IE 361 Module 20
R&R for 0/1 (Go/No) Inspection Part 2
(Point Estimates)

Reading: Section 2.6.2 *Statistical Methods for Quality Assurance*

ISU and Analytics Iowa LLC
Still thinking of a single fixed part, we’ll let

\[ \hat{p}_j = \frac{\text{the number of "non-conforming" calls made by operator } j}{m} = \frac{X_j}{m} \]

and define the (sample) average of these

\[ \bar{\hat{p}} = \frac{1}{J} \sum_{j=1}^{J} \hat{p}_j \]

It is possible to argue that

\[ \mathbb{E}\hat{p} = \pi \]

and a plausible estimate of \( \sigma_{\text{R&R}}^2 \) is then

\[ \hat{\sigma}_{\text{R&R}}^2 = \bar{\hat{p}} (1 - \bar{\hat{p}}) \]
Then, since \( \hat{p}_j (1 - \hat{p}_j) \) is a plausible estimate of the "per call variance" associated with the declarations of operator \( j \), \( p_j (1 - p_j) \), an estimate of \( \sigma_{\text{repeatability}}^2 \) is

\[
\hat{\sigma}_{\text{repeatability}}^2 = \bar{\hat{p}} (1 - \bar{\hat{p}}) \quad \text{(the sample average of the } \hat{p}_j (1 - \hat{p}_j) \text{)}
\]

Finally, a simple estimate of \( \sigma_{\text{reproducibility}}^2 = \nu \) is

\[
\hat{\sigma}_{\text{reproducibility}}^2 = \hat{\sigma}_{\text{R&R}}^2 - \hat{\sigma}_{\text{repeatability}}^2
\]

\[
= \bar{\hat{p}} (1 - \bar{\hat{p}}) - \bar{\hat{p}} (1 - \bar{\hat{p}})
\]
What to do based on multiple parts (say I of them) is not completely obvious. For our purposes in IE 361, we will simply average estimates made one part at a time across multiple parts, presuming that parts in hand are sensibly thought of as a random sample of parts to be checked, and that this averaging is a sensible way to combine information across parts.

In order for any of this to have a chance of working, m is going to have to be fairly large. The usual Gauge R&R "m = 2 or 3" just isn’t going to produce informative results in the present context. And in order for this to work in practice (so that an operator isn’t just repeatedly looking at the same few parts over and over and remembering how he’s called them in the past) this seems like it’s going to require a large value of I as well as m.
Some Simple R&R Point Estimates for 0/1 Contexts

Example 20-1

Suppose that \( I = 5 \) parts are inspected by \( J = 3 \) operators, \( m = 10 \) times apiece, and that the table below lists sample fractions of "non-conforming" calls made by the operators and estimated per call variances.

<table>
<thead>
<tr>
<th>Part</th>
<th>Operator 1</th>
<th>Operator 2</th>
<th>Operator 3</th>
<th>Average</th>
<th>( \hat{p} (1 - \hat{p}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1</td>
<td>.2</td>
<td>.16</td>
<td>.4</td>
<td>.24</td>
<td>.2</td>
</tr>
<tr>
<td>Part 2</td>
<td>.6</td>
<td>.24</td>
<td>.6</td>
<td>.24</td>
<td>.7</td>
</tr>
<tr>
<td>Part 3</td>
<td>1.0</td>
<td>0</td>
<td>.8</td>
<td>.16</td>
<td>.7</td>
</tr>
<tr>
<td>Part 4</td>
<td>.1</td>
<td>.09</td>
<td>.1</td>
<td>.09</td>
<td>.1</td>
</tr>
<tr>
<td>Part 5</td>
<td>.1</td>
<td>.09</td>
<td>.3</td>
<td>.21</td>
<td>.3</td>
</tr>
<tr>
<td>Average</td>
<td>.1</td>
<td>.09</td>
<td>.3</td>
<td>.21</td>
<td>.3</td>
</tr>
</tbody>
</table>

The final column of this table gives estimated per call repeatability variances for the 5 parts.
Some Simple R&R Point Estimates for 0/1 Contexts

Example 20-1

Continuing to use the $\hat{p}$ values from the previous slide, the next table provides estimated R&R and reproducibility variances for the 5 parts.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{p}$</th>
<th>$\hat{p} (1 - \hat{p})$</th>
<th>$\hat{p} (1 - \hat{p}) - \hat{p} (1 - \hat{p})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1</td>
<td>.267</td>
<td>.1956</td>
<td>.0090</td>
</tr>
<tr>
<td>Part 2</td>
<td>.633</td>
<td>.2322</td>
<td>.0022</td>
</tr>
<tr>
<td>Part 3</td>
<td>.833</td>
<td>.1389</td>
<td>.0156</td>
</tr>
<tr>
<td>Part 4</td>
<td>.100</td>
<td>.0900</td>
<td>0</td>
</tr>
<tr>
<td>Part 5</td>
<td>.233</td>
<td>.1789</td>
<td>.0089</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>.1671</td>
<td>.0071</td>
</tr>
</tbody>
</table>

So, to summarize

$$\hat{\sigma}^2_{R&R} = .1671, \hat{\sigma}^2_{\text{repeatability}} = .1600, \text{ and } \hat{\sigma}^2_{\text{reproducibility}} = .0071$$
Then, for example, a fraction of only

\[
\frac{.0071}{.1671} = 4.25\%
\]

of the inconsistency in conforming/non-conforming calls seen in the original data seems to be attributable to clear differences in how the operators judge the parts (differences in the binomial "success probabilities" \(p_j\)). Rather, the bulk of the variance seems to be attributable to unavoidable binomial variation. The \(p\)'s are not close enough to either 0 or 1 to make the calls tend to be consistent. So the variation seen in the \(\hat{p}\)'s in a given row is only weak evidence of operator differences.
Of course, we need to remember that the computations above are on the variance (and not standard deviation) scale. On the (more natural) standard deviation scale, reproducibility variation

$$\sqrt{.0071} = .08$$

and repeatability variation

$$\sqrt{.1600} = .40$$

are not quite so strikingly dissimilar.