Production planning in different stages of a manufacturing supply chain under multiple uncertainties

by

Goutham Ramaraj

A thesis for Master submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Industrial Engineering

Program of Study Committee:
Guiping Hu, Major Professor
Lizhi Wang
Stephen Vardeman

Iowa State University

Ames, Iowa

2017

`DEDICATION

In dedication to my family for their unconditional support.

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ACKNOWLEDGMENTS

I would like to take this opportunity express my profound sense of gratitude to those who have helped me throughout my research work and the writing of this thesis.

First and foremost, my major professor, Dr.Guiping Hu, for her guidance and continuous support throughout the completion on this thesis. Her timely advice, meticulous scrutiny, and scientific approach have helped me to a very great extent to accomplish this task. The conversations that we had during our weekly meetings not only helped to get invaluable feedback on my work but also provided me immense motivation for completing my graduate education.

I would also like to thank my committee members, Dr.Lizhi Wang and Dr. Stephen Vardeman for their guidance and support throughout the course of this research. I would like to thank Zhengyang Hu, my research partner, for taking out time from his busy schedule to meet with me and for providing valuable feedback on my work.

In addition, I would also like to thank my family of friends, and the department faculty for making my time at Iowa State University a wonderful experience. Most importantly, I would like to thank my parents, my brother and my fellow-cyclone, Aparna Sahajan, for your unconditional love, support and much needed encouragement. You guys have helped me learn that nothing is beyond the reach of our dreams.

ABSTRACT

This thesis focuses on designing stochastic programming models for production planning at different stages in a manufacturing supply chain under multiple sources of uncertainties. Various decision makers along the manufacturing supply chain often have to make planning decisions with embedded risks and uncertainties. In an effort to reduce risks and to ensure that the customer demand is met in the most efficient and cost effective way, the production plans at each stage need to be strategically planned. To assist production planning decisions, a two-stage stochastic programming model is developed with the objective of minimizing the total cost including production, inventory, and backorder costs. The proposed framework is validated with case studies in an automobile part manufacturer with real data based on literature. The results demonstrate the robustness of the stochastic model compared with various deterministic models. Sensitivity analysis is performed for the production capacity parameter to derive managerial insights regarding lot-sizing and scheduling decisions under different scenarios.

CHAPTER 1. GENERAL INTRODUCTION

Production planning is important in a manufacturing facility to ensure efficient, and effective utilization of resources. It typically involves sequencing and scheduling the production batches, determining the optimal batch quantities and prioritizing the batches. In reality, the planning of the production processes is often conducted under a variety of uncertainties, such as demand, machine availability, worker efficiency, etc. Therefore, it is important to take the uncertainties into consideration when making the production decisions.

There has been significant body of literature on production planning in manufacturing systems under uncertainty. Ho categorizes the uncertainties observed in manufacturing system into environmental and system uncertainties [1]. While environmental uncertainties include demand and supply uncertainties, production system uncertainties are related to the production processes itself, such as machine availability, operational yield, and production quality uncertainties. Different strategies for modelling the production planning processes under uncertainty have been developed and applications in a variety of industries have been discussed in the literature [2, 3, 4].

The modelling framework in the literature can be categorized into four classes of conceptual, analytical, artificial intelligence, and simulation models which was originally proposed by Giannoccaro and Pontrandolfo [5]. The analytical models used for production planning are based on different operations research techniques, mainly linear programming, stochastic programming, mixed integer programming, Markov decision process, and multi-objective programming. The areas of application, as identified by Mula et al., include capacity planning, manufacturing resource planning, inventory management and supply chain planning [6].

Production planning involves a number of stakeholders spanning across the manufacturing supply chain such as raw material suppliers, manufacturers, distributors, and customers. Figure 1 shows the schematic of the supply chain of a typical automobile manufacturer. All the stages are inter-connected that the uncertainties in a stage can influence the operations planning and decision-making in another stage. For example, an uncertainty in the quality or the lead time of the parts to be assembled could impact the production sequence due to parts availability. This disrupts the production planning at the assembly line feeding system as their operations are planned according to the initial production plan.

In the existing literature, production planning studies have been focused on the modelling approaches and the application areas, the stages along a manufacturing supply chain have not been studied carefully. In this thesis, we address the problem of production planning in different stages in the manufacturing system under a variety of uncertainties. We also highlight how the uncertainties propagate across different stages and highlight how they influence the business decisions in these stages.

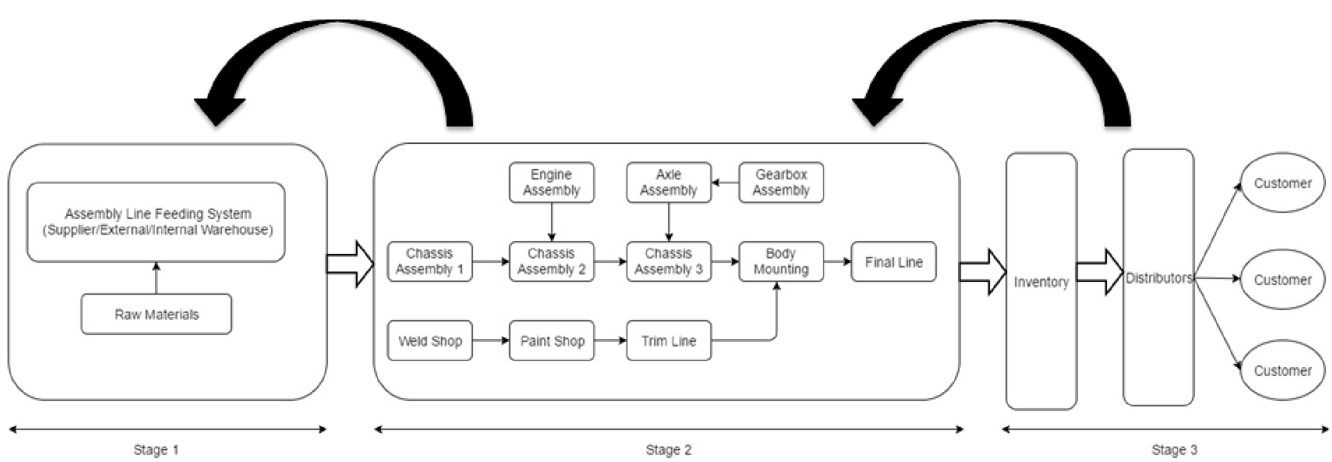

Figure 1: Supply chain of a typical automobile manufacturer

To gain a better understanding of production planning under uncertainty and to provide a foundation for the research, we briefly discuss some existing literature. Kitting system, which is a part of stage 1 of the supply chain, is one of the most widely used assembly line feeding system. Som et al. analyzed the kitting process of a multi-product, multi-level assembly system under uncertainty. They derived the processes that describe the input and output streams of the assembly system. The distribution of time between the kit completions was also derived [7]. Leung and Wu developed a robust optimization model to solve the aggregate production planning problem under uncertainty. They also presented the analysis of the tradeoff between solution and model robustness [8]. This study would apply to stage 2 in the manufacturing supply chain discussed above. Gupta and Maranas proposed an approach based on stochastic programming for managing demand uncertainty in supply chain planning. In their proposed framework, the decisions related to manufacturing (Stage 2) were the here and now decisions and the logistic decisions (Stage 3) were modeled as the wait and see decisions. The key features of their model were highlighted through a case study [9].

Many researchers have worked on both the deterministic and the stochastic versions of the decision-making models for production planning. Two-stage stochastic programming modeling approach has not been utilized extensively, which is a major motivation for this study. Modelling the production planning problems using a two-stage stochastic programming framework is one of the major contributions of our work. We are among the pioneers to adopt a two-stage stochastic programming framework to solve lot-sizing and scheduling problems at different stages along a manufacturing system. Through the proposed mathematical framework, we also highlight how the uncertainties affect the decision-making process in different stages of a supply chain.

In this thesis, we aim to fill in the gaps in studies related to production planning under uncertainty in a manufacturing environment. Through our study, we propose a two-stage stochastic programming framework for planning the production process at different stages in the supply chain. The objective is to minimize the total costs such that the downstream customer demands are met in the most efficient and cost effective way.

The remainder of the thesis is structures as follows: Chapter 2 provides a model for production lot-sizing and scheduling under demand and raw material quality uncertainties. A comparison of the implementation results from the deterministic and the stochastic models is also presented. In Chapter 3, we present a modelling approach to plan the production processes in an assembly line feeding kitting facility under demand and yield uncertainties. The influence of the uncertainties on the business decisions is also highlighted and discussed. Conclusions of the thesis and some future research directions are provided in Chapter 4.

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CHAPTER 2. A TWO-STAGE STOCHASTIC PROGRAMMING MODEL FOR PRODUCTION LOT-SIZING AND SCHEDULING UNDER DEMAND AND RAW MATERIAL QUALITY UNCERTAINTIES

Modified from a paper to be submitted to *Production Planning and Control*

Goutham Ramaraj, Zhengyang Hu and Guiping Hu

Abstract

Production planning and scheduling focus on efficient use of resources and are widely used in the manufacturing industry, especially when the system operates in an uncertain environment. The goal of this paper is to provide a two-stage stochastic programming framework for a multi- period, multi-product, lot-sizing and scheduling problem considering uncertainties in both demand and the quality of raw materials. The objectives are to determine the number of units to be produced and the production sequence so that the total production costs are minimized. The decisions made in the first stage include the basic production plan along with the production quantities and sequences, which are later updated with recourse decisions on overtime production made in the second-stage. To demonstrate the proposed decision-making framework, a case study for a manufacturing facility producing braking equipment for the automotive industry was conducted. The results show that the stochastic model is more effective in production planning under the uncertainties considered. The managerial insights derived from this study will facilitate the decision-making for determining optimal production quantities and sequences under uncertainties.

Keywords: Production planning, lot-sizing and sequencing, stochastic programming, demand uncertainty, quality uncertainty

2.1 Introduction

Production planning plays an important role in improving overall manufacturing system performance, especially when a system operates in an uncertain environment. Uncertainty in product demand, processing time, and quality of raw materials are among the common types of uncertainties that characterize production environments. To develop a robust production plan, it is important that the uncertain parameters are considered in the production planning process, because neglecting them will affect production efficiency and system performance [1]. This research focuses on a multi-period, multi-product, lot-sizing and scheduling problem under uncertainty.

The planning horizons for the decision-making process in production planning models are typically classified into three categories: long-term, medium-term, and short-term planning. Long-term planning primarily focuses on strategic long-term decisions such as equipment, product, and process choices whereas medium-term and short-term planning involve making decisions on material flow, and production lot-sizing and sequencing for optimizing overall performance. Typically, the time range for the short-term decisions is within a day [2]. In this paper, our focus is on the production lot-sizing and scheduling problem, which can be classified as short-term to medium-term production planning.

Many studies have been conducted by researchers over the years to solve the production lot-sizing and scheduling problem using a variety of techniques. The classical economic order quantity (EOQ) model marked the start of research on lot-sizing problems [3,4]. To bridge the gaps in the EOQ model, like stationary demand and no capacity constraints, other models like the economic lot scheduling problem (ELSP) and the Wagner-Whitin problem (WW) evolved [5]. While the ELSP considers an infinite planning horizon with capacity restrictions, WW assumes a finite planning horizon with dynamic demand. The latest models that have combined both the capacitated and the dynamic lot-sizing approaches include the discrete lot-sizing and scheduling problem (DSLP), the continuous lot-sizing and scheduling problem (CSLP), the proportional lot-sizing and scheduling problem (PLSP), and the general lot-sizing and scheduling problem (GSLP). DSLP works with the assumption that not more than one product can be produced per period which is not present in CLSP. PLSP overcomes the limitation of CLSP by using the remaining capacity for scheduling a second product within the same period. GSLP deals with the lot-sizing and scheduling of several products on a single capacitated machine [2]. Some of the characteristics, as identified by Karimi et al., that influences the modelling and the complexity of lot-sizing and scheduling models include number of levels in a production system, number of products manufactured, demand and capacity constraints [2]. To classify the different modeling approaches used for lot-sizing and scheduling problems, Guimaraes et al. presented a new framework as shown in Figure 2 [6]. The discrete time models for lot-sizing and scheduling are classified based on two main dimensions: technique and time structure. The different classes within these dimensions are defined by both the technique and time structure used. The two main approaches based on the first dimension are product oriented (PO) and sequence oriented (SO) formulations. Based on the dimension of time structure, the framework classifies the models into micro-period (mP) and macro-period models (MP). Multiple setups are allowed in MP models, while mP models allow only a single setup per micro-period. The framework further classifies the models into single lot (SL) and multiple lot (ML), based on the number of production lots of each product allowed to start within a time period. In our paper, the sequence of production lots in a machine is modeled as per the product oriented, single lot, macro-period model. To depict an example, consider the production sequence {1-4-3-2} as shown in Figure 3. If a macro-period model is used, the following setups will be selected to establish the production sequence: (1-4) (4-3) (3-2). Setup (1) was carried over from the previous period, and setup (2) will be carried over to the next period. Whereas for a micro-period model each setup state of (1) (2) (3) (4) would be captured separately to establish the changes in the sequences.

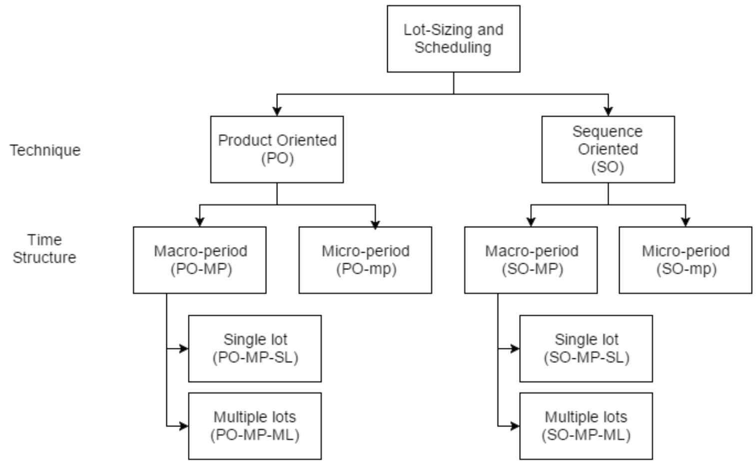

Figure 2: Classification framework for lot-sizing and scheduling problems [6]

There is significant body of literature on the topic production lot-sizing and scheduling. The solution techniques and their application on real-world problems have been discussed in various industries such as textiles, beverages, tobacco, paper, and pharmaceuticals. Araujo et al. developed a mixed-integer programming model for solving the lot-sizing and scheduling problem in a manufacturing setting considering the sequence-dependent costs and times [7]. Almada et al. proposed a mixed integer programming formulation to solve a short-term production planning and scheduling problem in a glass container industry. They used a Lagrangian decomposition based heuristic for generating good feasible solutions [8]. Silva and Magalhaes studied a discrete lot-sizing and scheduling problem found in the textile industry. They presented a heuristic approach for minimizing tool changeovers and the quantity of the product delivered after a due date [9]. In Gnoni et al. a hybrid modeling approach was used to solve a production planning problem of a manufacturing plant producing braking equipment. The developed hybrid model was comprised of a mixed-integer linear programming model and a simulation model [10]. Marinelli et al. proposed a robust optimization model for a capacitated lot-sizing and scheduling problem in a packaging company producing yoghurt. They developed an effective two-stage optimization heuristic for scheduling orders on a production line based on real data from a company [11]. In Diaby et al., a robust mixed-integer linear programming model was formulated and solved using Lagrangean relaxation to address the capacitated lot-sizing and scheduling problem [12]. Chen et al. proposed a robust optimization model for scheduling independent jobs on parallel machines to minimize makespan. Significant improvement from the existing algorithms was reflected in the computational tests performed [13].

