Applications of optimization under uncertainty methods on power system planning problems

by

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ABSTRACT

This dissertation consists of two published journal paper, both on transmission expansion planning, and a report on distribution network hardening.

We first discuss our studies of two optimization criteria for the transmission planning problem with a simplified representation of load and the forecast generation investment additions within the robust optimization paradigm. The objective is to determine either the minimum of the maximum investment requirement or the maximum regret with all sources of uncertainty explicitly represented. In this way, transmission planners can determine optimal planning decisions that are robust against all sources of uncertainty. We use a two layer algorithm to solve the resulting trilevel optimization problems. We also construct a new robust transmission planning model that considers generation investment more realistically to improve the quantification and visualization of uncertainty and the impacts of environmental policies. With this model, we can explore the effect of uncertainty in both the size and the location of candidate generation additions. The corresponding algorithm we develop takes advantage of the structural characteristics of the model so as to obtain a computationally efficient methodology. The two robust optimization tools provide new capabilities to transmission planners for the development of strategies that explicitly account for various sources of uncertainty.

We illustrate the application of the two optimization models and solution schemes on a set of representative case studies. These studies give a good idea of the usefulness of these tools and show their practical worth in the assessment of "what if" cases. We compare the performance of the minimax cost approach and the minimax regret approach under different characterizations of uncertain parameters. In addition, we also present extensive numerical studies on an IEEE 118-bus test system and the WECC 240-bus system to illustrate the effectiveness of the proposed decision support methods. The case study results are particularly useful to understand the impacts of each individual investment plan on the power system's

overall transmission adequacy in meeting the demand of the trade with the power output units without violation of the physical limits of the grid.

In the report on distribution network hardening, a two-stage stochastic optimization model is proposed. Transmission and distribution networks are essential infrastructures to modern society. In the United States alone, there are there are more than 200,000 miles of high voltage transmission lines and numerous distribution lines. The power network spans the whole country. Such vast networks are vulnerable to disruptions caused by natural disasters. Hardening of distribution lines could significantly reduce the impact of natural disasters on the operation of power systems. However, due to the limited budget, it is impossible to upgrade the whole power network. Thus, intelligent allocation of resources is crucial. Optimal allocation of limited budget between different hardening methods on different distribution lines is explored.

CHAPTER 1. INTRODUCTION

1.1 Overview

In this proposal, I present research during my PhD career. My research has been focusing on the application of optimization under uncertainty techniques on problems arise in power systems. Two main topics are covered in this proposal:

(1) The application of robust optimization on transmission expansion planning problems.

(2) The application of stochastic programming in the planning of the hardening strategies of distribution networks.

This chapter presents a brief introduction to the discussed topics. Chapter 2 and Chapter 3 contains my two papers on robust transmission expansion planning. My report for the third part of my research— power system resiliency planning, is presented in Chapter 4.

1.2 Optimization Under Uncertainty Methods

For many optimization problems, the input parameters of a developed model are assumed to be deterministic. However, in more and more real world applications, this is no longer true. In such cases, the optimal solution obtained by a deterministic model may not be optimal or even feasible. As such, methods of optimization under uncertainty have been garnering a lot of attention in fields including power system. Currently, two of the most popular ways to deal with uncertainty in mathematical programming is robust optimization and stochastic programming. Both methods have been widely applied in power system [61].

The problems solved in this dissertation all fall roughly into the category of two-stage optimization models. The two stages refer to the two decision making process before and after uncertainties are observed. If no uncertainty is considered, and suppose b denote the sources of uncertainty, x denote the first-stage decisions that should be made before uncertainty realization and y denote the second-stage decisions that can only be made after uncertainty is observed, a typical deterministic linear programming model can be formulated as follows:

$$\min_{x,y} \quad c^\top x + d^\top y \tag{1.1}$$

$$\text{s.t.} \quad x \in \mathcal{X} \tag{1.2}$$

$$Ax + By \leq \bar{b} \tag{1.3}$$

where $\bar{b}$ represent the mean of the uncertainty parameters.

In robust optimization, it is usually assumed that uncertain parameters in an optimization model can be described by polyhedral sets [11, 25]. Such uncertainty sets can be constructed by using information like the lower and upper bounds of the uncertain parameters. Then the objective value under the worst-case scenario is optimized. Although robust optimization is relatively conservative and can lead to worse performance in average, it can guarantee the feasibility of the optimal solution and provide a bound on the objective value. Typically, robust optimization can be formulated as follows:

$$\min_{x \in \mathcal{X}} \quad c^\top x + \max_{b \in \mathcal{U}} \min_{y \in \mathcal{Y}(x,b)} d^\top y \tag{1.4}$$

where $\mathcal{U}$ is the uncertainty set and $\mathcal{Y}(x,b) = \{y | Ax + By \leq b\}$ is the feasible set for second-stage decision variables. From this formulation we can see that the uncertainty parameter is treated as a variable to identify the worst-case scenario.

On the other hand, stochastic programming assumes the accurate probability distribution of the uncertain parameters can be obtained. The expected value of the objective function over the aforementioned distribution is optimized. Scenarios are usually generated based on the probability distributions to represent the uncertain data. In order to guarantee feasibility and good performance, a reasonable number of scenarios should be generated, which leads to large problem size. Many algorithms can be applied to solve such large-scale optimization problems, including Benders' decomposition and progressive hedging [43]. Stochastic programming models can be formulated as follows:

$$\min_{x\in\S} +\mathbb{E}[\mathcal{Q}(x,\tilde{b})] \tag{1.5}$$

where

$$\begin{aligned} \mathcal{Q}(x,b) = \quad & \min \quad c^\top y && (1.6) \\ & \text{s.t.} \quad Ax + By \leq \tilde{b} && (1.7) \end{aligned}$$

Both robust optimization and stochastic programming have their strengths and weaknesses. In this dissertation they are applied to two different types of problems based on the requirements of the applications.

1.3 Transmission Expansion Planning

Transmission planning (TP) is one of the key decision processes in the power system production pipeline. Electricity transmission networks are responsible for reliably and economically delivering power from generators to consumers. Thus, a robust and resilient transmission network is essential to the operation of power systems for decades to come. Good transmission investments have many benefits, including satisfying increasing demand, promoting social welfare, improving system reliability and resource adequacy, etc. Transmission planning is also very challenging due to various sources of uncertainty that planners need to consider. Besides uncertainty sources, such as demand variations and renewable energy intermittency faced by system operators in short-term scheduling, planners also need to take into consideration uncertainty of policy changes, technological advancements and natural disasters [61]. Such sources of uncertainty cannot be characterized in terms of analytical probability distributions. For example, to cope with challenges of climate change, the power generation industry is facing increasing pressure to reduce greenhouse gas emissions. In addition, a large amount of coal plants are anticipated to retire in response to the implementation of the EPA clean power plan [19], and their replacement in part by gas-fired plants. Moreover, after the restructuring of the power system, certain behavior by generation companies introduces additional uncertainty in power system planning in general and TP in particular. However, research on the impacts of uncertain future generation developments on TP is scarce in the decentralized decision making environment in competitive market. As such, the study of this topic is warranted.

Conventional transmission planning methodologies are typically deterministic and some make use of ad-hoc approaches to deal with uncertainty. These methodologies have served the industry relatively well in the vertically-structured industry. Present realities brought about, by the open access regime and the advent of competitive electricity markets, increased wind and solar resource outputs. Their highly time varying, uncertain and intermittent patterns, combined with the more dynamically varying and uncertain loads and a world-ranging environmental policy initiative have resulted in a more volatile utilization of transmission facilities. Critically needed is the improved planning methodologies that can effectively accommodate the realities of the new environment. Transmission planning, by its very nature, is subject to a wide range of sources of uncertainty that must be taken into account. The new regime indicates many additional sources of uncertainty and numerous complications that must be considered.

Over a 10-20 year planning horizon, major sources of uncertainty faced by transmission planning decision makers include load growth, fuel costs, climate change events, atmospheric conditions, variable generation outputs, generation expansion/retirement events, regulatory and legislative developments, technology breakthroughs and market outcomes. When the impacts of transmission plans are assessed, such studies must be carefully performed with the explicit representation of the sources of uncertainty, in light of the overarching consequences on the power system. These sources of uncertainty may be classified into two categories: aleatoric and epistemic sources of uncertainty. Sources of aleatoric uncertainty cannot be replaced by more accurate measurements but they can be statistically quantified. For instance, the coincidental uncertainty with respect to the electrical system state and network topological state can be both probabilistically and stochastically modeled and quantified if the relevant sets of input are made available. On the other hand, sources of epistemic uncertainty are due to information we may in principle know, but do not in actual practice. Often, such sources are also called sources of systematic uncertainty. For instance, it is hard to assign meaningful probabilities to sources of uncertainty associated with government policies, technology breakthroughs, and investment in renewable energy generation.

Electricity demand is expected to grow continually and steadily for decades to come. In the EIA Annual Energy Outlook 2013, forecasts indicate that electricity consumption will increase by 28% from 2011 to 2040 at an average of about 1% per annum rate[24]. To accommodate such growth in electricity demand, new capacity of electricity generation must be built at a pace that meets or exceeds the growth of demand and retirement of old generators to maintain power system reliability. The additional demand and capacity require commensurate transmission capacity.

With the additional sources of uncertainty and complications facing transmission planners and the need to accommodate increasing demand as well as more variable generation capacity, there is a critical need for new TP methodologies that can address the aforementioned concerns. The first two papers in this dissertation address this problem.

1.4 Power System Resiliency Planning

In addition to transmission expansion planning, resilience planning is also an important step to improve the security of power systems. In the United States, there are more than 200,000 miles of high voltage transmission lines and numerous distribution lines that span the whole country. Such vast networks are vulnerable to disruptions caused by natural disasters including seismic activities and extreme climate events. Due to climate change, frequency of natural disasters such as floods, droughts, hurricanes and other weather activities is expected to rise. Improving power system security and resilience not only would reduce the probability of disrupted operations, but also could be instrumental to the recovery of communities from catastrophes.

Resilience is defined as a combination of two capabilities: the inherent ability of a system to cope with disruption with the networks topology and redundancies, and its ability to recover from disruptions. Both abilities need to be considered in order to improve system resilience. One illustrative example is underground power lines, even though they are invulnerable to many natural disasters including hurricanes, wild fires and snow storms, they are more difficult to repair. In a region prone to earthquakes, even with low probability, undergrounding power lines may not be a good choice.

According to [54], hardening is one of the most effective approaches to improve the resilience of power grids. Methods of hardening include upgrading power poles and structures with stronger materials, vegetation management and undergrounding utility lines. In addition to hardening, investment can also be devoted to reduce response time and expedite infrastructure recovery after the occurrence of disruptions. Options include response planning, training of personnel and improved outage notification ability. Such investments are usually constrained by a limited budget. Thus, hardening of the entire grid simultaneously could be prohibitively expensive and therefore is unrealistic. It is crucial to allocate the limited resources to the most vulnerable components or to the power lines with the largest impact if affected. The last paper uses stochastic programming to solve the budget allocation problem in power system hardening.

CHAPTER 2. ROBUST OPTIMIZATION FOR TRANSMISSION EXPANSION PLANNING: MINIMAX COST VS. MINIMAX REGRET

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Abstract

Due to the long planning horizon, transmission expansion planning is typically subjected to a lot of uncertainties including load growth, renewable energy penetration, policy changes, etc. In addition, deregulation of the power industry and pressure from climate change introduced new sources of uncertainties on the generation side of the system. Generation expansion and retirement become highly uncertain as well. Some of the uncertainties do not have probability distributions, making it difficult to use stochastic programming. Techniques like robust optimization that do not require a probability distribution became desirable. To address these challenges, we study two optimization criteria for the transmission expansion planning problem under the robust optimization paradigm, where the maximum cost and maximum regret of the expansion plan over all uncertainties are minimized respectively. With these models, our objective is to make planning decisions that are robust against all scenarios. We use a two layer algorithm to solve the resulting tri-level optimization problems. Then, in our case studies, we compare the performance of the minimax cost approach and the minimax regret approach under different characterizations of uncertainties.

2.1 Nomenclature

Sets and indices

$\mathcal{U}$	The polyhedron uncertainty set of demand and new generation capacity profile
$\mathcal{I}$	Set of nodes
$\mathcal{L}$	Set of existing transmission lines
$\mathcal{N}$	Set of candidate transmission lines
$\mathcal{T}$	Set of years in the planning horizon
$\mathcal{M}$	Set of load blocks
$\mathcal{K}$	Set of technology types

Parameters

$P_{i,k,t}$	Capacity of existing generator of technology k at node i at time t
$c^{\mathrm{T}}_{ij,t}$	Cost of building the new transmission line ij at time t
$c^{\mathrm{L}}_{i,t}$	Cost of load curtailment at node i at time t
$c^{\mathrm{P}}_{i,k,t}$	Cost of power production of technology k at node i
$f^{\mathrm{C}}_{k,t,m}$	Average capacity factor of generation technology k at year t load block m
$F^{\max}_{ij}$	The maximum power flow on transmission line ij
B_{ij}	Susceptance of transmission line ij
M	A big constant used to linearize the power flow constraint
$\bar{d}_{i,t,m}$	The average amount of demand at year t load block m at node i
$\bar{P}^{\mathrm{N}}_{i,k,t}$	The average amount of generation expansion of technology k at node i at time t
$\theta_{\min}$	The lower bound of voltage angles
$\theta_{\max}$	The upper bound of voltage angles
λ	Market interest rate (Inflation included)
$P^{\mathrm{N,min}}_{i,k,t}$	Minimum amount of new generation at a node
$P^{\mathrm{N,max}}_{i,k,t}$	Maximum amount of new generation at a node
$P^{\mathrm{N,min}}$	Lower bound on the total amount of generation at all the nodes
$P^{\mathrm{N,max}}$	Upper bound on the total amount of generation at all the nodes

Decision Variables

x_{ij}	Binary variables indicating whether a transmission line is built
$P^{\mathrm{N}}_{i,k,t}$	The amount of new generation capacity of technology k at node i at time t. $P^{\mathrm{N}}_{i,k,t}$ is negative in the case of power plant retirement
$f_{ij,t,m}$	Power flow from node i to node j at year t load block m
$p_{i,k,t,m}$	Power production of technology k at node i at year t load block m
$r_{i,t,m}$	The amount of load shedding at node i at year t load block m
$\theta_{i,t,m}$	Voltage angle at node i at year t load block m
$d_{i,t,m}$	Demand at year t load block m at node i

2.2 Introduction

Transmission expansion planning is very challenging due to various uncertainties planners need to consider. Besides typical high-frequency uncertainties including demand variations and renewable energy intermittency faced by system operators in short-term scheduling, planners also need to take into consideration low-frequency uncertain events like policy changes, technological advancements, natural disasters, etc. [61]. Such uncertainties cannot be characterized by probability distributions. For example, to cope with challenges of climate change, the power generation industry is facing increasing pressure to reduce greenhouse gas emissions. In addition, a large amount of coal plants are anticipated to retire in response to government regulations and fuel price changes [19], replaced partially by gas-fired plants. Many of such retirements could be announced on relative short notice and unexpected by the system operator. Moreover, after the deregulation of the power system, strategic behavior of generation companies in generation expansion becomes an uncertain factor as well.

Currently, the most common practices in dealing with uncertainties in optimization include stochastic programming and robust optimization. In stochastic programming, scenarios are generated based on a certain probability distribution of the uncertain data. The weighted sum of the total cost under different scenarios is usually optimized. Stochastic programming has been successfully applied to power system capacity expansion planning problems. In [59, 36, 44, 26], uncertainties including load prediction inaccuracies, transmission line and generator outages, generation and transmission line capacity factors are considered. All of those uncertainties can be described by probability distributions and can be effectively modeled with stochastic programming techniques. However, most of those works only focus on uncertainties in the operation phase and do not address uncertainties in the planning phase. The reason is that it is difficult to obtain probability distributions of the non-random [17] or epistemic uncertainties caused by factors including policy changes, investment behavior of market players, etc. In this paper, besides load uncertainty, we also take into consideration uncertain generation expansion behavior of generating companies and coal power plants retirement.

As an alternative tool to address uncertainties, robust optimization [11, 25, 14] can avoid some of the difficulties arising from stochastic programming approaches. With robust optimization, uncertainty is described by parametric sets, which contain an infinite number of scenarios. Such uncertainty sets can be constructed with information like the lower and upper bounds of a random variable, which are much easier to derive than probability distributions. This approach can identify a set of decisions that is robust under the worst-case scenario contained in the uncertainty sets, which is desirable in planning problems where reliability is important. The conservativeness of the solution can be adjusted by changing the uncertainty sets [13], depending on how much uncertainty is desired to capture. Robust optimization has been applied to many problems in power systems. In [62], robust unit commitment with the $n-k$ security criterion is studied. The problem is then reformulated into a single-level problem with the help of dual variables. In [12, 32], the two-stage robust unit commitment problems under uncertainty are studied. Both papers propose to use Bender's Decomposition to solve the problem. A similar model is used in [74] where demand response is considered. A robust minimax regret model is proposed in [31] to solve the unit commitment problem under uncertainty. However, all those works study operation problems. In long term power system planning problems where more uncertainties need to be considered, application of robust optimization is limited. In [29], a robust transmission expansion planning model is proposed considering load and generation uncertainty. Bender's decomposition is also used.

In this paper, we propose two robust optimization models to address two main sources of uncertainty: load and generation expansion behavior of generating companies. Two criteria, minimax cost (MMC) and minimax regret (MMR), are used as the objective of our models. The MMC criterion has been used widely in robust optimization applications [12]. The MMR criterion is considered in [31] for the unit commitment problem. In comparison with the MMC criterion, it is concluded that MMR outperforms MMC for certain unit commitment problems. However, the same conclusion may not apply to transmission expansion planning problems due to the different structures of such problems. In [46], regret is considered as one of the objectives in a multi-objective optimization framework. It is applied to handle non-random uncertainties in [23, 7]. Both criteria use the performance of a decision under the worst possible scenario as the objective for optimization, but their main difference is how the "worst scenario" is defined. The MMC criterion focuses on the cost associated with a decision under a scenario, so the scenario that results in the highest cost is identified as the worst scenario. On the other hand, the MMR criterion defines the worst scenario as the one that leads to the highest regret for the decision maker. For a given decision d^0 and a given scenario s^0, the regret is the highest potential cost savings had the decision maker known that scenario s^0 would occur and made a decision accordingly. More rigorously,

$$R(d^0, s^0) = C(d^0, s^0) - \min_{d \in \mathcal{D}} C(d, s^0),$$

where $C(d^0, s^0)$ is the cost associated with d^0 and s^0, $\mathcal{D}$ is the set of all feasible decisions, and $R(d^0, s^0)$ is the regret associated with d^0 and s^0. Using these notations, the MMC and MMR criteria can be respectively formulated as

$$\min_{d \in \mathcal{D}} \left\{ \max_{s \in \mathcal{U}} C(d, s) \right\} \text{ and}$$

$$\min_{d \in \mathcal{D}} \left\{ \max_{s \in \mathcal{U}} R(d, s) \right\} = \min_{d \in \mathcal{D}} \left\{ \max_{s \in \mathcal{U}} \left\{ C(d, s) - \min_{d' \in \mathcal{D}} C(d', s) \right\} \right\}.$$

We use two simple examples in Tables 2.1 and 2.2 to demonstrate the differences between MMC and MMR. In the first example, under MMC, D_2 is the optimal decision because its worst scenario cost, \$8, is lower than that of D_1, \$9. Under MMR, D_1 is the optimal decision because its worst scenario regret, \$1, is lower than that of D_2, \$5. The argument for MMR is that since scenario S_1 is a "bad" scenario anyway because D_1 and D_2 both lead to higher costs in S_1 than S_2, the difference between the costs associated with the two decisions, which is the regret, may provide more information for decision making than the absolute value of the cost itself. In the second example, decision D_3 is obviously a bad choice because of its high

Table 2.1 Motivating Example for the Minimax Regret Model

Cost / Regret	Decision D_1	Decision D_2
Scenario S_1	\$9 / \$1	\$8 / \$0
Scenario S_2	\$2 / \$0	\$7 / \$5

cost in scenario S_4. Decision D_5 will be selected under MMC because its worst cost, \$18, is lower than that of D_3, \$40, and D_4, \$19. Under MMR, decision D_4 will be selected since its worst-case regret is \$13 while the regret of D_5 is \$14. We argue the MMC solution D_5 is better in this example because it is only slightly worse than the MMR decision D_4 in terms of regret in scenario S_3 only because of the existence of decision D_3, which cannot be selected anyways, but has a much lower cost in scenario S_4.

