen as 1.

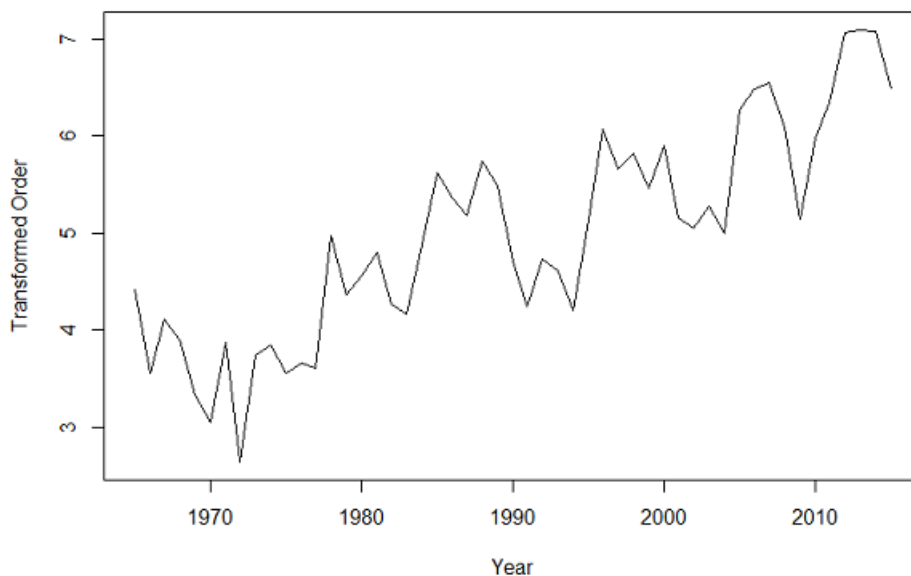


Figure 2.6 Transformed data

Through fixing $d = 1$, it reduces the size of parameter space significantly. By restricting p and q parameters in a reasonable region (typically less than 12), we computed the AIC for all combinations of parameters. We chose ARIMA (0,1,1) as the final model which has the lowest AIC. The MLE returned the best model for transformed data is:

$$S_t - S_{t-1} = Z_t - 0.3344Z_{t-1} \quad (2.8)$$

where $\{Z(t)\} \sim WN(0, 0.3243)$. Once we predicted the mean of annual order for next 20 years based on ARIMA model, we transformed it back to original scale. Then, delta method was used to obtain 95% confidence interval in original scale. The 95% confidence interval of $X(t)$ can be expressed as follow:

$$e^{S(t)} \pm z_{0.05}^* \frac{e^{S(t)} \sigma_S}{\sqrt{n}} \quad (2.9)$$

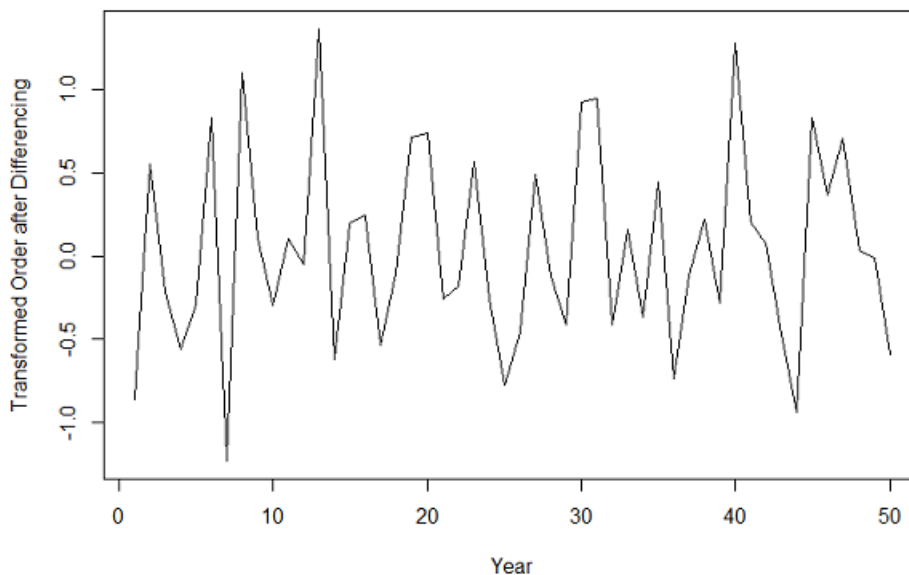


Figure 2.7 Differencing of transformed data

The forecasting result is shown in Figure 2.8 with prediction (red circles), 95% confidence interval of mean (green cross) and 90% prediction interval (blue triangles). The drift in the graph is generated by the inverse of non-zero mean differencing. The confidence interval in ARIMA is much smaller than modified Brownian motion because it which shows the chance to capture true mean but probability interval in modified Brownian motion which presents plausible scenarios. The mean prediction from ARIMA over 20 years ranges from 703 to 2,638. It is higher than the median prediction in modified Brownian motion. The width of confidence interval increases mildly with time compared to previous model. On the contrary, the 90% prediction interval is much wider than modified Brownian motion. The upper limit at year 20 is over 10^{13} and the lower limit is 0 most of time. The prediction interval seems unrealistic. Those extreme numbers are caused by the reverse of transformation.

Overall, for the expected or median prediction modified Brownian motion and ARIMA have different behaviors. If we project the prediction of ARIMA back to year 1965, it would look like a smooth convex curve. The prediction from ARIMA looks like a non-linear fitting to the historical data. However, it does not tell us anything about the

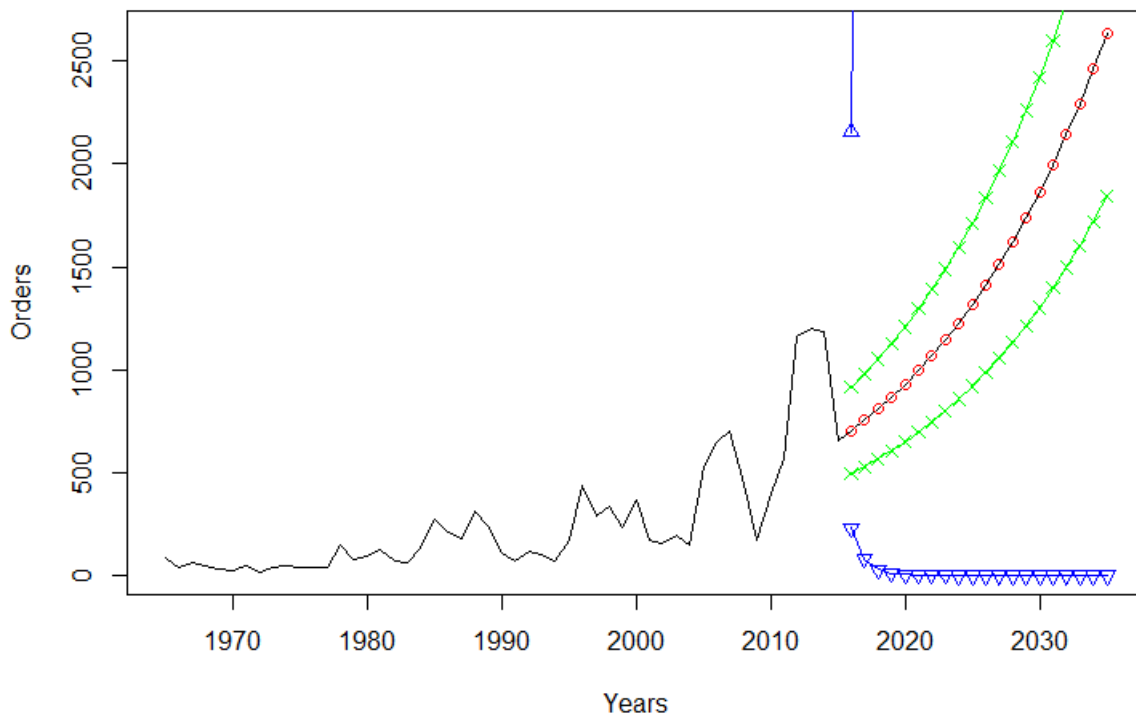


Figure 2.8 Twenty-year demand forecast for the 737 by ARIMA

variability, in particular, heteroscedasticity of the original data. By contrast, the probability interval in modified Brownian motion captures the variability and the increasing uncertainty in future. Therefore, we consider that the result from modified Brownian motion is more informative than mean prediction from ARIMA model in long-term decision making under uncertainty.

2.3.2 777 Airplane

Figure 2.9 shows the annual orders for the 777 from 1990 to 2015, and Figure 2.10 displays the difference in annual orders between each of the adjacent years. Similar to the 737, the annual orders for the 777 exhibit an upward trend and increasing variance over time. Figures 2.11 and 2.12 depict the ACF and PACF of ratio of annual order for the 777. Since the ACF and PACF plots for the 777 demonstrate that the ratio of orders is linearly independent, ARIMA model is no longer under consideration. Either

Brownian motion or the GBM could be an appropriate model depending on whether the data appear normally distributed.

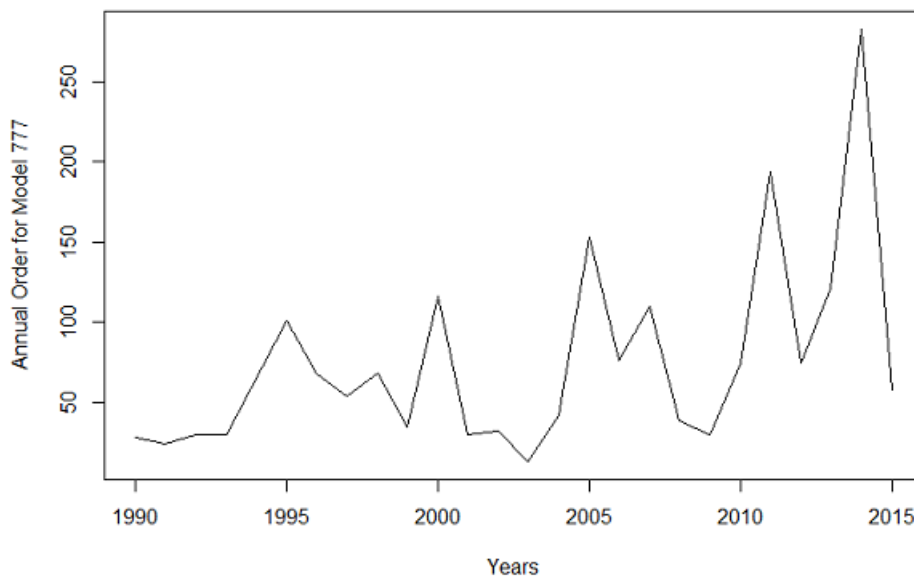


Figure 2.9 Annual orders for the 777

A Q-Q plot (Figure 2.13) without a log transformation for the 777 shows that the data for the annual orders for the 777 do not follow a normal distribution. A Q-Q plot of the log transformation of the original data is depicted in Figure 2.14. We did the Shapiro-Wilk test on log transformed data with p-value 0.7, so we failed to reject the null hypothesis: the original data follow a lognormal distribution. The annual orders for the 777 appear to satisfy the GBM normality assumption, and the GBM can be used to forecast annual orders for the 777.

The ratio is computed from the exponential of historical annual orders $Y(t)$, where $R(t) = \frac{Y(t+k)}{Y(t)}$ and the lag $k = 1$. A lognormal distribution was fitted for $R(k)$. The estimated mean $\hat{\mu}$ is the drift for GBM and the estimated variance $\hat{\sigma}^2$ is the variance for GBM. Table 2.2 shows all estimated parameters that are used in GBM for the 777.