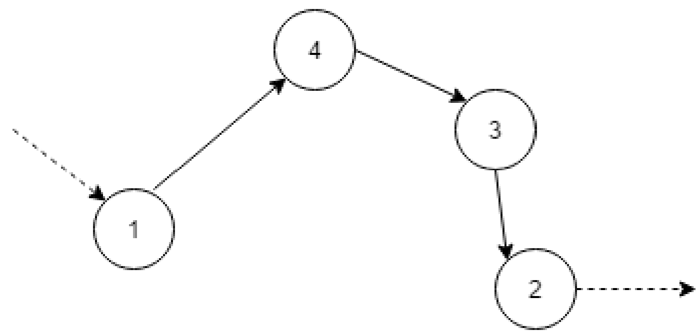

Figure 3: Example of production sequence path [6]

Application of deterministic and stochastic versions of mathematical models have been popular among the researchers studying production lot-sizing and scheduling problem. But very few have discussed the application of stochastic programming to these problems under uncertainty. Escudero et al. solved a multi-period, multi-product production planning problem with random demand using a multi-stage stochastic model [14]. Leung and Wu developed a robust optimization model to determine optimal production loading plan and workforce level for an aggregate production planning problem in an uncertain environment. The costs involved with production, labor, inventory, hiring and layoff were considered for the study [15]. In the study conducted by Bakir and Byrne a stochastic linear programming model based on a two-stage deterministic equivalent problem was used for addressing a multi-period, multi-product production planning problem with stochastic demand [16]. Hu and Hu studied the application of a two-stage stochastic programming framework for solving a lot-sizing and scheduling problem in an automotive part manufacturing plant [17]. Unlike the study conducted by Hu and Hu, our study models the dynamics of multiple uncertainties in a manufacturing environment, which is more practical. The two uncertainties are modelled separately and integrated into the two-stage stochastic programming model to aid better decision-making. We also discuss how the recourse actions gets influenced by the presence of multiple uncertain parameters in the model. Brandimarte developed a multi-stage mixed-integer stochastic programming model for solving a multi-item capacitated lot-sizing problem with uncertain demand. They also tested the advantages of a stochastic model over a deterministic one by conducting computational experiments [18]. Khor et al. studied the capacity expansion problem in petroleum refinery under uncertainty using a two-stage stochastic programming model and robust optimization models [19].

In the literature on production planning under uncertainty, typically only one uncertain factor is considered. In practice, there can be more than one uncertain factor, each with their own dynamics and behavior over time that can have an impact on the recourse decisions made in a manufacturing environment. Kazemi et al. proposed a multi-stage stochastic programming approach for a production planning problem with uncertainties in both demand and the quality of the raw materials. A hybrid scenario tree was developed by integrating the demand and the yield scenarios to formulate a stochastic programming model with full recourse for demand and simple recourse for yield [20]. Unlike our problem, the study conducted by Kazemi et al. does not include determining the overtime production quantities and the optimal production sequence. Although Mukhopadhyay and Ma developed a two-stage stochastic model to understand how a firm's procurement and production decisions are influenced by demand and quality uncertainties, their study did not involve any decision-making regarding production lot-sizing and scheduling [21]. The novelty of this study lies in integrating two different uncertainties (demand and quality of raw materials) to be used in a two-stage stochastic programming model to solve a production lot-sizing and scheduling problem in a manufacturing setting. The application of two-stage stochastic programming framework to tackle the lot-sizing and scheduling problem under uncertainty can be regarded as another major contribution of this paper as not many studies have discussed this application.

The remainder of the paper is organized as follows. In the next section, the problem statement for the lot-sizing and scheduling is presented. In section 3 we describe a deterministic linear programming model and the two-stage stochastic programming model for production lot-sizing and scheduling with random demand and raw material quality. In section 4, the application of the developed framework to a case study for an automotive part manufacturer is presented along with the comparison of computational results from both models. Our concluding remarks and further research directions are discussed in section 5.

2.2 Problem Statement

Production planning and scheduling are important tools that provide decision support for production activities in the manufacturing industry. The presence of uncertainties, which are common in most production environments, make the decision-making processes complex and challenging.

A typical manufacturing system schematic is shown in Figure 4. Raw materials are collected and accumulated at internal/external warehouses of the manufacturing plant. They are then shipped to the assembly line to be processed and assembled to produce final products for customers. These finished products are then transported to downstream sites or directly to customers.

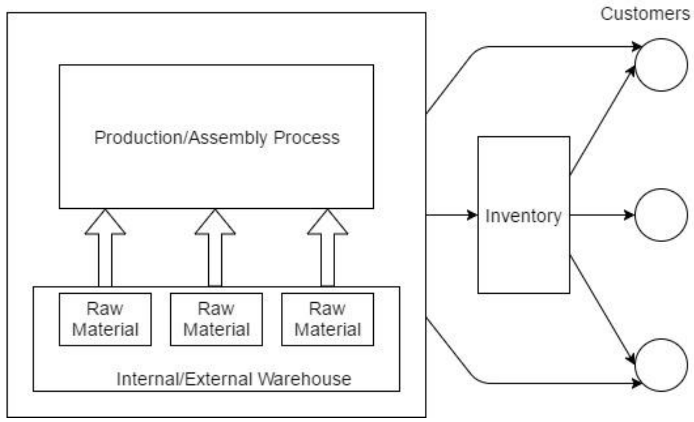

Figure 4: Manufacturing system under study

As Ho [22] discussed, the uncertainty that affects production processes in the real word can be both system and environmental. Environmental uncertainty is related to uncertainties beyond the production process, such as demand and supply uncertainty. System uncertainty includes uncertainties within the production process, such as lead time, quality, and yield uncertainties. The planning of the production process in a manufacturing plant is highly dependent on the demand for manufactured products from customers. The availability of quality raw materials from suppliers is also of vital importance to strike a balance between the inbound materials and outbound products. Neglecting these uncertainties in production planning will result in unsatisfactory and inefficient production plans.

The goal of this paper is to provide a two-stage stochastic programming framework for the production lot-sizing and scheduling problem considering uncertainties in both demand and the quality of raw materials. The results from this paper will facilitate the decision-making for determining optimal production quantities and sequences under uncertainties.

2.3 Model Formulation

In this section, the deterministic and the stochastic versions of the model for the production planning problem studied are introduced. The objective is to determine the number of units to be produced and their production sequence so that total costs are minimized. A deterministic mathematical formulation is first described and then extended to a stochastic setting, assuming product demand and quality of raw material to be uncertain with known probability distributions. The stochastic programming model has a two-stage structure to accommodate the decision-making process for planning production activities. The decision-maker makes certain decisions in the first stage, after which a random event occurs that affects the outcome of the first stage decisions. A recourse decision can then be made in the second stage, after the actual values of the first stage decision variables are realized. The second-stage decisions can compensate for the non-optimal effects of the first stage decisions [23]. In this paper, we extend the two-stage stochastic programming model originally developed by Hu and Hu to address the uncertainty from raw material quality along with the uncertainty in demand among the customers [17].

2.3.1 Mathematical Notations

The mathematical notations for the model formulation are included in

Table *1*. A production facility with a set of products I and a planning horizon consisting of T periods is considered. It should be noted that both I and J are used to indicate the same sets of final products as they are used to model the change-overs between the products.

2.3.2 Deterministic Model

The deterministic model assumes all parameters in production lot-sizing and the scheduling problem are known with complete certainty.

Before the model formulations are discussed, we introduce the assumptions made to clearly define the problem we investigate.

- Backorders are allowed, so the demand does not need to be fulfilled all the time.
- Initial inventory is assumed to be zero and the inventory level is calculated at the end of each planning period.
- The demand of a product in a specific time period is independent of its demand in the previous period.
- The quality of the raw material in a particular time period is independent of its quality in the previous period.
- Both the uncertainties are product independent; that is, the uncertainty of one product does not have any effect on the uncertainty of another product.
- There is a limit on the resources available for regular time and over time production and the same setup can be used for both.
- Setups are carried over to adjacent periods; that is, the last setup in the previous period will become the first setup in the following period.
- The uncertainties in demand and the quality of raw materials are independent of each other.
- The maximum number of setup changes allowed per product per time period is one.

Objective Function

The objective function minimizes the total cost involved in the production process, which is defined as the sum of the production, inventory, backorder, and raw material costs. The cost from the regular time production is $\sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^r * X_{i,t}$. The term $\sum_{i=1, i \neq j}^{I}\sum_{j=1}^{j}\sum_{t=1}^{T-1} SC_{i,j} * Y_{i,j,t}$ is the setup cost for changeover from product *i* to product *j*. No setup cost is incurred between identical products. The terms $\sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^o * O_{i,t}$, $\sum_{i=1}^{I}\sum_{t=1}^{T-1} h_i * I_{i,t}$, *and* $\sum_{i=1}^{I}\sum_{t=1}^{T-1} b_i * B_{i,t}$

are the overtime production cost, the inventory cost, and the backorder cost, respectively. Lastly, $\sum_{i=1}^{I}\sum_{t=1}^{T-1} M * m_i * (X_{i,t} + O_{i,t})$ is the cost of the raw materials consumed in the production process.

Table 1: Notations for the deterministic model

Subscripts		
i	1, 2….N	Products
j	1, 2….N	Products
t	1, 2….T	Time periods
Parameters		
$d_{i,t}$	Demand for product i in period t	
h_i	Inventory holding cost per unit of product i for one period	
b_i	Backorder cost per unit of product i for one period	
cap_t	Time capacity of the machine in period t	
p_i	Processing time of one unit of product i	
p_i^r	Regular time processing cost per unit of product i	
p_i^o	Overtime processing cost per unit of product i	
$q_{i,t}$	The maximum regular time production quantity of product i in period t	
$SC_{i,j}$	Setup cost for changeover from product i to product j	
$ST_{i,j}$	Setup time for changeover from product i to product j	
α	Maximum overtime ratio	
N	Number of products	
M	Cost of raw material per unit	
m_i	The units of raw material consumed per product i	
Decision Variables		
$I_{i,t}$	Inventory level of product i by the end of period t	
$B_{i,t}$	Backorder level of product i by the end of period t	
$X_{i,t}$	Regular time production quantity of product i in period t	
$O_{i,t}$	Over time production quantity of product i in period t	
$Y_{i,j,t}$	1 if a changeover from product i to product j is performed in period t. Binary Variable	
$Z_{i,t}$	1 if a setup of product i is carried over from period $t-1$ to period t. Binary Variable	
$V_{i,t}$	Production order of product i in period t. Integer variables start from 1.	

In sum, the objective function can be formulated as follows:

Minimize Z =

$$\sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^r * X_{i,t} + \sum_{i=1,i\neq j}^{I}\sum_{j=1}^{j}\sum_{t=1}^{T-1} SC_{i,j} * Y_{i,j,t} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^o * O_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} h_i * I_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} b_i * B_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} M * m_i * (X_{i,t} + O_{i,t})$$

All the regular time production resources are used before starting overtime production, since the unit production cost is lower for regular time production than for overtime production. Also, backorders are allowed only when the inventory is reduced to zero. Another thing to notice in this model is that only the first *T-1* periods are considered as there are no production resources in the last period. Other than inheriting a setup from a previous period, no production happens in the last period. This is done to hold a positive demand for the flow balance constraint of the model.

Constraints

Inventory balance constraints (1) and (2) satisfy demand either from inventory, backorder, or production within the current period. Neither the inventory $I_{i,t}$ nor the backorder $B_{i,t}$ can be positive at the same time even though they can both be zero at the same time. If one is positive, then the other must be zero. Inequality (3) is a production quantity constraint that limits the maximum quantity that can be produced during regular time in a particular time period. In this constraint, the setups are represented by either $Z_{i,t}$ = 1, which denotes that a setup is carried over from previous period *t-1*, or by $\sum_{j\neq i}^{J} Y_{j,i,t}$ = 1, which denotes that the setup is taken over in period *t*. The product will not be produced in that time period if neither of these occurs. According to our assumptions, $Z_{i,t}$ and $\sum_{j\neq i}^{J} Y_{j,i,t}$ will not be equal to one simultaneously, as a setup cannot be carried over from the previous period if it is already taken over in a period.

$$X_{i,t} + O_{i,t} = d_{i,t} + I_{i,t} - B_{i,t} \qquad \forall i, t = 1 \tag{1}$$

$$I_{i,t-1} - B_{i,t-1} + X_{i,t} + O_{i,t} = d_{i,t} + I_{i,t} - B_{i,t} \qquad \forall i, t = 2 \ldots T \tag{2}$$

$$X_{i,t} \leq q_{i,t} * \left(Z_{i,t} + \sum_{j \neq i}^{J} Y_{j,i,t} \right) \qquad \forall i, t \tag{3}$$

Constraint (4) ensures that the sum of total production time and the required setup time does not exceed the available capacity. The limitation on the overtime production quantity is set by constraint (5). Overtime production is regulated by both government and company policies to be not more than α percent of regular production time. Therefore, the overtime production quantity is restricted to be less than or equal to a fraction of the regular time production quantity.

$$\sum_{i=1}^{I} p_i * X_{i,t} + \sum_{i=1}^{I} \sum_{i \neq j\, j=1}^{J} ST_{i,j} * Y_{i,j,t} \leq cap_t \qquad \forall t \tag{4}$$

$$O_{i,t} \leq \alpha * X_{i,t} \qquad \forall i, t \tag{5}$$

Constraint (6) enforce an initial setup to be taken over at the beginning of each period. Constraint (7) ensures a balanced flow of setups and is applied through the first *T-1* periods, as the last period is a dummy period with zero demand. The left-hand side is the sum of setups directed towards product *i*, and the right-hand side is the sum of the setups directed away from product *i*. If there are no setup changes in period t, the machine product setup is carried over to period *T + 1*. For a product *i*, $Z_{i,t} = 1$ when the setup of the product is carried over from period *t – 1* and $\sum_{j \neq i}^{J} Y_{j,i,t} = 0$ if only one setup of the product is used in that period. If product *i* is the only product that we want to produce in that period, then $Z_{i,t+1} = 1$, as this setup will be carried over to the next period. Otherwise, a setup change must be performed, making $\sum_{j\neq i}^{J} Y_{i,j,t} = 1$. Through this constraint, the setup of the machine is traced and also ensures that both the left- and right-hand sides do not exceed 1.

$$\sum_{i=1}^{I} Z_{i,t} = 1 \qquad \forall t \qquad (6)$$

$$Z_{i,t} + \sum_{j\neq i}^{J} Y_{j,i,t} = Z_{i,t+1} + \sum_{j\neq i}^{J} Y_{i,j,t} \qquad \forall i, t = T-1 \qquad (7)$$

Constraint (8) ensures that there is no production in the last period, which is a dummy period that is added to the planning horizon. Unless there is an extremely high demand that can cause backorders to be carried out to the last period, the inventory in the last period will always be zero as long as the demand can be met in the normal production planning horizon. In practice, there might be situations of subtours in production sequences that start and return to the same setup state without connecting all the nodes. Subtours that form a perfect loop and that can happen in single lot model are classified as simple disconnected subtours. Other types of subtours like the α subtours and complex disconnected subtours require more than one identical setup per period [24].