Table 2.2 Motivating Example for the Minimax Cost Model

Cost / Regret	Decision D_3	Decision D_4	Decision D_5
Scenario S_3	\$4 / \$0	\$16 / \$12	\$18 / \$14
Scenario S_4	\$40 / \$34	\$19 / \$13	\$6 / \$0

From the previous two examples, we can see that there are no clear cut answers as to which criterion is superior. Each of them has advantages and disadvantages. The examples above can shed some light on which criterion may perform better in what situations. In the first example, both decisions perform better in one scenario and worse in the other. In this case, it makes sense to use MMR as the criterion because the MMC criterion is too conservative and does not consider non-extreme scenarios. On the other hand, in the second example, there exists a very risky decision that performs well under one scenario and extremely poorly under the other. In a planning problem where risks should be controlled, such decisions are usually not desirable, but they may affect the maximum regret of other decisions and distort the final decision.

The two-stage structure of our robust optimization models can capture both the planning and operation stages of the transmission expansion planning problem very well. They can be formulated as special cases of trilevel optimization problems. However, due to their non-linear, non-convex structure, they are very difficult to solve. In previous researches [12, 32], the authors use Bender's decomposition to reformulate the problem into a master problem and a bilinear subproblem, which is then solved with outer approximation. However, the outer approximation approach cannot handle the binary variables in the subproblem when the MMR criterion is used. In [31], statistical upper bounds are used to complement the outer approximation approach. We propose a two-layer algorithm where we decompose our problem into a master problem and a bilevel subproblem. The master problem is updated with a branch and cut type procedure, where new constraints and variables are iteratively generated and then solved as a mixed-integer program. This algorithm is a special case of the bilevel optimization algorithm [67]. Similar algorithms are proposed in [74, 73]. It works faster than the traditional Bender's decomposition approach with the use of primal information instead of dual variables. The subproblem is a mixed-integer bilevel optimization problem, which is more difficult to solve. In [49], the difficulty of solving a bilevel linear optimization program is discussed and several heuristics are proposed. We use the Karush-Kuhn-Tucker (KKT) conditions [15] to reformulate the bilevel problem into a single level problem with complementarity constraints, which is then reformulated into a mixed-integer programming problem [28].

The contribution of this paper can be summarized as follows. Firstly, we propose two robust optimization models that use two criteria to assess decisions. Both load uncertainties and generation expansion uncertainties are considered. Then, effective algorithms are proposed to solve the resulting trilevel optimization problems. Finally, in our case studies, the two models and their corresponding algorithms are tested with a modified IEEE 118-bus test system. We then analyze the results and compare the performances of the expansion plans under different scenarios.

The rest of the paper is structured as follows. Section 2.3 presents the mathematical formulation of the transmission expansion planning problems. In section 2.4, we present the trilevel optimization algorithm. Numerical results are presented in section 2.5. Finally, section 2.6 concludes this paper.

2.3 Model Formulation

Transmission expansion planning problems are usually modeled as two-stage problems to account for the long planning horizon, where the expansion planning decisions are made in the first stage when there is limited information on uncertain parameters and the operational decisions are made in the second stage after uncertainty realizations are observed. In this section, we first present the deterministic model, and then introduce two robust optimization models under the MMC and MMR criteria.

2.3.1 Deterministic model

In the deterministic model, consideration of uncertainty is avoided by assuming perfect information for all parameters. For example, in the following deterministic model, the load is fixed as $\bar{d}$ and the new generation capacity is fixed as $\bar{P}^{\mathrm{N}}$.

$$\min \sum_{ij,t} c^{\mathrm{T}}_{ij,t} x_{ij} + \sum_{i,k,t,m} (1+\lambda)^t (c^{\mathrm{P}}_{i,k,t} p_{i,k,t,m} + c^{\mathrm{L}}_{i,t} r_{i,t,m}) \tag{2.1}$$

$$\text{s. t.} \sum_{k} p_{i,k,t,m} + \sum_{j} f_{ji,t,m} - \sum_{j} f_{ij,t,m} = \bar{d}_{i,t,m} - r_{i,t,m}, \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M} \tag{2.2}$$

$$f_{ij,t,m} - B_{ij}(\theta_{i,t,m} - \theta_{j,t,m}) - (1 - x_{ij})M \leq 0, \forall ij \in \mathcal{N} \tag{2.3}$$

$$B_{ij}(\theta_{i,t,m} - \theta_{j,t,m}) - f_{ij,t,m} - (1 - x_{ij})M \leq 0, \forall ij \in \mathcal{N} \tag{2.4}$$

$$f_{ij,t,m} = B_{ij}(\theta_{i,t,m} - \theta_{j,t,m}), \forall ij \in \mathcal{L} \tag{2.5}$$

$$f_{ij,t,m} \leq F^{\max}_{ij} x_{ij}, \forall ij \in \mathcal{N} \tag{2.6}$$

$$-f_{ij,t,m} \leq F^{\max}_{ij} x_{ij}, \forall ij \in \mathcal{N} \tag{2.7}$$

$$f_{ij,t,m} \leq F^{\max}_{ij}, \forall ij \in \mathcal{L} \tag{2.8}$$

$$-f_{ij,t,m} \leq F^{\max}_{ij}, \forall ij \in \mathcal{L} \tag{2.9}$$

$$p_{i,k,t,m} \leq f^{\mathrm{C}}_{k,t,m}(P_{i,k,t} + \bar{P}^{\mathrm{N}}_{i,k,t}) \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M}, k \in \mathcal{K} \tag{2.10}$$

$$\theta_{\min} \leq \theta_{i,t,m} \leq \theta_{\max}, \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M} \tag{2.11}$$

$$x \text{ binary} \tag{2.12}$$

The objective function (2.1) is the transmission capital investment cost and total operational cost (including cost of power production and load shedding) over the planning horizon. This model is a static model, in which the total operational cost over the planning horizon is estimated by extrapolating from $|\mathcal{T}|$ years. A similar approach has been used by several other related studies [36, 26, 35]. Constraint (2.2) requires that the net influx at a node should be equal to the net outflow. Constraints (2.3) and (2.4) are equivalent to the equation $f_{ij,t,m} = x_{ij}B_{ij}(\theta_{i,t,m} - \theta_{j,t,m})$, which is nonlinear and complicates the model. We introduce the constant M to linearize this equation [35]. When $x_{ij} = 1$, then the two constraints are reduced to $f_{ij,t,m} = B_{ij}(\theta_{i,t,m} - \theta_{j,t,m})$, where the value of M does not matter. When $x_{ij} = 0$, then we need to make sure that M is large enough so that no additional constraints are imposed. On the other hand, if M is too large, it may cause computational difficulties. In our experiments, we set it to be ten times the largest value of $F_{ij}^{\max}$. Equation (2.5) calculates the power flow on existing transmission lines. Constraints (2.6)- (2.9) dictate that the power flow on transmission lines does not exceed their limits. Constraint (2.10) specifies the generation capacity on each node. Constraint (2.11) limits the range of phase angles at a node.

To facilitate algorithmic development and simplify the notations, we abstract the deterministic model as follows:

$$\min_{x,z} c^\top x + b^\top z \tag{2.13}$$

$$\text{s.t.} \quad Ax + C_1 z \leq g_1 \tag{2.14}$$

$$B\bar{y} + C_2 z \leq g_2 \tag{2.15}$$

$$Jz = \bar{d} \tag{2.16}$$

In this more concise abstract formulation, we use x to represent the binary variable indicating whether or not a transmission line should be built, y to represent the amount of new generation and z to represent operational variables including power production, phase angles, power flow and load curtailment. Vectors c and b represent coefficients of variables in the objective function. Matrices A, B, C_1, C_2, J are the coefficients of variables in the constraints. Vectors g_1, g_2 are the right-hand-side parameters in the constraints. Constraint (2.14) corresponds to equations (2.3)-(2.11). Constraint (2.15) corresponds to (2.10). Constraint (2.16) corresponds to (2.2).

2.3.2 The MMC model

In the two-stage MMC model, given the first-stage decisions, the second-stage problem is commonly known as the recourse problem [38], where the optimal operation decisions are identified. The feasible set of the recourse problem is defined as follows:

$$\mathcal{Z}(x,d,y) = \{z : C_1 z \leq g_1 - Ax, C_2 z \leq g_2 - By, Jz = d\}$$

The uncertainty set is defined as

$$\mathcal{U} = \{(d,y) : Q_1 d \leq q_1, Q_2 y \leq q_2\}$$

The matrices Q_1, Q_2 are the coefficients of d and y in the uncertainty set. Vectors q_1, q_2 are the right-hand-side parameters. They can contain information including the lower and upper bounds of the uncertain parameters, the lower and upper bounds of the linear combination of the uncertain parameters, etc. Such information can be obtained from historical data or statistical tests on historical data. In this paper we consider both the uncertainty caused by load forecast and the uncertainty caused by future generation expansion. In addition, other types of uncertainties can be easily plugged into the model without affecting the algorithm.

The MMC model can be formulated as:

$$\min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \min_{z\in\mathcal{Z}(x,d,y)} b^\top z \right\}. \tag{2.17}$$

2.3.3 The MMR model

Unlike the MMC model, the MMR model aims to minimize the worst-case regret under all possible scenarios. Before presenting the MMR model, we first define the feasible set of the perfect information solution $\mathcal{G}(d,y)$ and the perfect information cost $G(d,y)$ as follows:

$$\mathcal{G}(d,y) = \{(\hat{x},\hat{z}) : A\hat{x} + C_1\hat{z} \leq g_1, C_2\hat{z} \leq g_2 - By, J\hat{z} = d\}.$$

$$G(d,y) = \min_{\hat{x},\hat{z}\in\mathcal{G}(d,y)} \left\{ c^\top \hat{x} + b^\top \hat{z} \right\}.$$

We can see that $G(d, y)$ is only dependent on the uncertain parameters (d, y) and can only be known after the uncertainty realizations are observed. We call it the perfect information solution because $G(d, y)$ can only be achieved if perfect information about the uncertainties is available.

Then we can define the MMR model as follows:

$$\min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - G(d, y) \right\} \right\}. \tag{2.18}$$

Comparing the MMC model and MMR model side by side, their similarities are very noticeable. The difference between them lies in their definition of the "worst-case scenario". With the MMC criterion, the worst-case scenario is defined as the scenario with the highest cost, while the MMR criterion defines the worst-case scenario where regret is the highest.

To shed more light on which criterion is more appropriate under different situations, we can classify scenarios into two categories: regretful vs. regretless. We use x^{C} to denote the MMC solution and x^{R} to denote the MMR solution. $\mathcal{R}(x, s)$ is the regret of decision x under scenario s. If $\mathcal{R}(x^{\mathrm{C}}, s) \geq \mathcal{R}(x^{\mathrm{R}}, s)$, then we call scenario s a regretful scenario for decision x^{C}. Otherwise, we say it is regretless. When MMR is used, regret is redistributed among the scenarios. The regretful ones become less regretful and the costs in the regretless scenarios increase. As an uncertainty set consists of both regretful scenarios and regretless scenarios, it is unpractical to predict accurately which type of scenario will occur in the future. However, the classification of scenarios compares and illustrates the advantages of the MMR and MMC approaches for decision-makers to choose the criterion more appropriately for their specific problem. For example, if they are more confident that regretful scenarios will occur, they can choose the MMR criterion. Otherwise, they may choose the MMC criterion.

2.4 Algorithm Development

In this section, we develop a new trilevel optimization algorithm to solve the two robust optimization problems, which we decompose into two levels: the master problem and the subproblem. We first present the algorithm for the MMC model. This algorithm is then modified for the MMR model. Since we use a cutting plane procedure that does not require duality information, we can reformulate the sub-problem as a mixed-integer linear programming problem. In [31], the worst-case scenarios are identified via statistical upper bounds with Monte Carlo simulation. In contrast, our algorithm provides a theoretical global optimality guarantee to find the worst-case scenarios as the entire problem is solved as a mixed-integer linear programming problem after reformulating the sub-problem.

2.4.1 The master problem for the MMC model

The master problem is designed to provide a relaxation of the MMC model (2.17), in which the search for the worst-case scenario is restricted to be within a given finite set of scenarios, $\mathbf{\Omega}^{\mathrm{C}} = \{(d^i, y^i), \forall i = 1, ..., |\mathbf{\Omega}^{\mathrm{C}}|\}$, rather than the complete set of scenarios, $\mathcal{U}$. As such, the master problem yields a lower bound of the MMC model (2.17). We denote the master problem as $\mathbf{M}^{\mathrm{C}}(\mathbf{\Omega}^{\mathrm{C}})$, and it is formulated as the following single level mixed integer linear program.

$$\min_{x,\xi,z^i} \quad c^\top x + \xi \tag{2.19}$$

$$\text{s.t.} \quad \xi \geq b^\top z^i \qquad \forall i = 1, \dots, |\mathbf{\Omega}^{\mathrm{C}}| \tag{2.20}$$

$$Ax + C_1 z^i \leq g_1 \;\; \forall i = 1, \dots, |\mathbf{\Omega}^{\mathrm{C}}| \tag{2.21}$$

$$By^i + C_2 z^i \leq g_2 \;\; \forall i = 1, \dots, |\mathbf{\Omega}^{\mathrm{C}}| \tag{2.22}$$

$$Jz^i = d^i \qquad \forall i = 1, \dots, |\mathbf{\Omega}^{\mathrm{C}}| \tag{2.23}$$

$$x \text{ binary.} \tag{2.24}$$

2.4.2 The subproblem for the MMC model

The subproblem is defined as the MMC model (2.17) with a given first-stage decision, x. As such, the subproblem yields an upper bound of the MMC model (2.17). We denote the subproblem as $\mathbf{S}^{\mathrm{C}}(x)$, and it is formulated as the following bilevel linear program.

$$\max_{(d,y)\in\mathcal{U}} \min_{z\in\mathcal{Z}(x,d,y)} b^\top z. \tag{2.25}$$

This model can be further reformulated as the following linear program with complementarity constraints (LPCC).

$$\max_{d,y,z,\alpha,\beta,\gamma} \quad b^\top z \tag{2.26}$$

$$\text{s.t.} \quad Q_1 d \leq q_1 \tag{2.27}$$

$$Q_2 y \leq q_2 \tag{2.28}$$

$$0 \leq g_1 - Ax - C_1 z \perp \alpha \geq 0 \tag{2.29}$$

$$0 \leq g_2 - By - C_2 z \perp \beta \geq 0 \tag{2.30}$$

$$Jz = d \tag{2.31}$$

$$C_1^\top \alpha + C_2^\top \beta + J^\top \gamma + b = 0 \tag{2.32}$$

LPCC problems can be solved by several algorithms ([28] Branch-and-Bound, Bender's, Big-M). The big-M approach [28] was found to be one of the most computationally efficient in our computational experiments. This approach reformulates (2.26)-(2.32) as the following mixed-integer linear program (MILP).

$$\max_{d,y,z,\alpha,\beta,\gamma,\omega} b^\top z \tag{2.33}$$

$$\text{s.t.} \quad \text{Constraints (2.27), (2.28), (2.31), (2.32)} \tag{2.34}$$

$$0 \leq g1 - Ax - C_1 z \leq M\omega_1 \tag{2.35}$$

$$0 \leq \alpha \leq M(1 - \omega_1) \tag{2.36}$$

$$0 \leq g_2 - By - C_2 z \leq M\omega_2 \tag{2.37}$$

$$0 \leq \beta \leq M(1 - \omega_2) \tag{2.38}$$

Here, M is a sufficiently large constant (big-M) and ω_1 and ω_2 are auxiliary binary variables that are introduced to enforce the complementarity conditions in (2.29) and (2.30).

2.4.3 Algorithm for the MMC model

The proposed algorithm for the MMC model, which we call Alg^{MMC}, is an iterative one, in which the master problem is solved to provide an increasing series of lower bound solutions, and then the subproblem is solved to provide a series of decreasing upper bound solutions using the solution from the master problem, x, as an input. The input for the master problem, $\boldsymbol{\Omega}^{\mathrm{C}}$, is iteratively enriched by the solutions from the subproblem until the gap between the lower and upper bounds falls below a tolerance, ϵ. Detailed steps of this algorithm are described as follows:

$\underline{\mathrm{Alg}^{\mathrm{MMC}}(c, b, A, B, C_1, C_2, g_1, g_2, J, Q_1, q_1, Q_2, q_2)}$

Step 0 : Initialization. Create $\boldsymbol{\Omega}^{\mathrm{C}}$ that contains at least one selected scenario. Set $LB = -\infty$, $UB = \infty$, and $k = 1$. Go to Step 1.

Step 1 : Update $k \leftarrow k + 1$. Solve the master problem $\mathbf{M}^{\mathrm{C}}(\boldsymbol{\Omega}^{\mathrm{C}})$ and let (x^k, ξ^k) denote its optimal solution. Update the lower bound as $LB \leftarrow c^\top x^k + \xi^k$ and go to Step 2.

Step 2 : Solve the sub-problem $\mathbf{S}^{\mathrm{C}}(x^k)$ and let (d^k, y^k, z^k) denote its optimal solution. Update $\boldsymbol{\Omega}^{\mathrm{C}} \leftarrow \boldsymbol{\Omega}^{\mathrm{C}} \cup \{(d^k, y^k)\}$, and $UB \leftarrow c^\top x^k + b^\top z^k$.

Step 3 : If $UB - LB > \epsilon$, go to Step 1; otherwise return (x^k, d^k, y^k, z^k) as the optimal solution to (2.17) and LB as the optimal value.