We ran 100,000 simulations of the GBM over a 20-year period. The median (red circles) and a 90% prediction interval (blue triangles) are shown in Figures 2.15. Because of the log transformation, the 90% probability intervals for the 777 are too wide to be

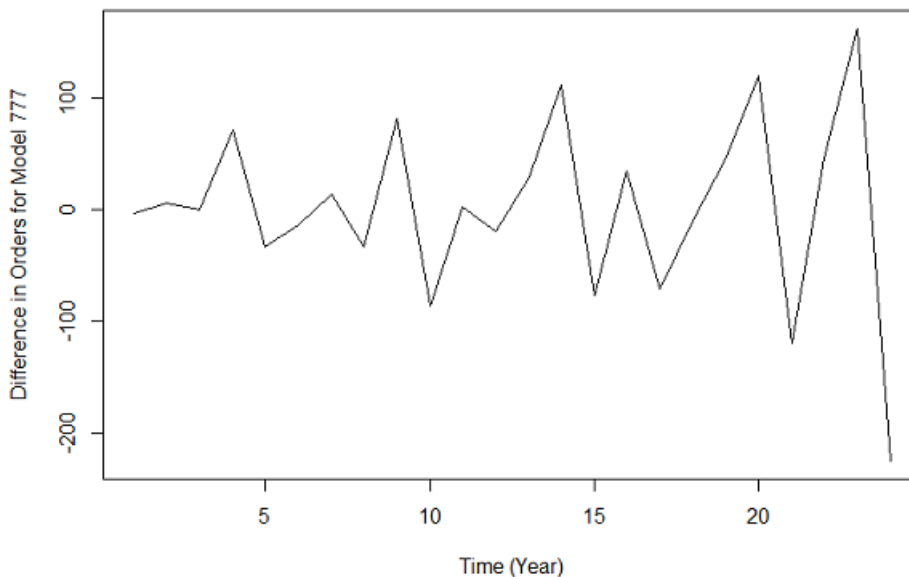


Figure 2.10 Difference in orders between adjacent years for the 777

shown on the graph. The upper bound for the 777 after three years is larger than 500 planes and increases to more than 10,000 planes in years 2025 and beyond. As we can see from the graph, the 90% prediction interval is too narrow for the first a few years. Before 2020, the 90% prediction is under 100 which is only one-third of the actual order in year 2014. Moreover, the median prediction seems to be too conservative. Even in 2035 the median prediction of annual order is under 150. These numbers seems to be unrealistic.

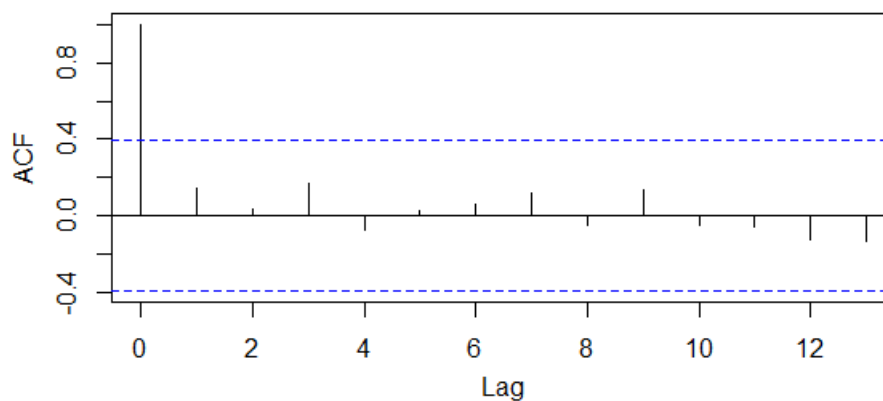


Figure 2.11 Autocorrelation of annual orders for the 777

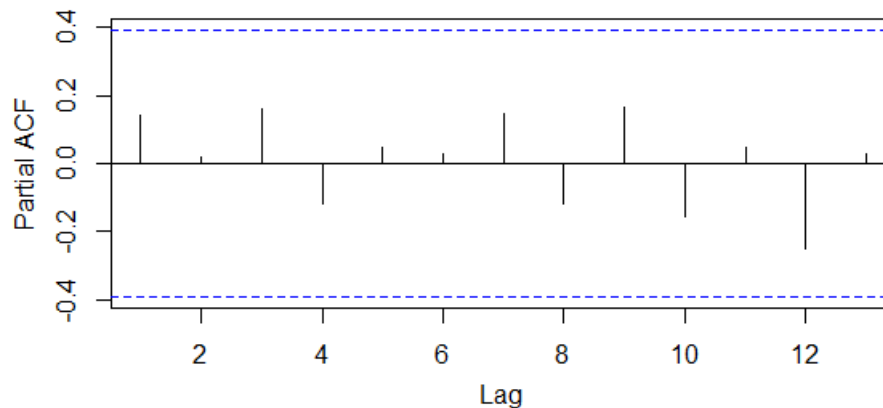


Figure 2.12 Partial autocorrelation of annual orders for the 777

Table 2.2 GBM parameters (traditional method) for the 777

Method	Drift (μ)	Sigma (σ)	Baseline (b)
Tradition	0.030	0.847	3.635

Because of the poor performance and large uncertainty of prediction with the traditional GBM model, we use the modified GBM as discussed earlier where $R(t) = \frac{Y(t)}{Y(0)}$. For each year t , $R(t) = \frac{Y(t)}{Y(0)}$ has a lognormal distribution with mean μt and variance $\sigma^2 t$. The MLE method is used to obtain the most likely μ and σ from the sum of log-likelihood function of lognormal distribution with $R(t)$. Table 2.3 shows all GBM parameters that were estimated using the alternative method.

Table 2.3 GBM parameters (alternative method) for the 777

Method	Drift (μ)	Sigma (σ)	Baseline (b)
Alternative	0.056	0.191	3.635

Figure 2.16 shows the median forecast values (red circles) and the 90% probability interval (blue triangles) of the modified GBM in which $t = 0$ corresponds to 1990. Note that the initial probability interval is very small which may not capture the uncertainty well. The estimated standard deviation (σ) is much smaller than the estimation from

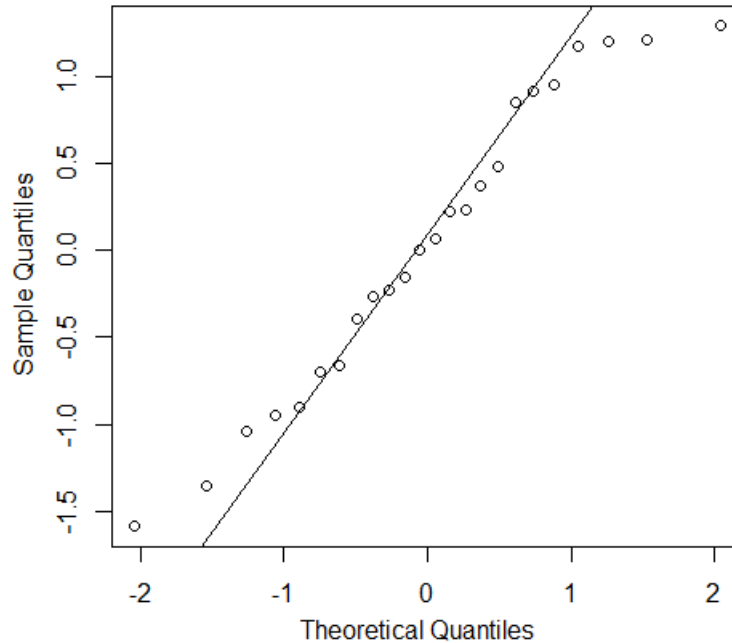


Figure 2.13 Normal Q-Q plot for the GBM normality check for the 777

the traditional method. On the contrary, the estimated drift is higher for alternative method. This is because the alternative method chooses the parameters which maximize the likelihood function with entire historical data (by beginning at $t = 0$) under the GBM framework.

We can move the initial point backwards and forwards along the time axis. For example, Figure 2.18 shows what the estimation would be if we choose year 2000 as a restart point for the 777, and Figure 2.17 shows the estimations for the 777 if the model starts at year 2016. However, it is important to mention that even when we start the estimation in the middle of the timeline, the GBM parameters are the same as in Table 2.3.

All data points on Figure 2.16 are inside 90% probability interval when we fitted the time dependent lognormal distribution with all historical data. The median prediction seems plausible and the unstable prediction points after 2030 is likely due to simulation error (more replications needed). Nonetheless, the high end 95% is too high to be realistic. It means that there is more than 500 orders for the 777 at year 2016 with 5% chance.

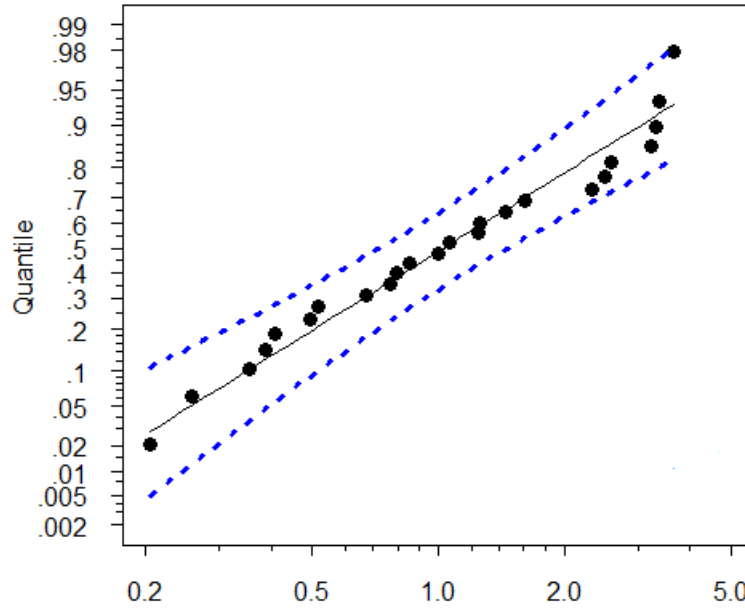


Figure 2.14 Lognormal probability plot for the GBM normality check for the 777

It is very unlikely because that would be double of the highest order at year 2014. Considering the prediction starts at year 2016, the median prediction in Figure 2.17 seems too conservative. Even at year 2035, it predicts less than 150 annual orders which is only half of the demand in year 2014. In addition, the initial probability interval is too narrow to be true. Given the high orders in previous years, the probability to have more than 100 orders in 2016 is clearly higher than 5%.