Out of the many approaches have been proposed to prevent subtours, we use the one developed by Hasse, where a decision variable is used to capture the order of processing the production lots in each time period [25]. In our model constraint (9) is used for subtour elimination.

$$X_{i,t} = 0 \qquad \forall i, t = T \qquad (8)$$

$$V_{j,t} \geq V_{i,t} + 1 - N * (1 - Y_{j,i,t}) \qquad \forall i, j \neq i, t \qquad (9)$$

2.3.4 Two-stage stochastic programming model

Demand of the products and raw material quality are among the influential uncertain parameters in a manufacturing context. To assist the decision-making in a stochastic setting, these uncertainties must be incorporated into the modeling framework.

In this study, product demand and quality of the raw materials are selected as the uncertain parameters to be investigated. Determining appropriate representation of the uncertain parameters is among the most important steps in incorporating uncertainties into production planning problems. Two distinct methods for representing uncertainties can be identified, the *scenario-based* approach and the *distribution-based* approach [26]. In this study, the uncertainties are represented by a set of discrete scenarios capturing how the uncertainty might play out in the future. Each scenario is a discrete value of demand, or the number of defective raw material parts per million (ppm) that is associated with a probability. Several such scenarios are generated to represent the known continuous distribution of the uncertain parameters.

The two-stage stochastic programming model aims to determine the optimal production plan to meet uncertain demand and uncertain raw material quality. We use a subscript *s* to represent scenarios of uncertain demand and uncertain quality of raw material that are associated with a probability Pr_s. Since we assume independence among the two uncertainties, the probability of the scenarios Pr_s is obtained by multiplying the independent probabilities of uncertain demand and uncertain quality scenarios. The two-stage stochastic programming problem is formulated as follows:

Minimize Z =

$$\sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^r * X_{i,t} + \sum_{i=1,i\neq j}^{I}\sum_{j=1}^{j}\sum_{t=1}^{T-1} SC_{i,j} * Y_{i,j,t}$$

$$+ \sum_{s=1}^{S} \Pr_s \left\{ \sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^o * O_{i,t,s} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} h_i * I_{i,t,s} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} b_i * B_{i,t,s} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} M * m_{i,s} * (X_{i,t} + O_{i,t,s}) \right\}$$

Subject to

Constraints $(3), (4), (6), (7), (8),$ and (9)

$$X_{i,t} + O_{i,t,s} = d_{i,t,s} + I_{i,t,s} - B_{i,t,s} \qquad \forall i, t = 1 \qquad (10)$$

$$I_{i,t-1,s} - B_{i,t-1,s} + X_{i,t} + O_{i,t,s} = d_{i,t,s} + I_{i,t,s} - B_{i,t,s} \qquad \forall i, s, t = 2 \ldots T \qquad (11)$$

$$O_{i,t,s} \leq \alpha * X_{i,t} \qquad \forall i, s, t \qquad (12)$$

The actions that must be made before the uncertainties are realized are the first stage decision variables. The recourse decisions that are made in the second stage after the uncertainty is realized are called the second stage decision variables. In this production planning model, the first stage decision variables include $X_{i,t}, Y_{i,j,t}, Z_{i,t}, and\ V_{i,t}$, which define the baseline production plan and determine regular time production quantity and the sequence of production. The second stage decision variables ($I_{i,t}, B_{i,t}, and\ O_{i,t}$) determine the inventory level, backorder level, and overtime production quantity, respectively. Constraints $(3), (4), (6), (7), (8),$ and (9) are the first stage constraints; they remain the same as in the deterministic model, and they are the same in all scenarios. The constraints $(1), (2),$

and (5) that involve the second stage variables change based on the stochastic scenarios. Therefore, they are replaced with constraints $(10), (11),$ and (12).

In this study, we use a moment matching method to generate the scenarios to be used in the model. It is one of the most commonly used methods for scenario generation. The main aim is to generate a set of outcomes and their associated probabilities so that the statistical properties of the approximating distribution match the specified statistical properties. To achieve this, the differences between the statistical properties of the constructed distribution and the known specifications are minimized, subject to nonnegative probabilities that sum to one [27]. The number of scenarios generated determines the computational effort in solving scenario-based optimization problems. The huge number of scenarios generated as a result of time-dependent uncertain parameters limits tractability [28]. Thus, in this paper we use a scenario reduction technique for the demand scenarios to approximate the original scenarios with a smaller subset that can approximate the original scenario set well. The scenario generation and reduction methods will be discussed in detail in section 4.

2.4 Case Study

To highlight the proposed framework for managing uncertainties in demand and quality of raw materials in production planning, the model is applied to a manufacturing system producing braking equipment for the automotive industry. Uncertainties in production planning tend to propagate to the upstream and the downstream entities in a supply chain and usually increases the variance of costs to the company, increasing the likelihood of decreased profit [29].

In this study, we consider a facility that manufactures three types of hydraulic braking actuators (P1, P2, and P3) as required by the customers of the original equipment marketplace. The facility also carries out assembly of the raw materials P1B and P2B, received from an upstream supplier, required for manufacturing the final products. For most of the production planning models the planning horizon is divided into a small number of long time periods that in most cases represents one week or one month [6].For this study, we consider a multi-product and a multi-period problem with a six-month planning horizon that is partitioned into time slots representing one month each. The main goal is to minimize the total costs involved in production by determining an optimal production plan. All the input parameters, except demand and the quality of raw materials, are assumed to be known with certainty.

2.4.1 Data Sources

As discussed earlier, the manufacturing facility assembles three types of hydraulic braking actuators (P1, P2, and P3) whose demands are uncertain and can vary according to probability density functions as shown in Table 2. The demand for all three products are Weibull distributed with their respective shape and scale parameters. The historical monthly demand data for a three-year period are fitted to obtain the distributions [10].

Table 2: PDF of monthly demand

Monthly Demand			
	P1	P2	P3
PDF	Weibull	Weibull	Weibull
Scale	518	38	169
Shape	1.51	2.76	2.27
Mean	467.25	33.82	149.7
Variance	99422	175.4231	4877.8
Skewness	1.06	0.25	0.47
Kurtosis	4.35	2.78	2.98

Probability distribution functions of the quality of the raw materials are generated by analyzing the survey data of 91 suppliers of Japanese and US automakers [30]. The ppm values of the defective raw materials for all the three products are assumed to be normally distributed with the moments as shown in Table 3. We assume that the demands and the quality of raw materials of the products are independent of each other [28, 29].

Table 3: PDF of raw material quality

	Quality Defects (ppm)		
	P1	P2	P3
PDF	Normal	Normal	Normal
Mean	254	254	254
Variance	6581	6581	6581
Kurtosis	3.00	3.00	3.00

The changeovers from one setup to another involve a cost and require significant setup times, which makes the lot-sizing and the production scheduling problem more complex. Thus, it is important to evaluate different combinations of production sequences to optimize the total cost of production. Setup times and operation times of the three products are listed in Table 4 and

Table *5*, respectively [10]. Setup costs are proportional to the setup times by a specified factor, which is set to be 0.2805 [33].

Table 4: Setup times

	Setup Times (min/setup)		
	P1	P2	P3
P1	0	270	90
P2	180	0	270
P3	90	180	0

Table 5: Operation Times

Operation Times (min/unit)			
	P1	P2	P3
Operation Time	6	6.6	7.2

The inventory costs and the time capacities available (including failure and repair time) are listed in Table 6 and

Table *7*. The regular time production costs are in listed Table 8. Overtime production costs and backorder costs are established as proportional to the regular time production costs with the factors set at 1.5 and 2, respectively [31 ,32].

Table 6: Inventory costs

Inventory Costs($/unit)			
	P1	P2	P3
Inventory Cost	0.16	0.15	0.38

Table 7: Time capacities

Time Capacities (min)	
Month	**Capacity**
1	6087
2	5367
3	6087
4	6087
5	4407
6	4407

Table 8: Regular production costs

Regular Time Production Costs ($/unit)			
	P1	P2	P3
Production Cost	254.08	254.08	254.08

The number of units of raw materials that are consumed per product are given in Table 9. The unit cost of raw materials is considered to be $8 [36].

Table 9: Raw material requirement per product

Number of units of raw materials required per product	
	Units
P1	4
P2	4
P3	5

To handle the demand in peak periods, employees might need to work for a longer time, exceeding the overtime limit set by government and company policies. This would result in fatigue and reduced employee efficiency and is therefore not considered in the production planning model. Therefore, for this study, the overtime production limit is set at 20% of the regular time production [34].

Along with inventory holding cost, setup cost, and production costs, it is important to determine the optimal production batch quantity for the proper evaluation of a production planning model [37]. Many researchers have carried out sensitivity analysis of the input parameters used in their mathematical model, to study the effects they have on the decisions made in the production planning process. Park investigated the effectiveness of integrating the production and distribution planning in a manufacturing setting. He also conducted sensitivity analysis on input parameters, including production capacities, to check the effectiveness of the integration [38]. In this study, we examine the solutions obtained by varying the parameter $q_{i,t}$, which is the maximum regular time production quantity for a product in a particular time period. We consider four different production quantities of 90%, 100%, 110% and, 120% of the mean values of demand. The insights obtained will help in evaluating the tradeoffs and in making managerial decisions for the production planning process.

2.4.2 Scenario generation and reduction

Determining how to represent the uncertainties involved in the multi-stage stochastic programming problem is one of the major challenges. The presence of random variables with multi-dimensional and continuous distributions in the model makes the problem computationally difficult to solve. In such cases, the method of scenario generation is applied to replace the distribution with a set of discrete outcomes and associated probabilities [27]. However, time dependent uncertainties in the model result in a huge number of scenarios that make it intractable to solve the mathematical program. Thus, it is important and necessary to reduce the original scenario set further to a smaller subset that still represents reasonably good approximations [4,13].

Scenario Generation

For this study, the moment matching method is used to generate the scenarios. To present the model used, we introduce the following notation. The statistical properties of the random variable are first defined and described. They are denoted by *m* and are present in a set *S* that consists of all specified statistical properties. The specified value of the statistical property *m* in *S* is defined by VAL_m. In this model, w_m is the weight and $f_m(x, p_r)$ is the mathematical expression of the statistical property *m* in *S*. We consider all the statistical properties to be equally weighted. M represents a matrix of zeros and ones, whose number of rows equals the length of Pr and whose number of columns equals the number of nodes in the scenario tree [27]. To maintain the essential properties, each property *m* creates multiple realizations of uncertainty x_m with respective probabilities Pr_m. For instance, if we want to capture the variance of the distribution, then VAL_m will be the value of the variance that is given as an input parameter to the model, and $f_m(x, Pr)$ will be the mathematical expression of the variance which is $Pr_m * (x_m - (\sum_m x_m * Pr_m))\text{^}2$. The constraints (13) and (14) of the model make sure that the probabilities add up to one and are each non-negative. The realizations of the uncertainties are generated in such a way that there is a match between the statistical properties of the approximating distributions and the specified statistical properties. This is done by minimizing the difference between them.

$$\min_{x,Pr} \sum_{m \in S} w_m * \left(f_m(x, Pr) - VAL_m\right)^2$$

$$\sum \mathrm{Pr} * M = 1 \qquad (13)$$

$$Pr \ \geq 0 \qquad (14)$$

For demand uncertainty, the four moments of mean, variance, skewness, and kurtosis are used for moment matching whereas for the quality uncertainty, the mean, variance, and kurtosis are used. The model is rerun until an objective value that is zero or close to zero is obtained, as this shows a good match between the statistical properties of the generated outcomes and the specified properties [27].

For this study we assume that the uncertainties are product independent: that is, the uncertainties in both the demand and quality of the raw material of one product have no effect on the other products [31]. Apart from this, we also assume that the two uncertainties are independent of each other, and that the realization of the uncertainties in each time period does not depend on the outcomes of the previous periods. According to these assumptions, the realizations of the uncertainties will be exactly the same in every time period as the value of the specified statistical properties does not change with time.

Using the moment matching method described above, a 6-period scenario tree for demand uncertainty is generated for three products. As four moments are used for generating the scenarios, the total number of the specified statistical properties in the set *S* is 72 while the total number of *m* is 18. The minimum number of outcomes that is necessary to obtain a perfect match is determined using the formula $(m+1) * y \sim |S|$. Therefore, the minimum y that is required for 72 specified statistical properties is 4, and we use 5 as the number of outcomes in each time period as it gives a better representation of the probability distribution than the model using 4 outcomes. A total of 5^6 scenarios are generated by sampling the underlying distribution and solving the non-linear optimization problem using the General Algebraic Modeling System (GAMS). Table 10 shows the summary of the scenarios in the first period. There is a perfect match between the outcomes and the specified scenarios with the objective value turning out to be zero. The demand scenario tree generated is shown in Figure 5.