2.4.4 Algorithm for the MMR model

The MMR model can be solved using a similar algorithmic framework to $\mathrm{Alg}^{\mathrm{MMC}}$ after the following simplifying yet equivalent reformulation.

$$\begin{aligned}
&\min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - G(d,y) \right\} \right\} \\
= &\min_{x \text{ binary}} \left\{ c^\top x + \max_{(d,y)\in\mathcal{U}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - \min_{(\hat{x},\hat{z})\in\mathcal{G}(d,y)} \left\{ c^\top \hat{x} + b^\top \hat{z} \right\} \right\} \right\} \\
= &\min_{x \text{ binary}} \left\{ c^\top x + \max_{\substack{(\hat{x},\hat{z})\in\mathcal{G}(d,y) \\ (d,y)\in\mathcal{U}}} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - (c^\top \hat{x} + b^\top \hat{z}) \right\} \right\}
\end{aligned}$$

To solve this reformulation of the MMR model, which is structurally similar to the MMC model (2.17), only slight modifications to the master and sub-problems are required. The master problem is defined for a different set of input scenarios, $\boldsymbol{\Omega}^{\mathrm{R}} = \{(d^i, y^i, \hat{x}^i, \hat{z}^i), \forall i = 1, ..., |\boldsymbol{\Omega}^{\mathrm{R}}|\}$, in which the two additional variables, $\hat{x}^i$ and $\hat{z}^i$, represent the optimal investment and recourse decisions with hindsight of the uncertainty realization (d^i, y^i). We denote the master problem as $\mathbf{M}^{\mathrm{R}}(\boldsymbol{\Omega}^{\mathrm{R}})$, and it is formulated as the following single level mixed integer linear program.

$$\min_{x,\xi,z^i} \quad c^\top x + \xi \tag{2.39}$$

$$\text{s. t.} \xi \geq b^\top z^i - (c^\top \hat{x}^i + b^\top \hat{z}^i) \forall i = 1, \ldots, |\boldsymbol{\Omega}^{\mathrm{R}}| \tag{2.40}$$

$$Ax + C_1 z^i \leq g_1 \qquad \forall i = 1, \ldots, |\boldsymbol{\Omega}^{\mathrm{R}}| \tag{2.41}$$

$$By^i + C_2 z^i \leq g_2 \qquad \forall i = 1, \ldots, |\boldsymbol{\Omega}^{\mathrm{R}}| \tag{2.42}$$

$$Jz^i = d^i \qquad \forall i = 1, \ldots, |\boldsymbol{\Omega}^{\mathrm{R}}| \tag{2.43}$$

$$x \text{ binary.} \tag{2.44}$$

We denote the subproblem as $\mathbf{S}^{\mathrm{R}}(x)$, and it is formulated as the following bilevel linear program.

$$\max_{(d,y)\in\mathcal{U};(\hat{x},\hat{z})\in\mathcal{G}(d,y)} \left\{ \min_{z\in\mathcal{Z}(x,d,y)} b^\top z - (c^\top \hat{x} + b^\top \hat{z}) \right\},$$

which can be solved using the same big-M approach with the following MILP.

$$\max_{d,y,z,\hat{x},\hat{z},\alpha,\beta,\gamma,\omega} b^\top z - (c^\top \hat{x} + b^\top \hat{z}) \tag{2.45}$$

$$\text{s. t.} \qquad \text{Constraints (2.34)-(2.38)} \tag{2.46}$$

$$A\hat{x} + C_1 \hat{z} \leq g_1 \tag{2.47}$$

$$By + C_2 \hat{z} \leq g_2 \tag{2.48}$$

$$J\hat{z} = d \tag{2.49}$$

$$\hat{x} \text{ binary.} \tag{2.50}$$

With the new definitions of master and sub-problems, the same algorithm $\mathrm{Alg}^{\mathrm{MMC}}$ can be used to solve the MMR model with the following minor modification to Step 2, besides the apparent need to change the superscript "C" to "R":

Step 2 : Solve the sub-problem $\mathbf{S}^{\mathrm{R}}(x^k)$ and let $(d^k, y^k, z^k, \hat{x}^k, \hat{z}^k)$ denote its optimal solution. Update $\boldsymbol{\Omega}^{\mathrm{R}} \leftarrow \boldsymbol{\Omega}^{\mathrm{R}} \cup \{(d^k, y^k, \hat{x}^k, \hat{z}^k)\}$, and $UB \leftarrow c^\top x^k + b^\top z^k - (c^\top \hat{x}^k + b^\top \hat{z}^k)$.

2.5 Case Study

In this section, we present numerical experiments of our model and algorithm on an IEEE 118-bus test system, which consists of 186 transmission lines, 5 wind farms, 5 coal plants, 5 gas plants and 33 loads. The network data is available in [2].

We consider 10 candidate lines. The operation costs are calculated based on the data of 4 load blocks. We consider a planning horizon of 20 years, with the operation cost extrapolated from the cost of year 1. Then the operation cost is assumed to increase at the same rate each year. The characteristics of generation and candidate lines are summarized in Table 2.3 and Table 2.4 respectively. In our case study, generation capacity data in the system is set to be able to satisfy all demand levels if there is no network congestion.

Table 2.3 Generation Parameters

Bus NO.	Type	Current Capacity (MW)	Fuel Cost ($/MWh)	Min New Capacity (MW)	Max New Capacity (MW)
25	Wind	1,000	0.0	2,000	3,000
31	Wind	500	0.0	150	400
32	Coal	300	43.0	–160	–80
36	Wind	1,100	0.0	800	1,500
49	Wind	1,000	0.0	1,200	2,000
61	Coal	400	28.2	–200	–70
65	Coal	1,500	52.7	–800	–600
66	Coal	300	28.3	–150	–100
76	Gas	200	31.0	80	100
85	Gas	300	63.9	150	200
87	Wind	900	0.0	1,300	2,500
89	Gas	150	49.1	100	150
92	Gas	200	32.0	200	250
99	Coal	300	27.5	–150	–80
113	Gas	200	65.0	200	300
Total		8,350		6,000	8,500

Uncertainty in future generation capacity consists of two parts, expansion and retirement. For wind and natural gas fired plants, we set the lower and upper bounds for new capacity. For coal plants, the range of reduced capacity is also provided. We use negative capacity to depict coal retirement. In addition to bounds on individual plants, we also set the lower bound and upper bound on total new generation capacity to control the randomness of the uncertainty set. The mean of our demand ($\bar{d}$) and the capacity factor are modified based on the real data from

Table 2.4 Candidate Line Parameters

From	To	Susceptance (Ω^{-1})	Transmission Capacity (MW)	Construction Cost ($m)
25	4	30	390	40.60
25	18	30	390	32.48
25	115	30	390	40.60
32	6	30	390	50.75
36	34	30	390	28.42
36	77	30	390	44.66
70	25	30	390	97.44
86	82	30	390	36.54
87	106	30	390	62.93
87	108	30	390	52.78

WECC [58]. We generate four instances by changing the uncertainty sets. Their definitions are listed as follows:

$$\mathcal{U}_1 = \{0.95\bar{d}_{i,t,m} \le d_{i,t,m} \le 1.05\bar{d}_{i,t,m} \tag{2.51}$$

$$P^{\mathrm{N,min}}_{i,k,t} \le P^{\mathrm{N}}_{i,k,t} \le P^{\mathrm{N,max}}_{i,k,t} \tag{2.52}$$

$$P^{\mathrm{N,min}} \le \sum_{i,k,t} P^{\mathrm{N}}_{i,k,t} \le P^{\mathrm{N,max}}\} \tag{2.53}$$

$$\mathcal{U}_2 = \{0.85\bar{d}_{i,t,m} \le d_{i,t,m} \le 1.15\bar{d}_{i,t,m} \tag{2.54}$$

$$\text{Equations}(2.52) - (2.53)\} \tag{2.55}$$

$$\mathcal{U}_3 = \{0.95\bar{d}_{i,t,m} \le d_{i,t,m} \le 1.05\bar{d}_{i,t,m} \tag{2.56}$$

$$[1.25 - 0.5\,\mathrm{sgn}(P^{\mathrm{N,min}}_{i,k,t})]P^{\mathrm{N,min}}_{i,k,t} \le P^{\mathrm{N}}_{i,k,t}$$

$$\le [0.75 + 0.5\,\mathrm{sgn}(P^{\mathrm{N,max}}_{i,k,t})]P^{\mathrm{N,max}}_{i,k,t} \tag{2.57}$$

$$0.75P^{\mathrm{N,min}} \le \sum_{i,k,t} P^{\mathrm{N}}_{i,k,t} \le 1.15P^{\mathrm{N,max}}\} \tag{2.58}$$

$$\mathcal{U}_4 = \{0.85\bar{d}_{i,t,m} \le d_{i,t,m} \le 1.15\bar{d}_{i,t,m} \tag{2.59}$$

$$\text{Equations}(2.57) - (2.58)\} \tag{2.60}$$

where $(P^{\mathrm{N,min}}_{i,k,t}, P^{\mathrm{N,max}}_{i,k,t})$ and $(P^{\mathrm{N,min}}, P^{\mathrm{N,max}})$ are listed in the last two columns in Table 2.3, with $(P^{\mathrm{N,min}}, P^{\mathrm{N,max}})$ in the last row.

The load curtailment cost is set as 2000\$/MWh. The interest rate is set as 0.1. The experiment is implemented on a computer with Intel Core i5 3.30GHz with 4GB memory and

CPLEX 12.5. The computation time of each instance is around 9 hours. The numbers of iterations for solving each instance are summarized in Table 2.5.

Table 2.5 Number of Iterations for Each Instance

	$\mathcal{U}_1$	$\mathcal{U}_2$	$\mathcal{U}_3$	$\mathcal{U}_4$
MMC	3	3	2	2
MMR	3	6	2	3

The transmission expansion plans, investment costs and objective values of each criterion under the four uncertainty sets are summarized in Table 2.6 and 2.7. We then compare the performances of the MMC solution and the MMR solution under various scenarios in Tables 2.8 and 2.9, where we use D^c, D^r and D^d to denote the optimal MMC solution, the optimal MMR solution and the optimal deterministic solution. The lower cost and regret between D^c and D^r are highlighted. The deterministic solution is derived by setting the mean demand as the load levels and the median of new capacity as the future expansion plans. The scenarios are generated by our algorithms when solving the MMR problems and MMC problems. Each scenario corresponds to an optimal solution to a sub-problem at an iteration of our algorithm and is the worst-case scenario for the first-stage solution obtained at the same iteration. Scenarios $S_1 - S_3$ and $S_6 - S_{10}$ are generated by solving the MMR problems. Scenarios S_4, S_5, S_{11} and S_{12} are generated when solving the MMC problems. Those scenarios typically have very high costs or regrets, thus are representative of bad scenarios that robust optimization tries to hedge against. The investment and operational costs of both the MMC and MMR decisions under the above scenarios are summarized in Table 2.10.

From Table 2.6 and Table 2.7, we can see that as the uncertainty in demand increases, although the numerical value of the maximum regret and worst-case cost increases, the change in the transmission expansion plan is not very substantial. It means many of the candidate lines are necessary regardless of the demand levels with our unchanged depiction of generation capacity uncertainties. The reason is that those candidate lines connect regions with very high locational marginal price differences due to the presence of large amount of wind energy. On the other hand, when uncertainty in generation expansion is increased, although the total cost also increases, fewer lines are actually built with the MMC criterion. That is because in the

Table 2.6 Expansion plan of the MMC approach

Lines (from bus, to bus)	Uncertainty Set			
	$\mathcal{U}_1$	$\mathcal{U}_2$	$\mathcal{U}_3$	$\mathcal{U}_4$
Candidate Line $(25, 4)$	1	0	1	1
Candidate Line $(25, 18)$	1	0	1	1
Candidate Line $(25, 115)$	0	1	0	0
Candidate Line $(32, 6)$	0	1	0	0
Candidate Line $(36, 34)$	1	1	1	1
Candidate Line $(36, 77)$	1	1	1	1
Candidate Line $(70, 25)$	0	1	0	0
Candidate Line $(86, 82)$	1	1	0	0
Candidate Line $(87, 106)$	1	1	1	1
Candidate Line $(87, 108)$	1	1	1	1
Investment Cost ($m)	298	414	262	262
Maximum Cost ($m)	1,233	1,960	1,927	2,802

worst-case scenarios, the system contains less renewable energy capacity and the differences in the locational marginal prices between the otherwise connected regions are not substantial enough to justify new transmission lines. When the MMR criterion is used, however, the final expansion plan does not seem to be sensitive to the change of uncertainty in generation expansion. One possible explanation is that since the MMR criterion does not make decisions only based on the boundary scenarios, it is less sensitive to the changes in uncertainty sets.

From the more detailed comparisons of results from the MMC and MMR approaches in Tables 2.8 and 2.9, we can gain more insights about which criterion is more appropriate under different situations. Both robust optimization solutions outperform the deterministic solution under most of the scenarios. When the uncertainty set is $\mathcal{U}_1$, the MMC solution outperforms the MMR solution under most of the listed scenarios. When the uncertainty set is $\mathcal{U}_2$, on the other hand, the MMR solution has a lower total cost under more listed scenarios. The solutions of the cases when the uncertainty sets are $\mathcal{U}_3$ and $\mathcal{U}_4$ yield similar results. According to our definition at the end of section 2.3, scenarios $S_2 - S_5$ in Table 2.8 and scenario S_{12} in Table 2.9 are regretless scenarios for the MMC decision, while scenarios S_1 and $S_6 - S_{11}$ are regretful ones. Which criterion should be used depends on the decision-maker's perception on the uncertainty sets. In typical regretless scenarios for MMC decisions, there usually exists high demand and

Table 2.7 Expansion plan of the MMR approach

Lines (from bus, to bus)	Uncertainty Set			
	$\mathcal{U}_1$	$\mathcal{U}_2$	$\mathcal{U}_3$	$\mathcal{U}_4$
Candidate Line $(25, 4)$	1	1	1	1
Candidate Line $(25, 18)$	1	1	1	1
Candidate Line $(25, 115)$	1	0	1	0
Candidate Line $(32, 6)$	0	0	0	0
Candidate Line $(36, 34)$	1	1	1	1
Candidate Line $(36, 77)$	1	1	1	1
Candidate Line $(70, 25)$	0	1	0	1
Candidate Line $(86, 82)$	1	1	1	1
Candidate Line $(87, 106)$	1	1	1	1
Candidate Line $(87, 108)$	1	1	1	1
Investment Cost (\$m)	339	395	339	395
Maximum Regret (\$m)	89	234	110	235

Table 2.8 Comparison of the MMC, MMR and Deterministic Solutions for Uncertainty Set $\mathcal{U}_1$

Cost/Regret (\$m)	D^c	D^r	D^d
Scenario S_1	1,137 / 117	**1,109 / 89**	1,085 / 65
Scenario S_2	**1,039 / 0**	1,080 / 41	1,263 / 224
Scenario S_3	**803 / 0**	844 / 41	1,125 / 422
Scenario S_4	**1,126 / 0**	1,167 / 41	1,325 / 199
Scenario S_5	**1,233 / 27**	1,272 / 66	1,367 / 161

low renewable energy penetration. If decision-makers care more about such scenarios or believe they are more likely, then MMC should be used. Otherwise, choosing MMR might be better. Both criteria provide good upper bounds for the total costs under scenarios contained in an uncertainty set. The MMC criterion provides a smaller upper bound with higher average costs while the costs of MMR decisions are lower on average but have higher variability.

From the above results, it is obvious that both the future generation expansion behavior of generation companies and demand uncertainty play important roles in transmission expansion planning. In addition, we can also conclude that both criteria have their merits and can yield relatively reliable expansion plans that guarantee zero curtailment for uncertainty realizations contained in the uncertainty sets. However, depending on the characteristics of uncertainty sets and the preference of decision-makers, they may outperform each other under different

Table 2.9 Comparison of the MMC, MMR and Deterministic Solutions for Uncertainty Set $\mathcal{U}_2$

Cost/Regret ($m)	D^c	D^r	D^d
Scenario S_6	1,730 / 170	**1,597** / **37**	1,861 / 301
Scenario S_7	1,375 / 133	**1,354** / **112**	1,519 / 277
Scenario S_8	1,309 / 497	**1,046** / **234**	836 / 24
Scenario S_9	1,398 / 288	**1,266** / **156**	1,473 / 363
Scenario S_{10}	776 / 246	**701** / **171**	666 / 136
Scenario S_{11}	1,959 / 201	**1,862** / **104**	2,082 / 324
Scenario S_{12}	**1,960** / **31**	1,962 / 33	2,173 / 244

Table 2.10 Investment and Operational costs for scenarios ($m)

	D^c	D^r		D^c	D^r
Invest $S_1 - S_5$	298	339	Invest $S_6 - S_{12}$	414	395
Operation S_1	839	770	Operation S_6	1,316	1,202
Operation S_2	741	741	Operation S_7	961	959
Operation S_3	505	505	Operation S_8	895	651
Operation S_4	828	828	Operation S_9	984	871
Operation S_5	935	933	Operation S_{10}	362	306
			Operation S_{11}	1,545	1,467
			Operation S_{12}	1,546	1,567

situations. Thus, a comparative analysis of the MMC and MMR criteria can shed more light on better utilization of both approaches.

2.6 Conclusion

In this paper, we propose two robust optimization models for the transmission expansion planning problem under uncertainty, where we take into consideration both the high-frequency uncertainty caused by load forecast errors and the low-frequency uncertainty caused by future generation expansion and retirement. We use two criteria: minimax cost and minimax regret, and compare their performances. The uncertain parameters are described by a polyhedral uncertainty set. With this approach, we can derive an expansion plan that is robust under all scenarios. The resulting models can be formulated as trilevel mixed-integer problems. We use a branch and cut type mechanism to decompose the problem into a master problem and a subproblem. The subproblem generates scenarios and returns them to the master problem to cut off sub-optimal solutions. The bilevel mixed-integer subproblem is reformulated into a single level mixed-integer-programming problem with the KKT conditions to obtain the global optimal solution. Our model and algorithm are then tested on an IEEE 118-bus system, where we compare the results of our MMR and MMC models and analyze their differences. We conclude that the MMR and MMC criteria may outperform each other depending on the uncertainty set and decision-maker's preference. Interesting topics for future research include developing effective heuristics and parallel computing mechanisms to speed up our algorithms and implementing our algorithms on high performance machines to test larger systems with more candidate lines.

CHAPTER 3. ROBUST TRANSMISSION PLANNING UNDER UNCERTAIN GENERATION INVESTMENT AND RETIREMENT

Published in *IEEE Transactions on Power Systems*

Bokan Chen and Lizhi Wang

Abstract

Transmission planning is typically faced with a wide range of uncertainty including growth in demand, renewable energy generation and fuel price. The restructuring of the electric power industry, the drive for energy independence and the push for a cleaner environment have led to additional sources of uncertainty in all aspects of power system operations and planning. The more decentralized decision-making implicates uncertainty in future generation investment and retirement of existing units. Such uncertainties are among the biggest challenges in transmission planning in power systems. However, existing approaches in the literature do not fully address this problem. In this paper, we construct a new robust transmission planning model representing generation investment and retirement uncertainty more realistically to improve the quantification and visualization of uncertainty and the impacts of environmental policies. With this model, we can manage the impacts of uncertainty in the time, the size and the location of candidate generation additions as well as the retirement of units. The corresponding algorithm we develop takes advantage of the structural characteristics of the model so as to obtain a computationally efficient solution methodology. We present an extensive numerical study on the WECC 240-bus system [58] and illustrate the effectiveness of our approach.