The starting point was chosen as year 2000 where the prediction mostly captures the variability of historical data visually. The median prediction in Figure 2.18 matches expectation while the probability interval still contains most of data. Based on this modified GBM model, the forecast median demand for the 777 is 1,823 planes. By comparison, according to Boeings 20-year outlook (Boeing, 2015), the global medium wide-body airplane demand in the next 20 years is 3,520 airplanes. Assuming that Boeing captures 50% of the market, the demand for the 777 is 1,760 planes. A close prediction does not mean the model works all the time. The down side of this approach is lacking of theoretical justification and model interpretation. It requires subject judgment therefore

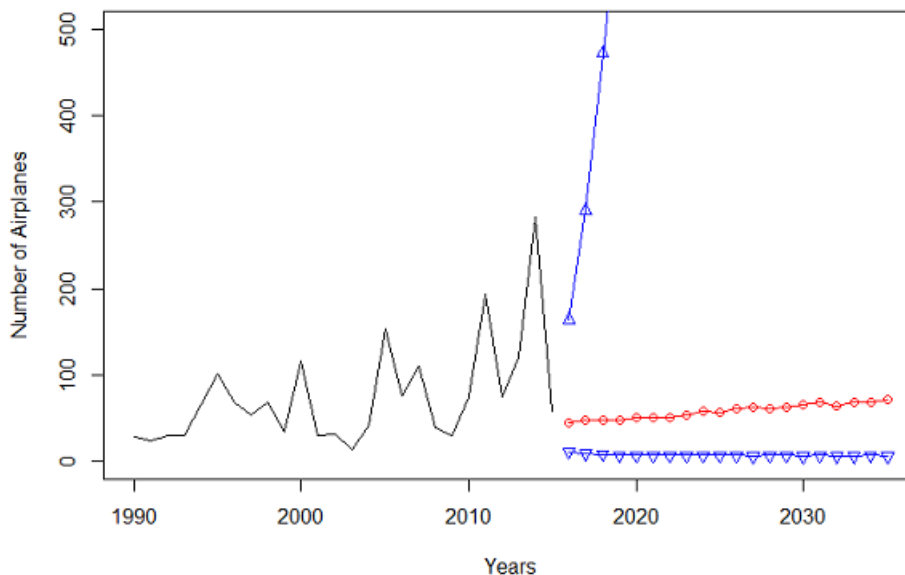


Figure 2.15 Twenty-Year demand forecast for the 777 (traditional GBM method)

may be influenced by the bias from decision maker as well. In addition, because of the high rate of variance increasing, GBM may not work well in long-term forecasting with large trend (μ). However, there is no correct answer for the forecasting. This approach allows the decision maker to explore various scenarios. So that the decision maker would have the chance to choose the scenario which captures the variability of data.

2.4 Conclusion

This research explores different methods for forecasting long-term demand based on historical data for Boeings airplanes. We have used Brownian motion and GBM models and have shown how these models need to be adjusted to fit the nature of the historical data. The median forecasted demand compares favorably to the 20-year Boeing demand forecast. In order to address the correlation in historical data, a modified Brownian motion model is proposed. The comparison between modified Brownian motion and ARIMA has been discussed. When precise prediction or expectation of demand is important, ARIMA is the top choice. However, the long-term extrapolation is still less

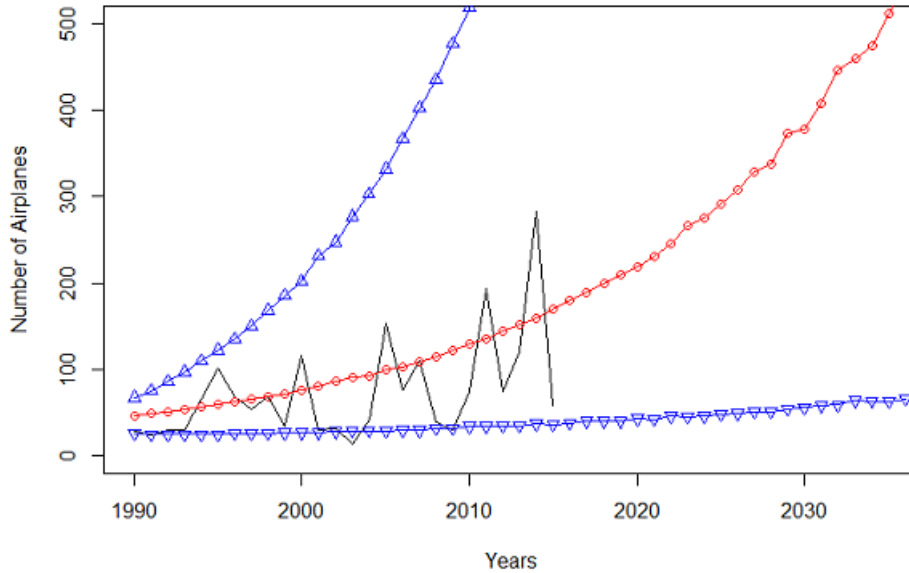


Figure 2.16 Fitted GBM for the 777 (alternative method)

warrant given a non-stationary series data. The probability interval information from modified Brownian motion model could be useful when the primary purpose of forecasting is strategy planning or demand satisfaction. In these situations, company is planning for the risk of extreme case which could cause a huge economic loss. When lag $k > 1$ and there is strong partial autocorrelation observes in timer series data, purposed model may no longer be suitable. The alternative approach in fitting GBM model is applied in forecasting of the 777. This approach provides a flexible way for decision maker to examine the variability of data. It is particularly useful when observations are limited and no correlation is exhibited. Both the Brownian motion model for the 737 and the GBM model for the 777 have significant uncertainty in the forecasts 15-20 years in the future. Although some reduction of uncertainty may be possible, we believe that accurately forecasting demand such a long time into the future will have a lot of uncertainty, and relying on models without such uncertainty could be exhibit overconfidence in our knowledge of the future.

These demand forecasts can serve as an input into a larger systems model that evaluates Boeings current production capacity for airplanes. Given a demand realization

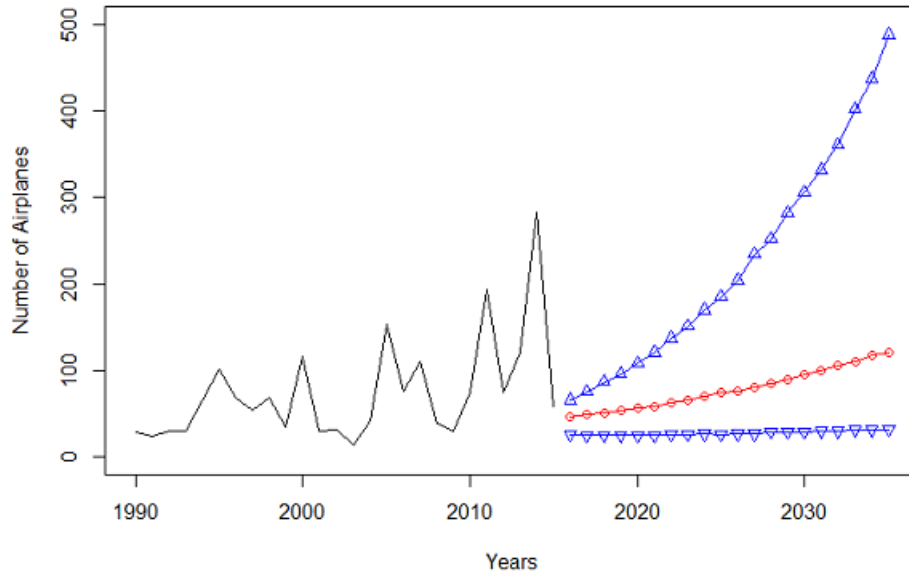


Figure 2.17 Fitted GBM starting at year 2016 for the 777 (alternative method)

based on the probabilistic models for the 737, 777, and other airplane models not discussed in this paper, a production planning model optimally schedules the painting of new airplane orders. The schedule determines how Boeing can most efficiently utilize its current painting capacity. Based on running this model with several demand realizations, we can calculate the probability that Boeings current painting capacity will be exceeded in any given year. The demand forecasts will be used by the systems model to assess if and when Boeing should expand its capacity for painting airplanes. Future research can develop a multi-variate model which incorporates other factors such as gross domestic product, fuel price, etc..

Acknowledgements

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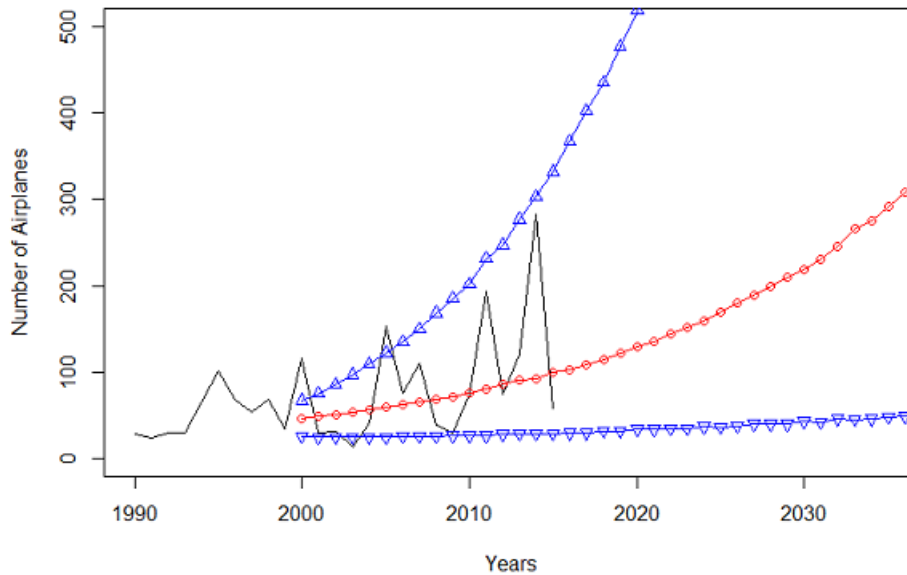


Figure 2.18 Fitted GBM starting at year 2000 for the 777 (alternative method)

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CHAPTER 3. ANALYZING DIFFERENT DECISION-MAKING METHODS FOR SITUATIONS WITH DEEP UNCERTAINTY

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Abstract

Making good risk-based decisions is especially difficult for situations with deep uncertainty that extend over time. Production planning problems may be especially prone to difficult uncertainties (such as demand, operations, supply chain) and yet firms need to plan for several years or decades into the future. Traditional decision-making theories such as subjective expected utility may be challenging to implement if probability distributions are difficult to ascertain. This paper compares different decision-making methods for a complex problem under the presence of long-term uncertainty. The decision-making methods are expected utility, robust decision making, and information gap. The purpose of this comparison is to explore under what circumstances and assumptions each method results in different recommended alternatives and what these results mean making good decisions with significant uncertainty in the long-term future.

3.1 Introduction

In general, uncertainty can be defined as a situation which involves imperfect and/or unknown information. Probability is the most common way to handle uncertainty. Deep uncertainty is a situation, as stated by Walker et al. (2013) "Uncertainties that cannot be treated probabilistically include model structure uncertainty and situations in which experts cannot agree upon the probabilities." Risks with significant, severe, or deep uncertainty that extend over time can be especially difficult to manage. Decisions in the manufacturing industry are especially prone to this type of uncertainty (Applequist, 2000; Brouthers, 2003). A typical approach to these problems is to develop a mathematical model that captures the essentials of the problem. Random variables governed by probability distributions can represent that uncertainty, and the model can usually be solved in order to determine the optimal alternative. If the model involves optimizing an objective function with uncertainty, stochastic programming has become a rich field to find the optimal alternative (Infanger, 1992; Ahmed, 2000; Santos, 2005).

According to Courtney (2001), uncertainty can be divided into 5 intermediate levels between complete certainty and total ignorance. For level 4 (multiplicity of futures) and level 5 (unknown future) uncertainty, it is not a easy task to assign probability distribution for problems with uncertainty far into the future. So, these types of uncertainty, they are usually referred as deep uncertainty (Walker et al., 2003). Based on some assumptions and trend-based scenarios, some practical problem such as demand forecasting and GDP growth could be categorized as situation with level 3 uncertainty (alternative futures with ranking). In Chapter 2, the development of demand forecasting model helps to reduce uncertainty for long-term capacity planning problem, so that it can be treated as situation with level 3 uncertainty. However, what if the assumptions we made in previous chapter are not true? In this chapter, we use different decision methods to address the capacity planning problem and test the sensitivity of underlying

assumptions in Chapter 2.