Table 10: Demand scenarios for first period

Outcome	Probability	Demand of P_1	Demand of P_2	Demand of P_3
1	0.223	64.305	30.729	132.143
2	0.233	439.817	54.927	54.700
3	0.177	574.24	28.886	199.126
4	0.100	1246.83	8.819	299.724
5	0.268	463.232	30.696	157.906

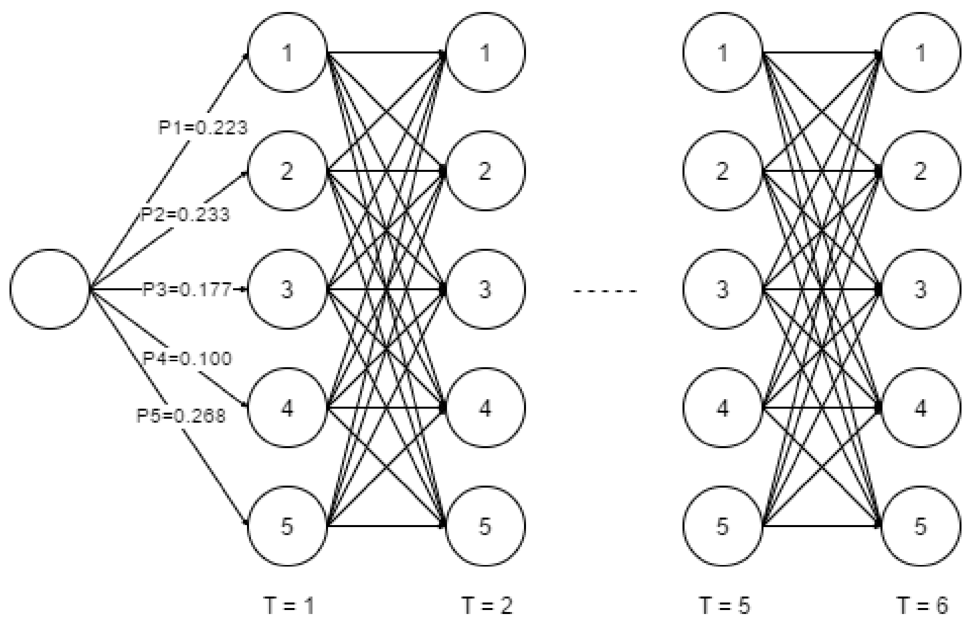

Figure 5: Demand scenario tree

A single period scenario tree is also generated for the raw material quality uncertainty. For this tree, the y value, which is the minimum number of outcomes in each period, is set as 3. A total of three scenarios are generated and the realizations of all three products remains the same as they follow the same probability distribution. Table 11 shows the summary of the quality scenarios generated. Since the uncertainty in quality is assumed to be product independent, all the combinations of product-wise quality scenarios are calculated to be used in the model. As a result, a total of 27 quality scenarios are obtained, as shown in Figure 6.

Table 11: Quality scenarios generated

Outcome	**Probability**	**Quality of P_1**	**Quality of P_2**	**Quality of P_3**
1	0. 33	243.393	243.393	243.393
2	0.12	272.453	272.453	272.453
3	0.55	256.338	256.338	256.338

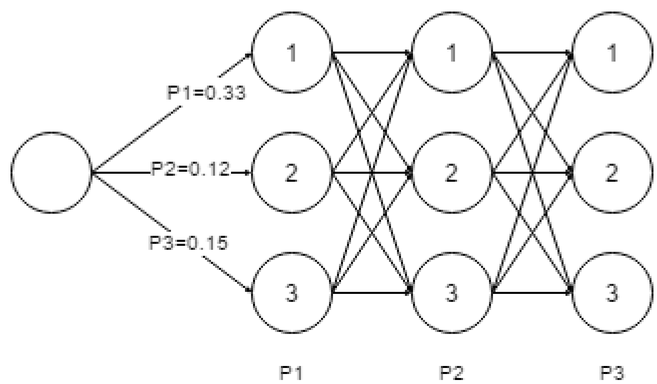

Figure 6: Quality scenario tree

Scenario Reduction

For this study, the common scenario reduction method of *fast forward selection (FFS)* is used to obtain a smaller subset of the original scenario set. The idea behind the FFS method is to select a subset of scenarios with a predefined cardinality in order to minimize the distance between the reduced and the remaining set of scenarios [40].

We briefly describe the general concept of the FFS method for reduction of scenarios. A scenario here is defined as α_k, with its corresponding probability p_k. The set of scenarios that remain unselected till the s^{th} iteration is denoted by $J^{[s]}$. The nomenclature used and the steps involved are discussed below.

Table 12: Nomenclature used for FFS

$\boldsymbol{S}$	**Number of scenarios in the original set**
$\boldsymbol{\alpha_k}$	Scenario k
$\boldsymbol{p_k}$	Probability of scenario k
$\boldsymbol{\eta(.)}$	Non-negative function of L_2 norm
$\boldsymbol{J^{[s]}}$	Unselected scenario set till the s^{th} iteration
$\boldsymbol{c_{k,u}^{[s]}}$	Distance between scenario k and u at s^{th} iteration
$\boldsymbol{Z_u^{[s]}}$	Total weighted distance of each scenario k with other scenarios at s^{th} iteration
$\boldsymbol{\Omega'}$	Selected subset from the original set of scenarios

Procedure of FFS:

Step 1. Compute the distance of the scenario pairs for $S = 1$:

$$c_{k,u}^{[1]} = \eta(\alpha_k, \alpha_u), \quad k, u = 1, \dots, S$$

Step 2. Compute the weighted distance of each scenario to the other scenarios:

$$Z_u^{[1]} = \sum_{k \neq u} p_k * c_{k,u}^{[1]}, \quad u = 1, \dots, S$$

Choose $u_1 = arg \min_{u \in \{1,\dots,S\}} Z_u^{[1]}$

Set $J^{[1]} = \{1, \dots, S\} \backslash \{u_1\}$

Step 3. Update the distance matrix using the scenario selected in the previous step.

Let $S = S + 1$; compute:

$$c_{k,u}^{[s]} = min\left[c_{k,u}^{[s-1]}, c_{k,u_{s-1}}^{[s-1]}\right], \quad k, u \in J^{[s-1]}$$

and

$$Z_u^{[s]} = \sum_{k \in J^{[s-1]} \setminus u} p_k * c_{k,u}^{[s]}, \quad u \in J^{[s-1]}$$

choose

$$u_s = arg \min_{u \in J^{[s-1]}} Z_u^{[s]}$$

set $J^{[s]} = J^{[s-1]} \setminus \{u_s\}$

Step 4. If the number of selected scenarios is less than the required number, return to Step 2.

Step 5. Add to the probability of each selected scenario the sum of the probabilities of all unselected scenarios that are near it; ie., $q_j = p_j + \sum_{i \in L(j)} p_i$, for any $j \in \Omega'$ and $L_{(j)} = \{i \in \Omega \setminus \Omega', j = j(i)\}$, $j_k = arg \min_{j \in \Omega'} \eta(\alpha_k, \alpha_u)$ for any $i \in \Omega \setminus \Omega'$ [39] [28].

After scenario reduction, a total of ten scenarios of demand and their associated probabilities were selected to be used in the model. These scenarios were combined with the 27 quality scenarios to obtain a total of 270 scenarios. The implementation results of using these scenarios in both the deterministic and stochastic models are discussed in the next two sub sections.

2.4.3 Analysis for the deterministic case

The deterministic lot-sizing and scheduling model was run for four different cases of maximum production quantities, and the results obtained are summarized in Table 13.

In the deterministic case, the overall cost decreases with the increase in the maximum production quantity allowed per product per time period ($q_{i,t}$). This is mainly because with more regular time production capacity, more products can be produced in the regular production time. There is no backorder cost for any of the cases, as the demand for the products can be met with overall production resources. Even though there is overtime production happening in the first case, it is no longer required in any of the other cases, as all the product demand can be met with regular time production. The overall cost remains the same for the second and third cases, showing that the maximum production quantity limit makes no contribution to the overall cost for these two cases. There is an inventory cost associated with the fourth case, showing that more products are manufactured in advance and kept in inventory, which can be used later to satisfy the demand and thus save some setup changeover cost. The raw material cost remains unchanged for all the four cases showing that the raw material quality is insensitive to the parameter ($q_{i,t}$). If there is no restriction on the maximum allowed production quantity, all the products P_2 will be produced in the first period followed by the required quantities of the products P_1 and P_3. This is because the demand for P_1 and P_3 are high, and producing them early and keeping them in the inventory will add to the inventory costs, which is undesirable.

Table 13: Summary of results from deterministic model (Cost in $)

	TOTALS					
Maximum Production Quantity: % of Mean Demand	**Regular time production cost**	**Overtime production cost**	**Setup cost**	**Raw material cost**	**Inventory cost**	**Overall cost**
90%	892877	148813	505	135473	0	1177888
100%	992086	0	505	135473	0	1128063
110%	992086	0	505	135473	0	1128063
120%	992086	0	429	135473	16	1128003

Even though the setup costs are identical for the first three cases, the sequences in which the products are manufactured are different. The production sequence obtained for the first case with the maximum production quantity as 90% of the mean demand is shown in

Table 14. In the first time period T_1, product P_3 is manufactured first, followed by products P_2 and P_1. As product P_1 is the last type of product to be manufactured in period T_1, this setup will be carried over and will become the first setup of period T_2. It is observed that the production quantity in regular time production is limited by the time capacity and the maximum allowed production quantity in each time period.

Table 14: Deterministic model production sequence

Time periods/ Products	T_1	T_2	T_3	T_4	T_5	T_6
P_1	3	1	2	3	1	2
P_2	2	3	1	2	3	1
P_3	1	2	3	1	2	3

2.4.4 Analysis for the stochastic case

The uncertainties considered in this study include the demand of the products and the quality of the raw materials required to manufacture the products. Similar to the deterministic case, four different cases of the maximum production quantities are investigated and the results are tabulated as shown in

Table *15*.

Different parameters are used to compare the results obtained from both the deterministic and the stochastic models. The deterministic solution, also called the EV (expected value) solution is obtained by using the expected values of the parameters from the stochastic scenarios in the deterministic model. The RP (recourse problem) solution is obtained from the stochastic model. The solution obtained by applying the decisions in the deterministic case to the stochastic environment is called EEV (expected results of EV). The VSS (value of stochastic solution) for a minimization problem is defined as VSS = EEV – RP, and it measures the effectiveness of the stochastic model over the deterministic one. The EVPI (expected value of perfect information) provides a benchmark for the value of collecting additional information. It estimates the value that the decision-maker is willing to pay for perfect forecasts of the future. This value can then be used to decide whether the methods of collecting more information should be pursued or not. The wait and see solutions (WS) are the solutions obtained for scenarios where the decision-maker makes no decision until all the random variables are realized.

The comparison of the test results for different values of maximum allowed production quantities are shown in Figure 7 and Figure 8. In the stochastic case, the value of RP decreases as the maximum allowed production quantities increase because of more flexible production capabilities and regular time production resources. It should be intuitively clear that the WS solutions are lower than the RP solutions for a minimization problem, as seen in Figure 7. The EVPI value increases with the increase in the maximum production quantity because, with increased flexibility and production resources, the decision-maker will be willing to pay more for getting accurate forecasts of the future. Some of the demand values of the products are extreme and getting accurate forecasts of them in a limited production resources setting does not add value. This is why the EVPI values are low for the cases with lower limits for the maximum production quantities. Apart from having a decreasing trend, the deterministic solution (EV solution) also appear to have the lowest values for total cost, as they do not consider any parameters used in the model to be uncertain. From Figure 7, EEV has the highest values for the total cost, as they are the expected results of EV, that is obtained by applying the deterministic case decisions to the stochastic environment.

Table 15: Summary of results from stochastic model (Cost in $)

Max Production Quantity- % of Mean Demand	**EV**	**WS**	**RP**	**EEV**	**EVPI**	**VSS**
90%	1234258	1723200	1745851	1751578	22651	5726
100%	1189012	1494400	1548738	1561211	54338	12473
110%	1167925	1339300	1412556	1490384	73256	77829
120%	1167858	1269700	1352675	1485102	82975	132427

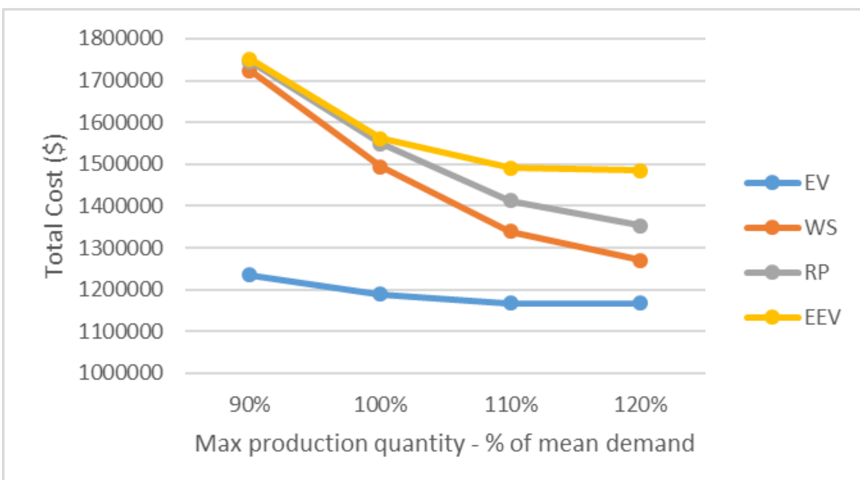

Figure 7: Comparison of test results for different values of production quantity

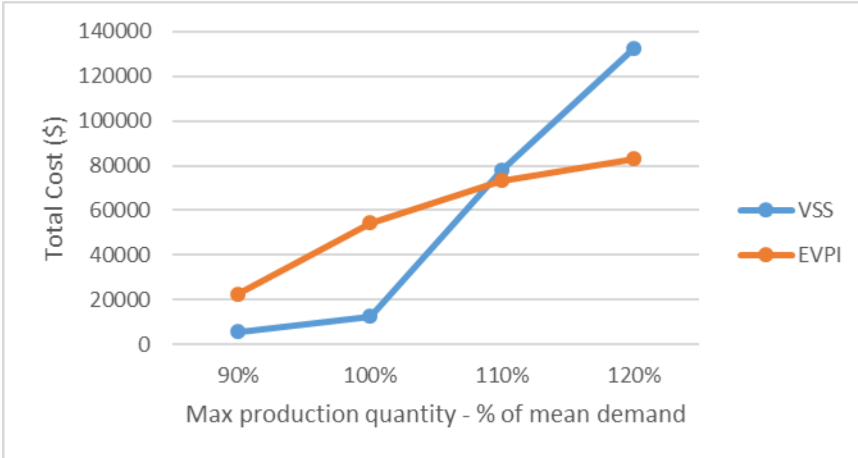

Figure 8: Comparison of VSS and EVPI values for different values of production quantity

The VSS values for the stochastic model also increase with the increase in the maximum allowed production quantities, indicating a corresponding increase in the criticality of the uncertain factors considered in the study. Even though the start is slow, the VSS values increase drastically as the maximum allowed production quantities increase, as seen in Figure 8. This is because, with more flexible production resources and capabilities, it becomes more beneficial for considering the uncertainties in the decision-making process. With a restriction on the maximum production quantities, the model is unable to quickly adapt to the uncertainties. However, with flexibility the model quickly reacts to the uncertainties and meets the extreme demands by producing extra products.

The production sequence obtained for the first case with the maximum production quantity as 90% of the mean demand is shown in Table 16. In time period T_6, product P_3 is manufactured first, followed by product P_1. Product P_2 is not manufactured in this time period. The deterministic and stochastic models resulted in different strategies for allocating the production resources and for sequencing the production activities. Therefore, considering uncertainty in the mathematical formulation for production planning in a manufacturing system would have effects on the production quantities as well as the production sequences. For the different production quantity limits considered in this study, the inventory costs mostly remain close to zero, showing that investing in production inventory control will not be effective in reducing the overall cost.