3.1 Nomenclature

Sets and indices

$\mathcal{I}$	Set of nodes, $i \in \mathcal{I}$
$\mathcal{Q}$	Set of all generators (including existing and candidate generators), $q \in \mathcal{Q}$
$\mathcal{W}$	Set of candidate generators, $q \in \mathcal{W}$
$\mathcal{L}$	Set of transmission lines existing before any new investment, $(i, j) \in \mathcal{L}$
$\mathcal{N}$	Set of candidate transmission lines, $(i, j) \in \mathcal{N}$
$\mathcal{T}$	Set of periods in the planning horizon, $t \in \mathcal{T}$
$\mathcal{M}$	Set of load blocks, $m \in \mathcal{M}$
$\mathcal{K}$	Set of generation technology types, $k \in \mathcal{K}$
$\mathcal{R}$	Set of renewable generation technology types, $k \in \mathcal{R}$

Parameters

$P_{q,i,k}$	Capacity of generator q of technology k at bus i
$c^{\mathrm{T}}_{i,j,t}$	Cost of building the new transmission line (i, j) at period t
$c^{\mathrm{L}}_{i,t}$	Cost of load curtailment at node i at period t
$c^{\mathrm{P}}_{q,i,k,t}$	Cost of power production by generator q of technology k at node i at period t
$F^{\mathrm{C}}_{k,t,m}$	Average capacity factor of generation technology k at period t for load block m
$F^{\max}_{i,j}$	Thermal capacity of transmission line (i, j)
$B_{i,j}$	Susceptance of transmission line (i, j)
M, M'	Big constants used to linearize the power flow constraints and the products between two variables
$d_{i,t,m}$	Demand at node i at period t for load block m
$\theta^{\min}$	The lower bound of voltage angles
$\theta^{\max}$	The upper bound of voltage angles
λ	Cash flow interest rate (with inflation)
$g_{q,i,k,0}$	Availability of generator q of technology type k at bus i at period 0

Variables

$x_{i,j,t}$	Binary variable indicating whether transmission line (i, j) is built at period t ($x_{i,j,t} = 1$) or not ($x_{i,j,t} = 0$)
$g_{q,i,k,t}$	Binary variable indicating whether a generator q of technology type k exists at bus i in period t ($g_{q,i,k,t} = 1$) or not ($g_{q,i,k,t} = 0$)
$f_{i,j,t,m}$	Power flow from node i to node j at period t for load block m
$p_{q,i,k,t,m}$	Power production by generator q of technology k at node i at period t for load block m
$r_{i,t,m}$	The amount of load shedding at node i at period t for load block m
$\theta_{i,t,m}$	Voltage angle at node i at period t for load block m
z	An aggregate variable representing all dispatch variables, including $f_{ij,t,m}$, $p_{q,i,k,t,m}$, $r_{i,t,m}$, and $\theta_{i,t,m}$
$C^{\mathrm{I}}(x)$	Net present value of the investment cost
$C^{\mathrm{O}}(z)$	Net present value of the operation cost

3.2 Introduction

Transmission planning is one of the key decision processes in the power system production pipeline. Electricity transmission networks are responsible for reliably and economically delivering power from generators to consumers. Thus, a robust and resilient transmission network is essential to the operation of power systems for decades to come. Good transmission investments have many benefits, including satisfying increasing demand, promoting social welfare, improving system reliability and resource adequacy. In addition, in the face of challenges of climate change, the power generation industry is under increasing pressure to reduce its carbon footprint, and to introduce more renewable energy into the power system. Transmission enables a large quantity of renewable energy to be integrated into the power system by bridging the distances between renewable energy resources and load centers.

Transmission planning is very challenging due to the long planning horizon and various sources of uncertainty planners need to consider. Traditionally, planners take into consideration sources of uncertainty such as demand variations and renewable energy intermittency [70, 3], forced outages of transmission lines and generators [40, 18], and transmission capacity factor [45]. Since the restructuring of the power system, generation expansion planning and transmission planning are conducted by separate entities, which makes the future generation investment uncertain to transmission planners. In addition, a large amount of coal plants are anticipated to retire in response to the EPA clean power plan, to be replaced in part by gas-fired plants. Such retirement could be unexpected by system operators. All of the above factors introduce additional sources of uncertainty to the transmission planning process. However, research on the impacts of uncertain future generation developments on transmission planning in the decentralized decision making environment is scarce in the literature.

In existing literature, the most popular methods to manage uncertainty in optimization include stochastic programming and robust optimization. Stochastic programming assumes that the uncertain parameters follow an estimated probability distribution and generates scenarios accordingly. It has been applied to many transmission planning problems [3, 40, 18, 45]. Ryan *et al.* [61] classify sources of uncertainties in the power system into two categories—high frequency and low frequency. Most operational level uncertainties are of high-frequency, whose probability distributions are readily available by utilizing historical data. Low frequencies uncertainties, such as climate change, natural disaster, technology breakthroughs and future generation investment and retirement, cannot be easily characterized by probability distributions. Stochastic programming, with its dependence on probability distributions, is more equipped to deal with high-frequency uncertainties. On the other hand, robust optimization uses parametric sets to describe uncertainty [74, 13], which can contain infinitely many scenarios without specific knowledge of the probability distributions. Therefore, robust optimization can also deal with low-frequency uncertainty. However, the focus in literature has been using robust optimization to ensure feasibility against any operational level high-frequency uncertainty realizations. In [29], a robust transmission planning model considering demand and renewable energy uncertainty is proposed. The model is solved by a two-layer Benders' decomposition algorithm. Ruiz and Conejo [60] propose a similar model with the uncertainty in equipment outages also represented. Robust optimization is also used in [50] with the explicit representation of the system with $n-k$ contingencies for transmission planning. In [5], uncertainty in construction costs and demand is considered. The cited references are limited to the management of operational level sources of uncertainty. The impacts of generation investment on transmission planning are usually explored with other methods.

Generation investment in transmission planning is usually studied with three different perspectives—deterministic, stochastic and game theoretic. The references cited in the previous paragraph fall into the deterministic category, which assume generation capacity available in the system to be known information throughout the transmission planning horizon. Stochastic methods include scenario analysis and robust optimization. Similar to stochastic programming, scenario analysis optimizes the average objective among several scenarios with equal weights that represent various sources of uncertainty. However, such scenarios do not incorporate any information on probability distribution. Munoz *et al.* [52] construct 3 scenarios based on EIA forecasts and some educated assumptions to describe different policy mandates and fuel costs. Generation investments are determined according to the three scenarios and are assumed to have the same objective as transmission investments. Buygi *et al.* [17] study several cases with different scenarios to explore the effect of low frequency uncertainties. In [42], multi-period scenario trees are constructed to describe the uncertainty in the size, location and types of renewable generation. The disadvantages of this approach is that only a limited number of scenarios are explored. In addition, the constructed scenarios are more of a "big picture" rather than detailed depictions of possible futures. Unlike scenario analysis, robust optimization can describe uncertainty with more details. Two robust transmission planning models with different optimization criteria—minimax cost and minimax regret are proposed in [20]. Demand and generation capacity uncertainty is considered, where generation capacity at specific locations is assumed to be in a polyhedral set. Limited by the way uncertainty is modeled, this work does not explore the impacts of location and time of generation investment and retirement.

In game theoretic models, generation investment is not a source of uncertainty, but the optimal behavior of generation companies (GENCOs), which can be anticipated based on market condition and the behavior of other players. A trilevel model is proposed in [57]. In the first level transmission investment decisions are made. In the second level, multiple GENCOs make generation investments in response to the transmission investment. Operational decisions are made in the third level. Perfect information competition between GENCOs is assumed. A similar trilevel model is proposed in [33] and [34], where a Cournot game between GENCOs is solved. Similar approaches are also used in [59, 51, 27]. Game theoretical models are useful in that they can provide us some insights into the behavior of different participants in the power market. However, some strong assumptions about the players are needed. One example is perfect information between different players, which usually is not possible. In addition, the high complexity of the models means that they are difficult to implement for large realistic-sized systems.

Given the limitations of the methods in the literature, there is a need for practical approaches to address the challenges faced by transmission planners in the current environment of the power electric industry. In this paper, we propose a new multi-period robust optimization framework for transmission planning with explicit representation of uncertainty in future generation investment and the retirement of existing units. According to the EIA forecast [24], the average annual demand growth in electricity is less than 1% in the next 20 years. Since the uncertainty in demand growth is relatively low, we do not consider demand growth uncertainty in this paper and focus on the uncertainty in future generation instead. However, demand uncertainty can be easily incorporated into the uncertainty set for future research. A customized solution method is developed for the model. The model and solution method is then tested on the WECC 240-bus system. The impacts of policy and natural gas price change is also explored in the case study.

Table 3.1 summaries the comparison between our new model and some previous works using robust optimization. The structure of our new model is more similar to the models in [20], but a few key aspects are different, including uncertainty modeling, number of decision periods and solution algorithm.

The contributions of this paper are summarized as follows:

1. A novel method to model generation investment and retirement uncertainty is proposed. With the more flexible uncertainty modeling, the uncertainty set can describe the uncertainty in not only the capacity of new generation investments but also their locations.

Table 3.1 Comparison with other robust optimization transmission planning models in the literature

Reference	Generation investment	Uncertainty	Case Study	# periods
[29]	none	load, renewable intermittency	IEEE 118-bus	1
[60]	none	load, renewable intermittency, equipment outage	RTS 24-bus	1
[50]	none	$n-k$ contingency	IEEE 300-bus	1
[5]	none	investment costs, demand	IEEE 118-bus	1
[20]	uncertainty	load, generation capacity	IEEE 118-bus	1
Our model	uncertainty	size, location and time of generation investment and retirement	WECC 240-bus	4

2. We presented a tri-level robust optimization model that contains multiple decision periods in the middle and bottom levels, representing when generation and transmission investments could be made. This feature allows a more detailed schedule of transmission line construction to be obtained by the model. It also adds the dimension of time into the description of generation investment and retirement uncertainty.

3. The proposed solution method is more efficient than the existing Karush-Kuhn-Tucker (KKT) reformulation method [35, 49], which made case studies on larger-sized systems with multiple decision periods more computationally tractable.

The rest of the paper is organized as follows. Section 3.3 describes a detailed formulation of the robust transmission planning model. Section 3.4 explains the solution method used in this paper. Section 3.5 presents the results of our numeric experiments. Finally, Section 5 concludes the paper.

3.3 Model Formulation

Transmission planning is a two-stage decision making problem. The two stages refer to the decision making processes before and after uncertainty realization. The first-stage decisions usually cannot be changed after uncertainty is observed. The second-stage decisions are

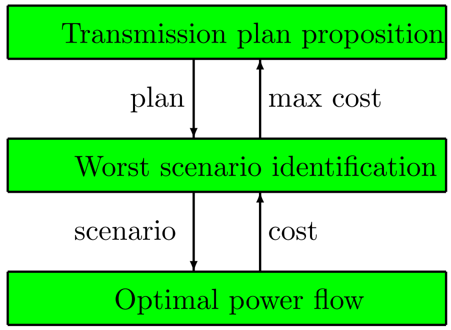

Figure 3.1 Summary of the robust optimization modeling framework

heavily influenced by the particular realization of uncertainty as well as the first-stage decisions. In our robust optimization model, new transmission lines are selected for construction with limited information on uncertainty during the first decision making stage. In the second stage, operation decisions are made after uncertainty realization. A summary of the two-stage modeling framework is shown in Figure 3.1.

In addition, multiple decision periods are captured in our model. Transmission planning usually spans a long planning horizon. Due to budget and other physical constraints, construction of all the needed transmission lines at the same time might be difficult or unrealistic. It stands to reason that the planned transmission lines may be available at different points along the planning horizon. In this model, we assume the planned transmission lines are available at the beginning of each decision period. The first-stage decision consists of transmission line construction plans for the multiple decision periods in the planning horizon.

The goal of robust optimization is to identify a schedule to construct new transmission lines that can achieve the lowest total cost, including construction cost and estimated operation cost. In this section, we present a two-stage multi-period robust transmission planning problem formulated as a trilevel optimization model.

$$\min_{x\in\mathcal{X}}\left\{C^{\mathrm{I}}(x)+\max_{g\in\mathcal{G}}\ \min_{z\in\mathcal{Z}(x,g)}\ C^{\mathrm{O}}(z)\right\} \tag{3.1}$$

where

$$\mathcal{X} = \Big\{ x | \sum_t x_{i,j,t} \leq 1, \forall (i,j) \in \mathcal{N},$$

$$x_{i,j,t} \text{ Binary}, \forall (i,j) \in \mathcal{N}, t \in \mathcal{T} \Big\} \tag{3.2}$$

$$C^{\mathrm{I}}(x) = \sum_{t,i,j} \frac{1}{(1+\lambda)^t} c^{\mathrm{T}}_{i,j,t} x_{i,j,t} \tag{3.3}$$

$$C^{\mathrm{O}}(z) = \sum_{i,t,m} \frac{1}{(1+\lambda)^t} \Big(\sum_q c^{\mathrm{P}}_{q,i,k,t} p_{q,i,k,t,m} + c^{\mathrm{L}}_{i,t} r_{i,t,m} \Big) \tag{3.4}$$

$$\mathcal{Z}(x,g) = \Big\{ z : f_{i,j,t,m} = B_{i,j}(\theta_{i,t,m} - \theta_{j,t,m}),$$

$$\forall t, m, (i,j) \in \mathcal{L} \qquad [\mu^{(1)}] \tag{3.5}$$

$$|f_{i,j,t,m} - B_{i,j}(\theta_{i,t,m} - \theta_{j,t,m})| \leq (1 - \sum_{k=1}^{t} x_{i,j,k}) M,$$

$$\forall t, m, (i,j) \in \mathcal{N} \qquad [\mu^{(1)}] \tag{3.6}$$

$$-F^{\max}_{i,j} (\sum_{k=1}^{t} x_{i,j,k}) \leq f_{i,j,t,m} \leq F^{\max}_{i,j} (\sum_{k=1}^{t} x_{i,j,k}),$$

$$\forall t, m, (i,j) \in \mathcal{N} \qquad [\mu^{(2)}, \mu^{(3)}] \tag{3.7}$$

$$-F^{\max}_{i,j} \leq f_{i,j,t,m} \leq F^{\max}_{i,j},$$

$$\forall t, m, (i,j) \in \mathcal{L} \qquad [\mu^{(2)}, \mu^{(3)}] \tag{3.8}$$

$$0 \leq p_{q,i,k,t,m} \leq F^{\mathrm{C}}_{k,t,m} P_{q,i,k} g_{q,i,k,t},$$

$$\forall q, i, k, t, m \qquad [\mu^{(4)}] \tag{3.9}$$

$$\theta^{\min} \leq \theta_{i,t,m} \leq \theta^{\max}, \forall i, t, m \qquad [\mu^{(5)}, \mu^{(6)}] \tag{3.10}$$

$$\sum_q p_{q,i,k,t,m} + \sum_j f_{j,i,t,m} - \sum_j f_{i,j,t,m} = d_{i,t,m}$$

$$- r_{i,t,m}, \forall i, t, m \Big\} \qquad [\mu^{(7)}] \tag{3.11}$$

and

$$\mathcal{G} = \Big\{ g | \mathbf{H} g \leq h, \; g_{q,i,k,t} \text{ Binary}, \forall q, i, k, t \Big\}. \tag{3.12}$$

From the model formulation (3.1), we can see the two-stage structure of the robust transmission planning model. Transmission construction schedule x is determined in the first stage. In the second stage, after generation investment and retirement g is observed, operational decision z is made. The objective is to minimize the investment cost and the projected operational cost under the most costly scenario. Investment and operation costs are represented by Equations (3.3) and (3.4) respectively. The operation cost includes generation cost and load curtailment cost (the two terms in the parentheses in Equation (3.4)). Constraint (3.2) means that a transmission line only needs to be built once in the planning horizon. The uncertainty set $\mathcal{G}$ is defined in Equation (3.12), which can be modified based on planners' belief on uncertain parameters. Thus the definition of the uncertainty sets is case-specific. The symbols H and h represent, respectively, the coefficient matrix of the constraints and the right-hand-side vector. The detailed formulation of the uncertainty set used in our case study can be found in Section 3.5.

Constraints (3.5)-(3.11) define $\mathcal{Z}(x, g)$, the set of feasible power flow solutions after new transmission and generation is fixed. Equation (3.5) defines the power flow on existing transmission lines, while Constraint (3.6) defines the power flow on candidate lines. It is equivalent to the constraint $f_{i,j,t,m} = (\sum_{k=1}^{t} x_{i,j,k})B_{i,j}(\theta_{i,t,m} - \theta_{j,t,m})$, which is nonlinear and can complicate the computation. By using the auxiliary parameter M, it is linearized. Constraints (3.7)-(3.8) impose power flow limits on candidate and existing transmission lines. Constraint (3.9) requires power generation to be within the capacity limit of generators after capacity factor is considered. Constraint (3.10) limits the range of difference between phase angles at each node and the phase angle of the reference node. Constraint (3.11) requires the net influx at a node to be equal to the net outflow. The symbols in the brackets represent the dual variables corresponding to the constraints. The loads here are modeled by load blocks. The dual variables for Constraints (3.5) and (3.6) only differ in the indices, so do the dual variables for (3.7) and (3.8). Constraints (3.7), (3.8) and (3.10) each have two dual variables, one for each side of the inequalities.

3.4 Solution Methods

From the decision making perspective, the proposed model is a two-stage multi-period model, where transmission investment decisions are made in the first stage and the operational decisions are made in the second stage. From the solution technique perspective, the model is also a trilevel robust optimization model, in which the two-stage decisions are made at the top and bottom levels and the middle level identifies the worst-case scenario of uncertainty realization. Trilevel optimization problems can be difficult to solve. In this section, we present a column and constraint generation (CCG) algorithm [74] to decompose the trilevel model into a master problem that makes the first level decisions and a subproblem that solves the middle and bottom levels. As such the subproblem is a bilevel optimization problem. The CCG method is similar to Benders' decomposition [12] in that both generate cutting planes iteratively. The difference is that Benders' decomposition relies on the dual formulation of the linear program subproblems to generate cuts, whereas the CCG method introduces both new cuts and new variables by solving the bilevel subproblem.

One of the most popular methods for solving bilevel optimization problems is reformulating the lower level as complementarity constraints using its KKT conditions and then linearizing it using the big-M method [20]. However, such method can be inefficient due to the large number of auxiliary binary variables introduced in the linearization step. In our solution method, we first dualize the subproblem by introducing bilinear terms, and then linearize them by exploiting the discrete structure of the uncertainty set. As such, the bilevel subproblem is reformulated as a single level mixed integer linear program. Since less binary variables are introduced, the solution method is more efficient, which will be demonstrated in Section 3.5.

3.4.1 Master problem and subproblem

The master problem is a relaxation of the robust transmission planning problem and provides a first-stage solution. The original trilevel problem requires the most costly scenario in the entire uncertainty set $\mathcal{G}$. In the master problem, the search for the worst-case scenario is restricted in a smaller set of scenarios $\mathcal{S} \subseteq \mathcal{G}$. We denote the master problem as $\mathbf{M}(\mathcal{S})$, and it is formulated as the following mixed-integer linear program:

$$\min_{x,z^s} C^{\mathrm{I}}(x) + \zeta \tag{3.13}$$

$$\text{s.t.} \quad x \in \mathcal{X} \tag{3.14}$$

$$\zeta \geq C^{\mathrm{O}}(z^s), \forall s \in \mathcal{S} \tag{3.15}$$

$$z^s \in \mathcal{Z}(x, g^s), \forall s \in \mathcal{S}. \tag{3.16}$$

When $\mathcal{S} = \mathcal{G}$, the master problem $\mathbf{M}(\mathcal{S})$ is equivalent to the trilevel formulation (3.1). However, due to the enormous scenario space in $\mathcal{G}$, it is unrealistic to search the entire uncertainty set. Instead, we search the subset $\mathcal{S}$. As such, the master problem provides a lower bound to the actual optimal objective value. New scenarios are iteratively added to the set $\mathcal{S}$ and the lower bound increases at each iteration.