Several decision-making methods have been developed and proposed to deal with uncertainty, including expected utility (Fishburn, 1970; Rabin, 2000), prospect theory (Tversky, 1992), interval analysis (Moore, 1979, 2003), mean-variance analysis (Epstein, 1985), chance-constrained programming (Charnes, 1959; Hogan, 1981; Charnes, 1983; Hogan, 1984), robust decision making (Lempert, 2003), information gap (Ben-Haim, 2004, 2006, 2015), preserving flexibility (Mandelbaum, 1990), and the precautionary principle (Steele, 2006). Most papers select one decision-making method, and little work has gone into exploring when these decision-making methods produce different results and what assumptions are necessary to implement a specific decision-making method. Lempert et al. (2007) compared robust decision making, expected utility and precautionary methods under a hypothetical environment problem. As the Info-gap method being proposed, Hall (2012) made a comparison between robust decision making and Info-gap for climate policies problem. One year later, similar comparison was done with the application in water resource system planning (Matrosova, 2013). However, all comparisons are in the area of environment and government policy. To the best knowledge of authors, no such comparison has been made for manufacturing capacity planning problem. This paper seeks to fill that gap by focusing on three popular decision-making methods: expected utility (EU), robust decision making (RDM), and information gap (Info-gap)

With its origins dating back to Daniel Bernoulli (Bernoulli, 1954) in 1738, EU is perhaps the most established and still one of the most popular methods for making decisions under uncertainty. EU chooses the alternative that maximizes the decision makers expected utility, and the decision makers utility incorporates his or her risk attitude. EU requires a probability for each potential outcome, and the probabilities represent the decision maker's subjective beliefs about the future.

It can be tricky to ascertain the decision makers utility function, but there are a

few papers provide some useful guidance (Samuelson, 1937; Parzen, 1962; Alt, 1971). Bell (1982) argues that incorporating regret into expected utility theory would improve the quality of decision making. Starting in the 1950s, as evidence from psychology and economic experiments discovered the violation of key axioms in utility theory, non-expected utility theory was proposed (Starmer, 2000). Prospect theory (Kahneman, 1979; Tversky, 1992) and rank-dependent expected utility (Hong, 1987; Quiggin, 2012) are notable models that seek to incorporate descriptive aspects of human behavior into decision making.

The idea of robust decision framework is first proposed by Jonathan Rosenhead (Mingers & Rosenhead, 2001). Then research in robust decision has emerged in areas such as politics (Groves et al., 2007), finance (Mahnovski, 2006) and operations research (Dimitris et al., 2006). In 2003, RDM framework was developed (Lempert, 2003). RDM is a natural method to apply when there is complex uncertainty that is not easily modeled with probability distributions. RDM is designed for situation under deep uncertainty, so it does not rely on prior probability which is a key input parameter in most decision-making models (Lempert et al., 2007). Even if the decision maker believes the uncertainty can be described by a probability distribution, there may be uncertainty around the parameters informing the probability distribution. RDM provides a solution to incorporate uncertainty in the parameter estimation. Generating all plausible scenarios remains a challenge in applying RDM. RDM resembles regret-based decision making in which a decision maker seeks to minimize the regret from a bad outcome. It tends to overweight on the worst scenarios and the best alternative may be very conservative or risk averse.

Info-gap is another method used to deal with severe uncertainty in decision making, especially when probabilities are difficult to assess (Ben-Haim, 2006). Similar to RDM, Info-gap uses sets of representors rather than a single probability distribution; considers the outcomes over a wide range of conditions; and provides a trade-off curve to decision

maker. RDM needs to define scenarios (i.e., distributions), but Info-gap constructs the uncertainty model first and then uses it to identify candidate strategies. The criteria for selecting an alternative are robustness and opportuneness which is the minimum and maximum reward. Info-gap has been criticized for not applicable under situations of "severe uncertainty" (Sniedovich, 2007). We have a discussion about it in this chapter.

This research compares these different decision-making methods by establishing a stochastic process over time and comparing the results obtained from EU, RDM, and Info-gap. Simulation is used to generate the results and compare among the methods. Sensitivity analysis is also conducted for these methods. We find that EU is still the best decision-making method when there is little uncertainty. Info-gap is applicable for level 2 or 3 uncertainty and it provides critical reward information which is especially useful in commercial industry. RDM is the best decision-making framework under deep uncertainty but it requires more effort on scenarios exploration and computational optimization. The rest of paper is structured as follows: section 3.2 introduces the capacity planning model in aviation industry and section 3.3 describes the behavior of the three methods and presents the result of sensitivity analysis. Comparison among these methods is summarized in section 3.4. The conclusion appears in section 3.5.

3.2 Decision-making in Aviation Industry with Deep Uncertainty

In the aviation industry, building new facilities for assembling and painting aircraft is expensive and the fixed operation cost of existing facilities is high. Capacity planning is an important strategic decision for manufacturers. Although we have high confidence that the demand of aircraft is likely to increase globally in future, it is very challenging to forecast demand for one particular manufacturer over a long time period. Economic growth rate, geographical location, global competition, and currency exchange rate all

influence demand. Given this large uncertainty in future demand, should a manufacturer build additional hangars? If they should build additional hangars, we want to know when the manufacturer should build them. In this section, we develop a model that captures the essential factors and variables in this decision problem.

3.2.1 Capacity Planning Model with Uncertain Demand

Table 3.1 and 3.2 lists all notations used in this chapter. A airplane manufacturer wants to plan when and if it should construct new hangars to paint airplanes. It can plan to construct hangars on an annual basis, and $I(t)$ is defined as the number of new hangars at time t , where t is an integer representing years. In this model, we only consider building new hangars and not removing hangars, so $I(t) \geq 0, \forall t$.

Table 3.1 Notation of functions

Notation	Definition	Description
A	Model	Maximum production capacity
D	Model	Demand
DC	Model	Depreciation cost
FC	Model	Fixed cost
G	Model	Profit function
\tilde{G}	Model	Total profit
H	Model	Capacity
I	Model	Investment decision
M	Model	Actual production
MC	Model	In-house painting cost
OC	Model	Outsourcing cost
P_s	RDM	Reward
R	Model	Revenue
RT_s	RDM	Regret
θ	RDM	Probability distribution
U	EU	Utility function
VC	Model	Variable cost

The demand of airplanes for the manufacturer in year t can be written $D(t)$, and we assume the manufacturer cannot influence demand. Based on previous analysis of

Table 3.2 Notation of variables and sets

Notation	Type	Definition	Description
α	Variable	Info-gap	Horizon of uncertainty
CE	Variable	EU	Certainty equivalent
g	Variable	EU	Additional profit beyond baseline
n	Variable	Model	Number of decision options
N	Variable	Model	Number of strategies
Φ	Set	Info-gap	Uncertainty space
r	Variable	EU	Risk tolerance
s	Variable	Model	Strategy
S	Set	Model	Strategy set
s_0	Variable	EU	Default strategy
t	Variable	Model	Time
\underline{t}	Set	Model	Time set
Θ	Set	RDM	Probability distribution set
μ_A	Variable	Model	Expected annual production
μ_D	Variable	Model	Expected annual demand
x	Variable	Model	State
X	Set	Model	State set
z	Variable	Model	Confidence level of best distribution

aviation industry in Chapter 2, we assume that demand follows Brownian motion with trend. Mathematically, demand is $D(t) \sim Nor(\mu_D, \sigma_D^2 t)$, where $\mu_D = \mu_{D0} + \mu_{D1}t$, μ_{D0} is the mean of demand at $t = 0$, μ_{D1} is the annual trend coefficient, and σ_D^2 is the variance in demand at $t = 1$. We assume the manufacturer's production of airplanes in time t equals the demand at time t .

We assume that all the manufacturer's airplanes can be painted in house or via outsourcing, and the revenue is computed as:

$$R(t) = aD(t) \tag{3.1}$$

where a is the selling price of the aircraft. We only consider one type of aircraft with a fixed selling price without adjusting for inflation.

We define $H(t)$ as the capacity—the number of hangars—in year t . Capacity only changes at the beginning of year and remains constant for the rest of that year. The

number of hangars in year t is given by:

$$H(t) = \sum_{i=1}^{t-1} I(i) + h_0 \quad (3.2)$$

where h_0 is the number of hangars at time $t = 0$. Notice that $H(t)$ is the number of total hangars up to time t and it never decreases if $I(t) \geq 0, \forall t$.

We use a straight-line depreciation method to compute the cost of new hangars at time t . The depreciation cost at time t $DC(t)$ is given by:

$$DC(t) = e * dr * [H(t) - h_0] \quad (3.3)$$

where e is the cost of a new hangar and $dr \leq 1$ is depreciation rate. If we want to depreciate capital cost evenly over time t , then $dr = \frac{1}{T}$ where T is the total number of years in the problem. Depreciating cost for this problem takes into account that hangars can be used beyond the total number of years examined in this problem. The manufacturer will not be penalized for building a hangar in year T due to the depreciation factor. In reality, a manufacturer that builds a hangar in year T would be able to use that hangar in years $T + 1, T + 2, \dots$

The maximum number of airplanes that can be painted during year t is $A(t)$, and $A(t)$ also has uncertainty. We assume that $A(t)$ follows a Gaussian distribution $A(t) \sim Nor(\mu_A, \sigma_A^2)$, where $\mu_{A(H)}$ is the average number of planes that could be painted given the number of hangars and σ_A^2 is the variance. The average number of planes painted in a year is $\mu_A = dH(t) + d_0$, where d and d_0 are positive parameters. Given the demand and maximum capacity, the actual number of planes at time t is given by:

$$M(t) = \min\{D(t), A(t)\} \quad (3.4)$$

The manufacturer needs to decide whether to paint an airplane in house or outsource to a third party. It is preferable to paint in house because of the lower cost and shorter lead time. However, if the actual demand exceeds the maximum capacity, the manufacturer will choose to outsource the painting operations. If we assume outsourcing capacity

is infinite, the outsourcing cost can be written as:

$$OC(t) = b \cdot \max\{[D(t) - M(t)], 0\} \quad (3.5)$$

where b is cost of outsourcing. The in-house painting cost is decomposed into two parts: fixed cost $FC(t)$ and variable cost $VC(t)$. Fixed cost is the maintenance cost of capital which is based on the number of hangars:

$$FC(t) = fH(t) + f_0 \quad (3.6)$$

where f and f_0 are fixed-cost coefficients. Variable cost is the operational cost based on the number of jobs.

$$VC(t) = kM(t) + k_0 \quad (3.7)$$

where k and k_0 are coefficients. The total in-house painting cost is the sum of the fixed cost and variable cost:

$$MC(t) = FC(t) + VC(t) = fH(t) + kM(t) + f_0 + k_0 \quad (3.8)$$

The profit function at time t is expressed as:

$$\begin{aligned} G(t) &= R(t) - MC(t) - OC(t) - DC(t) \\ &= aD(t) - fH(t) - f_0 - kM(t) - b \cdot \max\{[D(t) - M(t)], 0\} - e * dr * [H(t) - h_0] \end{aligned} \quad (3.9)$$

The manufacturer choose an investment strategy s in order to maximize its total profit over a period of T years. An investment strategy s is a unique collection of $I(\underline{t})$ where $\underline{t} = \{0, 1, \dots, T\}$. The objective function is calculated as:

$$\tilde{G}(T, s) = \sum_{t=1}^T G(t, s) \quad (3.10)$$

where $G(t, s)$ is the profit function in year t given an investment strategy s .

3.2.2 Decision Space

If a decision maker can choose n different alternatives in each year for T years, the number of strategies in the decision space is n^T . As the number alternatives or years increase, the number of available strategies increases dramatically. We assume that the the maximum number of new hangars can be built over T years is h_{max} and the hangars are not distinguishable. These assumptions reduce the number of strategies. The total number of strategies N in the decision space is calculated as:

$$N = \sum_{i=1}^{h_{max}} \binom{T+1}{i} \quad (3.11)$$

3.2.3 Decision-making Methods

In this section, we discuss the framework for the three decision-making methods: EU, RDM, and Info-gap.