Table 16: Stochastic model production sequence

Time Periods/ Products	T_1	T_2	T_3	T_4	T_5	T_6
P_1	1	2	3	1	2	2
P_2	3	1	2	3	1	0
P_3	2	3	1	2	3	1

2.5 Conclusion

Production planning is the process of the effective allocation and use of resources such as materials and production capacities to meet the requirements of customers. Due to the significance of the different production related costs, the planning of production lot-sizing and scheduling activities plays an essential role in optimizing the costs.

This paper provides a two-stage stochastic programming framework for a multi-period, multi-product lot-sizing and scheduling problem with uncertain demand and quality of raw materials. The first stage makes regular time production quantity and sequencing decisions while the second stage determines the use of overtime production resources including inventory and backlog. The optimization model facilitates decision-making for lot-sizing and sequencing decisions in a stochastic manufacturing setting.

The proposed approach was applied for production planning in a manufacturing company producing braking equipment under demand and quality uncertainties. The results indicated that the uncertain parameters play a significant role in production planning. It is observed that the parameter of maximum number of production quantity that is allowed to be produced in a particular time period has a significant impact on the production planning process. The results show that the stochastic model is more effective in production planning under the uncertainties considered especially with flexible production resources and capabilities. This is reflected in the increase in the VSS values as the maximum allowed quantity increases.

In summary, this paper provides a framework for making production lot-sizing and scheduling decisions under uncertainties. Although different parameters involved in production planning were reviewed, a need for further research is identified. Firstly, we assume that demand and the quality of raw materials are time independent. However, these factors may vary based on their previous values. Secondly, we consider only two sources of uncertainties and more uncertainty factors can be considered. Thirdly, sensitivity analysis of scenario generation, demand and quality parameters can be performed which might require a significant amount of meaningful raw data. Fourthly, the stability of the results can be tested by generating more scenario sets. Lastly, the quality of the scenario sets obtained through the scenario reduction techniques can be tested to determine how good their representation is of the actual scenario set. We shall address these limitations in our future research.

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CHAPTER 3. PRODUCTION PLANNING WITH A TWO-STAGE STOCHASTIC PROGRAMMING MODEL IN A KITTING FACILITY UNDER DEMAND AND YIELD UNCERTAINTIES

Manuscript to be submitted *Journal of Operations Management*

Goutham Ramaraj, Zhengyang Hu and Guiping Hu

Abstract

The assembly line feeding system is a major part of a manufacturing shop floor. The uncertainties in the production system pose a significant threat on the downstream operations as the decisions could impact the entire manufacturing supply chain. Therefore, it is vital to plan the production at the assembly line feeding system to satisfy the downstream operations and customer demands effectively. The main objective of this study is to develop a two-stage stochastic programming framework for lot-sizing and scheduling the production activities at a kitting facility to support a manufacturing plant. The demand of the kits and the yield of the kitting workers are the two sources of uncertainties considered in this study. The first-stage decisions include the baseline production schedule and the workforce requirement, while the second-stage makes recourse decisions on overtime production. The proposed decision-making framework is validated on a multi-period, multi-product case study involving a kitting facility supporting a manufacturing plant producing braking equipment. The uncertainties are introduced as discrete scenarios that are generated using a scenario generation method. These scenarios are reduced to a smaller subset of scenarios to improve the computational tractability without losing the probabilistic representation. The main conclusion of the study is that uncertainties have significant impacts on kitting planning decisions and that the proposed two-stage stochastic programming model was robust in determining optimal production plans under uncertainty.

Keywords. Production planning, kitting, stochastic programming, demand uncertainty, yield uncertainty

3.1 Introduction

The assembly line feeding system constitutes one of the major pillars of a manufacturing process. It controls and ensures the conveyance of materials from the external/internal warehouses to the work-centers in an assembly line in right quantities and at the right time. The two widely used line feeding systems are line side stocking and kitting [1]. Line side stocking stores all the required components in bulk quantities along the line side storage locations that are replenished frequently from an internal supermarket within the manufacturing facility. As the parts at the supermarkets get depleted, they are replenished by an external warehouse facility using Kanban-based policies. A typical line side stocking system is shown in Figure 9.

In a kitting system, as shown in Figure 10, all the components that are required to assemble a product are collected in a kit container and are delivered to an assembly line in accordance with the production schedule. Kits are usually assembled in an external kitting facility or at an internal kitting area within the facility therefore no pallet/box inventories are stored at the line side storage locations for these parts. Both systems have their own advantages and disadvantages and a decision to select the right system for a manufacturing plant needs to be made depending on the level of customization of the products manufactured and the storage space constraints within the facility.

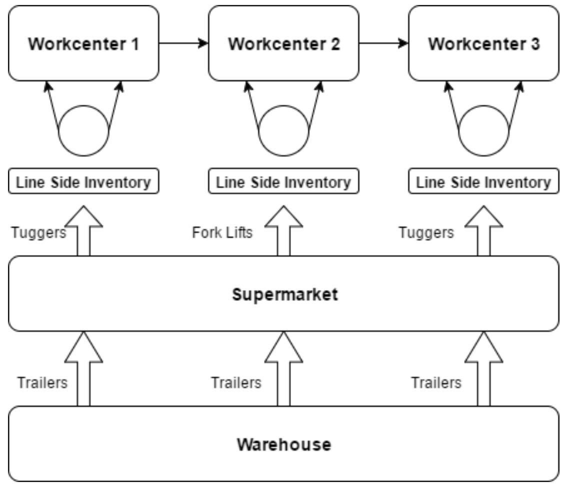

Figure 9: An Example of Line Side Stocking

Among product customization, the number of variant parts in the product manufactured is one of the major parameters that need to be considered before the assembly line feeding system is designed. Variant components are the parts that have multiple variants of the same product that needs to be assembled on a product based on the variant selected by the customer. This would require the product to be delivered to the line right on time for the assembly of the product requested by the customer. Space constraints at line side locations is another factor that influences the decision of selecting the best line feeding system. The process is complicated by the size of the parts assembled, ranging from small to bulk parts, and their rate of consumption, low, medium to high runners. Storing a full pallet of each part to be assembled near work-centers requires a very large production area and causes inconvenience to the operators in travelling large distances to obtain the parts/components required for the assembly process. The adoption of kitting system in many industries has helped to address the parts storage space issues and to better streamline the flow of material to the assembly line.

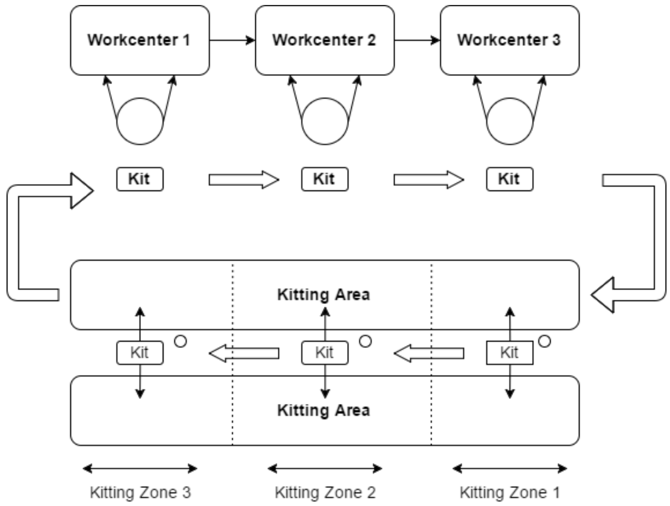

Figure 10: An Example of Kitting System

As per Choobineh and Mohebbi, a kit is collection of components/parts required for the assembly/production of a product [2]. The advantages of kitting, as stated by Bozer and McGinnis, include easy handling of the materials to workstations, increased productivity, easy changeover of product type, and removal of non-value adding activities from the assembly process [3]. Due to the inherent system and environmental uncertainties within a manufacturing plant, the planning of the kitting operations plays a crucial role in determining the level of productivity and the efficiency of the assembly activities of the final product. Some of the common uncertainties in a manufacturing environment include demand, yield and quality uncertainties [4]. Some of these uncertainties within the manufacturing plant tend to affect the upstream operations like the line feeding operations. For example, the change in the production sequence due to issues like part quality and availability creates demand change for the kits that in turn affects the upstream kitting operations. Apart from this, the uncertainties within the kitting facilities like the worker yield uncertainty further makes the operations planning process more complex and challenging. Despite of these challenges, the existing literature suggests that the kitting based line feeding system has been widely adopted by many industries. Toyota has set up a new kitting process called the Set Pallet System (SPS), at their manufacturing facility at San Antonio. This new system has helped the company to eliminate the involvement of the operators in the part picking process. Some of the other benefits include more value-added time by the operators, cleaner work areas and visual control, and fewer part selection errors [5].

In spite of the popularity of the kitting based line feeding system for manufacturers, there appear to be very few studies on kitting production planning. Ramachandran and Delen analyzed the dynamics involved in a simple kitting process of a stochastic assembly system where two independent streams feed into an assembly process. The findings from their study provide manufacturing system designers variety of control parameters to effectively analyze the system performance [6]. Caputo et al. developed a detailed descriptive mathematical model for kitting operations planning, allowing resources planning and valuation of system's economic performances [7]. Selcuk and Bulent proposed a mathematical model to design a kitting system by obtaining the optimum values for the design parameters. The tour period, the number of kitting workers, and the quantities of the kits are the parameters used in their study [8]. Gunther et al. proposes a heuristic solution procedure to solve the component kitting problem faced by electronics manufacturers. Their close to optimum results show that heuristic based approach is computationally efficient [9]. De Souza et al. conducted a study on how to pack parts in available containers to meet the assembly line workstation requirements with minimum cost over the entire planning horizon. An integer programming model was used to solve the line feeding problem [10]. Brynzer and Johansson carried out a number of case studies on the design and performance of kitting and order picking systems.

Some of these studies conducted were in the terms of location of the order picking activity, work organization, picking method, information system and equipment [11]. Limère et al. proposed a mathematical model to evaluate the allotment of parts to one of the two main material supply systems: kitting or line side stocking. The results show that a hybrid system with some parts kitted and other stored near line side is preferred over the exclusive use of any one material supply system [12]. Hanson and Medbo conducted a study to determine how kitting affects the time efficiency in a manual assembly process. Four cases studies of automobile assembly were conducted to determine the parts that needs to be kitted and the order in which the assembly operations needs to be performed [13].

In practice, production planning of kitting operations is most likely to be performed in an uncertain environment as the demand of the kits and the yield of the kitting workers are often not known for sure. Even though many researchers have extensively studied the planning of production activities in a kitting system, very few have considered the stochastic nature of the manufacturing environment. Our study takes into consideration the uncertainties in a kitting process which can be regarded as one of the major contributions of this paper. Choobineh and Mohebbi conducted a study for material planning for production kits under demand and procurement lead time uncertainty [2]. A comprehensive simulation study was conducted to investigate the impacts of demand and procurement lead time uncertainty on system performance. Unlike the simulation modeling approach, in our study we use a different modeling approach based on mathematical programming to assist the decision-making process in a kitting system under uncertainty. In terms of application, we are the pioneers to adopt a two-stage stochastic programming approach for a kitting specific production planning problem. Even though production planning under uncertainty is one of the major areas of application for optimization tools using mathematical modeling, not many studies discuss the use of two-stage stochastic programming framework for solving problems. In addition, this study considers two sources of uncertainties simultaneously in in one decision-making framework since it is common to often encounter situations with multiple uncertain parameters. In the existing literature for production planning problems, majority of them discuss the effect of only one uncertainty in a manufacturing environment. This paper is among the few studies that consider the effects of multiple uncertainties, which is more practical in a manufacturing environment. This is another major contribution of this study.

As suggested by literature and industrial practitioners, business decisions made at a line feeding system is critical and need to be carefully planned as it involves a number of players across the entire manufacturing supply chain. The purpose of this study is to propose a two-stage stochastic programming approach for production planning for a kitting system under demand and yield uncertainties. The objective of the model is to minimize the total kitting cost by finding the optimum kitting schedule and lot size. The novelty of the study lies in modeling multiple uncertainties using two-stage stochastic programming to solve a kitting specific production planning problem in a manufacturing setting.

The remainder of the paper is structured as follows: Section 2 provides the problem statement including the assumptions in the study. Section 3 presents the deterministic model and the stochastic model for the production planning problem. Discussion of the results from a case study based on an automotive part manufacturer are provided in Section 4 and finally, conclusions are provided in Section 5.

3.2 Problem statement

Production planning is a crucial step in a manufacturing environment especially when the system operates in a stochastic environment. Efficient production planning helps in optimizing the use of plant resources and in streamlining the conveyance of material to the assembly line. The schematic of the manufacturing system considered for this study is shown in Figure 11. Some of the uncertainties within the manufacturing system tend to propagate across the manufacturing supply chain, affecting the operations in the upstream and downstream facilities. For example, the uncertainty in customer demand affects the planning of the assembly process inside the manufacturing plant. Likewise, the uncertainties within the plant like the parts quality uncertainty and lead time uncertainty affects operations planning at the line feeding system. Whenever a change is made to the production sequence, it disrupts the kitting operations as they are planned based on the production sequence. This uncertainty in the demand of raw material/kits can lead to delay in the delivery of the kits to the manufacturing facilities, which can lead to line shut down and thus economic loses. Apart from this, the uncertainties within the line feeding system like the worker yield uncertainty further makes decision-making complicated and challenging.

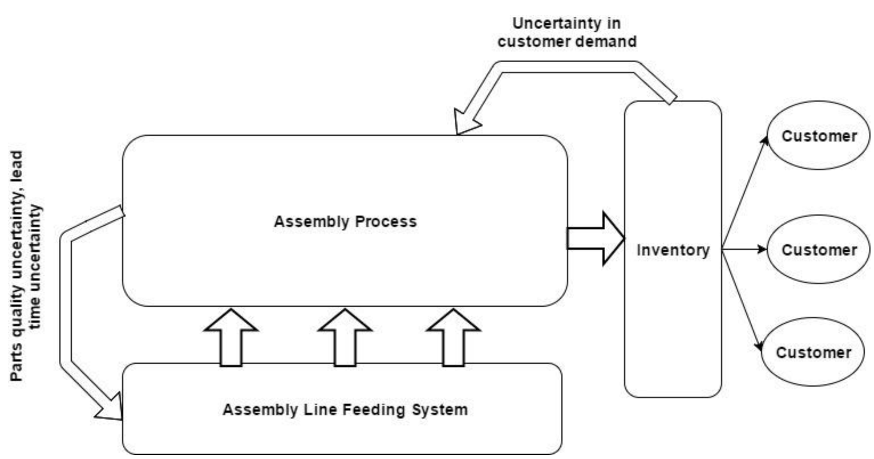

Figure 11: Manufacturing System Studied

In this paper, a mathematical framework based on two-stage stochastic programming is developed to assist decision-making in an assembly line feeding kitting system under uncertainty. The objective is to find the optimal decisions on kit production quantities and kitting sequences. The results from this paper can provide guidance and managerial insights to make decisions related to production lot-sizing and scheduling in a kitting facility under demand and worker yield uncertainties. The proposed framework can contribute to address and manage the uncertainties in a kitting facility.