The subproblem identifies the worst-case scenario given a first-stage decision x. Since the performance of such solution can only be worse than the optimal first-stage solution, the subproblem provides an upper bound to the optimal objective value for the original trilevel problem. We denote the subproblem as $\mathbf{R}(x)$ and it can be formulated as the following bilevel linear program:

$$\max_{g \in \mathcal{G}} \min_{z \in \mathcal{Z}(x,g)} C^{\mathrm{O}}(z) \tag{3.17}$$

3.4.2 Solving the subproblem

The subproblem defined in (3.17) is a bilevel optimization problem. In order to solve it, we write out the dual of the operation cost minimization problem $\min\limits_{z \in \mathcal{Z}(x,g)} C^{\mathrm{O}}(z)$, and the subproblem $\mathbf{R}(x)$ can be replaced by the following formulation.

$$
\begin{aligned}
\max_{\mu,g} \sum_{t,m} \Bigg[& -\sum_{(i,j)\in\mathcal{L}\cup\mathcal{N}} F_{i,j}^{\max}(\mu_{i,j,t,m}^{(2)} + \mu_{i,j,t,m}^{(3)}) \\
& - \sum_{q,i,k} F_{k,t,m}^{\mathrm{C}} P_{q,i,k} g_{q,i,k,t} \mu_{q,i,k,t,m}^{(4)} \\
& - \sum_{i} (\theta^{\max}\mu_{i,t,m}^{(5)} - \theta^{\min}\mu_{i,t,m}^{(6)}) \\
& - \sum_{i} d_{i,t,m}\mu_{i,t,m}^{(7)} \Bigg]
\end{aligned} \tag{3.18}
$$

$$
\text{s. t.} \quad \mu_{q,i,k,t,m}^{(4)} + \mu_{i,t,m}^{(7)} \geq -(1+\lambda)^{-t} c_{q,i,k,t}^{\mathrm{P}}, \quad \forall q, i, k, t, m \qquad [p] \tag{3.19}
$$

$$
\mu_{i,t,m}^{(7)} \geq -(1+\lambda)^{-t} c_{i,t}^{\mathrm{L}}, \forall i, t, m \qquad [r] \tag{3.20}
$$

$$
\mu_{i,j,t,m}^{(1)} + \mu_{i,j,t,m}^{(2)} - \mu_{i,j,t,m}^{(3)} - \mu_{i,t,m}^{(7)} + \mu_{j,t,m}^{(7)} = 0, \quad \forall (i,j) \in \mathcal{L}\cup\mathcal{N}, t, m \qquad [f] \tag{3.21}
$$

$$
\begin{aligned}
& \mu_{i,t,m}^{(5)} - \mu_{i,t,m}^{(6)} - \sum_{(i,j)\in\mathcal{L}} B_{i,j}\mu_{i,j,t,m}^{(1)} \\
& + \sum_{(j,i)\in\mathcal{L}} B_{j,i}\mu_{j,i,t,m}^{(1)} - \sum_{(i,j)\in\mathcal{N}} B_{i,j}(\sum_{k=1}^{t} x_{i,j,k})\mu_{i,j,t,m}^{(1)} \\
& + \sum_{(j,i)\in\mathcal{N}} B_{j,i}(\sum_{k=1}^{t} x_{j,i,k})\mu_{j,i,t,m}^{(1)} = 0, \\
& \qquad \forall i, t, m \qquad [\theta]
\end{aligned} \tag{3.22}
$$

$$
\mu^{(k)} \geq 0, k = 2, 3, 4, 5, 6. \tag{3.23}
$$

In this formulation, $\mu^{(2)}$ to $\mu^{(6)}$ are the dual variables of Constraints (3.5)-(3.11). Note that because the first-stage decision variable $x_{i,j,t}$ is a parameter in the subproblem, we can replace (3.6) with its equivalent form $f_{i,j,t,m} = (\sum_{k=1}^{t} x_{i,j,k})B_{i,j}(\theta_{i,t,m} - \theta_{j,t,m})$ to simply the formulation. In addition, with the previous replacement, we can combine Constraints (3.7) and (3.8) by removing the term $\sum_{k=1}^{t} x_{i,j,k}$. This explains why we can use the same dual variables $\mu^{(2)}$ and $\mu^{(3)}$ for Constraints (3.7) and (3.8).

In the objective function (3.18), the terms $F_{k,t,m}^{\mathrm{C}} P_{q,i,k} g_{q,i,k,t} \mu_{q,i,k,t,m}^{(4)}$ are bilinear. Since they are multiplications between a continuous variable and a binary variable, we linearize them by replacing $g_{q,i,k,t}\mu^{(4)}_{q,i,k,t,m}$ with variable $\eta_{q,i,k,t,m}$ and add two sets of constraints to the subproblem. The constraints are as follows.

$$\eta_{q,i,k,t,m} \geq \mu^{(4)}_{q,i,k,t,m} - M'(1 - g_{q,i,k,t}), \forall q, i, k, t, m \tag{3.24}$$

$$\eta_{q,i,k,t,m} \geq 0, \forall q, i, k, t, m \tag{3.25}$$

Then the subproblem can be easily solved as a mixed-integer linear program.

3.4.3 Algorithm

The algorithm uses the column and constraint generation procedure to iteratively generate new scenarios to be included in the subset $\mathcal{S}$. The master problem is solved in each iteration to provide an increasing series of lower bounds and a first-stage solution. Given the aforementioned first-stage solution, the subproblem is solved to provide an upper bound and a worst-case scenario to be included in set $\mathcal{S}$. The algorithm terminates after the optimality gap (difference between the upper and lower bounds) falls below a user defined threshold ϵ. Since the uncertainty set is a finite discrete one, the algorithm converges in a finite number of iterations. More detailed theoretic proof of the correctness and convergence of the algorithm can be found in [28]. A flow chart of the algorithm is summarized in Figure 3.2. Detailed steps of this algorithm are described as follows:

Step 0 : Initialize by creating $\mathcal{S}$ that contains at least one selected scenario. Set lower bound $LB = -\infty$, $UB = \infty$, and $s = 1$. Go to Step 1.

Step 1 : Update $s \leftarrow s+1$. Solve the master problem $\mathbf{M}(\mathcal{S})$ and let (x^s, ζ^s) denote its optimal solution. Update the lower bound as $LB \leftarrow \zeta^s$. Go to Step 2.

Step 2 : Solve the subproblem $\mathbf{R}(x)$ and let (g^s, z^s) denote the optimal solution. Update $\mathcal{S} \leftarrow \mathcal{S} \cup \{g^s\}$, and $UB \leftarrow C^{\mathrm{O}}(z^s)$.

Step 3 : If $UB - LB > \epsilon$, go to Step 1; otherwise return x^s as the optimal investment plan and $C^{\mathrm{I}}(x^s) + \zeta^s$ as the optimal value.

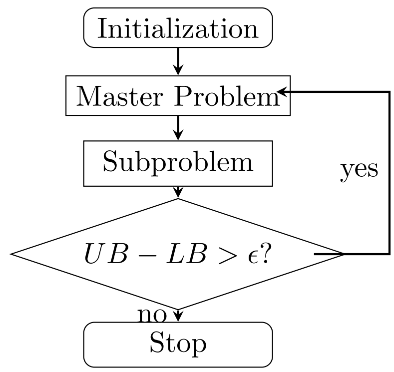

Figure 3.2 Flow chart of the decomposition algorithm

3.5 Case Study

In this section, we present a case study of our model and algorithm on the modified WECC 240-bus test system [58]. The topology of the test system is shown in Figure 3.3, which is from [52]. The system has 240 buses, 448 transmission lines, 139 load centers and 124 generation units, including coal, gas-fired, nuclear, hydro, geothermal, biomass, wind, solar and other renewable generators. The definition for generator type "renewables" can be found in [58].

We consider 18 candidate lines. The planning horizon of 20 years is divided into four five-year periods. As such, the solution space consists of 5^{18} possible solutions. We select the load of two representative hours from the typical week market data to be the baseline of our demand data. An annual growth rate of 1% is assumed.

We assume all of the 17 coal power plants will retire over the study period. Uncertainty lies in the time of their retirement. We also consider 53 candidate gas-fired units, wind and solar farms, and geothermal generators at various locations in the system, a subset of which will receive investment and become available during the planning horizon. When and which generators will be built is uncertain. The total number of generators with uncertainty adds up to 70. As such, the uncertainty set consists of $4^{17} \times 5^{53}$ possible scenarios. In the uncertainty set, we specify several requirements for the uncertain generators. Firstly, 50% of the new capacity needs to be included at the end of the planning horizon. Secondly, the total additional capacity at a decision point cannot be lower than 50% of the total additional capacity at the next decision point. Finally, we require that renewable energy generation account for at least

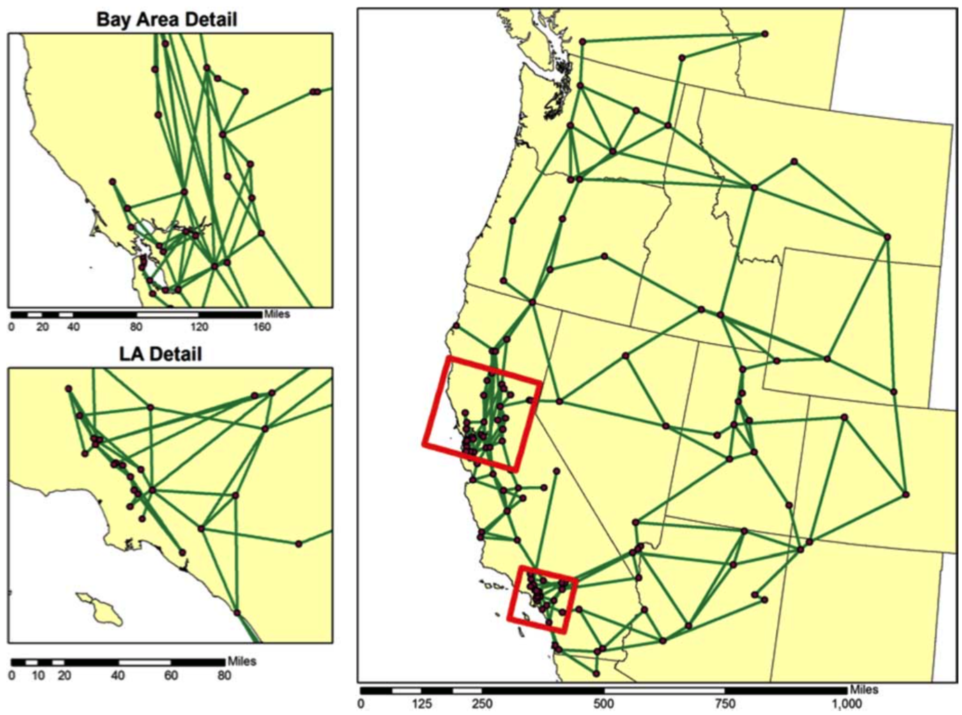

Figure 3.3 WECC 240-bus test system topology from [52]

a certain percentage of the total additional capacity. These restrictions on the uncertainty set can be adjusted based on available information as well as planners' belief on future generation profiles. The load curtailment cost is set as \$2000/MWh. The discount rate is set to be $\lambda = 10\%$.

We visualize the test system as well as the transmission planning problem in Figure 3.4. The 240 buses are arranged in a circle, with bus #1 starting at the three o'clock position and going counter-clockwise; bus #240 also comes back to the three o'clock position. Although the spatial information of the buses is distorted by this arrangement, it allows for more informative visualization of the big picture for the transmission planning problem. The 448 transmission lines are represented by black line segments connecting the corresponding bus pairs. The relative lengths of the line segments in the figure do not necessarily represent the actual lengths of the transmission lines. The 18 candidate lines are also plotted in green. These lines do not exist in the test system yet, and it is up to the planner to decide which (if any) lines should be built and when. The squares surrounding the inner circle represent the 139 load centers, with the areas of the squares proportional to the magnitudes of the loads at the corresponding nodes. The colorful dots surrounding the squares represent the existing 124 generators, with the areas of the dots proportional to the capacities of the generators at the corresponding nodes. The colorful stars represent the potential new generators that may be added to the corresponding nodes. The color map on the left sub-figure is for both existing and potential new generators. Multiple generators at the same nodes are plotted at different orbits to avoid overlap. The uncertainty set used in this case study is defined in Equation (3.26)-(3.31). Equation (3.26) means that once a coal power plant is retired, it remains retired throughout the rest of the planning horizon. Equation (3.27) means all coal power plants should be retired by the end of the planning horizon. Equation (3.28) means once a new generator is brought into the system, it remains there. Equation (3.29) means a certain percentage, denoted as w_1, of the total new generation capacity should be available by the end of the planning horizon. Equation (3.30) requires that the capacity change in the system be gradual, where w_2 is the highest allowed ratio between the variable capacity in the system in one period and that in the previous period. Equation (3.31) means certain percentage of the investment should be renewable resources, where w_3 is the percentage. In this case study, we set $w_1 = 0.7$, $w_2 = 0.6$, $w_3 = 0.2$ or 0.4.

$$\mathcal{G} = \Bigg\{ g | g_{q,i,k,t} \geq g_{q,i,k,t+1}, k \in \{\text{Coal}\} \forall q, i, t \tag{3.26}$$

$$g_{q,i,k,t^{\max}} = 0, k \in \{\text{Coal}\},\ \forall q, i \tag{3.27}$$

$$g_{q,i,k,t-1} \leq g_{q,i,k,t}, k \in \mathcal{K} \setminus \{\text{Coal}\}, \forall q, i, t \tag{3.28}$$

$$\sum_{q,i,k} g_{q,i,k,t^{\max}} P_{q,i,k} \geq w_1 \sum_{q \in \mathcal{W}, i, k \in \mathcal{K}} P_{q,i,k} \tag{3.29}$$

$$\sum_{q,i,k} g_{q,i,k,t} P_{q,i,k} \geq w_2 \sum_{q,i,k} g_{q,i,k,t-1} P_{q,i,k}, \forall t \tag{3.30}$$

$$\sum_{q,i,k \in \mathcal{R}} g_{q,i,k,t} P_{q,i,k} \geq w_3 \sum_{q,i,k \in \mathcal{K} \setminus \{\text{Coal}\}} g_{q,i,k,t} P_{q,i,k}, \forall t \Bigg\}. \tag{3.31}$$

We define four futures with respect to different scenarios of natural gas prices and policy requirement on renewables in order to analyze the sensitivity of the transmission planning solutions from our model.

- Future 1: High natural gas prices (double the current price) with a mandate of 20% additional renewables.

- Future 2: Low natural gas prices (at the current level throughout the next twenty years) with a mandate of 20% additional renewables.

- Future 3: High natural gas prices with a mandate of 40% additional renewables.

- Future 4: Low natural gas prices with a mandate of 40% additional renewables.

The methodology was implemented on a desktop computer with Intel Core i7 3.4GHz CPU, 8 GB memory and CPLEX 12.5. We set the optimality tolerance threshold ϵ as 10^{-3}, which means that the solution we found was at most \$0.001 more costly than the global optimal solution. The algorithm usually terminated within 3 to 7 iterations, with each iteration taking approximately two hours. In comparison, we also tried solving the case study using the KKT based algorithm from [49], but it was not able to terminate within days.

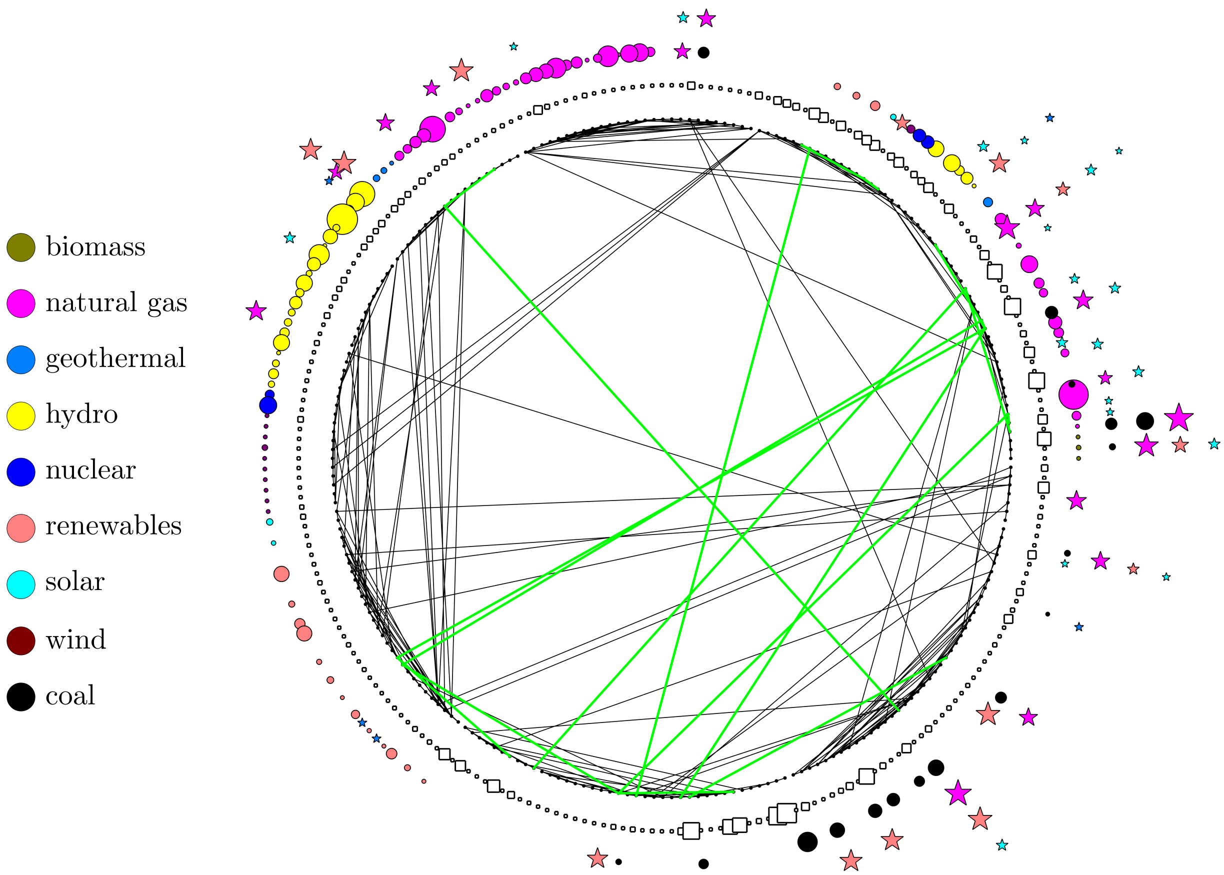

Figure 3.4 Visualization of the transmission planning problem

We obtained six transmission plans. Plans 1 to 4 are the optimal transmission plans under Futures 1 to 4, respectively using the proposed robust optimization model. Plans 5 and 6 are two other transmission investment plans that were constructed to represent non-optimal planning decisions. Plan 5 represents the under-investment case when the planner fails to add sufficient new transmission capacity, whereas Plan 6 represents the over-investment case, which is to invest in 15 out of the 18 candidate lines at once in the first period. These six transmission plans are visualized in Figure 3.5. Each column represents a plan, and each row represents a five-year period in the planning horizon. The green lines are to be added to the system in the period according to the transmission plan. Although the added new lines are cumulative, only new additions are shown in green. It can be seen that the first four transmission plans are similar and all build new lines gradually in the first three periods. Plan 5 has only a small number of lines, and plan 6 adds 15 out of 18 candidate transmission lines at once in the first period.

A by-product of the algorithm from Section 3.4 is a set of scenarios. We have collected twelve of these scenarios under the four futures and use them to illustrate the robustness performances of the six plans under these scenarios. The scenarios are shown in Figure 3.6. The four orbits of the stars indicate the periods in which the new generators are added to the generation portfolio. The higher the orbit, the later in time. The timing of the retirement of the coal power plants is not visualized in those scenarios. The label 'W1' is for the worst-case scenario (with highest cost) for Plan 1, 'B2' for the best-case scenario (with lowest cost) for Plan 2, and so on. It is noticeable in the figure that in the worst-case scenarios new generators tend to emerge later and in smaller amounts, most of which are expensive renewables; whereas in the best-case scenarios more of them start to serve the demand earlier in time, and they include more natural gas generators, which are less costly.