3.2.3.1 Expected Utility

In this case, we assume a single decision exhibits a risk-averse or risk-neutral risk attitude. Given the large uncertainty in this problem, a risk-averse decision is very realistic. Exponential utility function is used to compute the utility of profit. The general form of exponential utility function is:

$$U(g) = a_1 - b_1 \exp(-g/r) \quad (3.12)$$

where $r > 0$ is the risk tolerance; a_1 and b_1 define the scale of utility function; $g = \tilde{G}(T, s) - E[\tilde{G}(T, s_0)]$ which is the additional profit over T years given strategy s after removing the baseline expected profit. The baseline profit $E[\tilde{G}(T, s_0)]$ is the expected profit over T years given strategy s_0 (no additional hangars). If the decision maker is indifferent between obtaining the expected baseline profit and a p probability of gaining an additional m million dollars and $1 - p$ probability of losing m million dollars, then

the decision maker's risk tolerance r can be calculated:

$$\tilde{r} = \left(\frac{p}{1-p}\right)^{\frac{1}{m}} \quad (3.13)$$

$$r = \frac{1}{\ln \tilde{r}} \quad (3.14)$$

The parameters a_1 and b_1 can be ascertained by defining two values for utility, such as $u(0) = 0$ and $u(\$m \text{ million}) = 1$, and solving for a_1 and b_1 . The decision maker should choose the strategy that maximizes his or her expected utility. Once the expected utility is obtained, the certainty equivalent (CE) which is the inverse of utility function and is in units of dollars could be computed for comparison and judgment:

$$CE = U^{-1}(E[U(g)]) \quad (3.15)$$

3.2.3.2 Robust Decision Making

RDM incorporates various uncertainty into the model to support decision making. Uncertainty is represented as "a set of multiple, plausible future states of the world" (Hall et al., 2012). For example, the state x can be interpreted as the parameters μ_{D1}, σ_D in demand model. RDM assumes three sets: strategy set S , a plausible future state set X , and a probability distribution set Θ . The expected regret of strategy $s \in S$ contingent on distribution $\theta_i(x) \in \Theta$ is given by (Lempert et al., 2007):

$$\overline{RT}_{s,i} = \int_x RT_s(x) \theta_i(x) dx \quad (3.16)$$

where $RT_s(x) = \text{Max}_{s'}[P_{s'}(x)] - P_s(x)$ is the regret of strategy s in state x and i is the index of the probability distribution in set Θ . The reward function $P_s(x)$ is the expected utility of profit $E[U(g)]$ given state x .

Given a strategy s , there is one probability distribution $\theta_{best}(x)$ in set Θ which minimizes the expected regret $\overline{RT}_{s,best}$. Similarly, a probability distribution $\theta_{worst}(x)$ yields the maximum expected regret $\overline{RT}_{s,worst}$. The true expected regret given the true probability distribution, which is unknown, should lie in the interval $[\overline{RT}_{s,best}, \overline{RT}_{s,worst}]$.

RDM suggests a way to trade off between the optimal performance and model sensitivity. Mathematically, the trade-off is written as a weighted average of the best and worst expected regret:

$$V_s = z\overline{RT}_{s,best} + (1 - z)\overline{RT}_{s,worst} \quad (3.17)$$

where $0 \leq z \leq 1$.

The parameter z can be interpreted as the level of confidence in the probability distribution $\theta_{best}(x)$. According to RDM, the decision maker should select the strategy s that minimizes V_s given value of z . For example, if decision maker has 100% confidence on that $\theta_{best}(x)$ is the exact representation of truth, then $z = 1$ and the result turns out to be the same as expected utility. Because the expected regret of one strategy is the difference between its expected utility and the maximum expected utility over all strategies. Conversely, if $z = 0$, the decision maker believes there is high uncertainty in the probability distribution over the future state X and should prepare for worst case. Preparing for the worst-case corresponds to the well-known mini-max decision rule.

3.2.3.3 Information Gap

Both EU and RDM assume that uncertainty can be measured with probabilities, the uncertainty in Info-gap model is treated as a family of nested sets (Ben-Haim, 2004). In RDM, we use state x (recall it includes the parameters μ_{D1}, σ_D in the demand model) in set X to represent uncertainty.

Unlike RDM, which tries to measure a fixed uncertainty set with a branch of plausible probability distributions, the Info-gap model has a dynamic uncertainty set Φ and does not assume any probability distribution over that uncertainty set. The dynamic uncertainty set Φ is defined by a variable α . Given a fixed α , the set $\Phi(\alpha, \hat{x})$ states a degree of variability around \hat{x} which is interpreted as the most likely state. The parameter α is called the "horizon of uncertainty" (Ben-Haim, 2015) and explains the variability of x . The greater the value of α , the larger the size of set $\Phi(\alpha, \hat{x})$ and the higher variation. If

$\alpha = 0$, there is no uncertainty in the model. Several types of uncertainty models $\Phi(\alpha, \hat{x})$ exist, and a fraction error model is one of the most common (Hayes et al., 2013). The fraction error model creates an interval based on an initial estimation for each uncertain parameter $(\mu_{D1}, \sigma_D \in X)$ in the demand model:

$$\begin{aligned} \Phi(\alpha, \hat{x}) &= \Phi(\alpha, (\hat{\mu}_{D1}, \hat{\sigma}_D)) \\ &= \{(\mu_{D1}, \sigma_D) : \left| \frac{\mu_{D1} - \hat{\mu}_{D1}}{\hat{\mu}_{D1}} \right| \leq w_1\alpha, \left| \frac{\sigma_D - \hat{\sigma}_D}{\hat{\sigma}_D} \right| \leq w_2\alpha\} \end{aligned} \quad (3.18)$$

where weight parameter $w_1, w_2 \in [0, 1]$ and $(\hat{\mu}_{D1}, \hat{\sigma}_D)$ are initial estimates. Thus, Φ represents the uncertainty space in this problem.

Similar to RDM, the decision space is defined as the strategy set S . A reward function $P_s(x)$ measures the expected utility given the strategy s and state x . The decision maker selects p_c , which is the minimum requirement for the reward function. In the painting decision problem, p_c is the required profit. In the Info-gap model, robustness is defined as the maximum α that still maintains critical requirement for a strategy s , and opportuneness is defined as the the minimum α (Ben-Haim, 2006). The opportuneness function focuses on sweeping success, which might not be appropriate for situation examined in this paper. Hence, we focus on the robustness function $\hat{\alpha}(s, p_c)$ which calculates the greatest level of uncertainty that satisfies the minimum profit requirement.

$$\hat{\alpha}(s, p_c) = \max\{\alpha : \min_{x \in \Phi(\alpha, \hat{x})} P_s(x) \geq p_c\} \quad (3.19)$$

The decision maker should select the strategy s that meets the critical requirement with largest $\hat{\alpha}(s, p_c)$.

3.3 Application of Different Decision-making Methods

This section applies EU, RDM and Info-gap to the decision problem of when to build hangars. The purpose of this section is to analyze how optimal decisions differs among these decision methods given the uncertainty in demand.

3.3.1 Model Settings and Interpretation

Table 3.3 depicts the values chosen for this model. The maximum number of hangars, h_{max} , that the manufacturer can build over the next $T = 20$ years equals 2. We assume that the investment decisions are made only once at year 0, so the decision is planning for the 20 years. From equation 3.11, the total number of strategies is $N = 231$. The baseline strategy of this model is not build any hangars. If $I(t) = 0, \forall t$, after 20,000 replications of a Monte Carlo simulation, the profit over 20 years averages \$7.5 million with a standard deviation of \$0.13 million. In general, a decision is more conservative if less hangars were built or they were built in later year. By contrast, if more hangars were built early, the decision is more aggressive.

3.3.2 Expected Utility

A risk-averse exponential utility function is constructed as following procedure. Let $p = 0.6, r = 2.4663, a_1 = b_1 = 1.0102$. For each strategy in decision space, we run 20,000 replications to find the expected utility given a strategy. We choose 20,000 replications because the 95% confidence interval of mean is small enough (only 0.07% of mean). In addition, the result seems to stable as it does not varies for multiple runs of simulation. Figure 3.1 shows the expected utility for each strategy. All strategies in decision space have positive utility and therefore it suggests that it would be profitable if more hangars are built. A summary of strategies in Figure 3.1 grouping by the number of new hangars is presented in Figure 3.2. The average of expected utility increases as more hangars are built. Figure 3.3 shows average expected utility of strategies grouped by the year of building first hangar. As we can see from the graph, the average expected utility starts to decrease after year 6. Similarly, grouping by the year of building second hangar (Figure 3.4), average expected utility is quite stable from the beginning and it starts to decrease after year 9. These two graphs explains that the best strategy (maximum EU = 0.389) is to build 2 hangars with first at year 6 and second at year 9.

Table 3.3 Model parameters

Notation	Definition	Value	Description
a	Model	1000	Price for 737
a_1	Model	1000	Coefficient of utility function
b	Model	800	Outsourcing price
b_1	EU	1.0102	Coefficient of utility function
d	Model	86.7	Coefficient of expected production function
d_0	Model	0	Coefficient of expected production function
dr	Model	0.05	Depreciation rate
e	Model	30000	Cost of new hangar
f_0	Model	0	Coefficient of fixed cost function
h_0	Model	9	Initial number of hangars
h_{max}	Model	2	Maximum new hangars allowed
k	Model	500	Coefficient of variable cost function
k_0	Model	0	Coefficient of variable cost function
m	EU	1	Coefficient in risk tolerance estimation
p	EU	0.6	Coefficient in risk tolerance estimation
p_c	Info-gap	7	Required profit (in million)
σ_A	Model	3	Standard deviation of production
σ_D	Model	10	Standard deviation of demand
T	Model	20	Total years
μ_{D0}	Model	679	Coefficient of expected demand function
μ_{D1}	Model	18.6	Coefficient of expected demand function
$w1$	Info-gap	0.5	Weight parameter for trend
$w2$	Info-gap	0.2	Weight parameter for standard deviation

Since there is significant uncertainty in demand over the next 20 years, we have little confidence in the parameters used in the demand distribution. For the Brownian motion process, trend μ_{D1} and standard deviation σ_D play an important role in the realization of demand and significantly impacts the firm's profit. We perform sensitivity analysis on μ_{D1} by letting it vary from 0 to 38 airplanes. The optimal strategy graph under different trends in Figure 3.5.

The horizontal axis in Figure 3.5 is trend (μ_{D1}) in the Brownian Motion and vertical axis is the year to build hangar. If the year to build hangar exceeds year 20, it means that the hangar would not be built. According to Figure 3.5, the manufacturer should

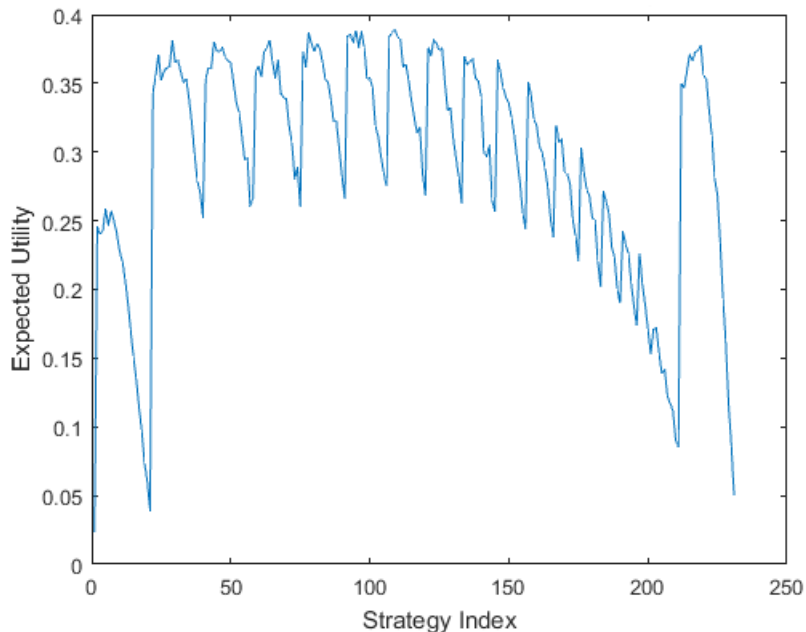


Figure 3.1 Expected utility for different strategies

build hangars earlier as the trend increases (more expected demand). The EU method suggests that the manufacturer should only build one hangar in next 20 years if the trend μ_{D1} is less than 2 airplanes per year. The optimal strategy changes significantly as the trend increases. The downside of making decision based on EU is that determining the optimal strategy is difficult under deep uncertainty (given we do not know the true trend).