3.3 Model formulation

In this section, we first present a deterministic mathematical formulation for the problem studied. Then the model is extended to a two-stage stochastic programming framework to address uncertainties in kit demand and worker yield in a kitting facility. A two-stage stochastic programming model is designed to accommodate flexible decision-making mechanism that can respond to events as they unfold. The decision maker has the option to compensate for the non-optimal effects of the first stage decisions through the recourse decisions made in the second stage of the decision-making process [14].

3.3.1 Mathematical notations

To describe the lot-sizing and scheduling problem addressed in thus study, we consider a kitting facility with N kits indexed by $i, j = 1, \ldots, N$ to be kitted over a time horizon of T periods, indexed by $t = 1, \ldots, T$. Both the indexes i and j are used to denote the same set of kits produced at the kitting facility to model the changeovers between the kits. The mathematical notations associated with the problem studied are included in Table 17.

Table 17: Notations for the mathematical formulation

Subscripts		
i	1, 2….N	Kits
j	1, 2….N	Kits
t	1, 2….T	Time periods
Parameters		
$d_{i,t}$	Demand for kit i in period t	
h_i	Inventory holding cost per unit kit i for one period	
b_i	Backorder cost per unit kit i for one period	
cap_t	Time capacity of a kitting worker in period t	
$p_{i,t}$	Processing time of unit kit i in period t	
p_i^r	Regular time processing cost per unit kit i	
p_i^o	Overtime processing cost per unit kit i	
$q_{i,t}$	The maximum regular time kitting quantity of kit i in period t	
$SC_{i,j}$	Setup cost for changeover from kit i to kit j	
$ST_{i,j}$	Setup time for changeover from kit i to kit j	
$\rho_{i,t}$	The units of kits i processed by a worker in time period t (yield of a worker)	
α	Maximum overtime ratio	
$\emptyset$	The monthly wage of a worker	
N	Number of kits	
Decision Variables		
K_t	The number of kitting workers in time period t	
$I_{i,t}$	Inventory level of kit i by the end of period t	
$B_{i,t}$	Backorder level of kit i by the end of period t	
$X_{i,t}$	Regular time production quantity of kit i in period t	
$O_{i,t}$	Over time production quantity of kit i in period t	
$Y_{i,j,t}$	1 if a changeover from kit i to kit j is performed in period t. Binary Variable	
$Z_{i,t}$	1 if a setup of kit i is carried over from period $t-1$ to period t. Binary Variable	
$V_{i,t}$	Production order of kit i in period t. Integer variables start from 1.	

3.3.2 Deterministic model

This model aims to determine the optimal production lot size and sequence to satisfy the customer demand effectively.

Before the modelling approaches are introduced, we present the assumptions to clearly define the problem considered for this study.

- Inventory is evaluated at the end of each planning period with the starting inventory level at the beginning of the planning horizon to be zero.
- The kit demand and the worker yield in a specific time period is independent from the previous time period.
- Both uncertainties are independent of kits and of each other.
- Limited resources are available for production, both for regular time and overtime production.
- The maximum setup for each product in a single time period is one. The same setup is used for both regular time and overtime production.
- The setup can be carried over to following time period.
- Backorders are allowed, so the kit demand does not need to be fulfilled for every time period.

Objective function and constraints

The objective function of the deterministic model captures the combined costs incurred in the kitting process. The costs include production, inventory, backorder and labor costs. The costs from regular time and over time production of kits are denoted by terms $\sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^r * X_{i,t}$ and $\sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^o * O_{i,t}$ respectively in the objective function. The terms $\sum_{i=1,i\neq j}^{I}\sum_{j=1}^{j}\sum_{t=1}^{T-1} SC_{i,j} * Y_{i,j,t}$, $\sum_{i=1}^{I}\sum_{t=1}^{T-1} h_i * I_{i,t}$, *and* $\sum_{i=1}^{I}\sum_{t=1}^{T-1} b_i * B_{i,t}$ are the setup changeover cost, inventory cost, and the backorder cost respectively. Lastly, the labor costs incurred for the kitting process is represented by the term $\sum_{t=1}^{T-1} \emptyset * (K_t)$ The decisions include the production quantity for both regular time and overtime production, production schedule, inventory and backorder levels and the kitting workforce requirement. Specifically, K_t, $I_{i,t}$, $B_{i,t}$, $X_{i,t}$, $O_{i,t}$, $Y_{i,j,t}$, $Z_{i,t}$, and $V_{i,t}$ constitute the decision variables used in the study and they clearly define the production and workforce requirements for the kitting process. The objective function is as follows:

Minimize Z =

$$\sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^r * X_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} p_i^o * O_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} h_i * I_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T-1} b_i * B_{i,t} + \; + \sum_{t=1}^{T-1} \emptyset * (K_t) + \sum_{i=1, i\neq j}^{I}\sum_{j=1}^{j}\sum_{t=1}^{T-1} SC_{i,j} * Y_{i,j,t}$$

Subject to

$$X_{i,t} + O_{i,t} = d_{i,t} + I_{i,t} - B_{i,t} \qquad \forall i, t = 1 \qquad (1)$$

$$I_{i,t-1} - B_{i,t-1} + X_{i,t} + O_{i,t} = d_{i,t} + I_{i,t} - B_{i,t} \qquad \forall i, t = 2 \ldots T \qquad (2)$$

$$X_{i,t} + O_{i,t} <= \rho_{i,t} * K_t \qquad \forall i, t \qquad (3)$$

$$X_{i,t} \leq q_{i,t} * \left(Z_{i,t} + \sum_{j\neq i}^{J} Y_{j,i,t} \right) \qquad \forall i, t \qquad (4)$$

$$\sum_{i=1}^{I} p_i * X_{i,t} + \sum_{i=1\, i\neq j}^{I}\sum_{j=1}^{J} ST_{i,j} * Y_{i,j,t} \leq cap_t \qquad \forall t \qquad (5)$$

$$O_{i,t} \leq \alpha * X_{i,t} \qquad \forall i, t \qquad (6)$$

$$\sum_{i=1}^{I} Z_{i,t} = 1 \qquad \forall t \qquad (7)$$

$$Z_{i,t} + \sum_{j \neq i}^{J} Y_{j,i,t} = Z_{i,t+1} + \sum_{j \neq i}^{J} Y_{i,j,t} \qquad \forall i, t = T-1 \qquad (7)$$

$$X_{i,t} = 0 \qquad \forall i, t = T \qquad (9)$$

$$V_{j,t} \geq V_{i,t} + 1 - N * (1 - Y_{j,i,t}) \qquad \forall i, j \neq i, t \qquad (10)$$

The decision variables in the model are constrained by a set of constraints represented with Eqs. (1) to (10). The balance between the production and demand of the kits are maintained by the inventory balance constraints Eqs. (1) and (2). The inventory from a previous period is considered to maintain the balance. Based on worker efficiency, the regular and overtime production quantity is subject to constraint Eq. (3). Eq. (4) restricts the total regular time production quantity irrespective of whether a setup change is performed or the setup is carried over from a previous period. A setup carryover happens when $Z_{i,t}$ = 1 and a setup changeover happens when $\sum_{j \neq i}^{J} Y_{j,i,t}$ = 1. Both these terms is restricted to be 1 at the same time. The constraint on time capacity is achieved through Eq. (5). The limitation on the overtime production quantity is defined through Eq. (6). Eq. (7) guarantees that the initial setup is performed at the beginning of each time period. The balance of flow of the setups is maintained by Eq. (8). It ensures that the setup is ready to be kitted either at the beginning of the period or at the beginning of the next period depending on the position of the product in the production sequence. Eq. (9) makes sure that there is no production in the last time period, which is considered as a dummy period for this study. The subtours in the production sequence is eliminated by Eq. (10).

This mathematical formulation takes a deterministic view of the lot-sizing and the scheduling problem by considering all the model parameters to be known with certainty. This assumption of complete knowledge of the parameter values, although desirable from the modeling point of view, is highly optimistic. In practice, most kitting and manufacturing environments are characterized by a variety of uncertainties. There have been lots of efforts to address uncertainties in the production planning problem in the existing body of literature. Kazaz proposed a two-stage stochastic programming model for olive oil production planning under yield and demand uncertainty [15]. Gurnani et al. studied supply management in an assembly system with random yield and random demand. They developed a cost function to determine the combined component ordering and assembly decisions for the firm [16]. The importance of incorporating these uncertainties into the production planning models has motivated our study. In Section 3.3, a two-stage stochastic programming framework considering demand and yield uncertainties is presented.

3.3.3 Two-stage stochastic programming model

Two-stage stochastic programming is an effective modeling approach for production planning under uncertainty [14]. The demand of the kits and the yield (efficiency) of the kitting workers are the uncertainties considered in our study. The probability distributions for these uncertainties are represented with discrete scenarios. We use a subscript s to represent the scenarios with a probability Pr_s, which is obtained by multiplying the individual probabilities of demand and yield uncertainties as they are assumed to be independent of each other. The two-stage stochastic programming model is formulated as follows:

Minimize Z =

$$\sum_{t=1}^{T-1} \emptyset * (K_t) + \sum_{i=1, i \neq j}^{I} \sum_{j=1}^{j} \sum_{t=1}^{T-1} SC_{i,j} * Y_{i,j,t}$$
$$+ \sum_{s=1}^{S} \Pr_s \left\{ \sum_{i=1}^{I} \sum_{t=1}^{T-1} p_i^r * X_{i,t,s} + \sum_{i=1}^{I} \sum_{t=1}^{T-1} p_i^o * O_{i,t,s} \right.$$
$$\left. + \sum_{i=1}^{I} \sum_{t=1}^{T-1} h_i * I_{i,t,s} + \sum_{i=1}^{I} \sum_{t=1}^{T-1} b_i * B_{i,t,s} \right\}$$

Subject to

Constraints (7), (8), and (10)

$$X_{i,t,s} + O_{i,t,s} = d_{i,t,s} + I_{i,t,s} - B_{i,t,s} \qquad \forall \mathrm{i, s, t} = 1 \qquad (11)$$

$$I_{i,t-1,s} - B_{i,t-1,s} + X_{i,t,s} + O_{i,t,s} = d_{i,t,s} + I_{i,t,s} - B_{i,t,s} \qquad \forall i, s, t = 2 \ldots T \qquad (12)$$

$$O_{i,t,s} \leq \alpha * X_{i,t,s} \qquad \forall i, s, t \qquad (13)$$

$$X_{i,t,s} + O_{i,t,s} <= \rho_{i,t,s} * K_t \qquad \forall i, s, t \qquad (14)$$

$$X_{i,t,s} \leq q_{i,t} * \left(Z_{i,t} + \sum_{j \neq i}^{J} Y_{j,i,t} \right) \qquad \forall i, s, t \qquad (15)$$

$$\sum_{i=1}^{I} \sum_{t=1}^{T-1} p_i * X_{i,t,s} + \sum_{i=1\, i \neq j}^{I} \sum_{j=1}^{J} ST_{i,j} * Y_{i,j,t} \leq cap_t \qquad \forall i, s, t \qquad (16)$$

$$X_{i,t,s} = 0 \qquad \forall i, s, t = T \qquad (17)$$

In the above formulation, the variables K_t and Y_{ijt}, Z_{it}, and V_{it} are the first stage decisions while the X_{its}, O_{its}, I_{its}, and B_{its} are the second stage, recourse decisions. The first stage decisions that include the baseline production schedule and the workforce requirement are made before the uncertainties in kit demand and worker yield are realized. The second

stage decisions, which include production quantities (regular time and overtime), inventory and backorder levels are made after the two sources of uncertainties are realized.
While some constraints are retained from the deterministic formulation, others are modified according to the scenarios. Eqs. (7), (8), and (10) are the first stage constraints and remain the same as the deterministic model. The remaining Eqs. (11) to (17) describe the second stage constraints.

One of the foremost steps to incorporate uncertainties into the production planning models is to determine the appropriate representation of the uncertain parameters. Gupta and Maranas identified two distinct methodologies for representing uncertainty, scenario-based approach and distribution-based approach [17]. While the scenario-based approach describes the uncertainty by a set of discrete scenarios each with an associated probability, distribution-based approach assigns a probability distribution to the continuous range of potential outcomes. For this study, we adopt the scenario-based approach. Each scenario is a discrete value of kit demand or yield of a worker. A scenario set is generated to represent the uncertain parameters.

Computational efficiency is among the most important characteristic to be considered when choosing the approach to decide the set of scenarios to represent uncertainties. In our study, we use a moment matching method to generate scenarios by discretizing the underlying distribution of uncertain parameters [18]. This is a widely-used scenario generation method, especially when the distribution functions of the marginals are unknown. In such cases the marginals are described by their moments (mean, variance, skewness, kurtosis etc.) instead [19]. To reduce the computational burden and to approximate the probability distribution with a smaller subset that is close to the original distribution, we make use of a scenario reduction technique [20, 21]. Both the scenario generation and scenario reduction techniques used in this study is discussed in detail in section 4.

3.4 Case study

In this section, the proposed approach for production planning under uncertain demand and yield is implemented for a kitting facility that supports a manufacturing plant producing braking equipment for the automotive industry. In the facility considered in this study, three types of hydraulic braking actuators, P1, P2, P3, are manufactured. The assembly process of these three actuators require three kits that are specific to the product manufactured. Based on the classification framework proposed by Guimaraes et al. for lot-sizing and scheduling problems, the planning horizon for the large bucket models is partitioned into small number of long time periods, representing, in most cases, a week or month [22]. For this study, we consider a multi-period multi-product problem with a six-month planning period that is divided into time slots of a month each. The main objective is to minimize the total costs involved in the kitting process to meet the needs of the upstream customers in the most cost effective way. All the input parameters used for this study, except the demand of kits and the yield of the kitting workers, are assumed to be known with complete certainty.

3.4.1 Data sources

As discussed earlier, the case study is based on a kitting facility that produces three types of kits (Kit1, Kit2, Kit3) for the assembly of three actuators (P1, P2, P3) in a manufacturing facility. Both the kit demands and the yield of kitting workers are uncertain and are distributed according to the probability density functions (pdf's) defined in

Table *18* and Table 19. The distributions for the monthly demand is assumed to be the same for the original equipment and its kits and is obtained by fitting the historical data of a three year period [23]. The data for yield of the kitting workers is obtained from the study conducted by Zhang et al. using real data from a manufacturing facility [24]. As many other studies in literature, we also assume the yield for all the three kits to be normally distributed with the moments as shown in Table 19 [25, 26, 27]. It is assumed that the demand of the kits and the yield of the kitting workers are independent of each other [27, 28].