Finally, the robustness of the six transmission plans under the twelve scenarios was evaluated with respect to the investment cost, the operations cost, and load curtailment (as a percentage of total load) over the planning horizon in Figure 3.7. The horizontal axis of each sub-graph is the 20-year planning horizon, containing four discrete 5-year periods. The blue horizontal lines in the first row represent the net present values of the total investments of the plans. The

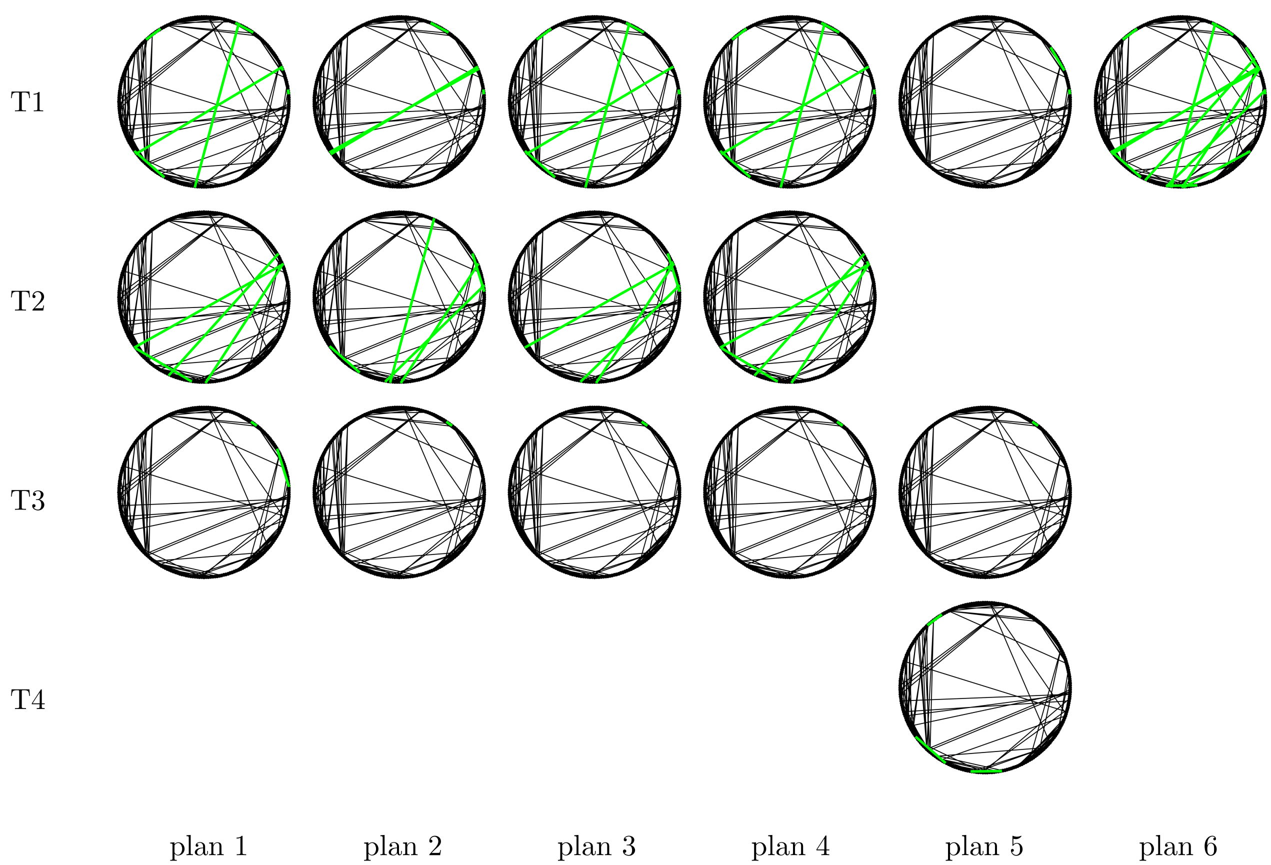

Figure 3.5 Six transmission plans.

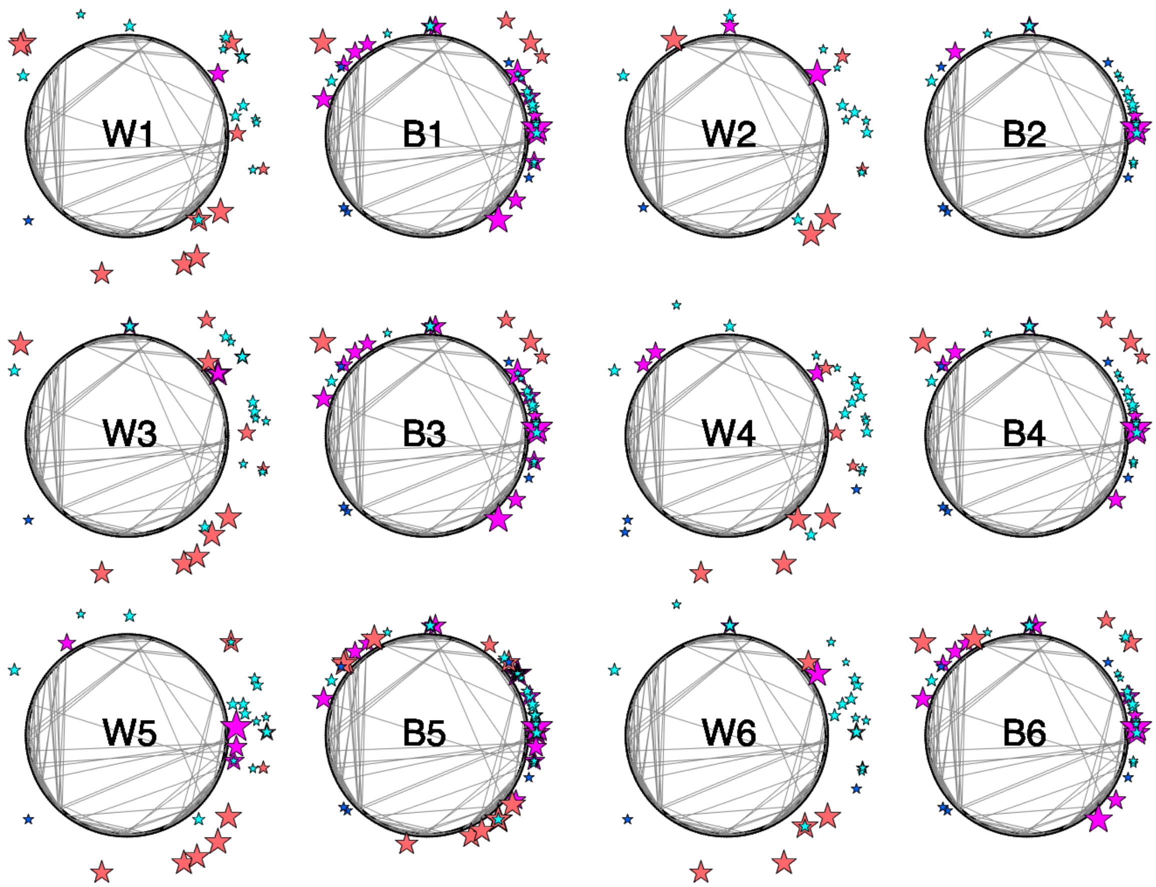

Figure 3.6 Twelve scenarios. The label 'W1' is for the worst-case scenario for Plan 1, 'B2' for the best-case scenario for Plan 2, and so on.

colored regions in the bottom two rows in Figure 3.7 are outlined by the upper and lower bounds performances under the twelve scenarios. The white curves inside the colored region are the intermediate scenarios. The figure suggests that the first four plans have similar investment costs, with Plan 4 being the least costly. Their performances in operational costs and load curtailments are also comparable. Plan 5 has a negligible amount of investment, which is a \$1.4B savings from Plan 4. However, as a consequence of the lack of enough investment, its operational cost is almost \$20B more than Plan 4 over the planning horizon, and its load curtailment in the worst case is also much severer than Plan 4. On the other extreme, Plan 6 makes about 20% more investment than Plan 4, but its performance in operations cost and load curtailment are similar, if not even worse than the cheaper counterpart. These results demonstrate the need for making enough and smart investment in transmission planning in order to reduce the long-term operations cost as well as enhancing the reliability of the grid.

Another observation we would like to make is the subtle differences among the first four plans. According to their performances in Figure 7, Plans 1 and 2 appear to be more similar and Plans 3 and 4 look more alike. This observation suggests that the transmission plans might be more sensitive to the renewable mandates (that set Plans 1 and 2 apart from 3 and 4) than they are to the natural gas prices. This observation is consistent with the intuition that transmission investment decisions should be driven by the availability of generation resources (or the lack of them) more than their fuel prices.

3.6 Conclusion

In this paper, we propose a multi-period robust transmission planning model focusing on the management of the impact of generation investment and retirement uncertainty. By using a multi-period model, a more detailed construction schedule for transmission lines can be derived compared to the static single period models. We propose a flexible modeling framework to represent generation investment and retirement uncertainty in more detail. Under the new modeling framework, uncertainty concerning the time, location and size of generation investment can be described in the uncertainty set. To solve the model, we use a column and constraint generation procedure to decompose the two-stage problem into a master problem

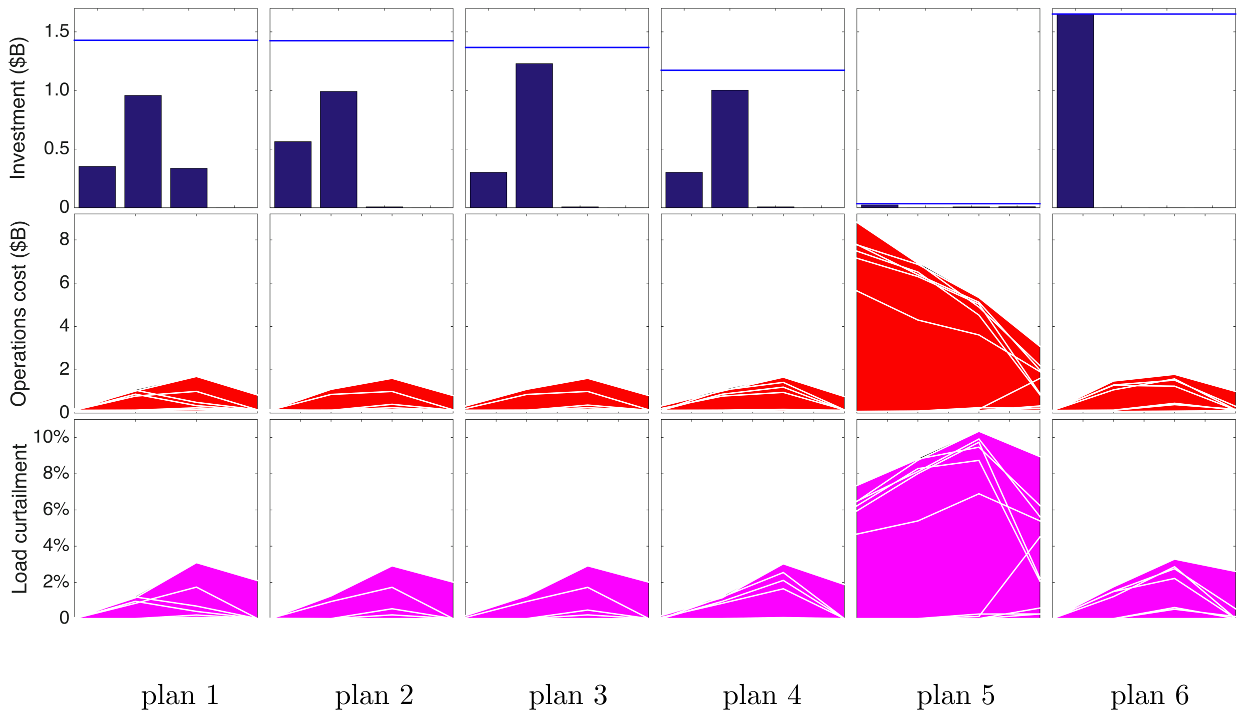

Figure 3.7 Performance of six plans. The horizontal axis of each sub-graph is the 20-year planning horizon, containing four discrete 5-year periods.

and a subproblem. By using the structural characteristic of our model, the bilinear subproblem can be reformulated as a mixed-integer linear problem without utilizing the KKT conditions, which also makes a larger test case more tractable. We apply our model and solution method to the WECC 240-bus test system. Then we compare the performance of the robust transmission plans derived by our model to the performance of plans derived from solving deterministic models or other heuristic rules under four different scenarios, which are obtained by adjusting natural gas price and renewable energy policy mandate. The results show that our transmission plans not only out-perform plans obtained from other means but also remain robust under different scenarios.

Several directions of possible extensions to this work are worth investigating for future research. One way to extend this work is to combine robust optimization and stochastic programming by modeling various sources of uncertainty with different methods. For example, we can generate scenarios for operational level uncertainty such as load and renewable energy generation and use uncertainty sets for other types of uncertainty. To solve such a problem, and to improve the computational efficiency of current models, new algorithms need to be developed. Another way to extend the work done to date is to incorporate additional details into the model, including explicit representations of various existing and possible policies, changes in demand caused by demographic development, climate change and recovery from natural disasters. As a caveat, our model was based on DC power flow, which is known to be less accurate than AC power flow model, although the latter would introduce non-linearity and non-convexity and make the model much harder to solve. It would be a meaningful future research to examine the extent to which the simpler DC power flow could compromise the quality of the transmission planning decisions. We also note that the proposed model is able to represent different types of transmission lines that differ in cost and thermal capacity; but other types of technological variations are not reflected.

CHAPTER 4. A STOCHASTIC PROGRAMMING FRAMEWORK FOR OPTIMAL DISTRIBUTION NETWORK HARDENING

Bokan Chen, Jianhui Wang, Lizhi Wang

Abstract

The power industry is paying more attention to improving system resilience by hardening the infrastructure in face of increasingly frequent natural disasters. In this paper, we propose a stochastic programming framework to optimally allocate limited budget to the hardening of the power distribution networks. The stochastic programming model is formulated as a two-stage problem, where first-stage decisions are investment strategy selections, and second-stage decisions include optimal power flow and load shedding. Different hardening methods and their impact on the failure probability of the distribution lines are considered. To consider all possible scenarios would make the stochastic program intractable. Sample average approximation is used to reduce the size of the problem. A reasonably large number of scenarios are simulated to improve the accuracy of the approximation. In order to solve the resulting large scale mixed-integer program, the L-shaped method (Benders Decomposition) is used to decompose the model into a master problem and a group of subproblems. A heuristic algorithm is also developed to get a reasonably good solution in a short time. The model and algorithms are tested on an IEEE 33-bus distribution system to demonstrate their effectiveness. The results are then validated through the single replication procedure.

4.1 Nomenclature

Sets and indices

$\mathcal{I}$	Set of nodes, $i \in \mathcal{I}$
$\mathcal{L}$	Set of distribution lines, $(i, j) \in \mathcal{L}$
$\mathcal{T}$	Set of time periods in consideration, $t \in \mathcal{T}$
$\mathcal{K}$	Set of hardening techniques, $k \in \mathcal{K}$
$\mathcal{S}$	Set of scenarios, $s \in \mathcal{S}$

Parameters

$c^{\mathrm{H}}_{i,j,k}$	Cost of hardening line (i, j) with technique k
B	Total hardening budget
$x_{i,j}$	Reactance of power line (i, j)
$r_{i,j}$	Resistance of power line (i, j)
$d^{\mathrm{P}}_{i,t}$	Real power demand at node i at time t
$d^{\mathrm{Q}}_{i,t}$	Reactive power demand at node i at time t
$P^{\max}_i$	Real capacity of distributed generator i
$Q^{\max}_i$	Reactive capacity of distributed generator i
$\tilde{z}_{i,j,t,k}$	Binary random variables indicating whether power line (i, j) hardened with strategy k is damaged at time t. We use $z^s_{i,j,t,k}$ to represent a sample of $\tilde{z}_{i,j,t,k}$. $\tilde{z}_{i,j,t,k} = 1$ means the power line is damaged; $\tilde{z}_{i,j,t,k} = 0$ otherwise
M	Large constant used to limit the power flow based on the condition of the distribution line
$V^{\min}$	The lower bound on voltage levels
$V^{\max}$	The upper bound on voltage levels
V_0	Reference voltage magnitude

Decision Variables

$f^{\mathrm{P}}_{i,j,t}$	Real power flow on line (i,j) at time t
$f^{\mathrm{Q}}_{i,j,t}$	Reactive power flow on line (i,j) at time t
$p_{i,t}$	Real power generated by generator at node i at time t
$q_{i,t}$	Reactive power generated at node i at time t
$v_{i,t}$	Voltage magnitude at node i at time t
$y_{i,j,t}$	Binary variable indicating whether power line (i,j) is operational at time t. $y_{i,j,t} = 1$ means the line is operational; $y_{i,j,t} = 0$ otherwise
$h_{i,t}$	The percentage of load not served at node i at time t
$u_{i,j,k}$	Binary variable indicating whether line (i,j) is hardened with strategy k. $u_{i,j,k} = 1$ means the line is hardened; $u_{i,j,k} = 0$ otherwise

4.2 Introduction

Electricity has become an integral part of people's day-to-day life. Transmission and distribution (T&D) networks are essential infrastructures for the power grid. In the United States, there are more than 200,000 miles of high voltage transmission lines and numerous distribution lines that span the entire country. The vast network of T&D lines enables the transportation and distribution of cheap electricity to countless consumers. In addition to the existing infrastructure, the networks are expanding to accommodate increasing demands and integration of renewable energy resources. However, such vast networks are vulnerable to disruptions caused by natural disasters including seismic activities and extreme climate events. The President's Council of Economic Advisers and the U.S. Department of Energy reported an estimated 679 widespread power outages related to severe weather conditions between 2003 and 2012, leading to an annual average of 18 to 33 billion dollars of cost to the economy [54]. Due to climate change, the frequency and intensity of natural disasters such as floods, droughts, hurricanes and other extreme weather activities are expected to rise [63]. Improving power system security and resilience not only would reduce the severity of operation disruptions and save the economy billions of dollars, but also could be instrumental to the recovery of communities from catastrophes.

The resilience concept includes two levels of capabilities: the inherent ability of a system to cope with disruptions and the ability to recover from such disruptions [48]. Both abilities of the power system could be improved to reduce the impact of natural disasters through investment in power grid modernization and resilience. One of the most effective ways to improve grid's ability to resist disruptions is by the hardening of existing components [54]. Investment in hardening can also help to reduce response time and expedite recovery after the occurrence of disruptions. Methods of hardening include upgrading power poles and structures with stronger material, vegetation management and undergrounding utility lines. Hardening of the entire grid simultaneously could be prohibitively expensive and therefore is unrealistic. Thus, such investments are usually constrained by a limited budget. It is crucial to allocate the limited resources to the most vulnerable components or to the power lines with the largest impact to system performance if affected.

Research on power system resilience planning is relatively limited in scope, with a lot of work focusing on transmission networks. Several previous studies use the Stackelberg game theoretic framework that models terrorists and natural disasters as adversaries aim to maximize damage on the power system. A bilevel programming approach was proposed in [6] to identify the most critical components in the power system. Other studies take the problem one step further by adding another level to the model that optimizes the preventive measures taken by the network planner before disruption occurs. Yao *et al.* [69] first proposed the defender-attacker-defender trilevel model for the resource allocation problem in power system defense. Alguacil *et al.* [4] proposed a two-stage trilevel model for power grid defense planning, where planners make moves before and after an attack occurs to minimize the maximum damage attackers can inflict. A similar model was proposed in [72] with improved solution methods using the column and constraint generation algorithm. These models can provide valuable insights on how to identify vulnerable components in a system and what to do with such information, but they do have some shortcomings. One drawback of the game theoretic models is that some strong assumptions (perfect information between players, for example) need to be made, which might not be true in some cases. In addition, due to the complexity of trilevel models, details giving nuances to the problem might be lost.

Designing adaptive network topology that can isolate the faulty components or reconfigure the larger network into smaller functioning networks is another popular direction in the research of improving power system resilience. Baran *et al.* [8] developed two power flow formulations for network reconfiguration to reduce loss and balance load. Chen *et al.* [21] proposed to break a large network into several small microgrids in the aftermath of disasters to pick up as much critical loads as possible. In [53, 56], intentional islanding was proposed as a method to minimize blackouts by creating a collection of operational or nonoperational islands to prevent blackouts in a larger scale. These works focus on corrective actions after damages to the network have been caused by natural disasters. The work in this paper, on the other hand, represents efforts aim to prevent such damages from occurring.