If trend μ_{D1} remains constant the variability in demand σ_D increases, the the manu-

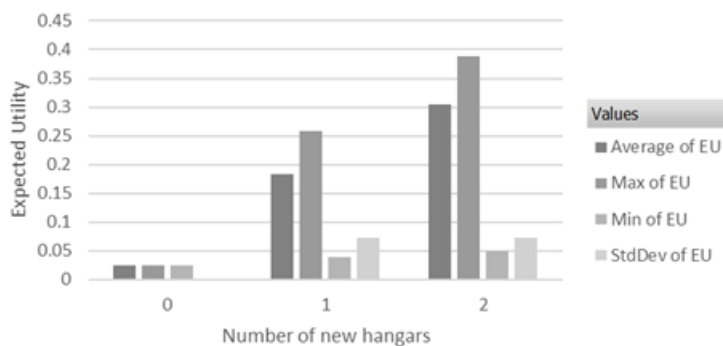


Figure 3.2 Summary of strategies by number of new hangars

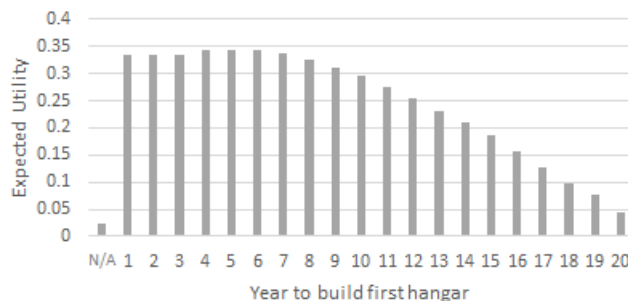


Figure 3.3 Summary of strategies by year of first hangar

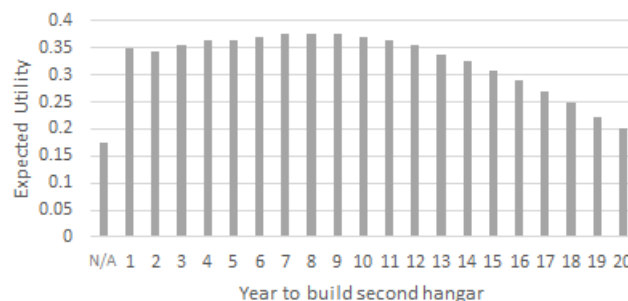


Figure 3.4 Summary of strategies by year of second hangar

facturer should also build hangars earlier and be more aggressive (Figure 3.6). If $\sigma_D = 0$, there is no variability in demand and it will keep steady increasing in next 20 years. EU suggests to build first hangar at year 7 and second at year 11 which is more conservative than the other scenarios with large σ_D . Intuitively, this recommendation makes sense because more variability increases the probability of large demand. The manufacturer should build hangars earlier in order to capture that possibility of larger demand. Under the assumption of risk averse decision maker, the decision will be more aggressive under higher estimation of μ_{D1} and σ_D .

For comparison, we plot the graphs of investment decision when decision maker has different risk tolerance (Figure 3.7). Recall p is probability when a decision maker is indifferent between obtaining the expected baseline profit and a p probability of gaining an additional m million dollars and $1 - p$ probability of losing m million dollars. So, a decision maker is risk seeking when $p < 0.5$, risk neutral when $p = 0.5$ and risk averse

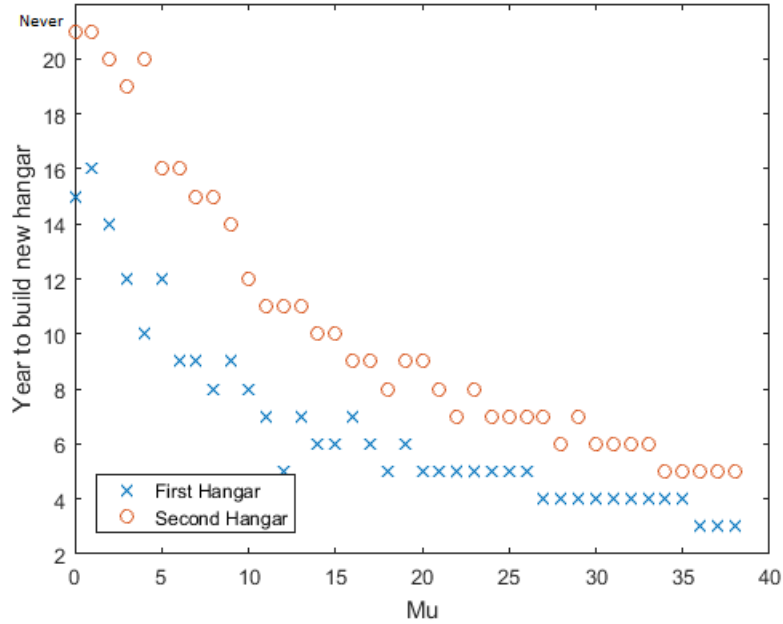


Figure 3.5 Optimal strategy by expected utility under different μ_{D1} of demand model

when $p > 0.5$. We can see that the optimal investment decisions do not varies a lot. For all the strategies in the graph, the expected utility under same p is fairly close. So, risk tolerance of decision maker has little influence to the suggestion made by EU in this problem.

3.3.3 Robust Decision Making

The strategy set in RDM is the same as in EU, but state set is different. EU assumes that μ_{D1} and σ_D are known, but RDM considers a wide range of possible values for μ_{D1} and σ_D . We initially assume $\sigma_D = 10$ and the true μ_{D1} is in range $[0,38]$. The states are the combination of μ_{D1} and σ_D within feasible region. We discretized the parameters to obtain finite states in state space for further numerical computation. Theoretically, an infinite number of probability distributions could characterize the probability of states within the state space. We assume four distributions are possible: uniform, right skew, left skew and a symmetric triangle. Figure 3.8 depicts the results after 20,000 replications, for different values of z . (Recall that z is the level of confidence on probability distribution

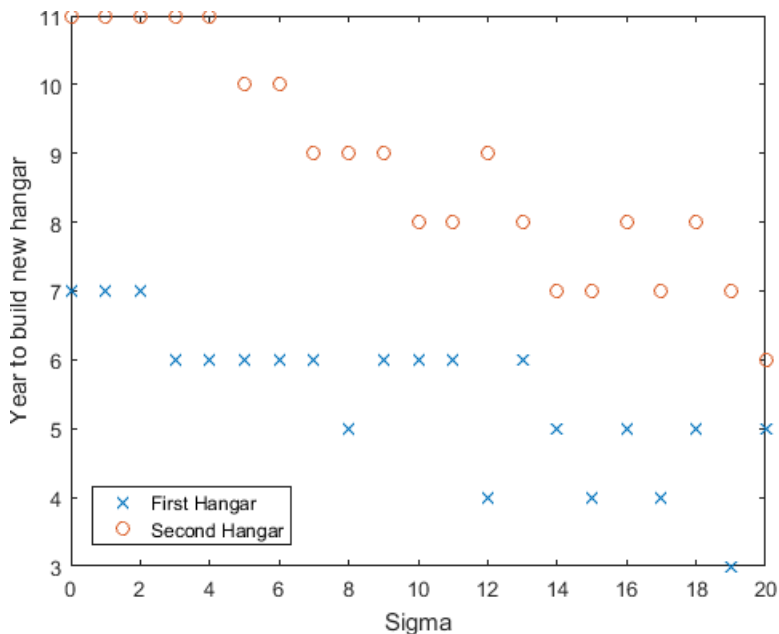


Figure 3.6 Optimal strategy by expected utility under different σ_D of demand model

which returns the best outcome.) RDM suggests the manufacturer should build two hangars in the next 20 years. For most of the values of z , RDM recommends to build that hangar closer to the end of the 20 years, except $z = 1$. If $z = 0$, which prepares for the worst-case, the manufacturer should build the first hangar in year 11 and the second hangar in year 17. If $z = 1$, the decision maker is confident that the best case distribution (left skew distribution with high probability in high demand) is the exact representation of truth. Since there is a high probability of large demand, intuitively, the decision maker should build the first hangar fairly early, in year 7 and second in year 11.

If $\mu_{D1} = 18.6$ remains constant and σ_D is the source of uncertainty in range $[0,20]$, the best-case distribution in this scenario is the right skew (high probability of a small σ_D) and the worst-case distribution is left skew. Figure 3.9 shows the results from the RDM when σ_D is uncertain for different values of z . If σ_D is variable, the decision maker should always build both hangars within the first 8 years. This result is similar to the EU result with large variability. The decision maker should build hangars early to take

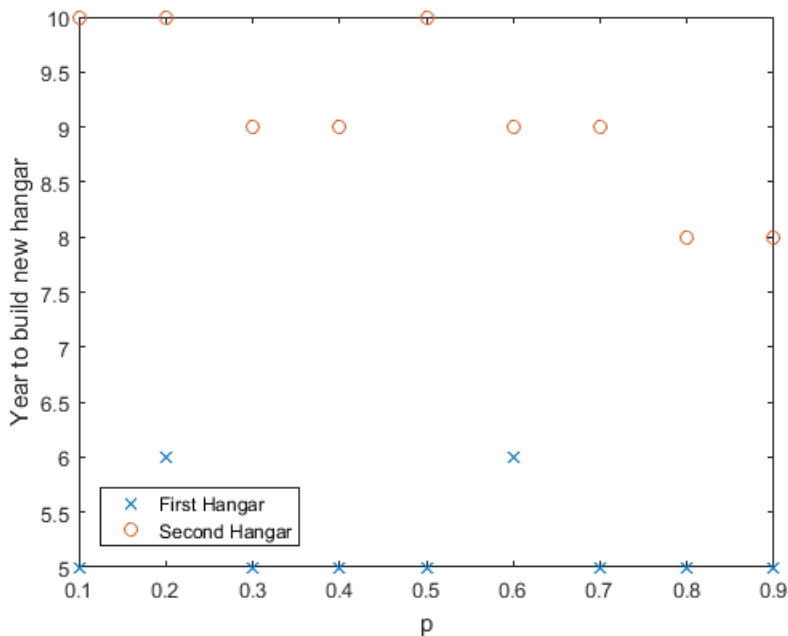


Figure 3.7 Optimal strategy under different p of demand model

advantage of the possibility that demand will be large.