Table 18: PDF of monthly demand

Monthly Demand			
	Kit1	Kit2	Kit3
PDF	Weibull	Weibull	Weibull
Scale	518	38	169
Shape	1.51	2.76	2.27
Mean	467.25	33.82	149.70
Variance	99422	175.42	4877.80
Skewness	1.06	0.25	0.47
Kurtosis	4.35	2.78	2.98

Table 19: PDF of worker yield

Product yield of workers			
	Kit1	Kit2	Kit3
PDF	Normal	Normal	Normal
Mean	60.69	51.59	43.70
Variance	9.10	9.10	9.10
Kurtosis	3	3	3

Setup costs for kitting include the costs of making changes to the kitting equipment, and moving materials or equipment. This would include the labor costs involved for making the changes and cost of lost opportunity of kitting a profitable output when the kitting operations were idle. The changeovers between the kits would also require significant times as the process and the equipment required for kiting would be different for different parts, making the lot-sizing and scheduling decisions more complex. Setup and operation times for the kitting process studied are listed in Table 20 and Table 21 [23]. The cost for the setup is set to be proportional to the setup time by a specified factor of 0.2805 [29].

Table 20: Setup Times

Setup Times (min/setup)			
	Kit1	Kit2	Kit3
Kit1	0	270	90
Kit2	180	0	270
Kit3	90	180	0

Table 21: Operation Times

Operation times (min/unit)			
	Kit1	Kit2	Kit3
Operation time	6	6.6	7.2

Error! Reference source not found. shows the time capacities available (including failure and repair time). The regular time kit production costs are shown in

Table *23*. Just like the setup costs, both overtime production and backorder costs are set proportional to the regular time production costs with the factors at 1.5 and 2, respectively [23, 24].

Table 22: Time Capacities

Time capacities (min)	
Month	Capacity
1	6087
2	5367
3	6087

4	6087
5	4407
6	4407

Table 23:Regular time production and Inventory costs

Regular time production and inventory costs ($/unit)			
	Kit1	Kit2	Kit3
Production cost	254.08	254.08	254.08
Inventory cost	0.16	0.15	0.38

The average daily production rate of a kitting worker is 2.89 kits. Their corresponding production rates (yields) for different levels of efficiency are given in

Table *24*. To calculate the labor cost of a worker in a time period, the regular time labor cost is set at $18 per hour and it is assumed that the company operates 8 hours a day and 21 days in a month [24].

Table 24: Kitting worker production rate

Worker Efficiency	**Production rate**
100%	2.89
85%	2.46
72%	2.08

For this study the maximum overtime ratio is set at 20% of regular time production to eliminate the possibility of a decrease in worker productivity due to exceeding the overtime limits set by the government and the company policies [24].

For a mathematical modeling problem it is important to conduct sensitivity analysis to understand the impact of the changes in the input sources on the system output. Jamal et al. identified batch size as a parameter that is equally important as other parameters like the inventory, holding, backorder and production costs. An optimal batch quantity depends mainly on the rate of production and the demand pattern of the final products [30]. Park also emphasizes the importance of production capacity parameters in production and distribution planning problems. He conducted sensitivity analysis of the input parameters used in his model to investigate the level of their effects on the problem studied [31]. Hu and Hu formulated a two-stage stochastic programming model to determine the optimal lot-size and production sequence under demand uncertainty. They conducted sensitivity analysis on the production capacity parameter to investigate the tradeoffs and to provide valuable insights on decision making under uncertainty [32]. Since the production capacities is one of the important parameters based on the literature, we conduct a sensitivity analysis for our study using the parameter $q_{i,t}$, which is the maximum regular time production quantity for a kit in a particular time period. We consider four different values for the allowed production quantities, 90%, 100%, 110% and, 120% of the mean values of demand. This will help understand how different values of production quantities impact the production planning decisions under uncertainty.

3.4.2 Scenario generation and reduction

The representation of the underlying stochastic process is one of the major concerns in stochastic programming [33]. Random variables represented by continuous or discrete distributions makes the computation process more tedious. To address this issue, the distributions are replaced with a set of discrete outcomes. In this study, we use a non-linear programming method proposed by Hoyland and Wallace to generate a limited number of discrete outcomes that meet specified statistical properties [34]. The number of scenarios generated has a direct influence on the computation requirements and accuracy when solving the mathematical model. So it is important to identify the right presentation of the probabilistic distributions [21]. Another common approach adopted is to use scenario reduction techniques that control the approximation's goodness-of-fit according to probability metrics [19]. In this study, we use the forward selection scenario reduction method to obtain a smaller subset of the generated scenarios. Both methods are discussed in detail in the subsections below.

Scenario generation

In our study, we use the moment matching method for scenario generation. The concept of moment matching is to minimize the distance between the generated scenarios with those of the observed data process. Following the notation presented by Hoyland and Wallace, define P as a set of all specified statistical properties and P_{VALi} as the observed value of the specified statistical property i from P [34]. Then let V be the number the number of random variables, T be the number of stages and R_t be the number of conditional outcomes in each stage t. Define the outcome vector x of dimension $V.R_1 + V.R_1.R_2 + \ldots + V.R_1.R_2....R_t$ that means there are $R_1.R_2....R_t$ outcomes of each variable $v = (1, \ldots, V)$ in stage $t = (1,\ldots,T)$. The probability vector p is of dimension $R_1 + R_1.R_2 + R_1.R_2...R_t$. The function $f_i(x,p)$ is the mathematical expression of the statistical property i in P and M is the matrix of zeros and ones whose number of rows is equal to p and number of columns equals the number of nodes in the scenario tree. Finally, let w_i, be the weight of statistical property i in P.

The vectors x and p are generated by solving the non-linear optimization problem:

$$\min_{x,p} \sum_{i \in P} w_i * (f_i(x,p) - P_{VALi})^2$$

$$\sum \mathrm{p} * M = 1 \qquad (13)$$

$$p \geq 0 \qquad (14)$$

For scenario generation with moment matching method, one can use as many moments and state-dependent statistical properties as desired. For the kit demand uncertainty in our model the four moments of mean, variance, skewness and kurtosis are used to generate the scenarios. Since the underlying distribution for yield uncertainty is a normal distribution, only mean, variance and kurtosis are used as the moments for scenario generation. The model has multiple optimal values since it a non-linear optimization problem. Therefore, it is run until a satisfactory objective value that is zero or close to zero is obtained [34].

The realizations generated with the method discussed above would be the same in every time period for both the uncertainties. This is because the value of the specified statistical properties remains the same for all time periods and due to the assumption that both the uncertainties are independent of each other and their realizations in each time period does not depend on their values in the previous periods. We also assume that the uncertainties are product independent, that is the kit demand and the yield of the kitting workers of one product has no influence on the corresponding parameters of other products [35].

For the kit demand uncertainty, a scenario tree with three products and six time periods is generated. The total number of specified statistical properties in the set *P* is 72 and the value of *i* is 18. We generate 5 scenario outcomes in each time period whose value is obtained using the formula $(i+1) * y \sim |P|$. The approximation of the probability distribution is observed to be better with 5 outcomes than with 4 outcomes that was obtained from the formula. The non-linear optimization problem was solved using General Algebraic Modeling System (GAMS) to generate a total of 5^6 scenarios for the kit demand uncertainty. The objective value obtained was zero indicating a perfect match between the specified properties and the generated scenarios. The kit demand scenarios for all the six periods are tabulated in Table 25 and the scenario tree generated is shown in Figure 12*: Kit demand scenario tree*.

Table 25: Kit demand scenarios for first period

	Time-Period		**1**	**2**	**3**	**4**	**5**	**6**
Kit Demand	Outcome1 P1= 0.223	Kit1	64.305	64.305	64.305	64.305	64.305	64.305
		Kit2	30.729	30.729	30.729	30.729	30.729	30.729
		Kit3	132.143	132.143	132.143	132.143	132.143	132.143
	Outcome2 P2= 0.233	Kit1	439.817	439.817	439.817	439.817	439.817	439.817
		Kit2	54.927	54.927	54.927	54.927	54.927	54.927
		Kit3	54.700	54.700	54.700	54.700	54.700	54.700
	Outcome3 P3= 0.177	Kit1	574.24	574.24	574.24	574.24	574.24	574.24
		Kit2	28.886	28.886	28.886	28.886	28.886	28.886
		Kit3	199.126	199.126	199.126	199.126	199.126	199.126
	Outcome4 P4= 0.100	Kit1	1246.83	1246.83	1246.83	1246.83	1246.83	1246.83
		Kit2	8.819	8.819	8.819	8.819	8.819	8.819
		Kit3	299.724	299.724	299.724	299.724	299.724	299.724
	Outcome5 P5= 0.268	Kit1	463.232	463.232	463.232	463.232	463.232	463.232
		Kit2	30.696	30.696	30.696	30.696	30.696	30.696
		Kit3	157.906	157.906	157.906	157.906	157.906	157.906

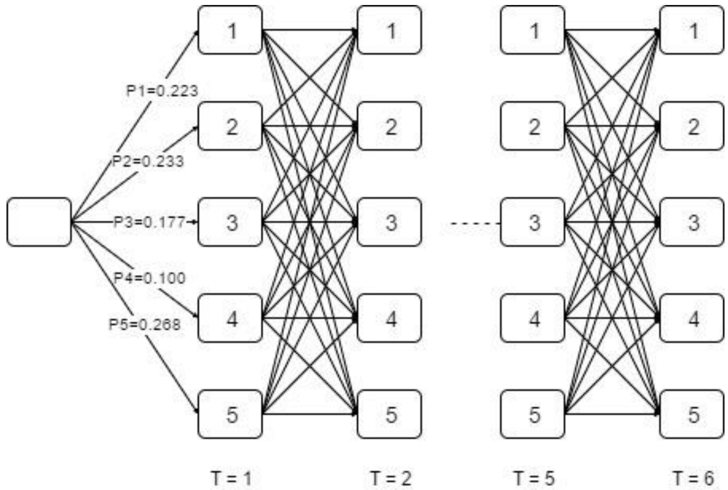

Figure 12: Kit demand scenario tree

Similarly, a scenario tree with 5^6 scenarios with five outcomes in each time period is generated for the kitting worker yield uncertainty. The summary of the yield scenarios generated for the first time period is shown in Table 26 and the scenario tree generated is shown in Figure 13.

Table 26: Yield scenarios for first period

	Time-Period		1	2	3	4	5	6
Worker Yield	Outcome1 P1= 0.152	Kit1	53.747	53.747	53.747	53.747	53.747	53.747
		Kit2	45.512	45.512	45.512	45.512	45.512	45.512
		Kit3	37.659	37.659	37.659	37.659	37.659	37.659
	Outcome2 P2= 0.298	Kit1	62.841	62.841	62.841	62.841	62.841	62.841
		Kit2	54.446	54.446	54.446	54.446	54.446	54.446
		Kit3	43.08	43.08	43.08	43.08	43.08	43.08
	Outcome3 P3= 0.196	Kit1	60.981	60.981	60.981	60.981	60.981	60.981
		Kit2	51.121	51.121	51.121	51.121	51.121	51.121
		Kit3	44.129	44.129	44.129	44.129	44.129	44.129
	Outcome4 P4= 0.120	Kit1	61.954	61.954	61.954	61.954	61.954	61.954
		Kit2	49.791	49.791	49.791	49.791	49.791	49.791
		Kit3	44.987	44.987	44.987	44.987	44.987	44.987
	Outcome5 P5= 0.234	Kit1	61.57	61.57	61.57	61.57	61.57	61.57
		Kit2	53.218	53.218	53.218	53.218	53.218	53.218
		Kit3	47.4	47.4	47.4	47.4	47.4	47.4

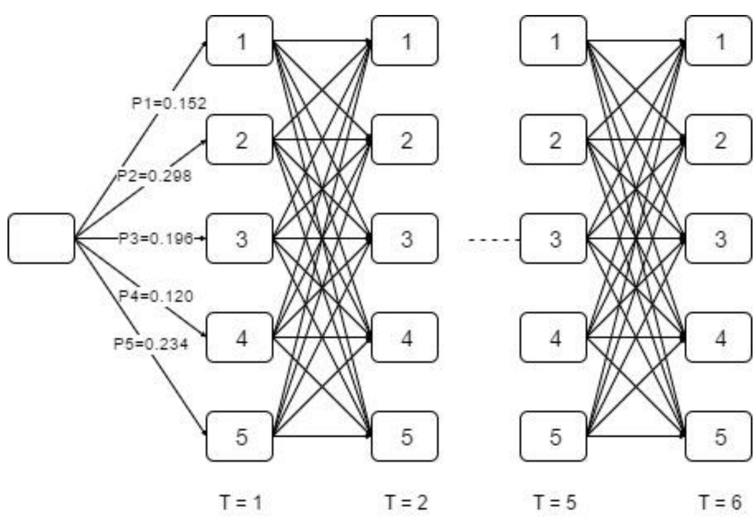

Figure 13: Worker yield scenario tree

Scenario reduction

To make our two-stage stochastic programming model computationally tractable, we make use of a scenario reduction technique based on fast forward selection (FFS) to accomplish this task. The FFS algorithm successively computes the distances between the generated scenarios to select the most representative ones.

We briefly review the FFS method used as follows [19, 36]. A scenario is defined as a path from the root node to the node in the last stage, denoted by s_i, with i = 1, 2,…, N. The probability of a particular scenario is represented by p_i , which the conditional probabilities of all the nodes over the entire path. The nomenclature used for the model and the steps involved in the process is discussed below:

Table 27: Nomenclature used for scenario reduction

$\boldsymbol{S}$	**Number of scenarios in the original set**
$\boldsymbol{s_i}$	Scenario k
$\boldsymbol{p_i}$	Probability of scenario k
$\boldsymbol{\eta}(.)$	Non-negative function of L_2 norm
$\boldsymbol{J^{[s]}}$	Unselected scenario set till the s^{th}iteration
$\boldsymbol{c_{i,u}^{[s]}}$	Distance between scenario k and u at s^{th} iteration
$\boldsymbol{Z_u^{[s]}}$	Total weighted distance of each scenario k with other scenarios at s^{th} iteration
$\boldsymbol{\Omega'}$	Selected subset from the original set of scenarios

Procedure of FFS:

Step 1. Compute the distance of the scenario pairs for $S = 1$:

$$c_{i,u}^{[1]} = \eta(\alpha_i, \alpha_u), \quad i, u = 1, \dots, S$$

Step 2. Compute the weighted distance of each scenario to the other scenarios:

$$Z_u^{[1]} = \sum_{i \neq u} p_i * c_{i,u}^{[1]}, \quad u = 1, \ldots\ldots, S$$

Choose $u_1 = arg \min_{u \in \{1,\ldots\ldots,S\}} Z_u^{[1]}$

Set $J^{[1]} = \{1, \ldots\ldots, S\} \backslash \{u_1\}$

Step 3. Update the distance matrix using the scenario selected in the previous step.

Let $S = S + 1$; compute:

$$c_{i,u}^{[s]} = min\left[c_{i,u}^{[s-1]}, c_{i,u_{s-1}}^{[s-1]}\right], \quad i, u \in J^{[s-1]}$$

and

$$Z_u^{[s]} = \sum_{i \in J^{[s-1]} \backslash u} p_i * c_{i,u}^{[s]}, \quad u \in J^{[s-1]}$$

choose

$$u_s = arg \min_{u \in J^{[s-1]}} Z_u^{[s]}$$

set $J^{[s]} = J^{[s-1]} \backslash \{u_s\}$

Step 4. If the number of selected scenarios is less than the required number, return to Step 2.