Due to the stochastic nature of natural disasters, uncertainty is an important facet of the resilience planning problem. As an optimization framework for decision making under uncertainty, robust optimization has been used to improve the resilience of power systems. The aforementioned research in the trilevel defender-attacker-defender models [4, 72], etc. can be seen as special cases of robust optimization. In addition, Yuan *et al.* [71] used robust optimization to solve the resilient distribution network planning problem. However, robust optimization uses polyhedral sets to describe uncertainty and optimize the objective function under the worst-case scenario. Due to the structure limitations of the model, it is very difficult to capture the dependency between the first-stage decisions and the uncertain parameters in a two-stage model. For example, in [71], it is assumed that hardened distribution lines would be invulnerable to the effect of natural disasters, which might not be true in many cases.

Stochastic programming, on the other hand, uses scenarios generated according to the probability distributions of uncertain parameters to characterize uncertainty, which is very suitable for the description of the impact of natural disasters on the power grid. Although its application in power system resilience is relatively limited, stochastic programming has been successfully applied to freight transportation network resilience planning [48, 22], which shares several similarities with power transmission and distribution networks. For example, both types of networks have high levels of interconnectedness that make them vulnerable to disruptions caused by natural disasters. Yamangil *et al.* [68] proposed a two-stage stochastic programming model for designing resilient electrical distribution grids by altering the topology of the networks, in which power flow is approximated by a multi-commodity network flow model. Damage scenarios from natural disasters are modeled as stochastic events to estimate the performance of the network in the aftermath of a disaster.

In this paper, we propose to use a stochastic programming approach to study the resilience planning problem. The effect of disasters on distribution networks can be easily described by probability distributions (Bernoulli distribution is used in this paper). The treatment of uncertainty in stochastic programming is more flexible than robust optimization. As a result, we are able to capture the fact that hardened lines are still susceptible to damages, albeit with lower probabilities. The resilience planning problem is modeled as a two-stage stochastic mixed-integer programming problem. The objective of this model is to identify the best strategy to allocate limited budget to critical links in the power grid and improve the resilience of distribution networks. In this problem, the first-stage variables are planning decisions on the selection of hardening strategies. The second-stage decisions include the optimal power flow, generation and load shedding. Sample average approximation and the L-shaped method (Benders decomposition) are used to reduce the size of the optimization model and speed up the computation. A heuristic algorithm based on the Knapsack problem is also developed to produce a reasonably good solution quickly. The model and algorithms are then applied to an IEEE 33-bus distribution system to test the effectiveness of the proposed methods. The contributions of this paper can be summarized as follows:

1) The proposed model can take the recovery time of the damaged components into consideration. In addition, the effects of hardening different components in the gird are considered.

2) The proposed approach models the dependencies between uncertainty parameters and decision variables.

3) A new heuristic method is developed to solve optimization problems with Knapsack constraints, which can identify a good solution quickly.

The rest of the paper is organized as follows. Section 4.3 presents the two-stage stochastic resilience planning model formulation in detail. Section 4.4 describes the sample average approximation and Benders decomposition methods used in this paper for computation. The single replication procedure and the Knapsack based heuristic is also presented. Section 4.5 outlines the data preparation (scenario generation) procedure for the case study and presents the numerical results. Finally, Section 4.6 provides a conclusion and some discussions on future research.

4.3 Model Formulation

The optimal resilience planning problem is formulated as a two-stage stochastic programming model. In the two-stage model, network hardening decisions need to be made before extreme weather events happens, without the information on the network damage status caused by such events. Then after the damage in the network is observed, the optimal operational decisions can be made.

One difficulty in the modeling process is taking the impact of investment decisions on the uncertain parameters into consideration. In most stochastic programs, uncertain parameters do not depend on decisions, which is not the case in this paper. Stochastic programs with decision dependent uncertainty is explored by Jonsbråten *et al.* [37] for a specific class of problems. In this paper, the decision-dependency of random variables is achieved by using binary variables to select the appropriate set of uncertainty realizations, which simplifies the solution process. The two-stage model can be formulated as follows:

$$\min_{u \in \mathcal{U}} \quad \mathbb{E}_{\tilde{z}}[Q(u, \tilde{z})] \tag{4.1}$$

where

$$\mathcal{U} = \left\{ u | \sum_{i,j,k} c^{\mathrm{H}}_{i,j,k} u_{i,j,k} \leq B \right. \tag{4.2}$$

$$\sum_{k} u_{i,j,k} = 1, \quad \forall (i,j) \in \mathcal{L} \tag{4.3}$$

$$\left. u_{i,j,k} \text{ Binary}, \quad \forall (i,j) \in \mathcal{L}, k \in \mathcal{K} \right\} \tag{4.4}$$

$$Q(u, \tilde{z}) = \min_{y,f,p,q,v,h} \quad \sum_{i,t} d^{\mathrm{P}}_{i,t} h_{i,t} \tag{4.5}$$

$$\text{s. t.} \qquad y_{i,j,t} = 1 - \sum_{k} \tilde{z}_{i,j,k,t} u_{i,j,k}, \quad \forall (i,j) \in \mathcal{L}, t \tag{4.6}$$

$$\sum_{j} f^{\mathrm{P}}_{i,j,t} = \sum_{j} f^{\mathrm{P}}_{j,i,t} + p_{i,t} - h_{i,t} d^{\mathrm{P}}_{i,t}, \quad \forall i, t \tag{4.7}$$

$$\sum_{j} f^{\mathrm{Q}}_{i,j,t} = \sum_{j} f^{\mathrm{Q}}_{j,i,t} + q_{i,t} - h_{i,t} d^{\mathrm{Q}}_{i,t}, \quad \forall i, t \tag{4.8}$$

$$\begin{aligned} -V^{\max}(1 - y_{i,j,t}) \leq v_{j,t} - v_{i,t} + \frac{r_{i,j} f^{\mathrm{P}}_{i,j,t} + x_{i,j} f^{\mathrm{Q}}_{i,j,t}}{V_0} \\ \leq V^{\max}(1 - y_{i,j,t}), \quad \forall (i,j) \in \mathcal{L}, t \end{aligned} \tag{4.9}$$

$$0 \leq p_{i,t} \leq P^{\max}_{i}, \quad \forall i, t \tag{4.10}$$

$$0 \leq q_{i,t} \leq Q^{\max}_{i}, \quad \forall i, t \tag{4.11}$$

$$-M y_{i,j,t} \leq f^{\mathrm{P}}_{i,j,t} \leq M y_{i,j,t}, \quad \forall (i,j), t \tag{4.12}$$

$$-M y_{i,j,t} \leq f^{\mathrm{Q}}_{i,j,t} \leq M y_{i,j,t}, \quad \forall (i,j), t \tag{4.13}$$

$$V^{\min} \leq v_{i,t} \leq V^{\max}, \quad \forall i, t \tag{4.14}$$

$$0 \leq h_{i,t} \leq 1 \quad \forall i, t \tag{4.15}$$

$$y_{i,j,t} \text{ Binary}, \quad \forall (i,j) \in \mathcal{L}, t \tag{4.16}$$

In this two-stage model, the objective is to minimize the expected value of the second-stage objective function with respect to the probability distribution of the random vector $\tilde{z}$, as expressed in Equation (4.1). In the first-stage, hardening strategies are selected from the feasibility set $\mathcal{U}$ defined by Equations (4.2)-(4.4). Equation (4.2) is the budget constraint. It means the investment on hardening distribution lines should be within a predetermined budget. Equation (4.3) means that exactly one hardening strategy should be selected. Doing nothing is also considered a strategy in this context.

The second-stage problem, which is defined by Equations (4.5)-(4.16), is a function of the first-stage decisions u and the uncertainty parameter z. Equation (4.5) represents the second-stage objective function—total load shedding. Equation (4.6) connects the operational status of power lines with the hardening strategy and the damaging status of power lines. It selects the appropriate status of a power line based on the first-stage solution and uncertainty realization.

Equation (4.7) and (4.8) means the real and reactive power flowing out of a node must be equal to the power flowing into a node. Equation (4.9) enforces the voltage differences caused by power flow between two connected nodes. Equations (4.7)-(4.9) are called DistFlow equations and have been widely used in distribution network planning and operation problems [66, 65]. Equations (4.10) and (4.11) are the generator capacity constraints. Equations (4.12) and (4.13) limit the power flow on distribution lines based on the operation status of lines (If a line is damaged, there would be no power flow on it). Equation (4.14) limits the range of voltage levels. Equation (4.15) limits the percentage levels of unserved load.

4.4 Solution Method

In the two-stage stochastic programming model represented by Equation (4.1), the random data vector $\tilde{z}$ has finite support. However, the possible number of scenarios for the uncertainty parameter is $2^{|\mathcal{L}|}$ and it is computationally intractable to solve the exact two-stage stochastic model. In this paper, we use sample average approximation (SAA) [41] to reduce the problem size through random sampling of the uncertainty parameters. In order for the approximation to achieve reasonable accuracy, a decent number of scenarios are needed. The resulting problem would still be large, which makes solving the problem directly difficult. We use the L-shaped method (based on Benders Decomposition) to further reduce the computation time.

4.4.1 Sample average approximation

The idea of SAA is rather simple. When SAA is applied to solve the two-stage stochastic program in Equation (4.1), instead of solving it directly, we estimate the optimal solution by sampling. More specifically, a sample $z^1, \ldots, z^{|\mathcal{S}|}$ of $|\mathcal{S}|$ realizations of the random vector $\tilde{z}$ is generated through Monte Carlo simulation. Assuming the sample is independent and identically distributed (i.i.d.), the expected value function $\mathbb{E}[Q(u, \tilde{z})]$ is approximated by the sample average function $\frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} Q(u, z^s)$. As a result, the original two-stage problem can be approximated by the following problem.

$$\min_{u \in \mathcal{U}} \quad \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} Q(u, z^s) \tag{4.17}$$

Given the scenario set $\mathcal{S}$, the model in Equation (4.17) is deterministic and can be solved with a deterministic optimization algorithm.

If we use $(\hat{u}, \hat{Q})$ to represent the optimal solution and objective value of the SAA model and use (u^*, Q^*) to represent the optimal solution and objective value of the original stochastic model in Equation (4.1), we have $\hat{Q} \to Q^*$ as $|\mathcal{S}| \to \infty$. Detailed theoretical discussion on SAA is available in [41].

4.4.2 Single replication procedure

SAA is a simulation based method. As the name suggests, it can only provide an approximation of the optimal solution. Thus, it is important to test the solution quality to see how good the approximation is. One good way to test the quality of stochastic programming solutions is the multiple replication procedure [10]. The drawback of the multiple replication procedure is that many sets of samples need to be generated (usually more than 30) and the corresponding stochastic program solved, which is undesirable when even solving one program takes a long time. To circumvent this problem, the single replication procedure outlined by Bayraksan *et al.* [9] is used instead.

Use $\hat{u}$ to denote the current optimal solution, which is the object for test; and $g(\hat{u})$ to denote the optimality gap between $\mathbb{E}[Q(\hat{u}, \tilde{z})]$ and $\min\limits_{u \in \mathcal{U}} \mathbb{E}[Q(u, \tilde{z})]$ The procedure is described as follows:

Step 1 : Generate a new sample of scenarios $\bar{\mathcal{S}}$ of the random vector $\tilde{z}$, with $\bar{z} = \{\bar{z}^1, \bar{z}^2, \ldots, \bar{z}^{|\bar{\mathcal{S}}|}\}$.

Step 2 : Solve the following stochastic program, and denote the optimal solution as u^*.

$$\min_{u \in \mathcal{U}} \frac{1}{|\bar{\mathcal{S}}|} \sum_{s \in \bar{\mathcal{S}}} Q(u, \bar{z}^s) \tag{4.18}$$

Step 3 : Calculate the sample mean of the optimality gap $\hat{g}$ and sample variance $s^2(u^*)$ as follows:

$$\hat{g}(\hat{u}) = \frac{1}{|\bar{\mathcal{S}}|} \sum_{s \in \bar{\mathcal{S}}} Q(u^*, \bar{z}^s) - Q(\hat{u}, \bar{z}^s) \tag{4.19}$$

$$\hat{\sigma}^2(\hat{u}) = \frac{1}{|\bar{\mathcal{S}}| - 1} \sum_{s \in \bar{\mathcal{S}}} \Big[(Q(\hat{u}, \bar{z}^s) - Q(u^*, \bar{z}^s)) - \hat{g}(\hat{u}) \Big]^2 \tag{4.20}$$

Step 4 : Calculate the one-sided confidence interval of the optimality gap $g(\hat{u})$ as $\big[0,\, \hat{g}(\hat{u}) + t_{|\bar{\mathcal{S}}|-1,\alpha}\hat{\sigma}(\hat{u})/\sqrt{|\bar{\mathcal{S}}|}\big]$, where $t_{|\bar{\mathcal{S}}|-1,\alpha}$ is the $1-\alpha$ quantile of the Students' t distribution with $|\bar{\mathcal{S}}|-1$ degrees of freedom.

This single replication procedure can provide an interval estimation of the optimality gap between the SAA approximate and the optimal solution, based on which we can judge the solution quality.

4.4.3 L-shaped method

Even with the reduced scenario number brought by SAA, the optimization problem can be very large and difficult to solve. Currently, the most frequently used algorithms to solve large scale stochastic programming include progressive hedging [43] and traditional L-shaped method [64] based on Benders decomposition. Since Benders decomposition relies on the dual information of the second-stage variables, the second-stage problem should be continuous. In this paper, the integer constraints on the second-stage variable y can be relaxed, which makes Benders decomposition a viable option. Another advantage of the L-shaped method is that it is relatively easy to implement and performs pretty well.

In the L-shaped method, the large scale optimization problem is decomposed into a master problem and a series of subproblems. The master problem and subproblems are formulated as follows:

Master Problem:

$$\min \quad \theta \tag{4.21}$$

$$\text{s.t.} \quad u \in \mathcal{U} \tag{4.22}$$

$$\theta \geq f^{\omega}(u), \forall \omega = 1, \ldots, \Omega \tag{4.23}$$

Subproblems:

$$Q(\hat{u}, z^s), \forall s \in \mathcal{S} \tag{4.24}$$

Optimality cuts represented by Equation (4.23) are iteratively generated until optimal solution is identified. The optimality cuts can be formulated as follows:

$$\begin{aligned}\theta \geq &\frac{1}{|\mathcal{S}|}\sum_{s}\Bigg[\sum_{i,j,t}[\alpha^{\omega}_{i,j,t,s}(1-\sum_{k} z^{s}_{i,j,k}u_{i,j,k}) - \mu^{-,\omega}_{i,j,t,s}V^{\max} \\ &- \mu^{+,\omega}_{i,j,t,s}V^{\max}] - \sum_{i,t}(\beta^{\omega}_{i,t,s}P^{\max}_{i} + \gamma^{\omega}_{i,t,s}Q^{\max}_{i} - \eta^{-,\omega}_{i,t,s} \\ &V^{\min} + \eta^{+,\omega}_{i,t,s}V^{\max} + \lambda^{\omega}_{i,t,s})\Bigg] \end{aligned} \tag{4.25}$$

where $\alpha^{\omega}_{i,j,t,s}$ is the dual variable for Equation (4.6) corresponding to scenario s at iteration ω. $\mu^{-,+,\omega}_{i,j,t,s}$ represents the dual variables of Equation (4.9). $\beta^{\omega}_{i,t,s}$ and $\gamma^{\omega}_{i,t,s}$ are the dual variables for Equations (4.10) and (4.11). $\eta^{-,+,\omega}_{i,t,s}$ and corresponds to Constraint (4.14). $\lambda^{\omega}_{i,t,s}$ is the dual variable for Equation (4.15).

The master problem is equivalent to the full problem if the optimality cuts in Equation (4.23) include cuts generated from all possible combinations of feasible dual variables from the subproblems. Since the optimality cuts are iteratively generated, the master problem represents a relaxation of the SAA approximation model and can provide an "optimistic" estimation (lower bound) of the optimal solution as well as a candidate first-stage solution $\hat{u}$. The candidate solution is then passed to the subproblems. After solving the subproblems, we can check if the estimation from the master problem is realistic or not. If it is not realistic, then the subproblems can send a feedback message in the form of optimality cuts in Equation (4.23) to the master problem. In addition, the average of the subproblem objective values provides an upper bound of the optimal solution. No feasibility cut is generated since the problem has complete recourse, which means the second-stage problems are always feasible regardless of which first-stage solution is selected.

Detailed steps of the algorithm are described as follows.

Step 0 : Initialize by setting lower bound $LB = -\infty$, $UB = \infty$, and iteration number $\omega = 0$. Go to Step 1.

Step 1 : Solve the master problem (4.21)-(4.23) and let $(\hat{u}, \hat{\theta})$ denote its optimal solution. Update the lower bound as $LB = \hat{\theta}$. Go to Step 2.

Step 2 : Solve the subproblems $Q(\hat{u}, z^s)$, $\forall s \in \mathcal{S}$. Update the iteration number $\omega \leftarrow \omega + 1$, and the upper bound $UB = \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} Q(\hat{u}, z^s)$. Go to Step 3.

Step 3 : If $UB - LB \geq \epsilon$, generate an optimality cut based on solutions of the subproblems and go to Step 1; otherwise return $\hat{u}$ as the optimal hardening strategy and UB as the optimal value.

4.4.4 Greedy search heuristic

With the Knapsack constraints, large number of scenarios and 24 hour operation consideration, this problem still takes a large amount of time to solve. In this section, we develop a heuristic method as an alternative to exact methods like the Benders Decomposition. In this algorithm, the two-stage model is simplified into a Knapsack problem, which then can be solved by dynamic programming [47]. The algorithm is briefly described as follows:

Step 0 : Normalize the budget and hardening costs: divide the budget and hardening costs by $\min c^{\mathrm{H}}_{i,j,k}$. Denote the normalized costs as $w_{i,j,k}$ and normalized budget as W.

Step 1 : Calculate the benefit of hardening each individual lines defined as $v_{i,j,k} = \frac{1}{|\mathcal{S}|} \sum_s [Q(0, z^s) - Q(u^0_{i,j,k}, z^s)]$, where $u^0_{i,j,k}$ means only line (i, j) is hardened with method k.

Step 2 : Solve the following Knapsack problem:

$$
\begin{aligned}
\max \quad & \sum_{i,j,k} v_{i,j,k} u_{i,j,k} && (4.26)\\
\text{s.t.} \quad & \sum_{i,j,k} w_{i,j,k} u_{i,j,k} \leq W && (4.27)\\
& \text{Equations}(4.3) - (4.4) && (4.28)
\end{aligned}
$$

This heuristic does not consider the additional benefits the system can obtain through coordination between different lines. Therefore the solution it yields is usually not optimal. However, it can provide a reasonably good solution almost instantly, which can be desirable when solving industry scale problems. In addition, this algorithm can be widely applied to problems with Knapsack constraints.

4.5 Case Study

In this section, we present our case study and the numerical results. The model and algorithm is tested on an IEEE-33 bus system. The performance of the distribution network in the 24-hour period during a hurricane is used to evaluate the hardening plans. As such, the effect of hurricane decay, different time points a component could fail, and component repair/recovery can be accounted for.

4.5.1 Scenario generation

For overhead power lines, severe weather poses the most significant threat [16]. In this paper, we focus on hurricanes for illustration. Note scenarios are treated as input data for the model. Other natural disasters can be added without affecting the model.

For power lines in distribution networks hit by a hurricane, failure typically occurs for several reasons:

- Distribution poles are toppled by strong winds.
- The conductors between distribution poles are damaged by direct wind force impact.
- The conductors between distribution poles are damaged by trees uprooted or broken by strong winds.