In both cases (uncertain μ_{D1} and uncertain σ_D), the result given $z = 1$ has large difference with other different value of z . For $z \neq 1$, the optimal strategies do not vary a lot for different values of z . The reason for this is the imbalance of variability and the magnitude of minimum/maximum expected regret ($\overline{RT}_{s,best} / \overline{RT}_{s,worst}$). The optimal strategy is selected by minimizing equation 3.17. Each strategy s has a corresponding minimum regret $\overline{RT}_{s,best}$ and a maximum regret $\overline{RT}_{s,worst}$. Figure 3.10 summarizes the mean and variability of both regret vectors for all the strategies when μ_{D1} is uncertain. Both the mean and variability of the maximum regret are much larger than minimum regret. As an affine function (equation 3.17) is used to combine maximum and minimum regret, one unit of regret from either $\overline{RT}_{s,best}$ or $\overline{RT}_{s,worst}$ is treated equally. The contribution from $\overline{RT}_{s,worst}$ will overshadow the contribution from $\overline{RT}_{s,best}$, which has a smaller mean and less variability. The optimal strategies (s^*) for different values of z are similar due to the fact that they all share values $\overline{RT}_{s^*,worst}$ which out-weighs the effect of z on V_s . One could argue that it is possible to put less weight on $\overline{RT}_{s,worst}$

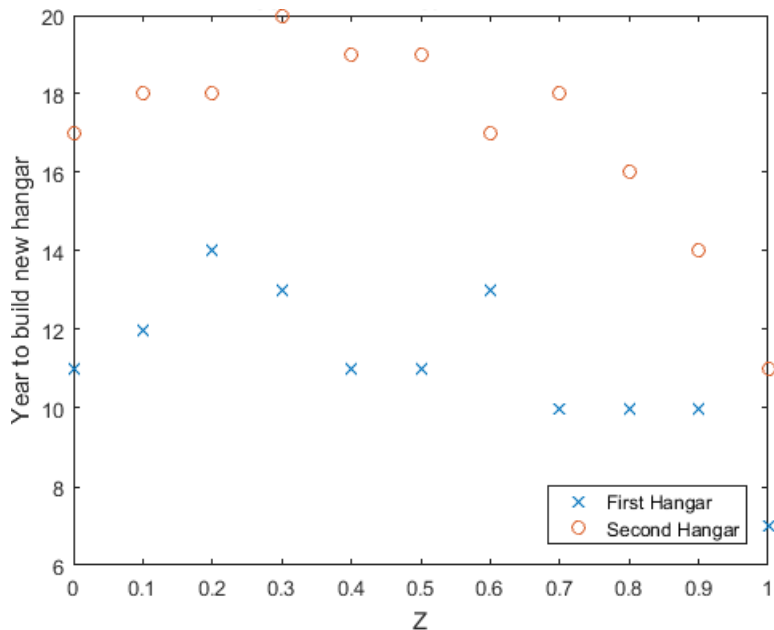


Figure 3.8 Suggested strategies for different z in RDM when μ_{D1} is uncertain

by properly choosing the value of z in order that the variability of $\overline{RT}_{s,worst}$ could be carefully rescaled to match the magnitude of $\overline{RT}_{s,best}$. Such an approach would make the choice of z to be challenging because the variability and magnitude will be different depending on specific problem setting. Moreover, there would be no clear interpretation of z since it is no longer the level of confidence on the best-case distribution as originally proposed.

In brief, our analysis shows that additional attention should be put on the variability and magnitude of the regret values when using RDM to make a decision. If the variability of $\overline{RT}_{s,worst}$ is larger than magnitude of $\overline{RT}_{s,best}$, the result of RDM behaves as if the decision maker is planning for the worst case.

3.3.4 Information Gap

The initial state in Info-gap model is the same as the state in EU which is the best estimation given the decision maker's current information. Thus, $\hat{\mu}_{D1} = 18.6$ and $\hat{\sigma}_D = 10$. The Info-gap algorithm continues to increase the value of α that expands the

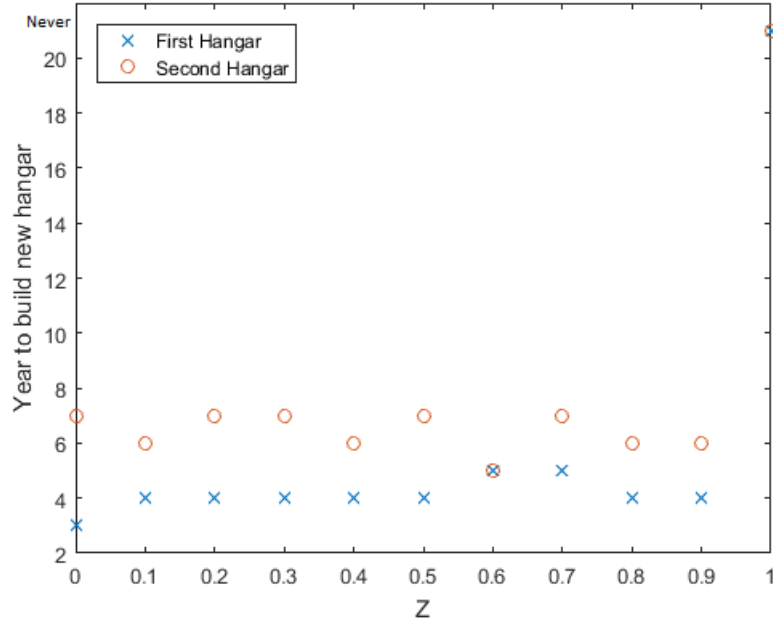


Figure 3.9 Suggested strategies for different z in RDM when σ_D is uncertain

state set around the initial state until no strategy which satisfies the required profit p_c given any state in set.

Figure 3.11 depicts the optimal strategies for different required profits according to the Info-gap model. The graph has a clear trend. As the required profit increases, the decision maker should become more aggressive. However, the corresponding $\hat{\alpha}$ decreases as the required profit increases (Figure 3.12). Decreasing $\hat{\alpha}$ means that less uncertainty around the initial estimate is allowed for larger required profits. If the required profit is greater than \$8 million, no feasible strategy could be found. If the required profit is less than \$6 million, the Info-gap model allows much more uncertainty. With such a small required profit, not building any hangars is the optimal alternative because it allows for the largest uncertainty. This strategy is similar to the optimal strategy for an extremely risk averse ($p=0.99$) decision maker in EU, which recommends only building a single hangar in the final year.

Info-gap theory has been criticized for overestimating the importance of the initial state while dealing with situation in deep uncertainty (Sniedovich, 2008, 2012, 2014). In

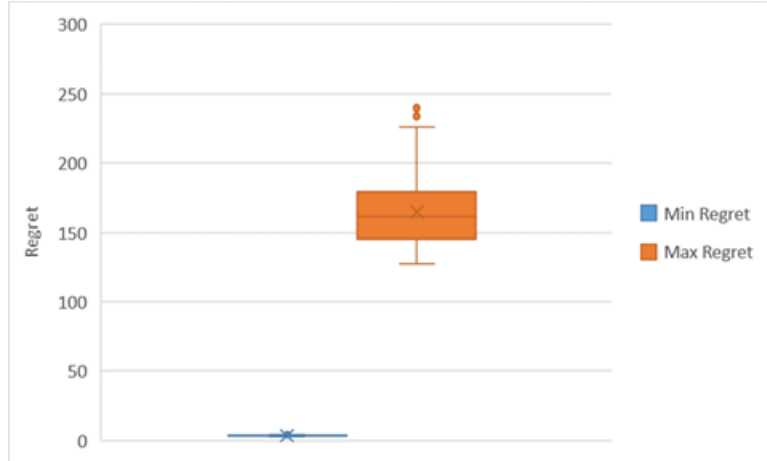


Figure 3.10 Boxplot for minimum regret and maximum regret for all strategies

order to check whether this critique applies to this application, we test the sensitivity of the initial state. We fix the required profit $p_c = 7.5$ million and change the initial state $\hat{\mu}_{D1}$, and Figure 3.13 depicts the result of the Info-gap model. The optimal strategies are similar for different initial states. The randomness in the simulation might explain why the optimal strategies are different. It appears that the results of the Info-gap are largely insensitive to the initial state for this situation. However, this problem has a limited state space with fairly well defined uncertainty. If the problem had more uncertainty such as in model uncertainty or uncertainty around several parameters, the Info-gap model might not be as insensitive to the initial state.

3.4 Comparison of Methods under Deep Uncertainty

If $\sigma_D = 10$, but uncertainty exists around the estimation of μ_{D1} , which ranges between 0 and 38. Figures 3.5, 3.8, and 3.11 depict the optimal strategies for each of the three decision-making methods. The optimal strategy in EU is very sensitive to the input parameter μ_{D1} . The optimal strategies vary from building 2 hangars beginning in year 3 to building 1 hangar at year 15. If the decision maker really has no knowledge about μ_{D1} except that it is between 0 and 38, EU provides little guidance unless the decision

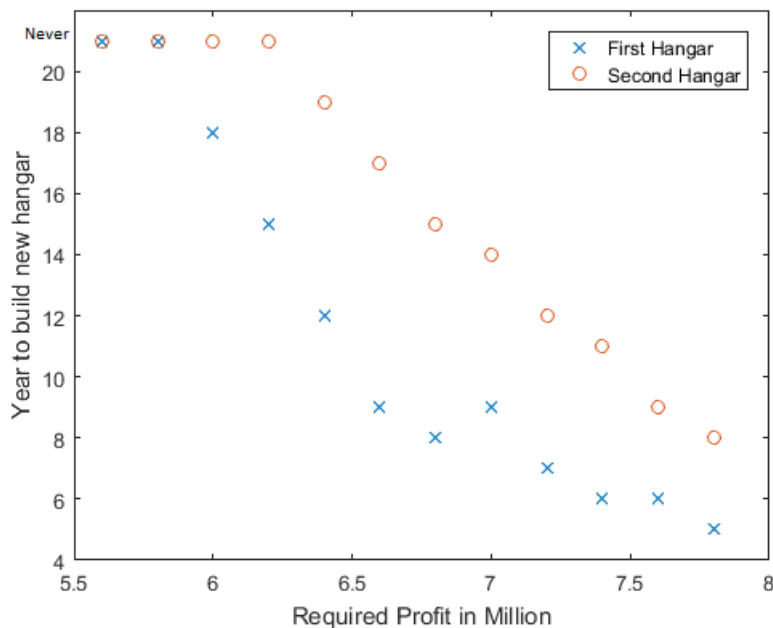


Figure 3.11 Optimal strategies for different required profit in Info-gap

maker wishes to assume a uniform distribution over μ_{D1} or choose $\mu_{D1} = 18.6$ as the expected average demand.

In order to overcome the drawback of EU under deep uncertainty, RDM and Info-gap are designed to handle uncertainty over model parameters. RDM recommends to build 2 hangars and later than that of EU. RDM seems to be more conservative. As discussed in the previous section, the variability and magnitude of the regret generates this conservative approach within RDM. Since RDM is seeking to minimize regret, RDM puts more weight on bad outcomes and is more sensitive to really bad outcomes. EU attempts to find the optimal alternative, but RDM is suitable when optimality is less important and possible bad outcomes have severe consequences.