Step 5. Add to the probability of each selected scenario the sum of the probabilities of all unselected scenarios that are near it; ie., $q_j = p_j + \sum_{i \in L(j)} p_i$, for any $j \in \Omega'$ and $L_{(j)} = \{i \in \Omega \backslash \Omega', j = j(i)\}$, $j_i = arg \min_{j \in \Omega'} \eta(\alpha_i, \alpha_u)$ for any $i \in \Omega \backslash \Omega'$.

Using the method discussed above, the 5^6 scenarios that were generated for demand and yield uncertainties were reduced to 10 most representative scenarios for each uncertainty considered in the study. The two-stage stochastic programming model was finally run for 100 combined scenarios of kit demand and worker yield uncertainties. The results and the findings obtained for both the deterministic and stochastic models are discussed in the sections below.

3.4.3 Analysis for the deterministic case

The deterministic model for the lot-sizing and scheduling problem was run for four different values of the maximum allowed production quantity. The implementation results are summarized in Table 28.

It can be observed that the overall kitting costs keep decreasing as the parameter of maximum allowed production quantity ($q_{i,t}$) is relaxed. This is because, as the maximum production quantities increase, more kits are produced during regular time production which is cheaper than overtime production. The lowest kitting cost is obtained for the case of 120% as in this case more kits are produced beforehand and stored in the inventory to meet the future customer demand. Even though this increases the inventory costs, it helps in saving more on the setup costs thus decreasing the overall costs. It can be observed that the overtime production cost is associated only with the first case as overtime production is no longer required as the maximum production quantity is made more lenient. There is no inventory or backorder cost involved when the maximum allowed production quantity equals the kit demand as more labors are employed to do the kitting process that increases the kitting cost. The regular time production cost remains the same for the first two cases showing that the parameter $q_{i,t}$ has no effect on the production costs for these two cases.

Table 28: Summary of results from the deterministic model (Cost in $)

Max production quantity- % of mean demand	Totals						
	Regular time production cost	Overtime production cost	Setup cost	Labor cost	Inventory cost	Backorder cost	Overall cost
90%	892877.27	144330.91	504.90	139104.00	24.43	5975.96	1182817.46
100%	992085.85	0.00	504.90	145152.00	0.00	0.00	1137742.75
110%	989097.87	0.00	504.90	139104.00	27.67	5975.96	1134710.40
120%	989097.87	0.00	429.17	139104.00	46.08	5975.96	1134653.07

Even though some of the costs of the four cases studied are similar, the kitting sequences that is resulted are different for different cases. The kitting sequence resulted for the first case with the parameter $q_{i,t}$ set to 90% of the mean kit demand is shown in Table 29. Product P_3 is manufactured first in the time period T_1, followed by products P_2 and P_1. The setup of product P_1 will be carried over and will become the first setup of period T_2, as it is the last type of product to be manufactured in period T_1 . The time capacity and the maximum allowed production quantity limits the regular time produc tion quantity.

Table 29: The kitting sequence from the deterministic model (90% of mean demand)

Time Period/ Kit	T_1	T_2	T_3	T_4	T_5	T_6
Kit1	2	2	2	3	1	2
Kit2	1	3	1	2	3	1
Kit3	3	1	3	1	2	3
Kt	8	8	8	7	8	7

3.4.4 Analysis for the stochastic case

Similar to the deterministic case, the two-stage stochastic programming model was analyzed for the four cases of the parameter $q_{i,t}$, which is the maximum production quantity allowed in the time period *t.* The summary of the results obtained is included in Table 30. The uncertainties that were considered for this study include kit demand uncertainty and kitting worker yield uncertainty. To evaluate the performance of the two-stage stochastic programming model and to compare the results from the deterministic and stochastic models, we use the following metrics: Expected Value solution (EV), Wait and See solution (WS), Recourse Problem solution (RP), Expected results of EV solution (EEV), Value of Stochastic solution (VSS), and Expected Value of Perfect Information (EVPI).

In wait and see situations the decision maker makes no decisions until all random variables in the model are realized. These solutions are called WS solution in literature. The stochastic programming solution is the RP solution. In real life problems, it is often important to evaluate the tradeoff between investing in better forecasting technology or to make decisions with the current information on hand. The EVPI metric is used for determining the worth on collecting additional information. It is the difference between the solutions RP and WS where the order of the metrics depends on whether the problem is a maximization or a minimization problem. The EV solution is obtained by using the expected values of the parameters in the stochastic scenarios as the numerical values in the deterministic case. The solution obtained by applying the decisions in the deterministic case to stochastic environment is the expected results of EV solution otherwise known as the EEV solution. The worth of using a stochastic model over a deterministic one is measured using the metric VSS. It is calculated using a four-step process. First the mean-value problem is solved to get the first stage solutions. Next, the problem is solved for all the scenario with the first stage decisions fixed. Thirdly, a weighted average of the optimal objective value of each scenario is taken to get the EEV solution. Finally, the difference between EEV and RP gives the VSS solution since the problem dealt with is a minimization problem.

The comparisons of the test results are shown in **Error! Reference source not found.** and **Error! Reference source not found.**. The values of the RP solution decreases as the parameter of $q_{i,t}$ is relaxed because of increased flexibility and availability of production resources. As expected, the WS solutions are observed to be lower than the RP solutions. There is an increasing trend in the EVPI value over the first three cases studied. This is because investing in better forecasting will be more worthwhile when the production resources are abundant. But a decrease in the value of EVPI is observed from the 3rd case to the 4th case, that shows that beyond a certain limit it is not worth to pay for more accurate information as the demand could be met with the available resources. From **Error! Reference source not found.**, EV solutions have the least cost among the four metrics it is compared with as its values are obtained by eliminating the uncertainties from the models. Also, it can be observed that the EEV solutions are having the highest cost as they are the expected value solutions of EV.

Table 30: Summary of the results from the stochastic model (Cost in $)

Max production quantity- % of mean demand	**EV**	**WS**	**RP**	**EEV**	**EVPI**	**VSS**
90%	1350134.15	1790520.44	1811583	1811583	21062.466	0
100%	1258534.04	1580985.66	1611270	1708847	30284.565	97577.1
110%	1227312.88	1407629.16	1446663	1607540	39033.86	160877
120%	1218498.42	1326440.77	1362434	1649444	35993.472	287010

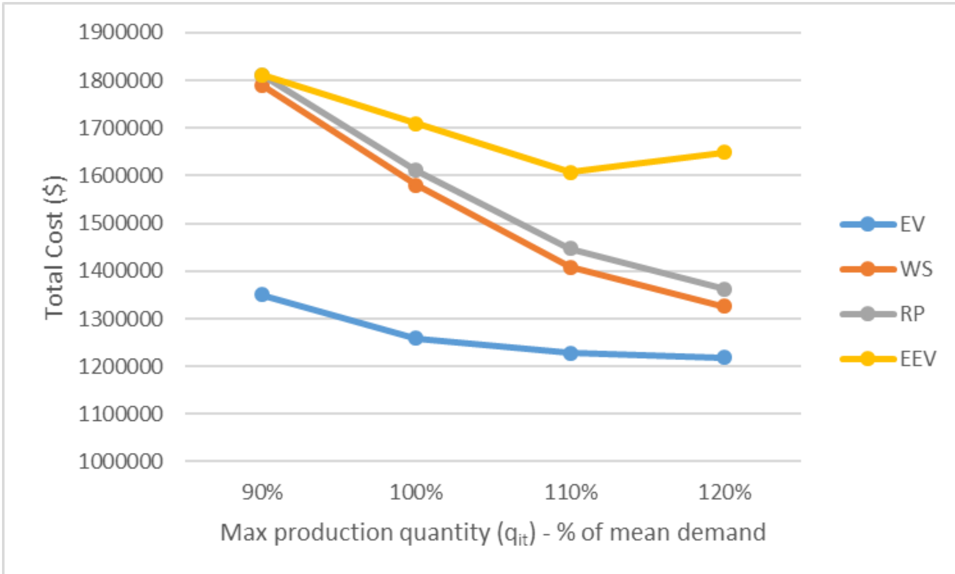

Figure 14: Comparison of test results for four cases of production quantities

From **Error! Reference source not found.**, the VSS values increases drastically as the maximum allowed production

quantities ($q_{i,t}$) increase. As the production capacities and resources increases and as more kits are manufactured, it makes more sense to consider the uncertainties to model the kit lot-sizing and scheduling problem. The kitting sequence and workforce requirement for each time period (k_t) obtained from the two-stage stochastic model for the first case with $q_{i,t}$ set at 90% of mean demand is shown in Table 31. The differences in the sequences resulted from the deterministic and stochastic models shows that the kitting sequence along with the kitting quantities is also sensitive to the uncertainties considered in this study.

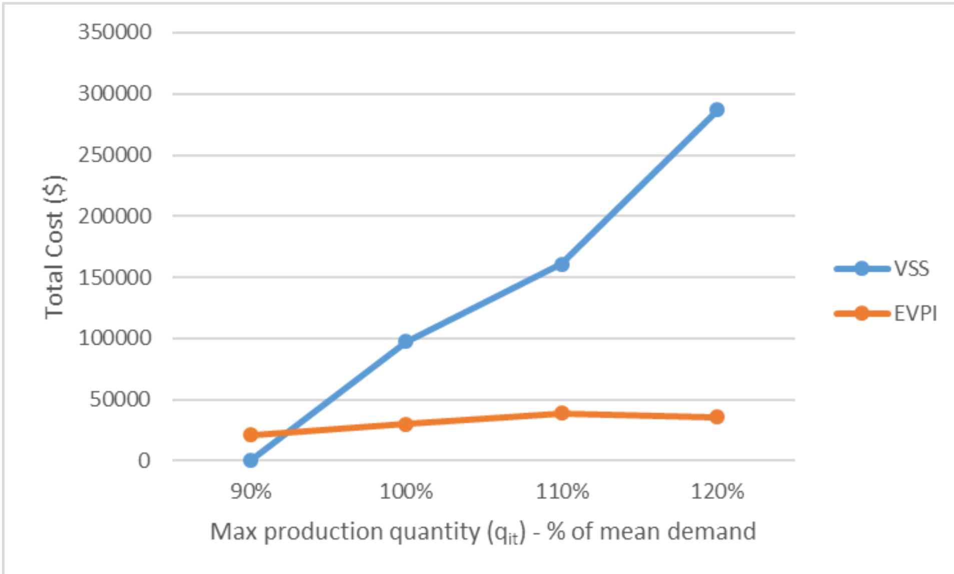

Figure 15: Comparison of VSS and EVPI for four cases of production quantities

Table 31: The kitting sequence from the stochastic model (90% of mean demand)

Time Period/ Kit	T_1	T_2	T_3	T_4	T_5	T_6
Kit1	2	2	2	3	1	2
Kit2	1	3	1	2	3	1
Kit3	3	1	3	1	2	3

Kt	9	9	9	9	9	8

3.5 Conclusion

An assembly line feeding system constitutes an important part of a manufacturing system. The decisions would impact many stakeholders across the manufacturing supply chain. Therefore, it is essential to plan the line feeding activities in the most efficient and cost effective way. In this paper, we presented a two-stage stochastic programming framework to address the problem of lot-sizing and scheduling in a kitting facility under uncertainties in kit demand and worker yield. Both uncertainties were modelled as a dynamic stochastic process and presented as a scenario tree. The first-stage decisions included the baseline production schedule and workforce requirement while the production, inventory, and backorder quantities were the recourse decisions at the second stage.

A case study was conducted to validate the model and derive managerial insights on production planning decision-making under uncertainty. It is observed that uncertainties have significant impact on kitting operations planning decisions and that the proposed two-stage stochastic programming model is robust when compared to the deterministic model, in determining optimal lot sizes and production schedules under uncertainties. Sensitivity analysis was conducted on production capacity parameter and impacts have been analyzed for a variety of scenarios. The insights derived from this study will help in making sensible managerial decisions in an assembly line feeding system supporting a manufacturing shop floor and aid in managing and mitigating the risk exposure of the company’s manufacturing through efficient planning of operations.

In summary, this study highlights the importance of integrating the uncertainties into the decision-making model for production planning under uncertainty. Although we aim to design the decision-making model to reflect the real decision-making scenario in a manufacturing environment, this study is still subject to a few limitations which suggest future research directions. Firstly, besides uncertainty in demand and yield, various other uncertainties exist in manufacturing, such as lead time, and quality uncertainty. More sources of uncertainties need to be considered to better reflect the reality. Secondly, we assume the two sources of uncertainties we considered are independent. In reality, there may be interactions between the uncertain factors, which may lead to dependency considerations. Lastly, the stability of the scenarios generated and the level of approximation of the scenarios obtained through the scenario reduction technique can be studied to investigate the efficiency and effectiveness. We shall address these in future studies.

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CHAPTER 4. CONCLUSION

Production planning is a vital part of a manufacturing process, but the presence of uncertainty makes the decision-making process very complicated. This thesis aims to develop an optimization model framework and apply it to production planning problems in two different stages in a manufacturing supply chain under multiple uncertainties. The proposed framework was demonstrated and validated with two case studies related to an automobile equipment manufacturer.

Our first study involved production planning in a manufacturing unit under multiple sources of uncertainty. The customer demand and raw material quality were the uncertainties considered to come up with optimal decisions on production lot-sizes and sequences. Even though this study captured the interactions between the stages 2 and 3 in a manufacturing supply chain, the impact of these uncertainties in facilities further downstream (stage 1) was not discussed. This extend of influence of these uncertainties on planning decisions made in downstream systems made us curious and motivated us to conduct our second study.

For our second study, we developed a two-stage stochastic programming framework for production planning in a kitting facility under multiple uncertainties. The model developed was applied to a kitting facility and the optimal business decisions were found by considering the demand of the kits and the yield of kitting workers as uncertain parameters. The implementation results obtained from both these studies showed significant difference between the deterministic and stochastic results. Hence, it is safe to conclude that the stochastic models offer solutions that are superior and robust when compared to the deterministic model solutions. It was also evident that the decisions are influenced by the parameter of the maximum allowed of regular time production quantity through the sensitivity analysis that was carried out.

In summary, it has been demonstrated that uncertainties in a manufacturing environment have significant impact on the business decision along the supply chain. It can be observed that considering the uncertainties explicitly in the decision-making models will lead to more robust production planning decisions. Even though an extensive study was conducted, there is still limitations for this study which suggest future research directions. First, the proposed models assume the uncertainties are independent of each other. However, in reality, there may be dependency between the uncertainties which may require new model formulations and solution technique studies. Second, other sources of uncertainties such as supplier lead time, and machine failure uncertainty can be considered when formulating the decision-making model. Lastly, more sophisticated scenario generation and scenario reduction methods for the stochastic programming model can better approximate and represent the uncertainties. Additional analysis can be conducted on uncertainty representation. We shall address these in the future studies.