Due to the lack of data, we only consider the first two reasons. In addition, since in this paper we mainly consider the hardening of distribution lines, damages to other components of a distribution networks including substations and distribution transformers are not considered.

We use p_{pole} and $p_{\text{conductor}}$ to represent the probability of failure caused by the aforementioned two reasons. Using the component fragility model proposed in [55] and [30], the probabilities can be formulated as follows:

$$p_{\text{pole}}=\min\{ae^{bw},1\} \tag{4.29}$$

$$p_{\text{conductor}}=\begin{cases} 0, & w \leq w_{\min} \\ \frac{w-w_{\min}}{w_{\max}-w_{\min}}, & w_{\min} \leq w \leq w_{\max} \\ 1, & w \geq w_{\max} \end{cases} \tag{4.30}$$

where w represents wind speed. (a, b) are parameters related to pole property. $w_{\max}$ represents the maximum wind speed the conductor material can withstand and $w_{\min}$ represents the minimum wind speed that can affect the conductor material. The restoration time for failed poles and conductors are assumed to be 6 and 4 hours respectively.

The model can handle multiple hardening strategies. However, to simplify computation and analysis, we only consider the following hardening strategy—updating the pole and conductor of a distribution line. Hardening strategies affect one or more parameters in the fragility model in (4.29)-(4.30), with the effect of reducing failure probability of the affected components. For example, with new conductor material, both $w_{\min}$ and $w_{\max}$ could increase.

Wind speed information can be obtained from the ASCE-7 windspeed website [1]. A distribution network typically covers a relatively small geographical area. Therefore in this paper, we assume the wind speed experienced by different segments of the entire distribution network is the same at a single moment. Based on the model proposed in [39], the wind speed of a hurricane decays after landing. The decay pattern is described by Equation (4.31).

$$V(t) = V_b + (RV_0 - V_b)e^{-\alpha t} \tag{4.31}$$

where $R = 0.9$ accounts for the sea-land wind speed reduction, $V_b = 13.75m/s$, $\alpha = 0.095h^{-1}$, and V_0 is the wind speed at the time of landfall.

With the wind speed information and component fragility probability, we can simulate the network failure occurrence via Monte Carlo simulation. For each pole or segment of conductor, if they are not damaged in the previous hours, then their failure statuses follow independent Bernoulli distributions with probabilities calculated from (4.29) and (4.30). If they are already damaged, then they will remain damaged until having been repaired. Since each distribution line in the test case consists of multiple poles and segments of conductors, their failure status can be obtained by aggregating the failure status of individual poles and conductors. The reparation of multiple components in the system is assumed to be conducted simultaneously. In addition, the repair time for each pole and conductor is assumed to be constant. However, due to the different failure time of the components in a distribution line, the repair time experienced by each line is variable. The typical damage condition scenario of a distribution

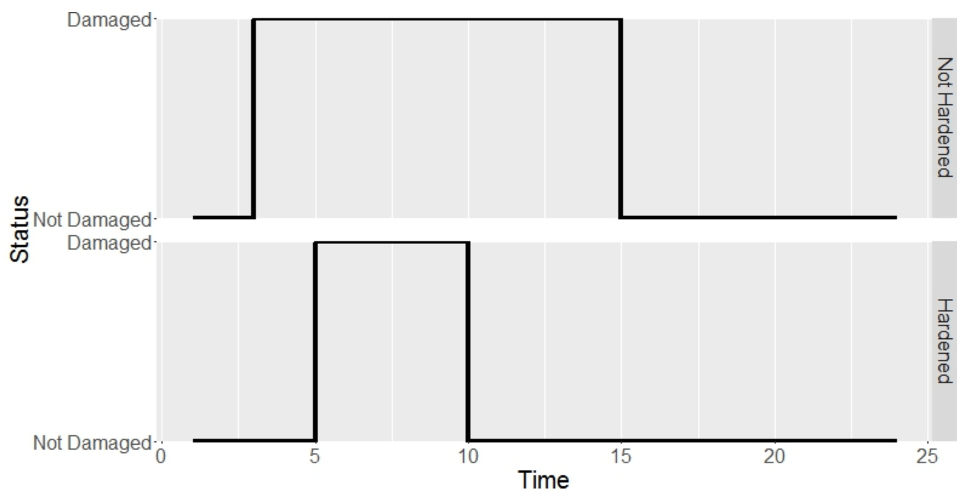

Figure 4.1 Typical Scenario

line in 24 hours is demonstrated in Figure 4.1. In this scenario, the line remains damaged for 13 hours if it is not hardened. If it is hardened, then it will only be damaged for 6 hours.

4.5.2 Experiment results

The IEEE 33-bus distribution system used in this paper is shown in Figure 4.2. In this system, there are 37 distribution lines, five of which are switchable tie lines. Detailed network and load data can be found in [8]. The length of each power line is assumed to be proportional to the resistance of the line. Then we can estimate the number of poles and conductors represented by each line in the network. Assuming the span between 2 poles is 150 feet, the cost of hardening each component is summarized in Table 4.1. With this information we can calculate the cost of hardening each line accordingly. The 24-hour load profile is displayed in Figure 4.3. In addition, we assume distributed generators (DG) exist at nodes 7, 10, 13, 19, 23, 26 and 30. The complete scenario space has 2^{37} scenarios. Based on the procedure outlined in Section 4.5.1, 100 scenarios are randomly generated. These scenarios represent the damage conditions of the power lines under different states of hardening.

The solution methods were implemented with AMPL and MATLAB on a desktop computer with Intel Core i7 3.4 GHz CPU, 8 GB memory. The Benders Decomposition algorithm takes several hours to converge, while the heuristic can yield a solution instantaneously.

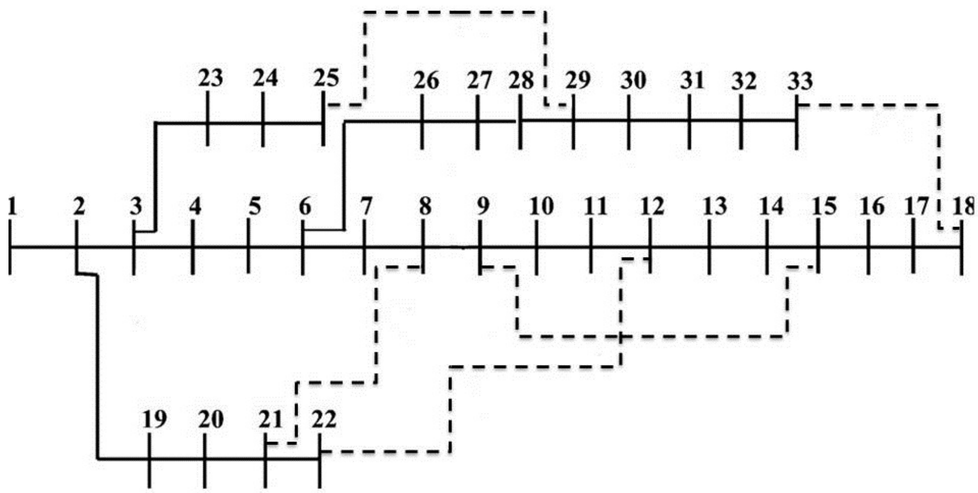

Figure 4.2 IEEE 33-Bus Test Case

Table 4.1 Hardening costs

Strategy	Cost ($/unit)	Unit
Update poles	6,000	N/A
Update conductors	700,000	mile
Update both	910,000	mile

When budget is 7 million dollars, 19 lines are hardened. The hardened lines are highlighted in Figure 4.4. We can see many lines on the main branch are strengthened. In addition, one tie-line connecting two branches is hardened, forming a large loop of strengthened lines. DGs also have noticeable effect on the selection of hardening strategies—most of the lines adjacent to DGs are hardened.

In Figure 4.5, the performances of three solutions under different scenarios are summarized. We call the three solutions Wait and See (WS) solution, Optimal Recourse Problem (RP) solution, and “None” solution. WS solution is obtained by solving the two-stage problem for each scenario and then taking the expected value of the objectives. WS solutions represent the best possible investment decisions a planner can make if the perfect information about future uncertainty is available. In other words, planners know exactly which lines are going to be damaged and when such damages would occur, and can change their investment decisions accordingly. This solution is unattainable and can only provide a lower bound on the objective

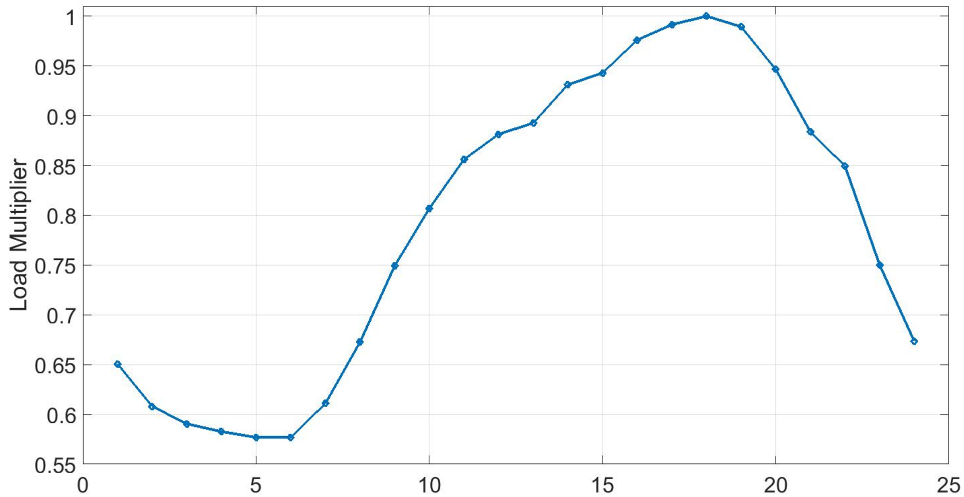

Figure 4.3 24-Hour Load Multiplier

value. The RP solution is obtained through the solution procedure outlined in Section 4.4. The "None" solution means no investment. We can see that although the RP solution has worse performances than the WS solutions, it is considerably better than no investment, which highlights the importance of investment in network hardening.

By varying the amount of available budget, we can examine the effect of available budget on the performance of resulting hardening plans. It is obvious that the amount of load shedding would decrease and budget increases. However, by looking at the trend, we can gauge the marginal benefit of increasing the investment budget. The performance of the hardening plans under different budgets are summarized in Table 4.2. The difference between WS and RP values is called the Expected Value of Perfect Information (EVPI). It represents the benefit of having access to perfect information about uncertainty. We can see EPVI is roughly decreasing with the increase of budgets. The heuristic solution is also included in the table. We can see the heuristic method can provide pretty good solutions.

4.5.3 Solution validation

In this paper, we test the quality of solution by calculating the confidence interval of optimality gaps under different budgets. The results are summarized in Table 4.3. The width of the confidence intervals vary, but are typically small (less than 6% of the objective value). This sug-

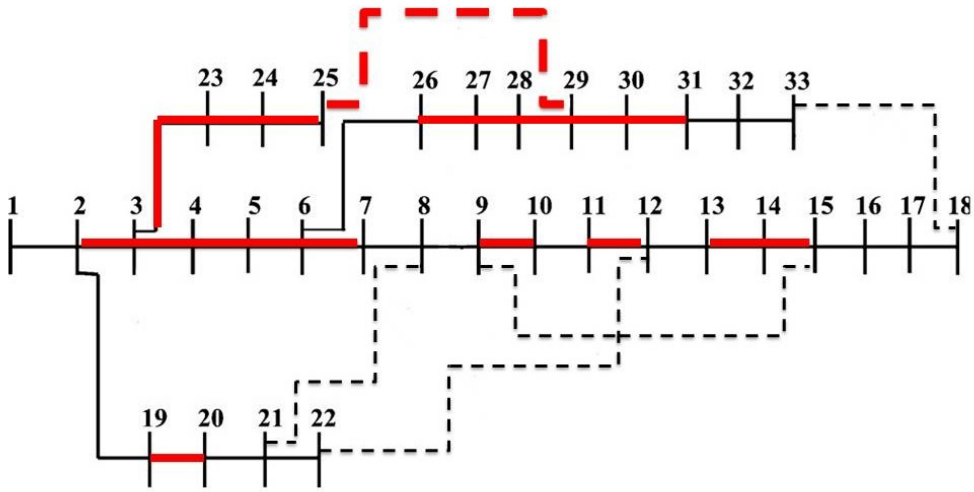

Figure 4.4 Hardening Plan (Budget = 7m$)

Table 4.2 Avergae Load Shedding under different budgets

Budget ($m)	WS	RP	EVPI	Heuristic
5	2.4445	3.0355	0.5910	3.1126
6	2.3105	2.9164	0.6059	2.9578
7	2.2029	2.8245	0.6216	2.8869
8	2.1187	2.6659	0.5472	2.6869
9	2.0548	2.5672	0.5124	2.5927
10	2.0050	2.4816	0.4766	2.5439
11	1.9753	2.4455	0.4702	2.5187
12	1.9577	2.4287	0.4710	2.5111

gests that with 100 scenarios, the SAA method can provide a reasonably good approximation to the optimal solution.

4.6 Conclusion

In this paper, a new two-stage stochastic programming model is proposed for improving the resilience of a distribution network with hardening. In this problem, the uncertain parameters concerning the condition of a network affected by natural disasters are considered. The average load shedding under different scenarios are optimized. To simplify the computation, sample average approximation is used to reduce the problem size. In addition, Benders decomposition is applied to speed up the computation. A heuristic method is developed to yield a good solu-

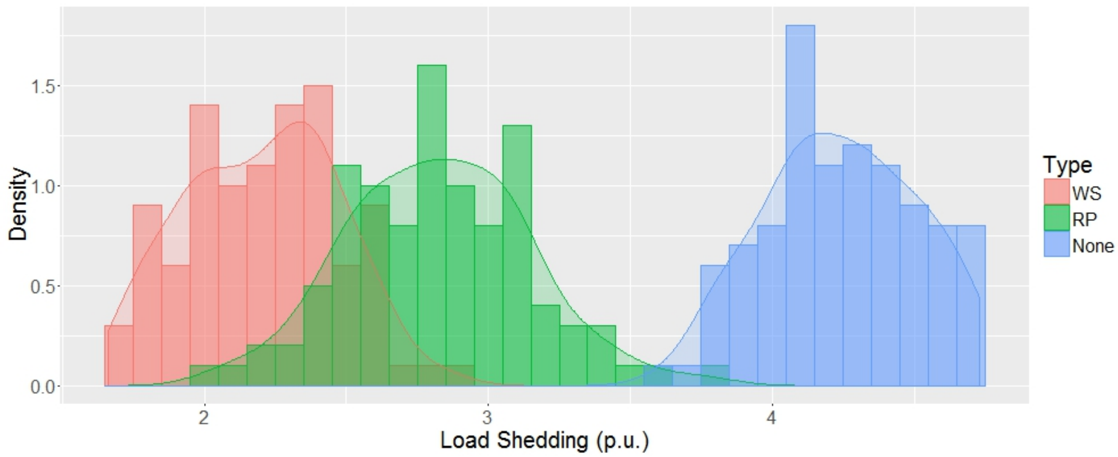

Figure 4.5 Histogram of Load Shedding Under Different Plans (Budget = 7m$)

Table 4.3 Confidence interval of the optimality gap under different budgets

Budget ($m)	95% C. I.	Budget ($m)	95% C. I.
5	[0, 0.1004]	9	[0, 0.0576]
6	[0, 0.0281]	10	[0, 0.0783]
7	[0, 0.0739]	11	[0, 0.0741]
8	[0, 0.1514]	12	[0, 0.1145]

tion quickly. A case study on the IEEE-33 bus distribution system is used to test the model and algorithms. The experiment results are then tested for quality with the single replication procedure. The solution validation procedure demonstrates that the SAA can provide a reasonably good approximation to the actual optimal solution.

For future research, we will focus on the following directions. Firstly, better scenario generation methods can be developed to take into consideration the dependencies between components and the randomness in the recovery time of damaged components. In addition, other disaster scenarios like earthquakes and floods can be explored. Secondly, more hardening strategies and preparedness measures such as training of personnel and emergency drill to improve the response time of repair crew can be included in the model for a more flexible solution. Thirdly, we can take into consideration other emergency operation measures such as intentional islanding to further improve the solution. Last but not least, more sophisticated solution methods might be needed to solve larger scale problems.

CHAPTER 5. CONCLUSION AND DISCUSSION

In my dissertation I present three applications of mathematical programming under uncertainty methods on different types of power system planning problems. The first two papers focus on the application of robust optimization to transmission expansion planning. The third paper presents an application of stochastic programming to power distribution network hardening planning. The detailed contributions of these works and conclusions are discussed as follows.

Transmission expansion planning facilitates generation expansion and is critical to the reliable and economic operations of power systems. The long planning horizon means there are a various sources of uncertainty. In the first two papers, robust optimization is used to address the challenges posed by generation expansion uncertainty. In the first paper, in addition to generation expansion and retirement, load forecast is also considered as a source of uncertainty. Two optimization criteria: minimax cost and minimax regret are compared under the robust optimization paradigm. With this approach, a transmission plan that is robust and has good performances under all scenarios can be derived. The resulting models are formulated as trilevel mixed-integer programming models. Column and constraint generation is used to decompose the model into a master problem and a subproblem, where new constraints and variables are iteratively generated by the subproblems to be fed back to the master problem. The subproblems themselves are bilevel mixed-integer programs, which are reformulated into single level mixed-integer programs with the KKT optimality conditions. The models and algorithm are tested on an IEEE 118-bus system. The results of the MMR and MMC models are analyzed and compared. We conclude that both criteria may outperform each other depending on the uncertainty set. Thus, the optimization criteria should be selected based on available data and decision-makers' preferences.

As for the second paper, a robust transmission expansion planning model that can explore the impact of generation expansion and retirement uncertainty in more detail is proposed. We use binary variables to describe whether a generator exists after generation expansion or retirement in the future. With the new modeling framework, the uncertainty set can contain detailed description of the uncertain parameters including the location, size and time of commission. The same column and constraint generation procedure is used to solve the trilevel model. However, by utilizing the special structure of the subproblem, the bilinear subproblem can be solved to optimality relatively quickly. A WECC 240-bus test system is used to demonstrate the effectiveness of the model and algorithm. The performance of the robust transmission plans is compared to that of the plans derived from the deterministic model and heuristic rules under different scenarios of fuel price and environmental policies. The results show that the robust plans not only out-perform plans obtained by other methods, but also remain robust under different scenarios.

In the third paper, a distribution network planning problem is solved with stochastic programming. To improve system resilience under extreme weather events, the power industry is investing to hardening the distribution networks by upgrading the infrastructure. With the limited budget, it is impossible to harden the entire network indiscriminately. A model is developed to optimally allocate the resources to the most critical or vulnerable components in the network. In this problem, the uncertainty lies in the condition of the network after it is affected by a natural disaster. Sample average approximation is used to reduce the problem size and Benders decomposition is used to accelerate the computation. A heuristic algorithm is also developed to yield a good solution quickly. A case study on the IEEE 33-bus system is used to test the model and algorithms. The solution is then validated via the single replication procedure, which shows that SAA can provide a reasonably good approximation to the optimal solution.

There exist several interesting directions for future work. In terms of transmission expansion planning, other sources of uncertainty and methods of dealing with them can be incorporated into the model. For example, since the probability distributions of load and renewable generation might be obtained from historical data, it is possible to combine stochastic programming and robust optimization by modeling different sources of uncertainty with different methods. Such a problem would be even more challenging to solve with the increased problem size. New algorithms and heuristics needs to be developed. In addition, more details including more explicit representations of various policies, demographic changes and climate changes can be incorporated into the existing model. In terms of distribution network hardening and resilience planning, more hardening strategies can be added to the model, including preparedness measures such as training of personnel and emergency drill to improve the response time of repair crew. Other disaster scenarios like earthquakes and floods can also be considered.

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