The Info-gap model structures the problem differently than RDM and EU. The uncertainty space (the state space in this problem) is no longer a fixed set. The size of set is determined by the uncertainty parameter $\hat{\alpha}$. Instead of directly defining that μ_{D1} is in the range $[0,38]$, the range of μ_{D1} dynamically changes in the Info-gap given different reward requirements. Due to the dynamic state set, results from Info-gap are not directly

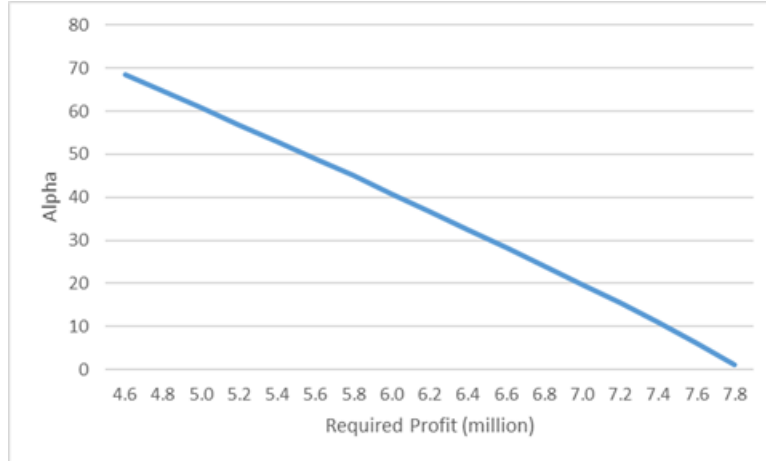


Figure 3.12 Relationship between required profit and maximum α

comparable to RDM and EU. Nevertheless, given the initial state $\hat{\mu}_{D1} = 18.6$ and that 2 units of increase in $\hat{\alpha}$ leads to 2 units of increase in the state set around the initial state, the strategy with \$6 million required profit covers approximately the same state space as RDM and EU. With this required profit, Info-gap recommends that the decision maker should only build one hangar in year 18 (Figure 3.11). Before compared to other decision-making methods, the effect of risk tolerance in utility theory needs to be considered. It seems that the result of Info-gap is more conservative than RDM when decision maker with slightly risk aversion (Figure 3.8 and 3.11). However, RDM for which the decision maker with extreme risk aversion ($p=0.99$) suggests to build first hangar from year 17 to 20 (Figure 3.14). It yields very similar result as Info-gap. Although RDM and Info-gap have different model structures, the experiment from this problem suggests that both methods encourage less aggressive, or more risk-averse, decisions. The advantage of Info-gap method is the additional information regarding to the reward which has real-world applicability for a decision maker. However, because of the previously mentioned flaw, Info-gap may rely too much on the initial state. Comparing the result from Info-gap with that of RDM could be a good way to check the robustness of decision.

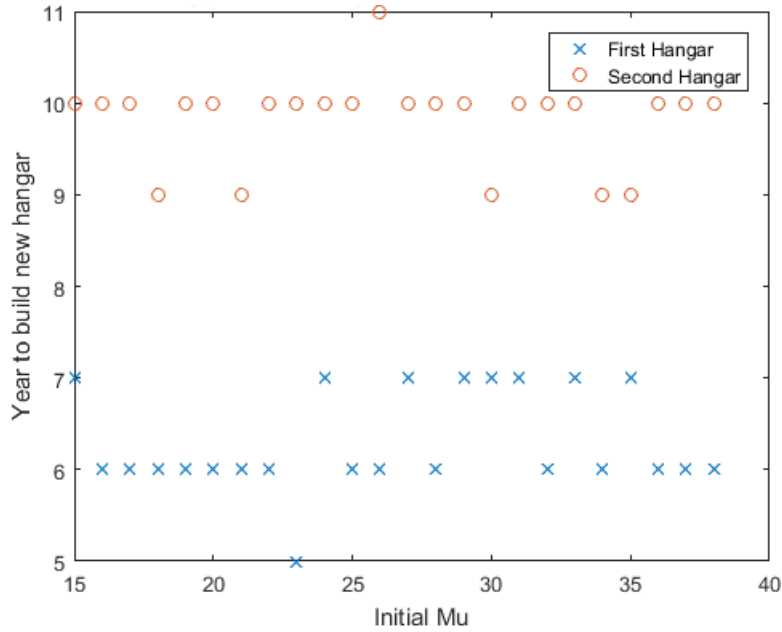


Figure 3.13 Sensitivity analysis of initial $\hat{\mu}_{D1}$

3.5 Conclusion

As perhaps the most popular method for making decisions with uncertainty, EU works well if the uncertainty can be modeled with probability distributions. According to the Bayesian or subjective theory of probability, a decision maker can always assign a probability to an uncertainty where the probability represents the decision maker's beliefs about the future. However, in situations with a severe lack of information, a decision maker might be challenged to select a specific probability distribution or may not feel comfortable assigning a probability. The optimal strategy for EU may vary a lot given different parameters for the distribution. If the probabilities are incorrect, especially if the worst outcomes are a lot more probable than assumed by the decision maker, the optimal strategy according to EU may expose the decision maker to significant risk. In the painting example in this paper, overestimating the trend parameter in the demand distribution will lead the decision maker to build hangars too early which will not be fully utilized. If the demand is underestimated, the painting cost may increase significantly

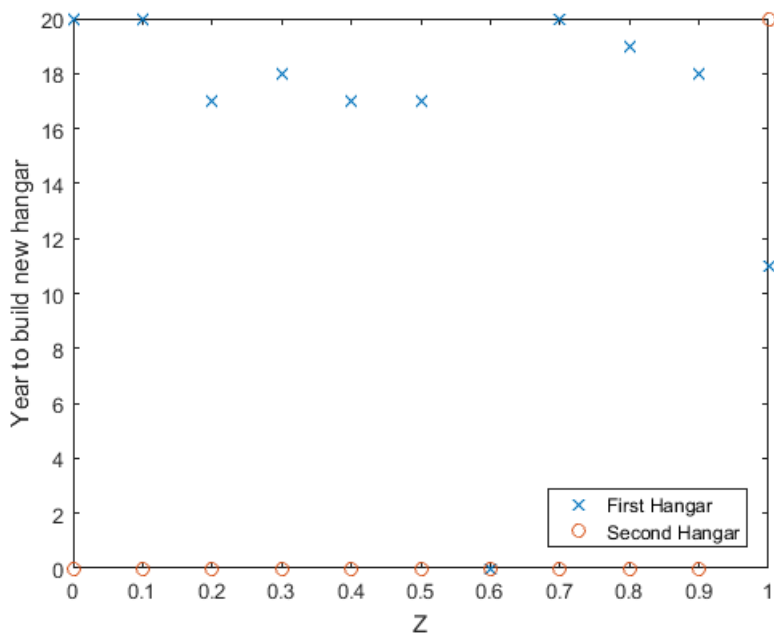


Figure 3.14 Suggested strategies in RDM with extreme risk aversion ($p=0.99$)

due to the high cost of outsourcing. Compared to RDM and Info-gap, EU does not provide any parameters to allow the decision maker to trade off between optimality and uncertainty. A trade-off curve can help a decision maker because the trade-off curve shows a picture of the problem rather than a single "optimal" point.

RDM can appear to be a pretty complex decision-making method. In one sense, RDM generalizes utility theory as it expands a single probability distribution in EU to a set of probability distributions. Uncertainty is incorporated in the state set. RDM searches the space to find the optimal strategy given the parameter z . Theoretically, all possible outcomes could be covered by establishing a large set. Nevertheless, more computational power and advanced optimization algorithms may be needed to search a large state set. One of the important advantages of RDM is the trade-off curve based on the level of confidence to the best-case versus worst-case distribution. In our experiment, however, the interpretation of z could be misleading if the variability of worst case regret is larger than the magnitude of best-case regret. Unfortunately, the distribution of regret over strategies is based on individual problem and is likely to be unknowable before

calculation. The actual interpretation of z may thus vary from case to case, and greater exploration of the interpretation of z may be a potential direction for improvement in RDM. In the painting example, RDM recommends more conservative decisions (build few hangars and later) than EU and the Info-gap model when decision maker is extreme risk averse. In the situation when decision maker is slightly risk averse, Info-gap gives the most conservative decisions.

One appealing feature of the Info-gap model is the minimum requirement which provides a direct way to incorporate profit into the decision-making process. This minimum requirement links the desired outcome and optimal decision directly without constructing a complex and abstract probabilistic model, which has likely increased its popularity with industry (Takewaki, 2005; Matrosoy, 2013; Soroudi, 2017). In our example, Info-gap and EU recommend a similar strategy decision given the same initial state and expected profit requirement. For instance, EU recommends to build 2 hangars in years 6 and 9, and Info-gap recommends building the hangars in years 6 and 10. Info-gap has been criticized because it searches for local optimality. The result of Info-gap might be misleading if there is model uncertainty or uncertainty around several parameters in the problem.

Based on previous discussion, we conclude the situations to apply different decision-making methods. When a probability distribution can be assigned to represent uncertainty with very high confidence level, EU is the best decision-making method. Info-gap is applicable when uncertain future could be represented as several probability distribution with rank and we have relative high confidence on our initial state estimation. Moreover, by choosing Info-gap, the underlying assumption is that the decision maker cares robustness more than optimality. In the situation of deep uncertainty, we have little information about the future, RDM is the only decision-making method applicable among the three.

In general, before choosing particular decision-making method, one important question to ask ahead is whether we want an optimal solution or a good enough feasible

solution. As the famous statement wrote by Box 1987, "All models are wrong, but some are useful." In the end, all of these decision-methods should be evaluated by their usefulness in making practical decisions.

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CHAPTER 4. GENERAL CONCLUSIONS

This research discusses demand forecasting and decision making for the purpose of long-term production planning. Although the analysis and discussions are tailed to aviation industry, the methodology could extend to other industries in industrial and manufacturing field.

In Chapter 2, several demand forecasting methods were studied and applied to Boeing's airliners (737, 777). We clarifies the assumptions of Brownian motion, geometric Brownian motion and ARIMA model. A modified Brownian motion model is purposed to address the correlation in historical orders of the 737. A comparison between purposed model and ARIMA is conducted. We conclude that ARIMA is applicable when high lag correlation is observed in data and accurate expected prediction is desired. Purposed Brownian motion model has more conservative expected prediction and shows wide range of possible future states. These information could be highly valuable when a long-term strategy decision needs to be made based on the forecasting. The application of GBM on forecasting of the 777 is also included. We shows that the traditional GBM fitting method does not work well under the present of high variability from small dataset. We demonstrate a new approach to fit GBM for the entire dataset. The starting position of fitted model could be adjusted according to variability of historical data. The advantage of this method is that it provides a flexible way for a decision maker to interpret the variability of data which could be highly dependent on domain knowledge. Same methods are applied to forecast other types of airliners (747, 767 and 787) which does not show on the paper. The predictions are shown in table 4.1.

Table 4.1 20 years forecasting of orders

Airplane Size	Airplane Model	Prediction (median values)
Single-Aisle	737	20,072
Small Wide-body	787,767	1,856
Medium Wide-body	777	1,823
Large Wide-body	747	354
Sum		24,105

In Chapter 3, a capacity planning which dedicates to investment decision is developed. As both non-linearity and uncertainty present in the model, simulation based method is used to find the solution. Three different decision making methods (Expected Utility, Robust Decision Making, Information Gap) are introduced and applied. We find the optimal strategy suggested by each method and test the sensitivity of model given different inputs. As expectation, expected utility method is highly sensitive to the input probability distribution. RDM and Info-gap which are designed for deep uncertainty, perform fairly stable and consistent outcomes. Detailed analysis and interpretation of result is conducted for each method. We find that the interpretation of z parameter in RDM is not exactly as the claim made by Lempert (2007). The actual effect of z primarily depends on problem which yields different minimum and maximum regret matrix. These three methods yield different suggested strategies under same setting. Compared to expected utility, the strategies made by RDM and Info-gap are more robust. In addition, RDM with extreme risk aversion and Info-gap have similar outcomes for this problem. We conclude the advantages and limitations of these decision making methods and specify the situations when they are applicable.

By combining demand forecasting and decision making together, we purpose an overall framework on long-term capacity planning. The research provides a quick overview of capacity planning problem by choosing proper models. It helps decision maker to understand the risk of decision with existing univariate data without putting enormous

amount of effort on collecting multivariate data and developing complex causal model. Therefore, the study is a good starting point for the initial estimation of risk in long-term capacity planning problem. In future research, more important data such as macroeconomics, fuel price could be collected. With the support of additional information, a careful designed multivariate model is likely to reduce the uncertainty further and yields high quality capacity planning strategy.

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